SIDE
Symmetries and Integrability
of Difference Equations

SIDE 14.2

Monday 19 June 2023 - Friday 23 June 2023

Faculty of Physics, University of Warsaw

Book of abstracts

Table of contents

Continued fractions and integrability / 56

TCD maps

Mr. AFFOLTER, Niklas [1]

[1] *TU Berlin*

Corresponding Author: affolter@posteo.net

A TCD map is a map from a triple crossing diagrams to projective space, satisfying an incidence requirement. We introduce dynamics on TCD maps based on Menelaus theorem and show multi-dimensionally consistency with Desargues theorem. To each TCD map we associate a hierarchy of dimer models, which provides local and global invariants. Conveniently, TCD maps include as special cases a large number of known maps, including Q-nets, Line complexes, Darboux maps, Desargues maps, dSKP lattices, t-embeddings, T-graphs, the pentagram map, integrable cross-ratio systems and others. Additionally, we show how to relate the resistor subvariety of the dimer model to dBKP reductions and the Ising subvariety to dCKP reductions.

Integrable birational maps / 13

A 3D generalisation of QRT and the construction of integrable birational systems

Dr. ALONSO, Jaume [1]; Prof. SURIS, Yuri [1]; Mr. WEI, Kangning [1]

[1] *Technische Universitaet Berlin*

Corresponding Author: jaume.alonsofernandez@tu-berlin.de

When completely integrable Hamiltonian systems are discretised, the resulting discrete-time systems are often no longer integrable themselves. This is the so-called "problem of integrable discretisation". Two known exceptions to this situation in 3D are the Kahan discretisations of the Euler top and the Zhukovski-Volterra gyrostat with one non-zero linear parameter β, both of degree 3. The integrals of these systems define pencils of quadrics. By analysing the geometry of these pencils, we develop a framework that
generalises QRT maps and QRT roots to 3D, which allows us to create new integrable maps as a composition of two involutions. We show that under certain geometric conditions, the new maps become of degree 3. We use these results to create new families of discrete integrable maps and we solve the problem of integrability of the Zhukovski-Volterra gyrostat with two β's.

This is a joint work with Yuri Suris and Kangning Wei.

. / 22

Deformations of the van Diejen model

ATAI, Farrokh [1]

[1] *University of Leeds*

Corresponding Author: f.atai@leeds.ac.uk

The (quantum) van Diejen model is an integrable many-body system defined by a family of mutually commuting analytic difference operators that is known to have hyperoctahedral symmetry in its variables and E_8 Weyl group symmetry in its parameters (under certain constraint).
In this talk, new generalizations of the van Diejen Hamiltonian - and some exact eigenfunctions - are presented. In particular, I present a Chalykh-Feigin-Sergeev-Veselov type deformation of the van Diejen Hamiltonian, which is given by an analytic difference operator with two shift parameters, along with the corresponding weight function and kernel function identities.
If time permits, I will also present how some (formal) eigenfunctions of this deformed van Diejen model are related to elliptic hypergeometric integrals of Selberg type.

Compatibility / 41

The principle of transfer and integrable discrete systems.

Mr. ATKINSON, James [1]

[1] *NA*

Corresponding Author: james.l.atkinson@gmail.com

There is Hesse's principle of transfer: the transfer of geometric assertions from one dimension into another, facilitated by the fact that the projective subgroup stabilising a normal curve is isomorphic in every dimension. When space is coordinatised via a normal curve, geometric assertions become SL2-invariant equations, as opposed to homogeneous equations if a simplex is used. Although examples of KP, KdV and Painleve type are unified in this way, the relation should not be confused with dimensional reduction via integrable constraints, which places the systems, and the dimensions, into a hierarchy. Rather it is a path to understand the invariant geometric origin of the integrability. I will explain the integrable multi-quadratic quad-equations from this geometric point of view.

Discrete differential geometry / 71

How discrete integrable systems helped to solve a classical problem in differential geometry

Prof. BOBENKO, Alexander [1]

[1] *TU Berlin*

Corresponding Author: bobenko@math.tu-berlin.de

We consider a classical problem in differential geometry, known as the Bonnet problem, whether a surface is characterized by a metric and mean curvature function. We explicitly construct a pair of immersed tori that are related by a mean curvature preserving isometry. This resolves a longstanding open problem on whether the metric and mean curvature function determine a unique compact surface. Discrete integrable systems and discrete differential geometry are used to find crucial geometric properties of surfaces. This is a joint work with Tim Hoffmann and Andrew Sageman-Furnas.

Discrete differential geometry / 78

About a movie: "Solving Bonnet problem: hands on adventure in 17 chapters"

Corresponding Author: bobenko@math.tu-berlin.de

Singularity analysis / 36

Singularity analysis and bilinear approach to some Bogoyavlensky differential difference equations

Dr. CARSTEA, Adrian Stefan [1]

[1] *Institute of Physics and Nuclear Engineering, Dept of Theoretica Physics, Magurele, Bucharest 077125, Romania*

Corresponding Author: carstea@gmail.com

We discuss singularity analysis and bilinear integrability of four Bogoyavlensky differential-difference equations. Three of them are completely integrable and the fourth is, to our knowledge, a new one. Blending the singularity confinement with Painlevé property reveals strictly confining and anticonfining (weakly confining) singularity patterns. The strictly confining patterns are useful because they provide different representations using tau functions and a possible extension of the so called "express method" for testing integrability. For the new proposed equation we get also the bilinear form and multisoliton solution, being a good candidate for a new integrable system. In addition, using bilinear formalism we recover the integrable time-discretisations of the first three systems.

Non-commutative / 46

Hamiltonian structures for nonabelian differential-difference systems

Author: Dr. CASATI, Matteo [1]

Co-Author: Prof. WANG, Jing Ping [2]

[1] *Ningbo University*

[2] *University of Kent*

Corresponding Author: matteo@nbu.edu.cn

Integrable nonabelian systems are equations of motion in which the field variables take values in a noncommutative algebra, as a matrix one. In a series of papers with Jing Ping Wang (Nonlinearity 2021, CMP 2022) we have investigated the Hamiltonian structure and recursion operators for hierarchies of differential-difference integrable equations, providing a geometrical interpretation that helps to shed some light onto the structure on nonabelian Hamiltonian systems in general.

I will present the notions of double multiplicative Poisson vertex algebra and of nonabelian functional polyvector fields, with the latter as the natural language to describe Hamiltonian systems in the familiar geometrical terms. As an application, I will discuss some results towards the classification of scalar Hamiltonian difference structures and present a list of such structures for the nonabelian generalization of well-known integrable systems such as Volterra, Toda and Kaup lattices.

