Saturation and Geometrical Scaling: from Deep Ineastic Scattering to Heavy Ion Collisions

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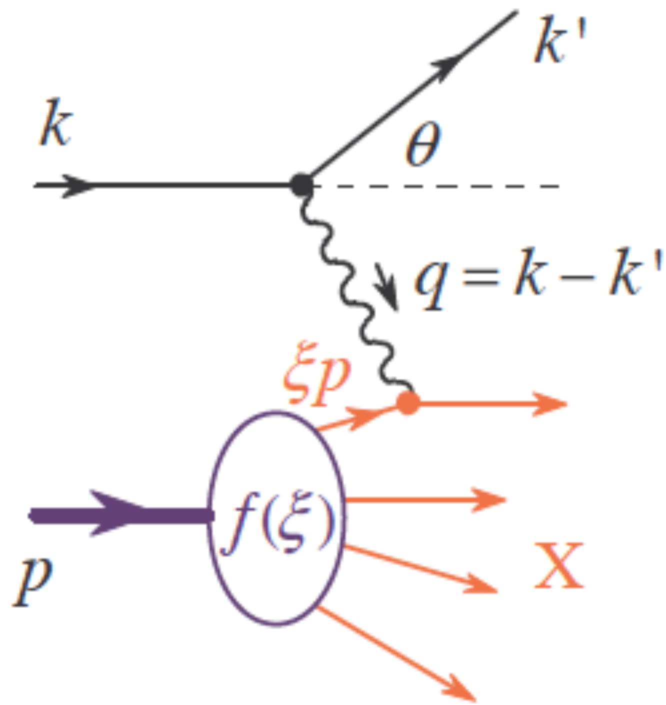
message to take away:

There exits an intermediate energy scale, called *saturation scale,* that, by dimensional arguments, deterimines inclusive and semi--inclusive observables in kinematical regions where no other energy (momentum) scales exist.

Bjorken Scaling



Nobel 1990:
Jerome Friedman (MIT)
Henry Kendall (MIT)
Richard Taylor (SLAC)

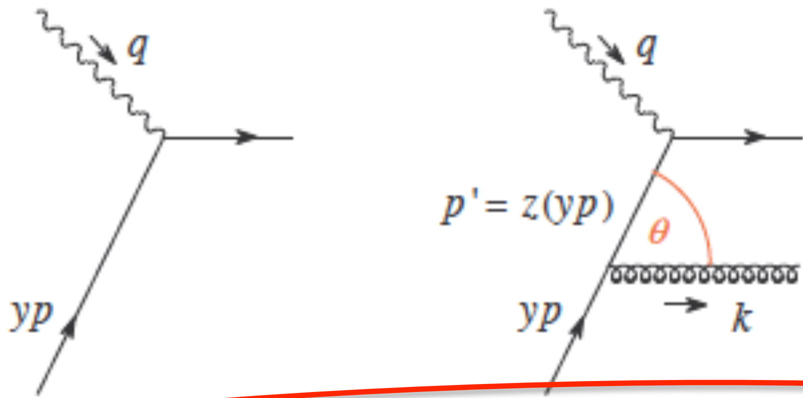
neglecting masses:

$$(\xi p + q)^2 = 0$$

$$2\xi\, pq + q^2 \; = \; 0$$

$$\xi \; = \; \frac{Q^2}{2pq} = x$$

DGLAP Evolution



Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

$$Q^2 \frac{d}{dQ^2} q_i(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[P_{qq} \otimes q_i(Q^2) + P_{qG} \otimes G(Q^2) \right]$$

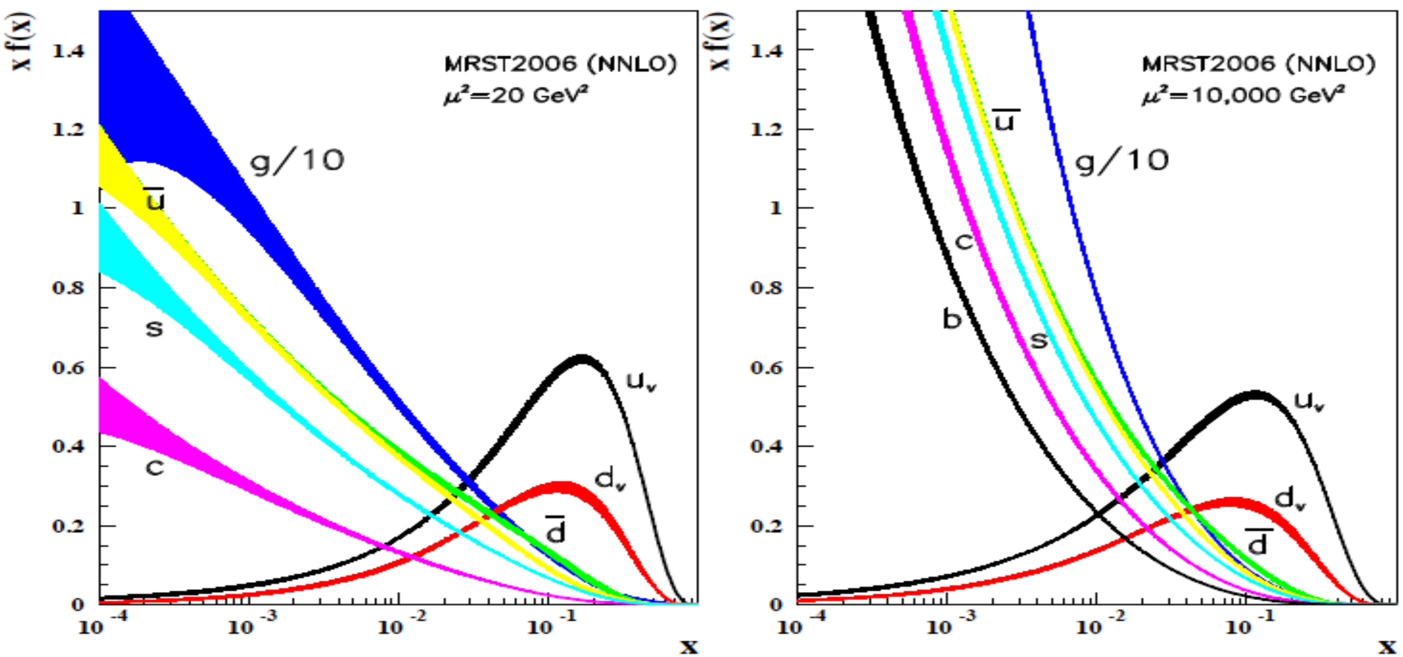
$$Q^2 \frac{d}{dQ^2} G(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[P_{Gq} \otimes \sum_i q_i(Q^2) + P_{GG} \otimes G(Q^2) \right]$$

$$P_{qq} \otimes q = \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) q(y) = \int_x^1 \frac{dz}{z} P_{qq}(z) q\left(\frac{x}{z}\right)$$

