Yukawa unification in extended GMSB models

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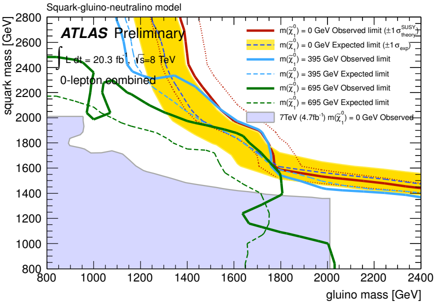
Outline

- extended GMSB: GMSB + messenger superpotential couplings
- messenger-matter mixing & $y_{t,b,\tau}$ running
- $y_t - y_b - y_\tau$ unification in $SO(10)$ inspired model

What do the LHC searches tell us about MSSM?

- no SUSY signal so far
- relevant exclusions only for 1st and 2nd family
- still $\widetilde{Q}_3, \ldots$ can be as light as 500 GeV



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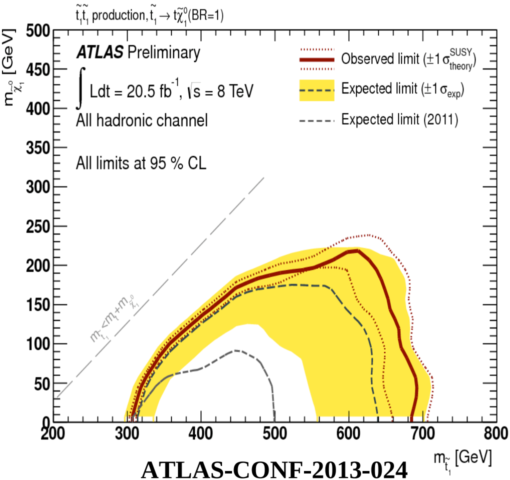
2. Limits on stop mass



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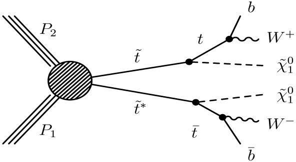

3. LHC vs. MSSM

What do the LHC searches tell us about MSSM?

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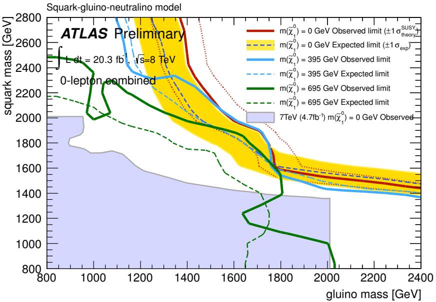

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BUT important information comes from Higgs mass measurement:

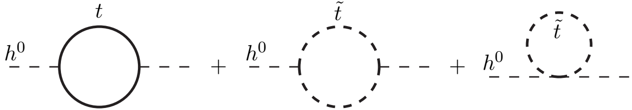
- $m \sim 125$ GeV $\to$ need for large loop corrections

ASSUME other MSSM Higgses are much heavier and masses of $\widetilde{Q}_{1,2}$ and $\widetilde{g}$ are bigger than 1.8 TeV.

4. 1-loop corrections to m_{h^0}

- dominant contribution from top quarks and stops (due to $y_t \sim 1$):

$\Delta(m^2_{h^0}) =$



$$m^2_{h^0} = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2}\left[\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right] \approx (125\,\mathrm{GeV})^2,$$

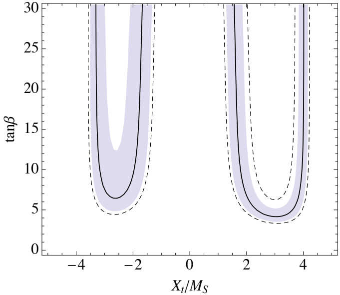
$M_S = \sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$

$X_t = A_t - \mu\cot\beta$

Large A-terms or heavy stops!