Darboux polynomials / 73

Generalization of the discrete gradient approach: improving accuracy and extending the scope of application

Author: CIEŚLIŃSKI, Jan [1]

Co-Author: Mr. KOBUS, Artur [1]

[1] *University of Bialystok*

Corresponding Author: j.cieslinski@uwb.edu.pl

The motivation of our research is to find structure preserving discretizations of dynamical systems by developing some ideas related to the method of discrete gradients [1].
First, we recall that the discrete gradient method can be improved in two different ways without losing the energy conservation property, either by increasing its order [2], or by so-called locally exact discretizations that become extremely accurate in the vicinity of stable equilibria [3].
Second, we present our recent results concerning systems with e.g. non-linear damping or amplification (like Van der Pol oscillator or chaotic Rössler system). The approach based on the so-called reservoirs and para-Hamiltonian formulation, developed by the co-author [4], makes possible to extend the discrete gradient methods to a general class of autonomous ODEs (no integrals of motion are required). Side effect of this approach is a whole slew of new geometric methods.

[1] R.I. McLachlan, G.R.W. Quispel, N. Robidoux: Geometric Integration using discrete gradients, Philosophical Transactions of the Royal Society A: Math. Phys. Eng. Sci. 357 (1998) 1021--1045.
[2] J.L .Cieśliński, B. Ratkiewicz, Discrete gradient algorithms of high order for one-dimensional systems, Computer Physics Communications 183 (2012) 617--627.
[3] J.L. Cieśliński: Locally exact modifications of discrete gradient schemes, Physics Letters A 377 (2013) 592-597.
[4] A. Kobus, J.L. Cieśliński: Para-Hamiltonian form for General Autonomous ODE Systems: Introductory Results, Entropy 24 (3) (2022) 338.

Continuous integrable systems / 55

Reduction of a bidimensional sine-Gordon system to the sixth Painlevé equation

Author: Mr. CONTE, Robert [1]

Co-Author: Mr. GRUNDLAND, A. Michel [2]

[1] *E'cole normale supérieure de Paris-Saclay, France*

[2] *Université du Québec à Trois-Rivières*

Corresponding Author: robert.conte@cea.fr

We establish all the reductions of a system of two coupled sine-Gordon equations introduced by Konopelchenko and Rogers to ordinary differential equations.
All these reductions are degeneracies of a master equation of Chazy,
"curious for its elegance", algebraic transform of the sixth Painlevé equation.

72

Hydrodynamic chain hierarchy and skew-orthogonal polynomials

Dr. DELL'ATTI, Marta [1]

[1] *University of Portsmouth*

Corresponding Author: marta.dellatti@port.ac.uk

The discrete integrable structure concealed by orthogonal matrix ensembles is the Pfaff lattice, built on a semi-infinite moment matrix defined for a skew-symmetric weight. The dependence on the infinitely many times is encoded at the level of the weight, and gives rise to an integrable hierarchy expressed as the collection of infinitely many Lax equations. The leading order of the continuum limit of the variables populating the lattice can be recast in the form of an integrable hydrodynamic chain for every time, producing a hierarchy of hydrodynamic chains. With all the times set to zero, the eigenfunctions for the Pfaff lattice are semi-infinite skew-orthogonal polynomials and can be mapped onto orthogonal polynomials, eigenfunctions for the Toda lattice. The Pfaff lattice structure is completely determined and it offers valuable insight for the form of the field variables with nonzero times.

Continued fractions and integrability / 2

Rational interpolation/approximation and integrability

Prof. DOLIWA, Adam [1]

[1] *University of Warmia and Mazury*

Corresponding Author: doliwa@matman.uwm.edu.pl

It is well known that the rational (or Padé) approximants are closely related with the discrete-time Toda equation. The structural connection with orthogonal polynomials provides a link between the theory of integrable systems and various classical results of applied mathematics and numerical analysis. Inspired by recent advances on symmetry and integrability of difference equations I would like to disuss several generalizations of the relation (some of them already known):

* the role of Hirota's discrete KP system;
* non-commutative versions of the classical results;
* approximation as a confluent limit of the interpolation.

[1] A. Doliwa, A. Siemaszko, Integrability and geometry of the Wynn recurrence, Numer. Algorithms doi: 10.1007/s11075-022-01344-5

[2] A. Doliwa, A. Siemaszko, Hermite-Padé approximation and integrability, arXiv:2201.06829

[3] A. Doliwa, Non-commutative Hermite-Padé approximation and integrability, Lett. Math. Phys. 112 68 (2022) doi: 10.1007/s11005-022-01560-z

[4] A. Doliwa, Non-autonomous multidimensional Toda system and multiple interpolation problem, J. Phys. A: Math. Theor. 55 (2022) 505202 (17 pp.) doi: 10.1088/1751-8121/acad4d

Discrete Painlevé / 38

Recurrence relations for the generalized Laguerre and Charlier orthogonal polynomials and discrete Painlevé equations

Dr. DZHAMAY, Anton [1]

[1] *University of Northern Colorado, USA and BIMSA, China*

Corresponding Author: anton.dzhamay@unco.edu

We consider two examples of certain recurrence relations, or nonlinear discrete dynamical systems, that appear in the theory of orthogonal polynomials, from the point of view of Sakai's geometric theory of Painlevé equations. Of particular interest is the fact that both recurrences are regularized on the same family of rational algebraic surfaces, but at the same time their dynamics are non-equivalent. This is a joint work with Galina Filipuk, Xing Li, and Da-jun Zhang.

Darboux polynomials / 52

Kahan-Poisson Maps

Author: Dr. EVRIPIDOU, Charalampos [1]

Co-Authors: KASSOTAKIS, Pavlos [2]; Prof. VANHAECKE, Pol [3]

[1] *Special Scientist*

[2] *University of Warsaw*

[3] *University of Poitiers*

Corresponding Author: cevrip02@ucy.ac.cy

The Kahan discretization of a Lotka-Volterra system leads to a rational map parametrized by the step size. When this map is Poisson with respect to the quadratic Poisson structure of the Lotka-Volterra system we say that this system has the Kahan-Poisson property. There is a well known family of Lotka-Volterra systems having the Kahan-Poisson property. Their underlying graph has n vertices 1, 2, ..., n and an arc from i to j precicely when i less than j. We prove that, modulo permutation of the variables and clonings, these are the only Lotka-Volterra systems having the Kahan-Poisson property.