DGLAP Evolution

solutions found by taking moments (Mellin transform):

$$M_n = \int_0^1 dx\, x^{n-1} P \otimes f = P_n f_n = \gamma^n f_n$$

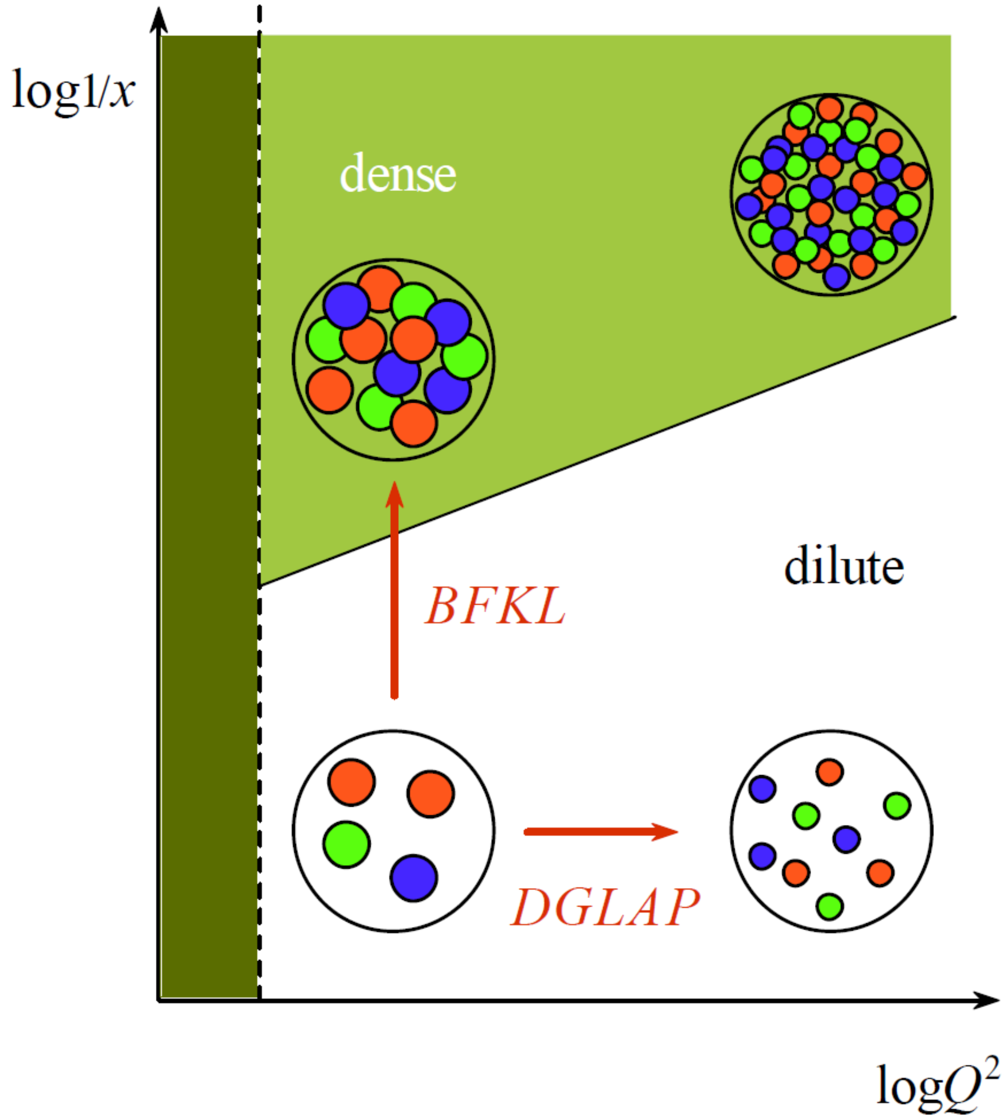


DGLAP vs BFKL Evolution

small x
large W

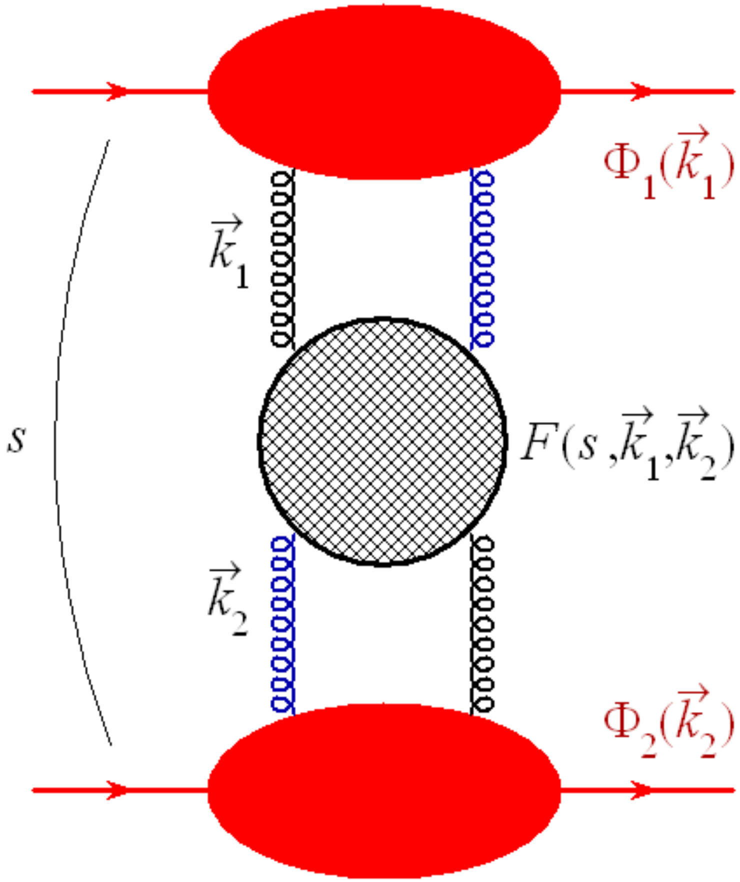
Balitsky, Fadin,
Kuraev, Lipatov

large x
small W



Forward BFKL Amplitude

$$\sigma_{\text{tot}} = \frac{C_R}{(2\pi)^4} \int \frac{d^2\vec{k}_1}{\vec{k}_1^2} \frac{d^2\vec{k}_2}{\vec{k}_2^2} \Phi_1(\vec{k}_1)\Phi_2(\vec{k}_2)\, F(s, \vec{k}_1, \vec{k}_2)$$