A-terms:

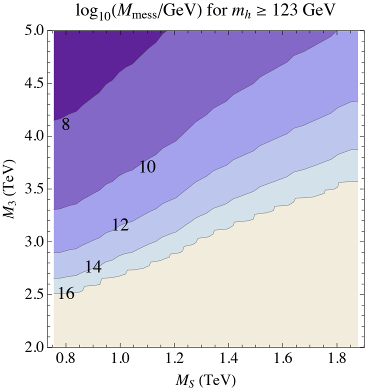
$$V_{\text{soft}} \supset y_t A_t H_u \widetilde{Q}_3 \widetilde{\overline{U}}_3 \longrightarrow y_t A_t h_0 \widetilde{t}_1 \widetilde{t}_2$$



Draper et al. 1112.3068

5. A-terms in GMSB

- in GMSB models A-terms $= 0$ at messenger scale



Draper et al. 1112.3068

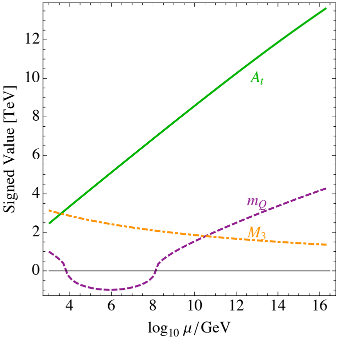
$$\mu \frac{dA_t}{d\mu} \sim y_t^2 A_t + g_3^2 M_3$$

- hard to reconcile
  - $m_{h^0} \gtrsim 123\,\mathrm{GeV}$
  - pure GMSB mechanism
  - light stops
  - $M_{\tilde{g}} \lesssim 2.5\,\mathrm{GeV}$

- large A-terms at M?

6. How to generate large A-terms?

- value of A-term gives initial condition for RGE evolution



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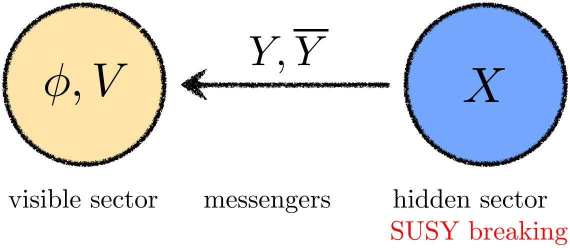
$$\mu \frac{dA_t}{d\mu} \sim y_t^2 A_t + g_3^2 M_3$$

- heavy $\tilde{g}$ and RGE evolution from $M \gtrsim 10^{14}$ GeV
- or large A-terms at M

- how to get A-terms in GUT model?

$\boxed{\text{Extended GMSB models (EGMSB)}}$

7. SUSY breaking mediation



- singlet $\langle X \rangle = M + \theta^2 F \rightarrow$ spontaneous SUSY breaking

$$\xi = \frac{F}{M} \sim 10^5 \text{ GeV}$$

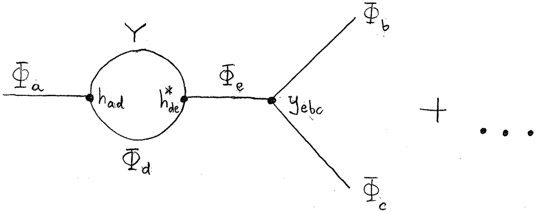
- messengers have large masses e.g. $M \sim 10^8 - 10^{14}$ GeV
- mediation = interactions between $Y, \overline{Y}$ and other fields
- assumption: all messengers couple to the spurion X in the same way

$$X Y_a \overline{Y}_a$$

and $M \gtrsim 10^8$ GeV $\longrightarrow$ 1-loop soft masses negligible

9. Trilinear terms in EGMSB models

$$W = \frac{1}{6} y_{abc} \Phi_a \Phi_b \Phi_c + \frac{1}{2} h_{ab} \Phi_a \Phi_b \mathsf{Y} + h_a \Phi_a \mathsf{YY} + \eta \mathsf{YYY}$$



$$V \supset T_{abc} \widetilde{\Phi}_a \widetilde{\Phi}_b \widetilde{\Phi}_c, \qquad T_{abc} = -\frac{\xi}{16\pi^2} \left[C_a h_{ad} h^*_{de} y_{ebc} + \ldots \right] + (a \leftrightarrow b) + (a \leftrightarrow c)$$