Continuous integrable systems / 3

On Hamiltonian structures of quasi-Painleve equations

Dr. FILIPUK, Galina [1]

[1] *University of Warsaw*

Corresponding Author: g.filipuk@uw.edu.pl

The quasi-Painleve property of a system of ordinary differential
equations, here meaning the condition that movable singularities reachable by analytic continuation along finite length curves are at worst algebraic branch points, is described in terms of global Hamiltonian structures on an analogue of Okamoto's spaces of initial conditions for the Painleve equations. This is a joint work with A. Stokes (Japan).

Ultra discrete equations / 58

A visual approach to ultra-discrete equations

Dr. GILSON, Claire [1]

[1] *University of Glasgow*

Corresponding Author: claire.gilson@glasgow.ac.uk

In this talk we look at some simple visual methods to analyse the solutions to ultra-discrete equations with real values for the dependent variable as opposed to integer values.
Using these visual techniques the number of solitons present can be determined and solitons can be easily removed or added to the system.

12

Coalgebra symmety and discrete integrable systems

Dr. GUBBIOTTI, Giorgio [1]; Dr. LATINI, Danilo [2]; Dr. TAPLEY, Benjamin [3]

[1] *Universita degli Studi di Milano*

[2] *The University of Queensland*

[3] *NTNU*

Corresponding Author: giorgio.gubbiotti@unimi.it

We introduce the concept of coalgebra symmetry for discrete systems. We use this powerful tool to prove (super)integrability, superintegrability of some discrete systems in arbitrary dimension. In particular, we study the systems obtained by the coalgebra symmetry with respect to special linear Lie--Poisson algebra in dimension 2.

Integrable birational maps / 11

Growth and integrability of some birational maps in dimension three

Dr. GRAFFEO, Michele [1]; Dr. GUBBIOTTI, Giorgio [2]

[1] *Scuola Internazionale Studi Superiori Avanzati*

[2] *Universita degli Studi di Milano*

Corresponding Author: giorgio.gubbiotti@unimi.it

Motivated by the study of the Kahan--Hirota--Kimura discretisation of the Euler top, we characterise the growth and integrability properties of a collection of elements in the Cremona group of a complex projective 3-space using techniques from algebraic geometry. This collection consists of maps obtained by composing the standard Cremona transformation in three dimensions with projectivities that permute its fixed points and the points over which it performs a divisorial contraction. More specifically, we show that three behaviour are possible: (A) integrable with quadratic degree growth and two invariants, (B) periodic with two-periodic degree sequences and more than two invariants, and (C) non-integrable with submaximal degree growth and one invariant.

Continuous integrable systems / 67

Painlevé-type delay differential equations in the complex plane

Prof. HALBURD, Rod [1]

[1] *University College London*

Corresponding Author: r.halburd@ucl.ac.uk

Several examples of delay differential equations (equations relating a function of a single variable to shifts and derivatives of the function with respect to that variable) have appeared in the literature that deserve to be called integrable. Some are known to be reductions of integrable differential-difference equations and possess Lax pairs as well as continuum limits to the classical Painlevé equations. We will describe the nature of some solutions of these equations in the complex domain, and also provide some new examples. This work has led to a number of general results and conjectures about the value distribution of meromorphic functions.

Ultra discrete equations / 63

Second main theorem in tropical Nevanlinna theory

HALONEN, Juho [1]

[1] *University of Eastern Finland*

Corresponding Author: juho.halonen@uef.fi

In the talk I will introduce a new version of the second main theorem in the tropical semiring and for tropical hypersurfaces. I will also present the inverse problem of tropical Nevanlinna theory. I shall show applications of tropical Nevanlinna theory for ultra-discrete Painlevé equations.

Degree growth of lattice equations and maps / 60

Degree growth of some lattice equations defined on a 3x3 stencil

Prof. HIETARINTA, Jarmo [1]

[1] *University of Turku*

Corresponding Author: hietarin@utu.fi

We study the growth of complexity, or degree growth, of one-component lattice equations defined on a 3x3 stencil. The 2x2 case was discussed in a previous talk by T. Mase. The equations studied here include two 7-point equations in Hirota bilinear form as well as 9-point Boussinesq equations of regular, modified and Schwarzian type. The initial values are given on a staircase or on a corner configuration and depend linearly or rationally on a special variable x, and we count the degree in x of the iterates. Known integrable cases have linear growth if only one initial values contains x, and quadratic growth if all initial values contain x. Even a small deformation of an integrable equation changes the degree growth to become exponential, because the deformation will change factorization properties and thereby prevent cancellations. The simplest case in which only one initial value contains x is sufficient to differentiate between integrable and non-integrable equations.

Continued fractions and integrability / 62

Discrete integrable systems and orthogonal polynomials from continued fractions in function fields

Prof. HONE, Andrew [1]

[1] *University of Kent*

Corresponding Author: a.n.w.hone@kent.ac.uk

It has been known for some years that there are deep connections between continued fractions and integrable systems: one of the earliest examples appears in Moser's work on solutions of the finite Kac-van Moerbeke (or Volterra) lattice, but there are many other examples e.g. in recurrence coefficients for orthogonal polynomials arising in random matrix theory, which satisfy (discrete and continuous) Painleve equations. In this talk we describe our recent work on continued fractions of Jacobi type (J-fractions) for a certain family of functions on hyperelliptic curves, based on a construction of van der Poorten related to Somos-4 sequences (corresponding to genus g=1). We explain how to interpret van der Poorten's result for all genera g, in terms of a family of discrete integrable systems. This not only leads to an elementary derivation of Hankel determinant formulae for Somos-4 found by Chang, Hu & Xin, but also provides a natural construction of higher genus analogues of Chebyshev polynomials, and produces genus g solutions of the infinite Toda lattice. The J-fractions naturally arise from even models of hyperelliptic curves, but recent work with John Roberts and Pol Vanhaecke also reveals another family of discrete integrable systems associated with continued fraction of Stieltjes-type (S-fractions) and odd models of the same curves, which yield solutions of the infinite Volterra lattice. In particular, we find that the g=2 S-fraction corresponds to an integrable map with 2 degrees of freedom, discovered in a recent classification of 4D maps with Lagrangian structure by Gubbiotti, Joshi, Viallet & Tran. Other 4D integrable maps found by the latter authors turn out to be connected with the modified Volterra lattice: our additional joint work with Federico Zullo has revealed that they arise from the same genus 2 S-fraction, but as BTs in the sense of Sklyanin.