transverse momenta

process dependent impact factors

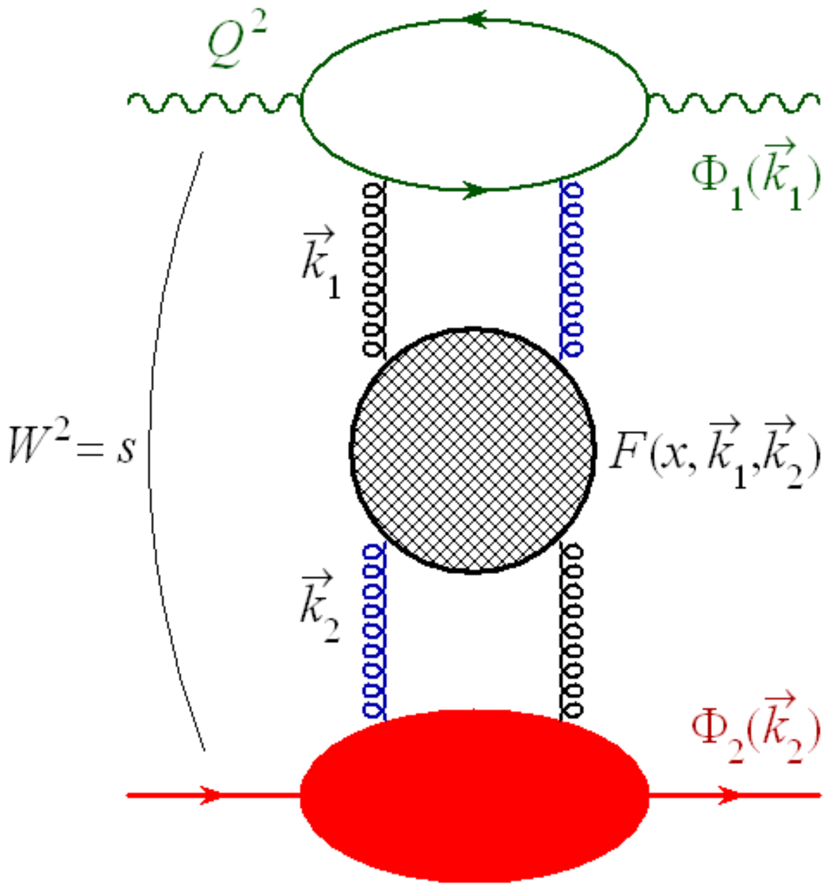
energy dependent universal amplitude

zeroth order „initial condition":

$$F(s_0, \vec{k}_1, \vec{k}_2) \sim \delta^{(2)}(\vec{k}_1 - \vec{k}_2)$$

Forward BFKL Amplitude

can be applied in DIS:

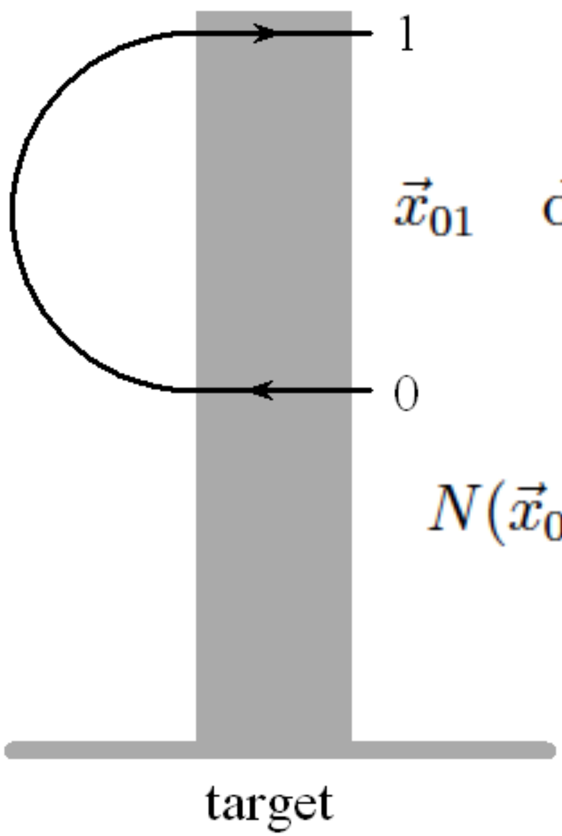


$$x = \frac{Q^2}{W^2 + Q^2} \simeq \frac{Q^2}{W^2} = \frac{s_0}{s}$$

Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller

A.H. Mueller and J.-w. Qiu,
Nucl. Phys. B 268 (1986) 427

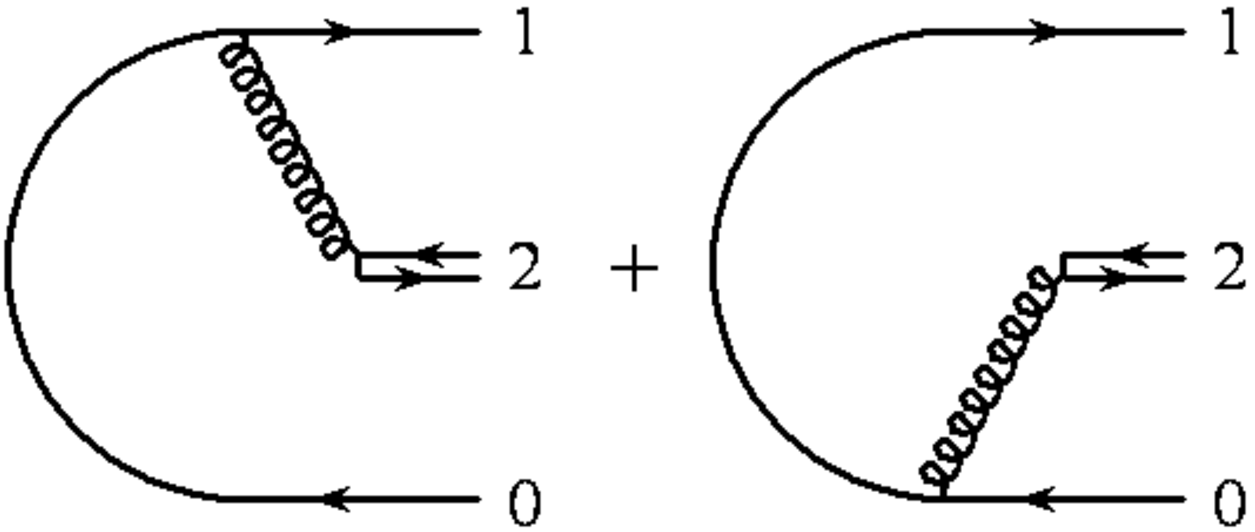


$\vec{x}_{01}$ dipole transverse size

$N(\vec{x}_{01}, Y)$ dipole-target forward amplitude

$$\tilde{N}(\vec{k}, Y) \sim \alpha_s \int F(x, \vec{k}, \vec{l})\Phi(\vec{l})\frac{d^2\vec{l}}{\vec{l}^2}$$