- T_{abc} are 'partially aligned' to MSSM Yukawas y_{abc}

10. A-terms in EGMSB models

$$V \supset H_u \widetilde{Q} (T_u) \overline{\widetilde{U}} + H_d \widetilde{Q} (T_d) \overline{\widetilde{D}} + H_d \widetilde{L} (T_e) \overline{\widetilde{E}}$$

- $(T_{u,d,e})_{33} =: y_{t,b,\tau} A_{t,b,\tau}$

$$A_{t,b,\tau} \approx -\frac{\xi}{16\pi^2} C^{(t,b,\tau)} |h|^2 \qquad \text{e.g. } C^{(t,b,\tau)} = 10, 12, 11$$

- A-terms
  - relevant to the m_{h^0}
  - may also lead to CCB when

$$A_f^2 > 3(m_{\widetilde{f}_L}^2 + m_{\widetilde{f}_R}^2 + \mu^2 + m_{H_u}^2)$$

  - affect sfermion masses $m_{\widetilde{f}_{1,2}}$

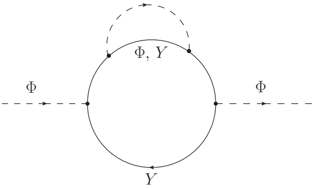
$$(\widetilde{f}_L^* \ \widetilde{f}_R^*) \begin{pmatrix} m^2_{\widetilde{f}_{LL}} & m_f(A_f - \mu \tan \beta^{\pm 1}) \\ m_f(A_f - \mu \tan \beta^{\pm 1}) & m^2_{\widetilde{f}_{RR}} \end{pmatrix} \begin{pmatrix} \widetilde{f}_L \\ \widetilde{f}_R \end{pmatrix}$$

$\rightarrow \widetilde{f}_1$ may be tachyonic

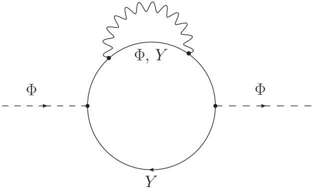
11. Soft masses in EGMSB models

- 2-loop contributions to soft masses

$$W_Y = h^{(I)}\Phi \mathsf{Y}\mathsf{Y} + h^{(II)}\Phi\Phi\mathsf{Y}$$



$\sim h^4, h^3 y, h^2 y^2$

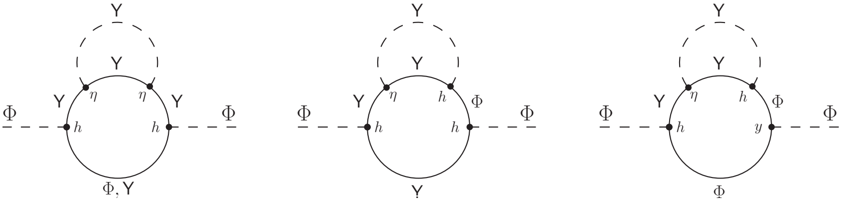


$\sim h^2 g^2$

$$m^2_{\tilde{\Phi},h} \sim \frac{\xi^2}{(4\pi)^4}(h^4 + h^3 y - h^2 y^2 - h^2 g^2)$$

12. 2-loop soft masses induced by YYY

$$W_Y = h_i^{(I)} \Phi_i \mathsf{YY} + h_{ij}^{(II)} \Phi_i \Phi_j \mathsf{Y} + \eta \mathsf{YYY}$$



$$m^2_{\widetilde{\Phi},\eta} \sim \frac{\xi^2}{(4\pi)^4} (\eta^2 h^2 + \eta h^3 + \eta h^2 y)$$

Remark: η are relevant only if a model contains both $5 + \overline{5}$ and $10 + \overline{10}$ messengers