32

A natural signed bijection between monotone triangles and shifted Gelfand-Tsetlin patterns

Mr. INOUE, Takuya [1]

[1] *Graduate School of Mathematical Sciences, The University of Tokyo*

Corresponding Author: inoue@ms.u-tokyo.ac.jp

The alternating sign matrices-descending plane partitions (ASM-DPP) bijection problem is one of the most intriguing open problems in integrable combinatorics. Recently, Fischer and Konvalinka have obtained a bijection between ASM(n) × DPP(n−1) and DPP(n) × ASM(n−1) using the notions of a signed set and a signed bijection and which involves an explicit construction of a signed bijection between alternating sign matrices and so-called shifted Gelfand-Tsetlin patterns. Their proof can be considered as a combinatorial proof of the enumeration of ASM. We define the notion of compatibility of a signed bijection to measure its naturalness and we use it to simplify said signed bijection between alternating sign matrices and shifted Gelfand-Tsetlin patterns, thereby providing a better combinatorial proof of the enumeration of ASM.

Non-commutative / 37

Fermionic description of K-theoretic symmetric functions of type A and C

Dr. IWAO, Shinsuke [1]

[1] *Keio University*

Corresponding Author: iwao-s@keio.jp

The term "K-theoretic symmetric function" refers to a family of symmetric functions representing Schubert varieties in the K-theory of flag varieties. (They are referred to as "Grothendieck polynomials" for type A and as "K-Q-functions" for type C.) In this talk, I will introduce a fermionic description of these K-theoretic symmetric functions in terms of the boson-fermion correspondence. This method generalizes the vertex operator presentation of Schur polynomials, which has been studied intensively in the field of classical integrable systems, particularly the KP equation. This characterization provides a unified approach for proving algebraic formulas, such as determinant formulas, Schur expansions, and plethysm. Additionally, I will introduce the 'K-theoretic non-commutative Schur operators,' which naturally act on the space of Grothendieck polynomials.

Non-linear waves / 16

A Truss Structure with Mechanical Optimality, Integrability and Artisiticity

Author: Prof. KAJIWARA, Kenji [1]

Co-Authors: Prof. YOKOSUKA, Yohei [2]; Dr. HAYASHI, Kazuki [3]; Dr. JIKUMARU, Yoshiki [4]; Dr. HAYAKAWA, Kentaro [3]

[1] *Institute of Mathematics for Industry, Kyushu University*

[2] *Kagoshima University*

[3] *Kyoto University*

[4] *Toyo University*

Corresponding Author: kaji@imi.kyushu-u.ac.jp

We report that a class of integrable discrete holomorphic functions can generate planar truss structures with a certain mechanical optimality called the Michell structure, well-known in the area of architecture. Further, the discrete planar curves formed by the edges are nothing but the discrete analogue of the logarithmic spiral which is a special case of the discrete log-aesthetic curves. Discrete log-aesthetic curves are integrable discrete analogue of the log-aesthetic curves, which is known as a class of planar curves with built-in aesthetic nature, and those curves are invariant curves with respect to integrable deformation of planar curves in similarity geometry.

Compatibility / 74

Consistency for 5-point lattice equations

Dr. KELS, Andrew [1]

[1] *UNSW*

Corresponding Author: andrew.p.kels@gmail.com

In this talk, we present some details of how to implement the consistency scheme for 5-point lattice equations broadly introduced in the talk by Wolfgang Schief. We will also present some other formulations of consistency for 5-point lattice equations, particularly for equations in a hexagonal lattice. Time permitting we will also present consistent multicomponent 5-point equations and consistency of related equations in lattices of dimension greater than 2.

Cluster algebras / 33

Integrable deformation of …

Mr. KIM, Wookyung [1]

[1] *Lancaster University*

Corresponding Author: w.kim@lancaster.ac.uk

An integrable deformation of a cluster map is an integrable Poisson map which is composed of a sequence of deformed cluster mutations, namely, parametric birational maps preserving the presymplectic form but destroying the Laurent property, which is a necessary part of the structure of a cluster algebra. However, this does not imply that the deformed map does not arise from a cluster map: one can use so-called Laurentification, which is a lifting of the map into a higher-dimensional space where the Laurent property is recovered, and thus the deformed map can be generated from elements in a cluster algebra. This deformation theory was introduced recently by Hone and Kouloukas, who presented several examples, including deformed integrable cluster maps associated to Dynkin types A_2,A_3 and A_4. In this talk, we will consider the deformation of integrable cluster map corresponding to the general even dimensional case, Dynkin type A_{2N}.

Cluster algebras / 48

Deformations and integrability of cluster maps

Dr. KOULOUKAS, Theodoros [1]

[1] *University of Lincoln*

Corresponding Author: tkouloukas@lincoln.ac.uk

We present parametric deformations of sequences of cluster mutations in the framework of cluster algebras, which destroy the Laurent property but preserve a presymplectic structure induced by the corresponding exchange matrix. We investigate the Liouville integrability of deformed parametric cluster maps associated with the A3 and A4 quivers by imposing suitable constraints on the parameters. We also study examples of more general types of deformed mutations associated with affine Dynkin quivers which correspond to periodic reductions of the discrete sine-Gordon equation.

Continuous integrable systems / 35

Classification of 3d Dupin Cyclidic Coordinates

Author: Dr. KRASAUSKAS, Rimvydas [1]

Co-Authors: Ms. HOXHAJ, Eriola [2]; Mr. MENJANAHARY, Jean Michel [3]

[1] *Vilnius University*

[2] *Johannes Kepler University, Linz, Austria*

[3] *Vilnius University, Vilnius, Lithuania*

Corresponding Author: rimvydas.krasauskas@mif.vu.lt

Dupin cyclidic (DC) coordinates in R^3 are triple orthogonal coordinates where all coordinate lines are circles or straight lines. Besides classical examples (spherical, cylindrical, conical, etc.) there are less-known DC coordinates constructed using two focal conics (ellipse and hyperbola, or two parabolas) in orthogonal planes, used by Darboux for separation of variables in the Laplace equation (see details in [1]).
We classify all possible DC coordinates up to conformal transformations of R^3. It appears that most general DC coordinates can be reduced to the canonical form where their singular set Sing is a union of up to 3 bicircular quartic curves on separate orthogonal planes. Such an arrangement of curves had already appeared in Darboux's book [2, p. 472] in the context of different coordinates (with coordinate lines definitely distinct from circles).
The full conformal classification of DC coordinates contains two big 2-parameter classes (with 3 and 2 orthogonal planes as coordinate surfaces) that are uniquely defined by their Sing sets and 1-parameter class, generated by offsets of a rotational cone.