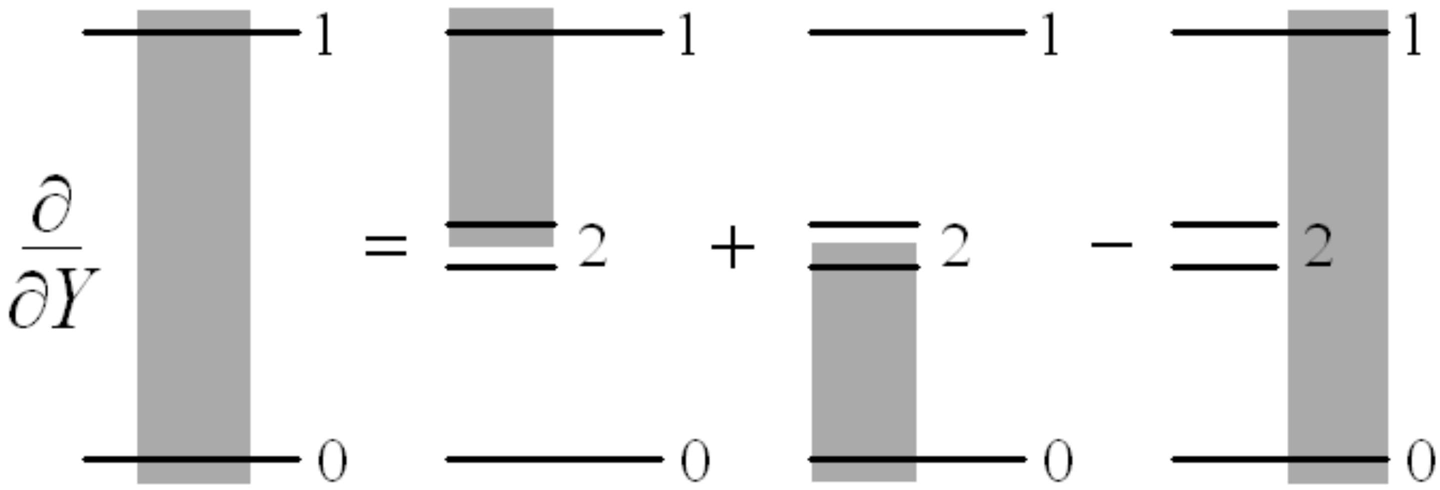
Gluon Emission in the Dipole Picture



- large N_c
- dipole emission kernel is very simple
- reproduces BFKL equation

$$\int d^2\vec{x}_2 \, \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

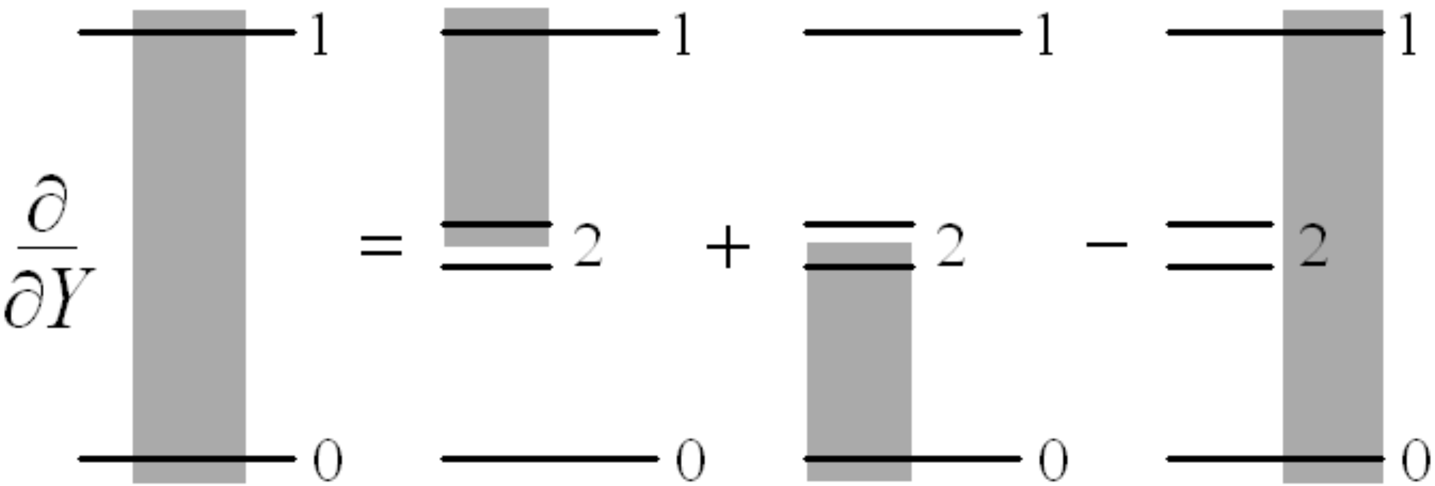
BFKL Eq. in the Dipole Picture



$$\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\overline{\alpha}_s}{2\pi} \int d^2\vec{x}_2 \, \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \left[N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) \right]$$

no singularities at *e.g.* 2 → 0 due to color transparency: $N(0,Y)=0$

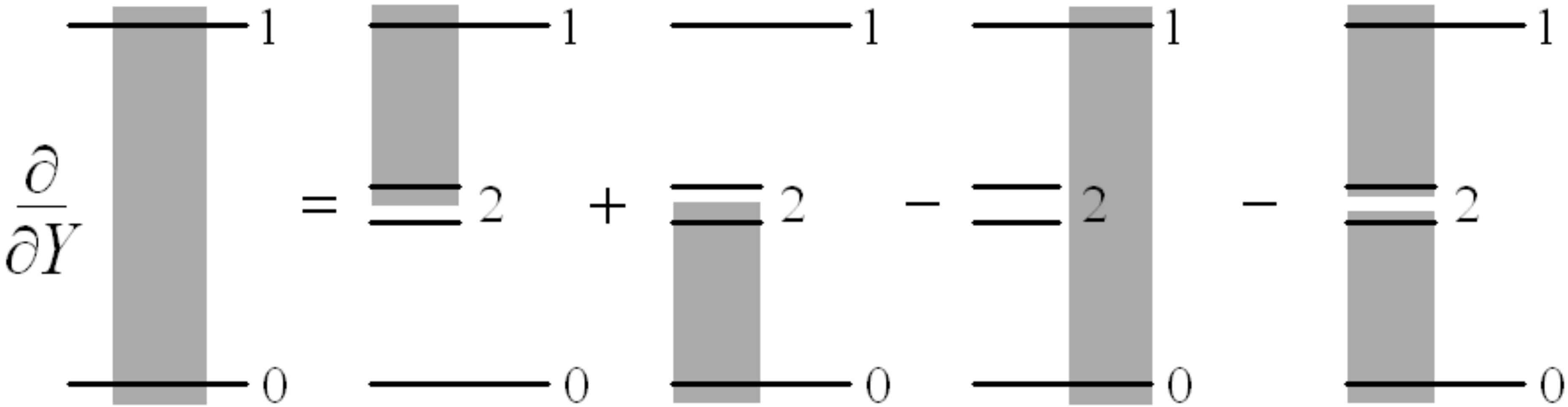
BFKL Eq. in the Dipole Picture



$$F(s,\vec{k}_1,\vec{k}_2) = \frac{1}{\pi}\frac{1}{\sqrt{\vec{k}_1^2\vec{k}_2^2}}\left(\frac{s}{s_0}\right)^{\overline{\alpha}_s 4\ln 2}\frac{1}{\sqrt{\pi\Delta(s)}}\exp\left(-\frac{\ln^2(\vec{k}_1^2/\vec{k}_2^2)}{\Delta(s)}\right)$$

I. Balitsky,
Nucl. Phys. B 463 (1996) 99
Y.V. Kovchegov,
Phys. Rev. D 60 (1999) 034008

BK Equation



$$\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\overline{\alpha}_s}{2\pi} \int d^2\vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \Big[N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) - N(x_{02}, Y) N(x_{12}, Y) \Big]$$

double scattering stops rapid growth of the amplitude with *Y*
note that [....] = 0 for *N* → 1