13. Kinetic mixing

- fields Y, ϕ with the same charges can mix: $\phi \leftrightarrow Y$

$$Q \leftrightarrow Y_Q,\ \overline{U} \leftrightarrow Y_{\overline{U}}, \ldots \quad \text{(in some models: } H_d \leftrightarrow L \leftrightarrow Y_L\text{)}$$

- superpotential and Kähler potential K at scale $t = \log\mu$

$$W = \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M_{ij}\Phi_i\Phi_j, \quad K = \Phi_i^\dagger Z_{ij}(t)\Phi_j, \quad Z = Z^\dagger,\ Z > 0$$

- couplings $\widetilde{\lambda}(t)$ and masses of canonically normalized fields $\widetilde{\Phi}_i = Z_{ij}^{-1/2}\Phi_j$

$$\widetilde{\lambda}_{ijk}(t) = \lambda_{i'j'k'}Z_{i'i}^{-1/2}Z_{j'j}^{-1/2}Z_{k'k}^{-1/2}, \quad \widetilde{M}_{ij}(t) = M_{i'j'}Z_{i'i}^{-1/2}Z_{j'j}^{-1/2}$$

- $\boxed{\text{RGE evolution of } Z \text{ (re)introduces mixing mass terms!}}$

  e.g.

$$W = \widetilde{M}_1\widetilde{Y}_R\widetilde{Y}_{\overline{R}} + \widetilde{M}_2\widetilde{\phi}_R\widetilde{Y}_{\overline{R}} + \ldots$$

- important for decouplings and running Yukawa (couplings between light states)!

14. Decoupling and running

$$W = \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M_{ij}\Phi_i\Phi_j, \quad K = \Phi_i^\dagger Z_{ij}(t)\Phi_j, \quad Z = Z^\dagger,\ Z > 0$$

- method 1 - rotate $\widetilde{\Phi} = Z^{-1/2}\Phi$ such that light fields are present
- method 2 - instead of computing $Z^{-1/2}$ and then rotating $\widetilde{\Phi}$ use Cholesky decomposition of Z:

$$Z = V^\dagger V, \qquad \widetilde{\Phi} = \begin{pmatrix} \widetilde{\phi} \\ \widetilde{Y} \end{pmatrix} = \underbrace{\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}}_{V} \begin{pmatrix} \phi \\ Y \end{pmatrix}$$

$$\widetilde{\lambda}_{ijk}(t) = \lambda_{i'j'k'} V^{-1}_{i'i} V^{-1}_{j'j} V^{-1}_{k'k}$$

- one can check that

$$\widetilde{\lambda}_{abc}(t) = \frac{\lambda_{abc}}{\sqrt{Z_{aa}Z_{bb}Z_{cc}}}, \qquad \phi_a \; - \; \text{light fields}$$

15. Evolution of Z from GUT scale t_{GUT}

- RGE for $Z(t)$ with boundary condition $Z(t_{GUT}) = 1$

$$\frac{d}{dt} Z_{ij} = -\frac{1}{8\pi^2} \left(\frac{1}{2} d_{kl} \lambda^*_{ikl} Z^{-1\,*}_{km} Z^{-1\,*}_{ln} \lambda_{jmn} - 2C^{(r)}_{ij} Z_{ij} g_r^2 \right)$$

  d_{kl} and C^r_{ij} - group theory factors

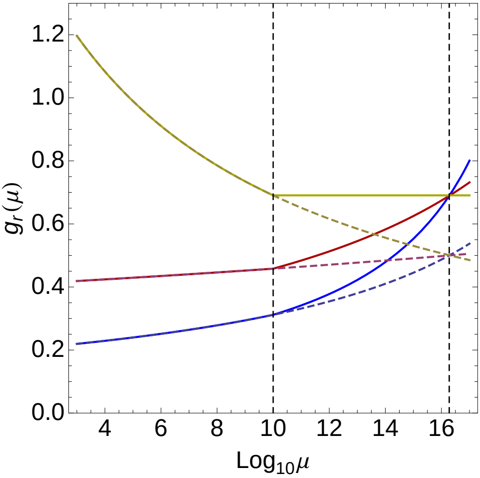
- solve numerically or use approximate solution:

$$Z_{ij}(t) = 1 + Z^{(1)}_{ij}(t - t_{GUT}) + \frac{1}{2!} Z^{(2)}_{ij}(t - t_{GUT})^2 + \ldots$$