[1] A. Sym, and A. Szereszewski, On Darboux's Approach to R-Separability of Variables, SIGMA 7 (2011).
[2] G. Darboux, Principes de Géométrie Analytique, Gauthier-Villars, Paris 1917.

Continuous integrable systems / 30

A Weighted and Elliptic extension of Fibonacci numbers

Author: Ms. KUMARI, Archna [1]

Co-Authors: Dr. SCHLOSSER, Michael [2]; Dr. BHATNAGAR, Gaurav [3]

[1] *Indian Institute of Technology, Delhi, India*

[2] *University of Vienna*

[3] *Visiting Professor, Ashoka University, India*

Corresponding Author: maz178305@maths.iitd.ac.in

We extend Fibonacci numbers with arbitrary weights and generalize a dozen Fibonacci identities. As a special case, we propose an elliptic extension which extends the q-Fibonacci polynomials appearing in Schur's work. The proofs of most of the identities are combinatorial, extending the proofs given by Benjamin and Quinn, and in the q case, by Garrett. Some identities are proved by telescoping.

Continuous integrable systems / 27

Peter-Weyl theorem for current and Iwahori groups.

Dr. MAKEDONSKYI, Yevhen [1]

[1] *BIMSA*

Corresponding Author: makedonskii_e@mail.ru

The famous Peter-Weyl theorem tells that the ring of functions on a simple Lie group has a direct sum decomposition with summands isomorphic to tensor products of irreducible representations. In the case of more general groups there is no direct sum decomposition. However we prove that there exists a filtration such that its subquotients are tensor products of some very natural representations for current and Iwahori groups.

Degree growth of lattice equations and maps / 21

Degree growth calculations for lattice equations

Author: Dr. MASE, Takafumi [1]

Co-Authors: Prof. HIETARINTA, Jarmo [2]; Prof. WILLOX, Ralph [1]

[1] *the University of Tokyo*

[2] *University of Turku*

Corresponding Author: mase@ms.u-tokyo.ac.jp

Integrability criteria that have been enormously successful for second order mappings, such as singularity confinement or zero algebraic entropy, are often applied to lattice equations as though the latter were mere mappings.
In this talk we will show that such a naïve approach can (and does) lead to all sorts of contradictions and that considerable care is needed when using such methods to investigate the integrability of a given lattice equation.

More precisely:
In this talk we show that the results of degree growth calculations for lattice equations strongly depend on the initial value problem that one chooses, either because of problems that arise in the past light-cone, or because of interferences in the future light-cone.
Among the examples we treat are initial value problems for dKdV, discrete Liouville and dToda, for which the degree growth becomes exponential, in contrast to the common belief that discrete integrable equations must have polynomial growth and that linearizable equations necessarily have linear degree growth, regardless of the initial value problem one imposes.
Finally, as a possible remedy for one of the observed anomalies, we also propose basing integrability tests that use growth criteria on the degree growth of a single initial value instead of all the initial values.

Reference:
J. Hietarinta, T. Mase & R. Willox: J. Phys. A: Math. Theor. 52 49LT01 (2019).

Ultra discrete equations / 59

Characterization of Orthogonal Polynomials on lattices

Dr. MBOUNA, Dieudonne [1]

[1] *University of Warsaw*

Corresponding Author: dmbouna@aims.ac.tz

We give positive answers to some conjectures due to Ismail contained in his monograph of 2005. This concerns characterizations theorems for orthgonal polynomials on lattices.

Non-commutative / 49

A novel approach to quantisation of dynamical systems

Prof. MIKHAILOV, Alexander [1]

[1] *University of Leeds*

Corresponding Author: a.v.mikhailov@leeds.ac.uk

We propose to revisit the problem of quantisation and look at it from an entirely new angle, focussing on quantisation of dynamical systems themself, rather than of their Poisson structures. We begin with a lift of a classical dynamical system to a system on a free associative algebra with noncommutative dynamical variables and reduce the problem of quantisation to the problem of studying of two-sided quantisation ideals, i.e. the ideals of the free algebra that define the commutation relations of the dynamical variables and are invariant with respect to the non-commutative dynamics. Quantum multiplication rules in the quotient algebra over a quantisation ideal are manifestly associative and consistent with the dynamics. We found first examples of bi-quantum systems which are quantum counterparts of bi-Hamiltonian systems in the classical theory. Moreover, the new approach enables us to define and present first examples of non-deformation quantisations of dynamical systems. The new approach also sheds light on the problem of operator's ordering.

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Yang-Baxter and tetrahedron equations / 34

Towards a factorised solution of the Yang-Baxter equation with Uq(sln) symmetry

Mr. MORRIS, Benjamin [1]

[1] *University of Leeds*

Corresponding Author: mmbajm@leeds.ac.uk

The talk will discuss the emergence of factorised solutions of the Yang-Baxter equation in terms of transposition operators acting in q-difference (differential) representations of the algebra Uq(sln) (U(sln)), respectively. I will focus on the q-deformed case, where all but one of the transposition operators are constructed explicitly and the proof of a crucial symmetric group relation introduces a surprising terminating q-series identity relating q-Lauricella series of differing rank. The inability to obtain an appropriate q-analogue of a known factorisation result for the U(sl_n) Lax operator for n>3, is discussed as an obstruction to finding the missing transposition operator.

Lagrangian multiform theory / 15

Lagrangian multiforms and discrete and semi-discrete KP systems

Prof. NIJHOFF, Frank [1]

[1] *University of Leeds*

Corresponding Author: frank.nijhoff@gmail.com

Recently I presented a Lagrangian 3-form structure for a generalised Darboux system. The original Darboux system arose in connection with the theory of conjugate nets for systems of orthogonal curvilinear coordinates. The generalised system, in which the relevant fields are labelled by continuous parameters which can be associated with lattice parameters of an underlying
3-dimensional integrable discrete system, amounts to a presentation of the KP
hierarchy in terms of Miwa variables, and can be thought of as a "generating PDE" system for that hierarchy. In connection with this result, the case of Lagrangian multiforms for fully discrete and semi-discrete KP type systems was left open, except in the case of the bilinear form of the discrete KP equation
the multiform structure of which was established in 2009. In contrast, I will present 3-form structures for the actual nonlinear forms of the (potential) discrete and semi-discrete KP equations.