BK Equation

$$\frac{\partial}{\partial Y} N(x_{01}, Y) = \frac{\overline{\alpha}_s}{2\pi} \int d^2\vec{x}_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \left[N(x_{02}, Y) + N(x_{12}, Y) - N(x_{01}, Y) - N(x_{02}, Y) N(x_{12}, Y) \right]$$

Y.V. Kovchegov, Phys. Rev. D 61 (2000) 074018

rewrite in terms of a Fourier transform: $N(x, Y) = x^2 \int \frac{d^2\vec{k}}{2\pi} e^{i\vec{k}\cdot\vec{x}} \tilde{N}(k, Y)$

$$\frac{\partial}{\partial Y} \tilde{N}(k, Y) = \overline{\alpha}_s \, \chi(-\partial/\partial \ln k^2) \, \tilde{N}(k, Y) - \overline{\alpha}_s \, \tilde{N}^2(k, Y)$$

here χ is a BFKL characteristic function related to the kernel $K(k_1, k_2)$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$$

there exists a theorem from the ‘30 (Fisher, Kolomogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions

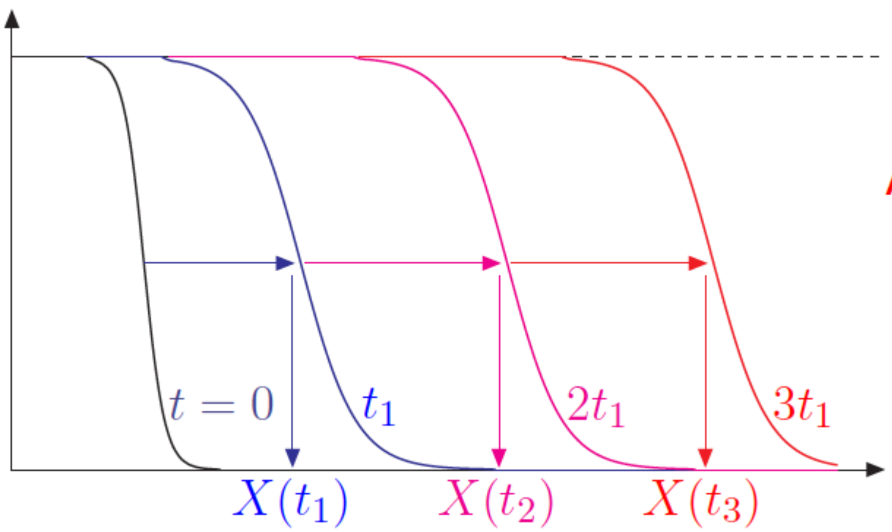
S. Munier, R.B. Peschanski
PRL 91 (2003) 232001
PRD 69 (2004) 034008

Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP) 1937



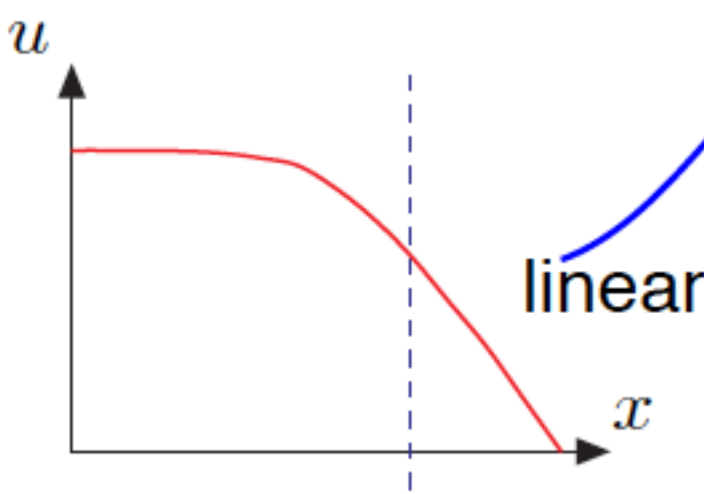
Asymptotic solution: travelling wave

$u(x,t) = u(x - v_c t)$

Position: $X(t) = X_0 + v_c t$

Travelling waves

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$



$$\partial_t u = \partial_x^2 u + u = \omega(-\partial_x) u$$

$$\omega(\gamma) = \gamma^2 + 1$$ same as $\overline{\alpha}_s \chi$

initial contition

velocity:

$$v = \frac{\omega(\gamma)}{\gamma}$$

$$\Rightarrow u(x,t) = \int \frac{d\gamma}{2i\pi} u_0(\gamma) \exp[\omega(\gamma)t - \gamma x]$$

Travelling waves

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

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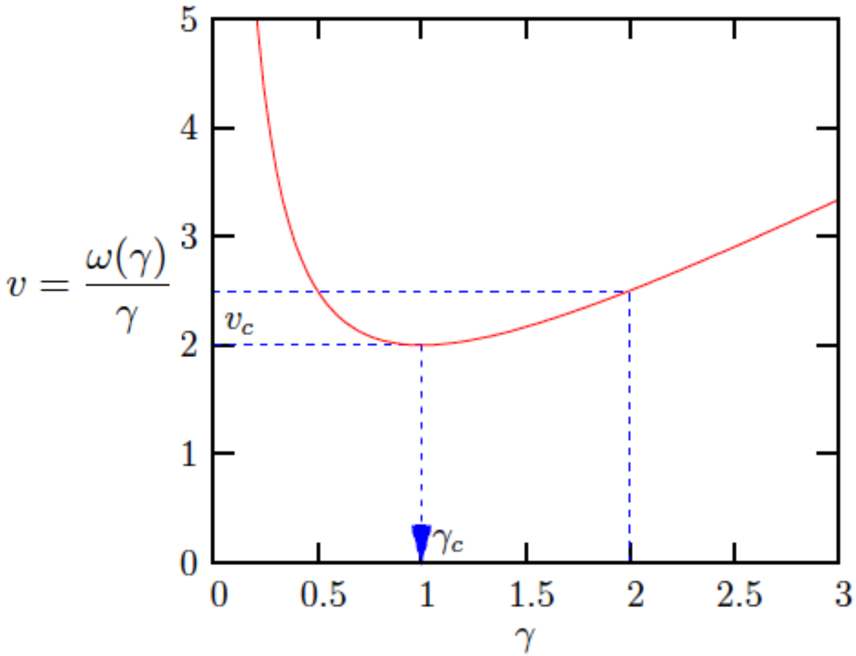
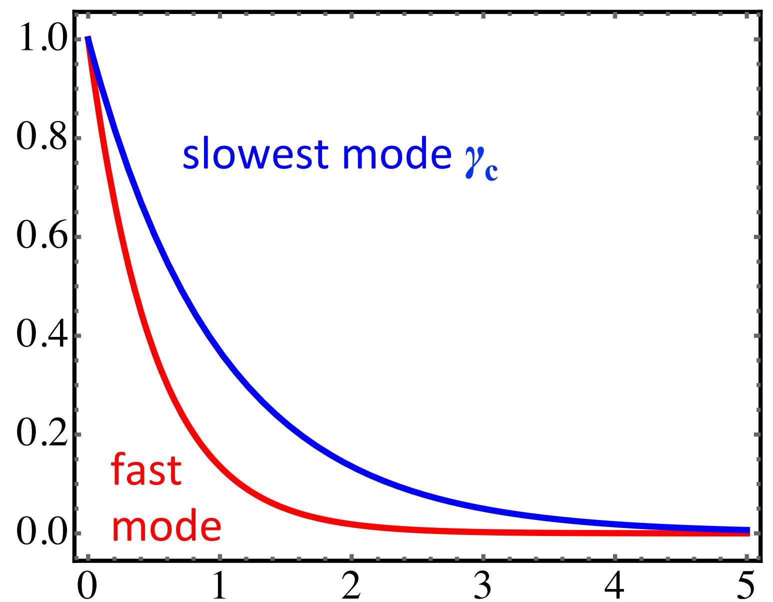




solution is driven by the slowest mode

Travelling waves in QCD

$$\frac{\partial}{\partial Y}\tilde{N}(k,Y) \;=\; \overline{\alpha}_s\,\chi(-\partial/\partial\ln k^2)\,\tilde{N}(k,Y) - \overline{\alpha}_s\,\tilde{N}^2(k,Y)$$