- to compute $Z^{(n)}$ one needs all $Z^{(k)}$, $k < n$
- $Z^{(n)}$ are expressed in terms of $\epsilon = \ln 10/16\pi^2$, λ_{ijk}, d_{kl}, g_{GUT} and $\beta_{g_r}(t_{GUT})$

16. Running of gauge couplings

$$W = y_t H_u Q\overline{U} + h_t(H_u Q Y_{\overline{U}} + H_u Y_Q \overline{U}) + M(Y_U Y_{\overline{U}} + Y_Q Y_{\overline{Q}}) + \dots$$



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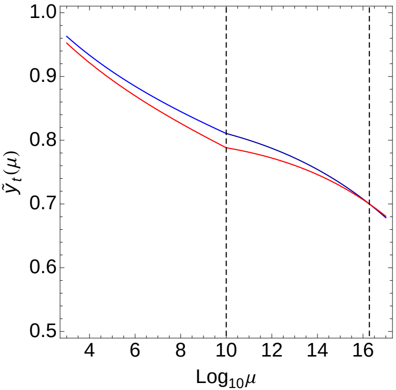
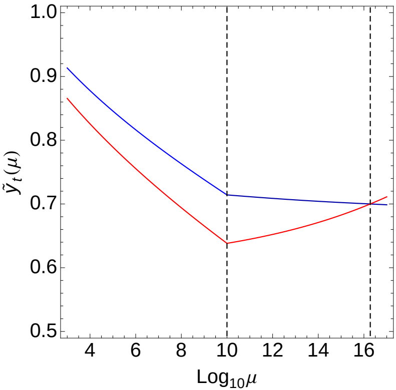
$y_t = 0.7,\ h_t = 0.4$



$y_t = 0.7,\ h_t = 0.9$



18. SO(10) inspired GUT model

- at $M_{GUT} \sim 10^{16}$GeV: $\quad SO(10) \to SU(5) \times U(1)_\chi \to \dots$
- chiral matter Φ

$$H_{10}: \quad 10 \to 5_2 + \overline{5}_{-2}, \qquad \phi_{16}: \quad 16 \to 10_{-1} + \overline{5}_3 + 1_{-5}$$

- messengers $\mathsf{Y} = (Y_{16}, Y_{\overline{16}})$

$$W = y H_{10}\phi_{16}\phi_{16} + h H_{10}\phi_{16}Y_{16} + \frac{1}{2} M Y_{16} Y_{\overline{16}} + \dots$$

  - $y = y_t(t_{GUT}) = y_b(t_{GUT}) = y_\tau(t_{GUT})$
  - $\phi_1 = N_R$, Y_1 and Higgs triplets masses $\sim M_{GUT}$
  - only couplings to 3rd generation

19. Z for $SO(10)$ inspired model

- RGE for $Z(t)$ with boundary condition $Z(t_{GUT}) = 1$

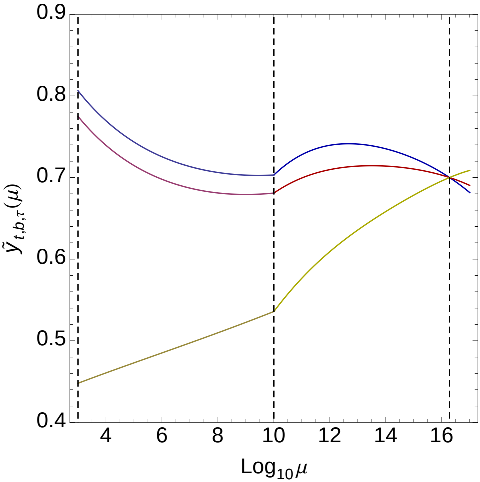
$$\frac{d}{dt} Z_{ij} = -\frac{1}{8\pi^2}\left(\frac{1}{2} d_{kl}\lambda^*_{ikl} Z^{-1\,*}_{km} Z^{-1\,*}_{ln} \lambda_{jmn} - 2C^{(r)}_{ij} Z_{ij} g^2_r\right)$$