Yang-Baxter and tetrahedron equations / 68

Entwinning Yang-Baxter maps and their extensions over Grassmann algebras

Dr. PAPAMIKOS, Georgios [1]

[1] *University of Essex*

Corresponding Author: g.papamikos@essex.ac.uk

I will present certain birational maps that are solutions of the parametric entwining Yang-Baxter equation. These maps are obtained via the refactorisation problem of certain Darboux transformations associated with the Lax operators of certain soliton PDEs. I will also present various dynamical properties of the derived maps, such as existence of invariants and associated symplectic or Poisson structures, and I will prove their complete integrability in the Liouville sense, where possible. Then I will describe the generalisation of such maps over Grassmann algebras using refactorisation of products of supermatrices, i.e. Darboux transformations with bosonic and fermionic entries. I will use the analogue of the characteristic polynomial, which in this non-commutative setting is the characteristic function, to define an analogue of a spectral curve. The latter can be used to obtain invariants of these maps involving Grassmann variables. New higher dimensional commutative maps can be obtained fixing the order of the Grassmann algebra Γ(n) and I will discuss integrability properties of these derived commutative maps.

Discrete Painlevé / 20

A 3×3 Lax form for the q-Painlevé equation of type E_6

Ms. PARK, Kanam [1]

[1] *Associate Professor*

Corresponding Author: kanami415@hotmail.co.jp

For the q-Painlevé equation with affine Weyl group symmetry of type E_6^{(1)}, a 2×2 matrix Lax form and a second order scalar lax form were known.
In this talk, we give a 3×3 matrix Lax form and a third order scalar equation related to it. They seems to be new.

Darboux polynomials / 53

Darboux Polynomials for Differential Equations and for Difference Equations

Prof. QUISPEL, Reinout [1]

[1] *La Trobe University*

Corresponding Author: r.quispel@latrobe.edu.au

We will discuss (mainly linear) Darboux polynomials for ODEs and for difference equations, and show their relation to the preservation of measures and first integrals, and in the construction of integrable systems.

Continuous integrable systems / 29

Expansion formulas for multiple basic hypergeometric series over root systems

Ms. RAI, SURBHI [1]; Dr. BHATNAGAR, GAURAV [2]

[1] *RESEARCH SCHOLAR, IIT DELHI*

[2] *VISITING ASSOCIATE PROFESSOR, DEPARTMENT OF MATHEMATICS, ASHOKA UNIVERSITY, INDIA*

Corresponding Author: surbhi.rai_tf@ashoka.edu.in

We extend the expansion formulas of Liu given in 2013 to the context of multiple series over root systems. Liu and others have shown the usefulness of these formulas in Special Functions and number-theoretic contexts. We extend Wang and Ma's generalizations of Liu's work which they obtained using q-Lagrange inversion. We use the A_n and C_n Bailey transformation and other summation theorems due to Gustafson, Milne, Lilly, and others, from the theory of A_n, C_n and D_n basic hypergeometric series. Our intent here is to provide several infinite families of extensions of Liu's key formulas to multiple basic hypergeometric series over the root systems.
This work was done in collaboration with Dr Gaurav Bhatnagar.

Keywords:
U(n +1)basic hypergeometric series
A_n and C_n basic hypergeometric series
A_n and C_n Bailey transform
q-Lagrange inversion

Lagrangian multiform theory / 43

Multiform Structures of Ordinary Difference Equations Satisfying Addition Formulae

Author: Mr. RICHARDSON, Jacob [1]

Co-Author: Prof. NIJHOFF, Frank [1]

[1] *University of Leeds*

Corresponding Author: mmjjr@leeds.ac.uk

In aiming to find simple nonlinear Lagrangian 1-forms that exhibit all the relevant features of a multiform structure, we investigate Lagrange structures arising from addition formulae. This results in non-quadratic Lagrangians and commuting maps with discrete and continuous interpolation flows. We discuss the relationship to integrable quad equations and applications to quantum propagators.

Darboux polynomials / 47

An ansatz concerning discrete Darboux polynomials

Prof. ROBERTS, John [1]

[1] *School of Mathematics and Statistics, UNSW Sydney Australia*

Corresponding Author: jag.roberts@unsw.edu.au

Celledoni et al [1,2] have illustrated the power of using so-called Darboux polynomials to search for preserved integrals and measures of a rational map L (including a map arising as a Kahan-Hirota-Kimura discretisation of a (Hamiltonian) ODE). For instance, integrals are obtained as the ratio of two Darboux polynomials where each Darboux polynomial satisfies an equation of the form P(x') = C(x) P(x), where C(x) is a rational function called a cofactor (and prime denotes the image of x under L).

In [1,2], C(x) is assumed to take a form involving factors from the numerator and denominator of the Jacobian determinant of L. In this talk, we confirm this "Jacobian factor ansatz" for C(x) using concepts from algebraic geometry and explore its further consequences.

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Discrete Painlevé / 39

On q-Painlevé VI and the Geometry of Segre surfaces

Prof. JOSHI, Nalini [1]; Dr. ROFFELSEN, Pieter [1]

[1] *The University of Sydney*

Corresponding Author: pieter.roffelsen@gmail.com

For each of the differential Painlevé equations, the Riemann-Hilbert correspondence maps its solution space onto an affine cubic surface. In recent work with Nalini Joshi, a q-analog was established for the q-Painlevé VI equation, where the associated algebraic surface is an affine Segre surface. In this talk, I will discuss this result and explain how the geometry of the Segre surface relates to different aspects of the q-Painlevé VI equation.

Non-linear waves / 64

The theory of periodic anomalous (rogue) waves in continuous and discrete NLS type equations

Author: Prof. SANTINI, Paolo [1]

Co-Authors: Prof. GRINEVICH, Peter [2]; Dr. COPPINI, Francesco [3]

[1] *University of Roma "La Sapienza"*

[2] *Landau Institute for Theoretical Physics RAS*

[3] *Università di Roma "La Sapienza*

Corresponding Author: paolo.santini@roma1.infn.it

Modulation instability and nonlinearity are the main causes of the appearance of anomalous (rogue) waves (AWs) in several physical contexts. The theory of periodic anomalous waves has been recently developed on the basic Nonlinear Schrödinger (NLS) model in 1+1 dimensions, adapting the finite gap method to the Cauchy problem for periodic initial perturbations of the homogeneous background solution of NLS [1]. This theory allows one to express the solution of the Cauchy problem, to leading order, in terms of elementary functions of the unstable part of the initial data, and has already been tested in the nonlinear optics of a photorefractive crystal [2]. Also a perturbation theory of AWs allowing one to study the leading order effects of small perturbations of the NLS equation on the dynamics of AWs has been constructed [3]. In this lecture we develop a lattice theory of AWs using as basic model the Ablowitz-Ladik lattice, integrable discretization of the NLS equation [4], [5].