Mellin transform: $$\tilde{N}(k,Y) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi} n_0(\gamma)\,\exp\left[\overline{\alpha}_s\,\chi(\gamma)\,Y - \gamma\ln k^2\right]$$

minimal velocity: $$v = \min \frac{\overline{\alpha}_s\,\chi(\gamma)}{\gamma} \quad\rightarrow\quad \gamma_c\chi'(\gamma_c) = \chi(\gamma_c) \qquad \begin{aligned}\gamma_c &= 0.6275\\ v_c &= 4.8834\bar{\alpha}\end{aligned}$$

Travelling waves in QCD

$$\frac{\partial}{\partial Y}\tilde{N}(k,Y) = \overline{\alpha}_s\,\chi(-\partial/\partial\ln k^2)\,\tilde{N}(k,Y) - \overline{\alpha}_s\,\tilde{N}^2(k,Y)$$

Mellin transform: $\tilde{N}(k,Y) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi} n_0(\gamma)\,\exp\left[\overline{\alpha}_s\,\chi(\gamma)\,Y - \gamma\ln k^2\right]$

minimal velocity: $v = \min \frac{\overline{\alpha}_s\,\chi(\gamma)}{\gamma} \quad \rightarrow \quad \gamma_c \chi'(\gamma_c) = \chi(\gamma_c)$ $\quad \gamma_c = 0.6275$, $\quad v_c = 4.8834\bar{\alpha}$

travelling wave condition:

$$\overline{\alpha}_s\chi(\gamma_c)Y - \gamma_c\ln(k^2/k_0^2) = -\gamma_c\ln\left[\left(\frac{1}{x}\right)^{-v_c}\frac{k^2}{k_0^2}\right] = -\gamma_c\ln\left[\frac{k^2}{Q_s^2(x)}\right]$$

saturation scale: $Q_s^2(x) = k_0^2\left(\frac{1}{x}\right)^{v_c}$

scaling variable

Travelling waves in QCD imply Geometrical Scaling

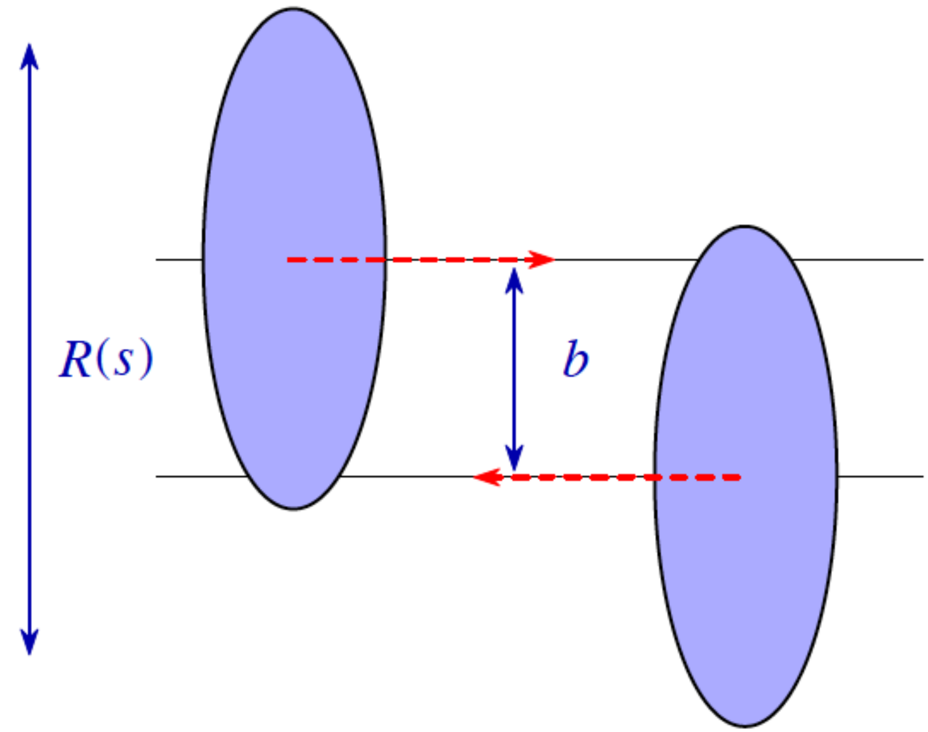
$$f(x, k^2) = \mathcal{F}\left(\frac{k^2}{Q_s^2(x)}\right)$$

$$Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

„Geometrical Scaling"

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;
J. Dias de Deus, Acta Phys. Polon. B 6 (1975) 613.

$$A(b,s) = A(b/R(s))$$



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Geometric scaling for the total γ^* p *cross-section in the low* x *region.*
A.M. Stasto, K. J. Golec-Biernat , J. Kwiecinski
Phys.Rev.Lett. 86 (2001) 596-599

$$\sigma_{\gamma^* p} \sim \frac{F_2(x,Q^2)}{Q^2} = \sigma_0 \mathcal{F}\left(\frac{Q^2}{Q^2_{\rm sat}(x)}\right)$$

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J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;
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L. McLerran, M. Praszalowicz: Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. Praszalowicz: Phys.Rev.Lett.106:142002,2011
M. Praszalowicz: Acta Phys.Polon. B42 (2011) 1557-1566
M. Praszalowicz, T. Stebel: JHEP 1303 (2013) 090