- $SO(10)$ inspired minimal model with superpotential

$$W = y H_{10}\phi_{16}\phi_{16} + h H_{10}\phi_{16} Y_{16} + M Y_{16} Y_{\overline{16}}$$

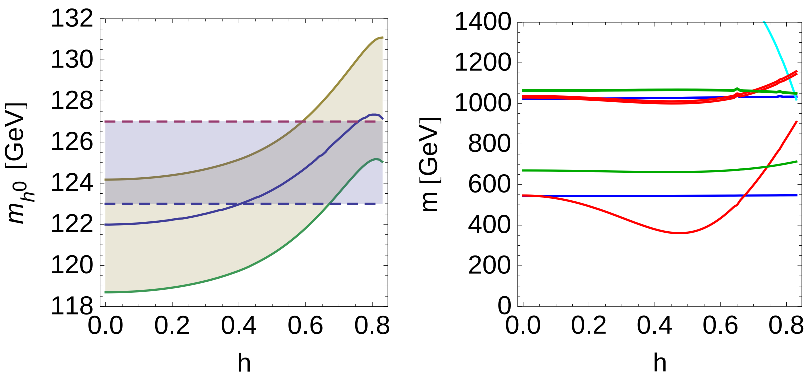
$$\begin{aligned} Z_{H_u H_u} &= 1 + \frac{6}{5}\epsilon[3g^2_{GUT} - 5(2h^2 - y^2)](t - t_{GUT}) \\ &+ \frac{24}{25}\epsilon^2[29g^2_{GUT} + 35(2h^2 + y^2) - 25(2h^4 + 4h^2y^2 + y^4)](t - t_{GUT})^2 \\ &+ \ldots \end{aligned}$$

20. $t - b - \tau$ unification



$$W = yH_{10}\phi_{16}\phi_{16} + hH_{10}\phi_{16}Y_{16} + MY_{16}Y_{\overline{16}}, \quad y = 0.7,\ h = 0.4,\ M = 10^{10}\ \mathrm{GeV}$$

$$\tan\beta = 35, \quad \xi = 10^5\,\mathrm{GeV}, \quad M = 10^{14}\,\mathrm{GeV}$$



22. Phenomenology of $SO(10)$ inspired GUT model

- scan over parameters

$$8 < t_M < 14, \quad 0.6 < y < 0.9, \quad 0 < h < 1.2$$

- check low-energy constraints

$$m_{h^0} \approx 125\,\mathrm{GeV}, \quad M_{\widetilde{g},\widetilde{q}_{1,2}} > 1.8\,\mathrm{TeV}, \quad \mathrm{UFB/CCB}, \quad a_\mu, \quad \ldots$$

- for moderate $\tan\beta \sim 20$: no tachyons, $\widetilde{\tau}$ is NLSP , but threshold corrections to $y_{b,\tau} \sim 200\%$ or more are needed

- to get $\sim 20\%$ threshold correction for y_b one has to fix $\tan\beta \sim 45 \rightarrow$ tachyonic $\widetilde{\tau}$

- to avoid instabilities of the potential one could extend spectrum or allow additional messenger couplings

23. Conclusions

- messenger couplings λ not only generate soft terms but can also lead to kinetic mixing
- wave-function renormalization Z is a handy tool to analyze RG flow of Yukawas; this method can be implemented in a similar way at 2-loop level
- phenomenology of the simplest $SO(10)$ model is spoiled by tachonic $\widetilde{\tau}$ $\rightarrow$ extend spectrum or allow additional couplings