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Compatibility / 65

Consistency of discrete equations on lattices of type D

Prof. SCHIEF, Wolfgang [1]

[1] *University of New South Wales*

Corresponding Author: w.schief@unsw.edu.au

Consistency of discrete equations on higher dimensional lattices constitutes a central element of integrable systems theory. The consistency of discrete equations defined on the squares and cubes of lattices of type B and the octahedra of lattices of type A have been studied extensively and with great success. However, it appears that the consistency of discrete equations naturally defined on lattices of type D or discrete equations which are defined on a larger number of vertices of a lattice has been explored to a significantly lesser degree. In this talk, we present some thoughts on this matter and illustrate them by considering linear and nonlinear (5-point) Laplace-type equations, a nonlinear 14-point equation and the 9-point (generalised) discrete Tzitzeica equation. Coincidentally, two Polish connections will be made.

Continuous integrable systems / 42

Multidimensional integrable systems from contact geometry

Prof. SERGYEYEV, Artur [1]

[1] *Silesian UNiversity in Opava*

Corresponding Author: artur.sergyeyev@gmail.com

In the present talk we address a longstanding problem of search for integrable partial differential systems in four independent variables, i.e. in the case most relevant for possible applications in physics, and show that such systems are significantly less exceptional than it appeared before: in addition to a number of previously known examples like the (anti)self-dual Yang--Mills equations there is a large entirely new class of such systems with nonisospectral Lax pairs of a novel kind related to contact vector fields. In particular, we will show that this class includes inter alia two new infinite families of such systems as well as e.g. the first example of an integrable system in four independent variables with a nonisospectral Lax pair which is algebraic, rather than rational, in the spectral parameter; please see the papers A. Sergyeyev, Lett. Math. Phys. 108 (2018), 359-376 & A. Sergyeyev, Appl. Math. Lett. 92 (2019), 196-200 for further details.

Non-linear waves / 45

Integral preserving discretization of 2D Toda lattice

Dr. SMIRNOV, Sergey [1]

[1] *Lomonosov Moscow State University*

Corresponding Author: ssmirnov1977@mail.ru

2D-Toda lattices corresponding to the Cartan matrices of simple Lie algebras are known to be Darboux integrable, i.e. they admit complete families of essentially independent characteristic integrals. During the last three decades various discrete analogs of these systems were obtained. In 2011 Habibullin proposed a systematic way to discretize 2D-Toda lattices. His approach was based on the
idea to look for semi-discrete systems such that they have the same characteristic integrals as their continuous analogs. Careful analysis of the systems corresponding to the Cartan matrices of the rank 2 allowed Habibullin and his collaborators to introduce semi-discrete and purely discrete analogs of 2D-Toda lattices and to conjecture that they are Darboux integrable for Cartan matrices
of arbitrary rank. After that some partial results on Darboux integrability of these systems were obtained, but the general claim remained unproved.

We prove that if function I is a y-integral of 2D-Toda lattice corresponding to some Cartan matrix, then this function is an n-integral of its semi-discrete analog. This implies the existence of a complete family of n-integrals for each of these systems. We use the concept of characteristic algebra to prove that these systems admit complete families of characteristic x-integrals as well.

Singularity analysis / 18

Full deautonomisation by singularity confinement as an integrability test

Author: Dr. STOKES, Alexander [1]

Co-Authors: Prof. WILLOX, Ralph [2]; Prof. GRAMMATICOS, Basile [3]; Dr. MASE, Takafumi [2]

[1] *The University of Tokyo*

[2] *the University of Tokyo*

[3] *Université Paris-Saclay and Université de Paris*

Corresponding Author: stokes@ms.u-tokyo.ac.jp

Since its introduction, the method of full deautonomisation by singularity confinement has proved a strikingly effective way of detecting the dynamical degrees of birational mappings of the plane.
This method is based on a conjectured link between two a priori unrelated notions: firstly the dynamical degree of the mapping and secondly the evolution of parameters required for its singularity structure to remain unchanged under a sufficiently general deautonomisation.
In this talk we will present a proof of this conjectured correspondence for a large class of birational mappings of the plane via the spaces of initial conditions for their deautonomised versions.
We show that even for non-integrable mappings in this class, the surfaces forming these spaces have effective anticanonical divisors and one can define a kind of period mapping, which provides a bridge between the evolution of coefficients in the deautonomised mapping and the induced dynamics on the Picard lattice which encode the dynamical degree.
We will illustrate the method in some examples and also discuss connections to the theory of rational surfaces associated with root systems of indefinite type.

Discrete Painlevé / 14

An affine Weyl group action on the basic hypergeometric series arising from the q-Garnier system

Prof. SUZUKI, Takao [1]

[1] *Kindai University*

Corresponding Author: suzuki@math.kindai.ac.jp

The Garnier system is an extension of the sixth Painlev\'e equation from a viewpoint of the isomonodromy deformation of a Fuchsian system.
Its q-difference analogue was proposed by Sakai as the connection preserving deformation of a linear q-difference system.
Recently, we formulated the q-Garnier system in a framework of an extended affine Weyl group of type $A^{(1)}_{2n+1}\times A^{(1)}_1\times A^{(1)}_1$.
On the other hand, the q-Garnier system admits a particular solution in terms of the basic hypergeometric series ${}_{n+1}\phi_n$.
Then it becomes the next problem to investigate an action of the extended affine Weyl group on ${}_{n+1}\phi_n$.
In this talk, we give an answer to this problem.
Namely, we give a left action of a subgroup of the extended affine Weyl group on a vector whose components are described in terms of ${}_{n+1}\phi_n$.
Hence q-contiguity relations and a linear q-difference equation for ${}_{n+1}\phi_n$ can be derived from the extended affine Weyl group systematically.

Yang-Baxter and tetrahedron equations / 70

Tetrahedron maps and symmetries of 3D integrable discrete equations.

Dr. TONGAS, Anastasios [1]

[1] *Department of Mathematics, University of Patras GR*

Corresponding Author: tasos.tongas@gmail.com

I will discuss a connection between the tetrahedron equation for maps and the consistency property of integrable discrete equations on $\mathbb{Z}^3$. The connection is based on the invariants of symmetry groups of the lattice equations, generalizing a method developed in the context of Yang-Baxter maps. The method will be demonstrated to certain octahedron type lattice equations, leading to some new examples of tetrahedron maps and integrable coupled lattice equations.