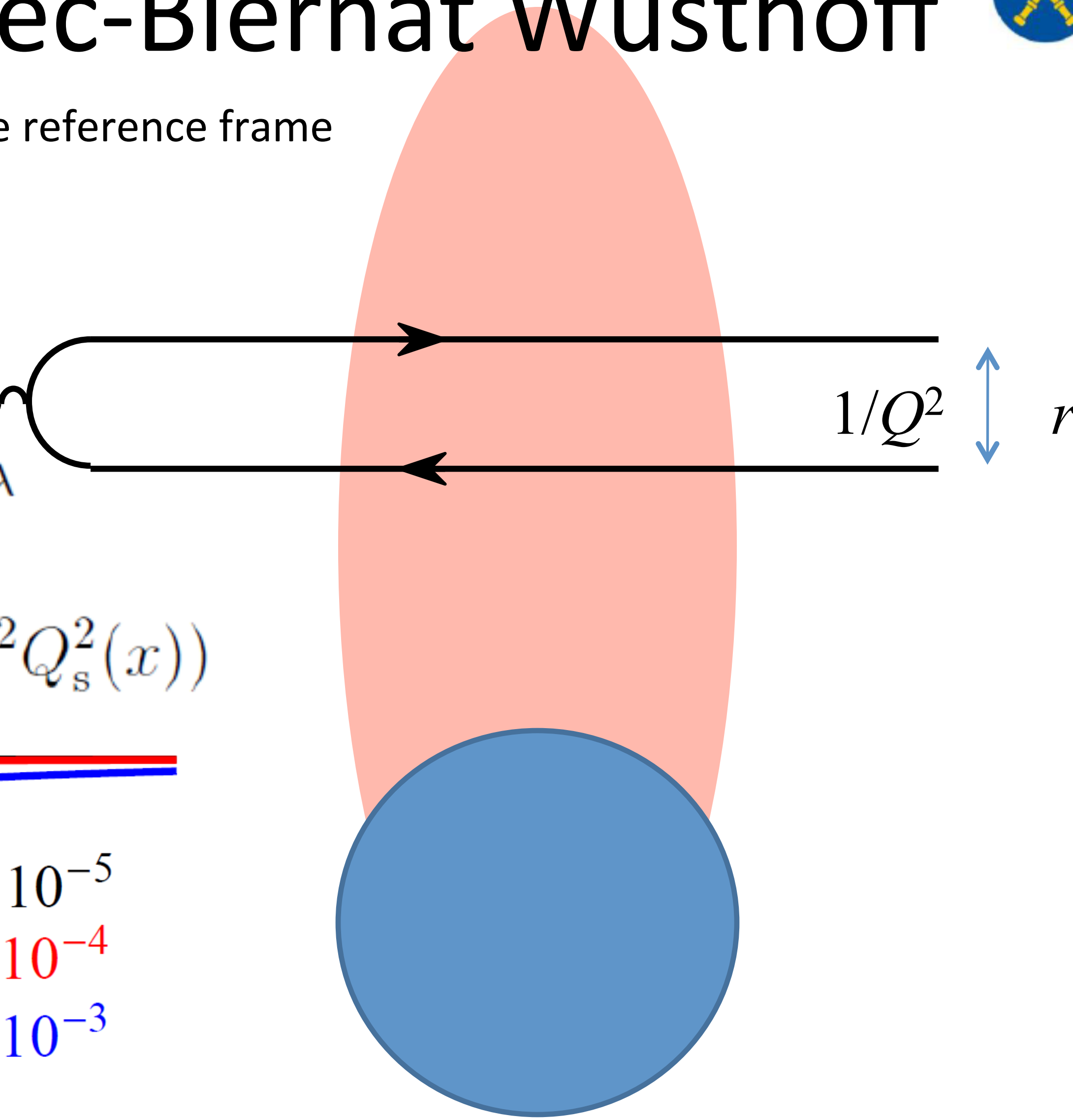
$$\frac{dN_{\rm ch}}{d\eta dp_{\rm T}^2}(s, p_{\rm T}) \quad = \quad \frac{1}{Q_0^2}\mathcal{F}\left(\frac{p_{\rm T}^2}{Q_{\rm sat}^2(s)}\right)$$