The talk is based on a joint work with Pavlos Kassotakis, Maciej Nieszporski and Vassilis Papageorgiou.

. / 76

Integrable maps; from algebraic construction to geometric understanding

Dr. VAN DER KAMP, Peter [1]

[1] *La Trobe University*

Corresponding Author: peterhvanderkamp@gmail.com

Integrable maps can be obtained as periodic or open reductions from lattice equations which satisfy consistency conditions.
An open reduction (this will be explained in the talk by Cheng Zhang) from Q1_0 was decomposed as a composition of Manin involutions on each curve of the invariant pencil.
However, one of the involution points appeared to be curve-dependent....

77

Features of discrete integrability

VIALLET, CLAUDE M. [1]

[1] *Centre Natonal de la Rechche Scientifique / Sorbonne Université*

Corresponding Author: claude.viallet@upmc.fr

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Degree growth of lattice equations and maps / 26

Computing degree growth of birational maps from local indices of polynomials

Dr. ALONSO, Jaume [1]; Mr. WEI, Kangning [2]; Prof. SURIS, Yuri [2]

[1] *Technische Universitaet Berlin*

[2] *TU Berlin*

Corresponding Author: weikangning12@gmail.com

One of the most important dynamical invariant associated to a birational map f is given by its dynamical degree, or equivalently, its algebraic entropy, which is defined via the rate of growth of the sequence deg(f^n). More concretely, the degrees deg(f^n), although not birationally invariant by themselves, are also of great interests in understanding the dynamics of the birational map.
We propose a general method for computing the degrees deg(f^n). More precisely, given a homogeneous polynomial P, we compute the iterated pullbacks of the polynomial by the map f. To do this, we perform a sequence of blowing ups and to each blowing up we associate a local index \mu(P) to a polynomial P. Together with the degrees deg(f^n), these local indices satisfy a recurrence relation which can be solved to obtain the degrees deg(f^n).
In two dimensional cases, we show that these indices are closely related to the intersection numbers. In principle, however, this method is applicable to birational maps in any dimension.

Singularity analysis / 19

The singularity structure of the discrete KdV and mKdV equations

Prof. WILLOX, Ralph [1]

[1] *the University of Tokyo*

Corresponding Author: willox@ms.u-tokyo.ac.jp

Although the notion of singularity confinement was first introduced for the discrete KdV (dKdV) equation, as of yet there is still no rigorous definition of the notion of `confinement' for lattice equations. In fact, somewhat ironically, it has taken nearly 30 years before an exhaustive study of the singularities of the dKdV equation finally revealed their intriguing properties and the full richness of the singularity structures they produce.
In this talk I will explain the basic classification of singularities for the dKdV and mKdV equations, which involves a novel `strip-like' type of singularity. These strips or bands can interact with the singularities of other types, producing singularity patterns of increasing complexity. Moreover, I will show that the interaction of these singularities with a particular type of singularity can be described using a special symbolic dynamics which turns out to be equivalent to the Takahashi-Satsuma Box dynamics.

Discrete differential geometry / 28

Systems of difference equations, symmetries and integrability conditions

Dr. XENITIDIS, Pavlos [1]

[1] *Liverpool Hope University*

Corresponding Author: xenitip@hope.ac.uk

In this talk we introduce a class of systems of difference equations defined on an elementary quadrilateral of the lattice and derive necessary conditions for their integrability. These conditions follow from the requirement that the system admits infinite hierarchies of symmetries, and can be used in the construction of the lowest order symmetries of the system. These considerations are demonstrated with the help of three systems from the class under consideration.

50

Geometrical view of the Lagrangian 1-from closure relation

Author: Dr. YOO-KONG, Sikarin [1]

Co-Author: Mr. KONGKOOM, Thanadon [1]

[1] *The Institute for Fundamental Study, Naresuan University*

Corresponding Author: sikariny@nu.ac.th

We will present a geometrical interpretation of the Lagrangian 1-form closure relation, inferred as the multi-dimensional consistency in time evolution on the space of independent variables. New mathematical objects, such as Lagrange vector field and Hamilton vector field defined on the the space of independent variables, will be used to capture the integrability condition.

. / 10

Integrable boundary conditions for quad-graph systems: classification and applications

Author: Dr. ZHANG, Cheng [1]

Co-Authors: Dr. CAUDRELIER, Vincent [2]; Dr. CRAMPE, Nicolas [3]; Dr. PETER, van der Kamp [4]; Mr. SUN, Pengyu [1]

[1] *Shanghai University*

[2] *University of Leeds*

[3] *University of Tours*

[4] *La Trobe University*

Corresponding Author: ch.zhang.maths@gmail.com

The notion of boundary conditions for quad-graph systems will first be introduced. The boundary conditions are naturally defined on triangles that arise as dualization of given quad-graphs with boundary. For three-dimensionally consistent quad-graph systems, the so-called integrable boundary conditions will be characterized as boundary conditions satisfying the boundary consistency condition that is a consistency condition defined on a half of a rhombic-dodecahedron. Based on these notions, three main results will then be presented: a classification of integrable boundary conditions for quad-equations of the ABS classification; Lax formulations of integrable boundary conditions; and the so-called open boundary reduction technique as systematic a means to construct integrable mappings from integrable initial-boundary value problems for quad-graph systems. This talk is based on [Caudrelier, Crampe, CZ, Sigma, 10(014), 2014], [Caudrelier, van der Kamp, CZ, IMRN, rnac207, 2021] and [Sun, CZ, IMRN, rnab188, 2022].

Lagrangian multiform theory / 31

tau function, vertex operator and linearization scheme associated with Lamé function

Prof. ZHANG, Da-jun [1]

[1] *Shanghai University*

Corresponding Author: djzhang@staff.shu.edu.cn

The talk contains two parts. In the first part I will describe a bilinear framework for elliptic soliton solutions (which are composed by the Lamé-type plane wave factors and expressed using Weierstrass functions). The framework includes tau functions in Hirota's form, vertex operators to generate such tau functions and the associated bilinear identities. These are introduced in detail for the KdV equation and sketched for the KP hierarchy. The second part I will report an elliptic direct linearisation scheme associated with discrete Lamé-type plane wave factors. This scheme allows us to have lattice KP equations and lattice Boussinesq equations that have elliptic soliton solutions. In both continuous can discrete cases, the so-called elliptic N-th roots of unity are needed to define plane wave factors and implement reductions. This talk is based on a joint work with Xing Li (arxiv: 2204.01240) and a joint work with Frank Nijhoff and Yingying Sun (1909.02948).