Deep Inealstic Scattering

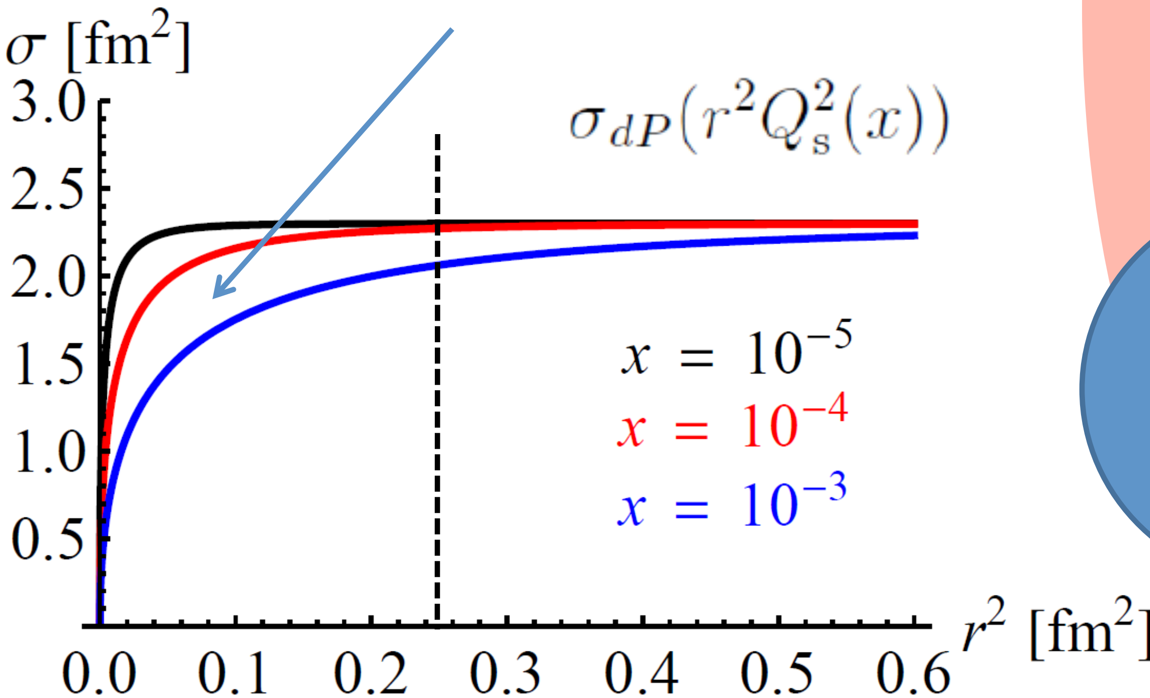
Model: Golec-Biernat Wüsthoff

Changing the reference frame

K.J. Golec-Biernat, M. Wusthoff
PRD 59 (1998) 014017
PRD 60 (1999) 114023



$$Q_s^2 = Q_0^2 (x_0/x)^\lambda$$



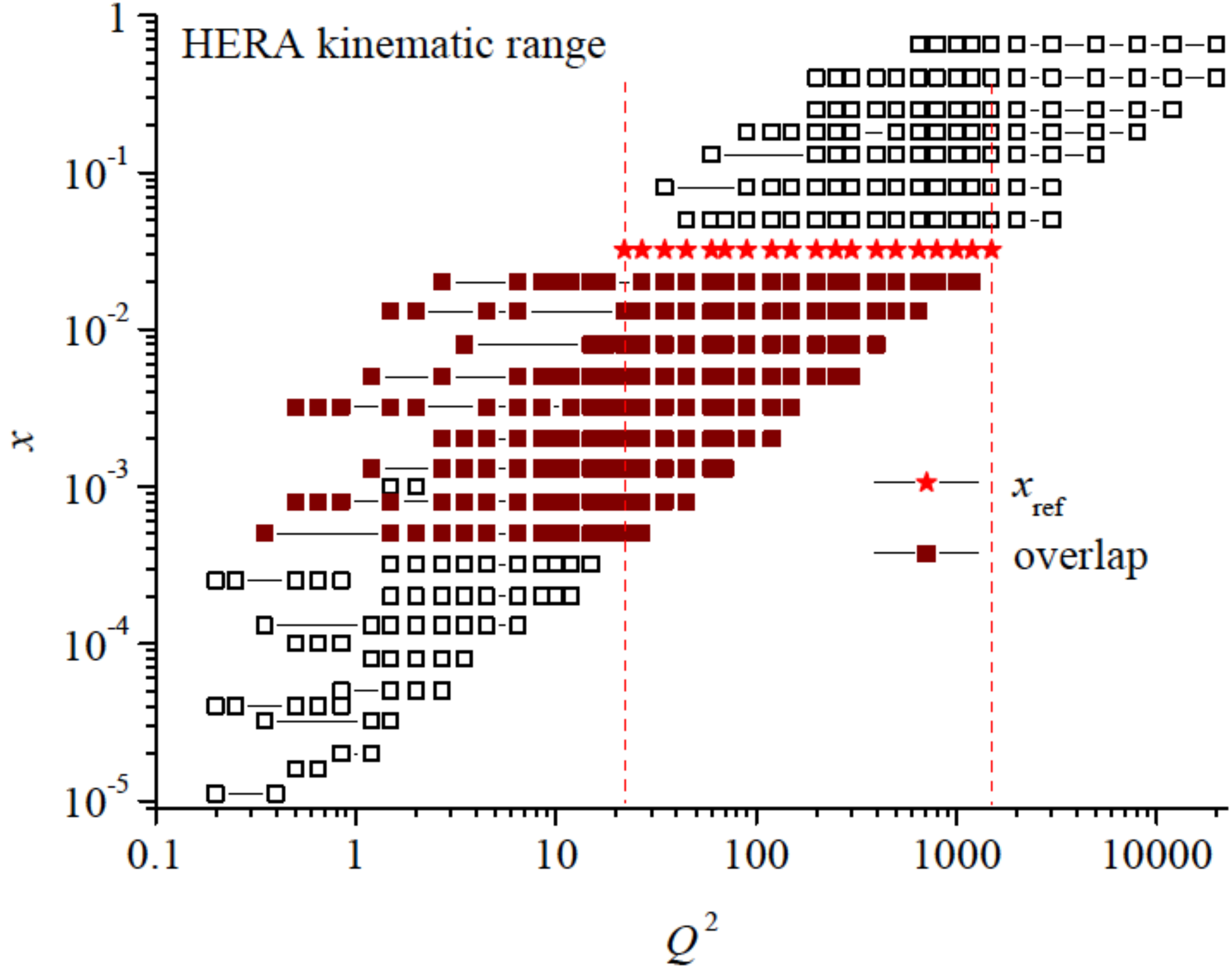
Geometrical Scaling

$$\sigma_{\gamma^* p} = \int dr^2 \left|\psi(r, Q^2)\right|^2 \sigma_{dP}(r^2 Q_s^2(x))$$

$$\sigma_{\gamma^* p} \quad = \quad \sigma_{\gamma^* p}\left(\frac{Q_s(x)}{Q}\right)$$

GS does not depend on the particular form of the dipole cross-section

$$Q_s^2 = Q_0^2(x_0/x)^\lambda$$



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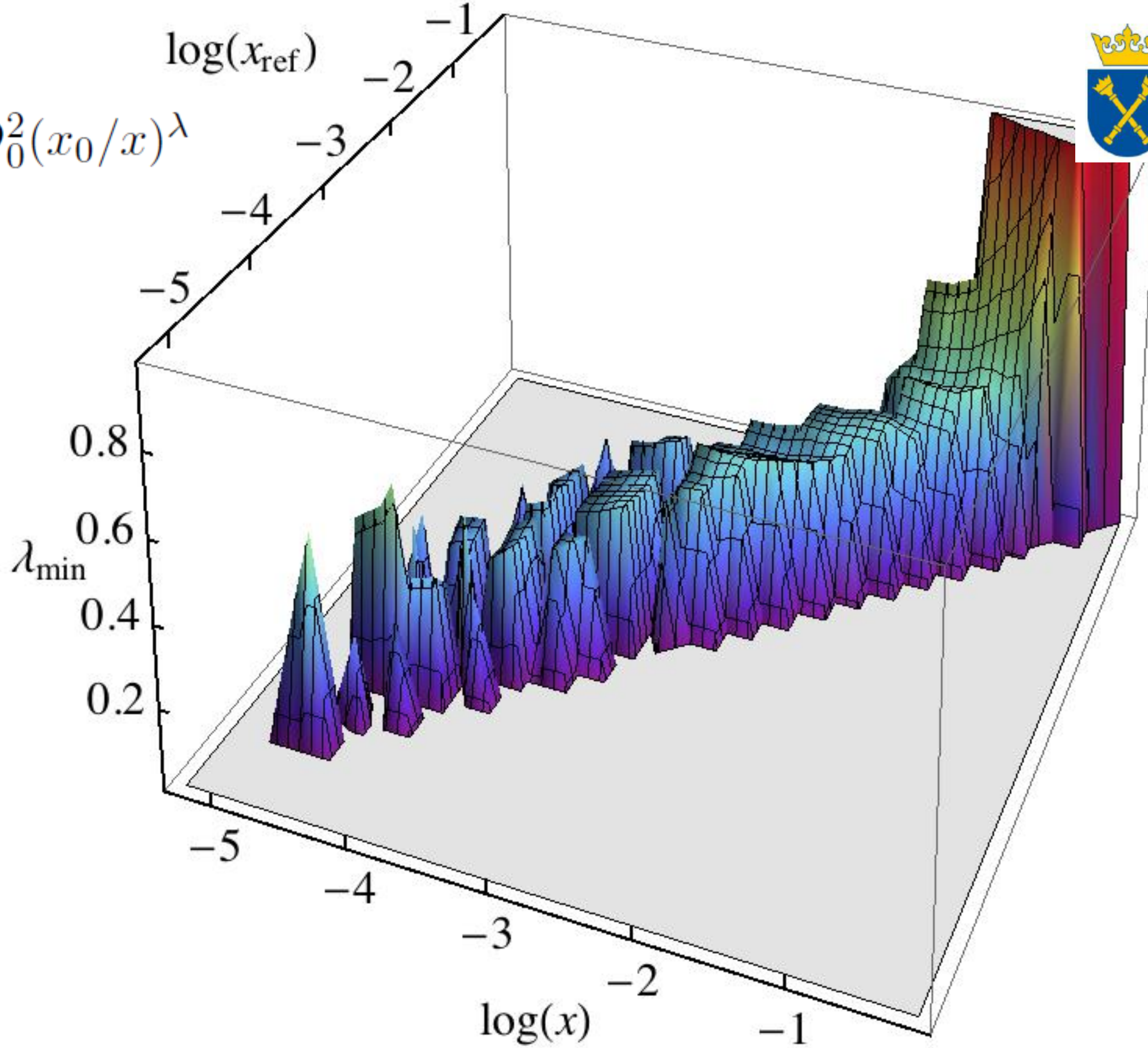
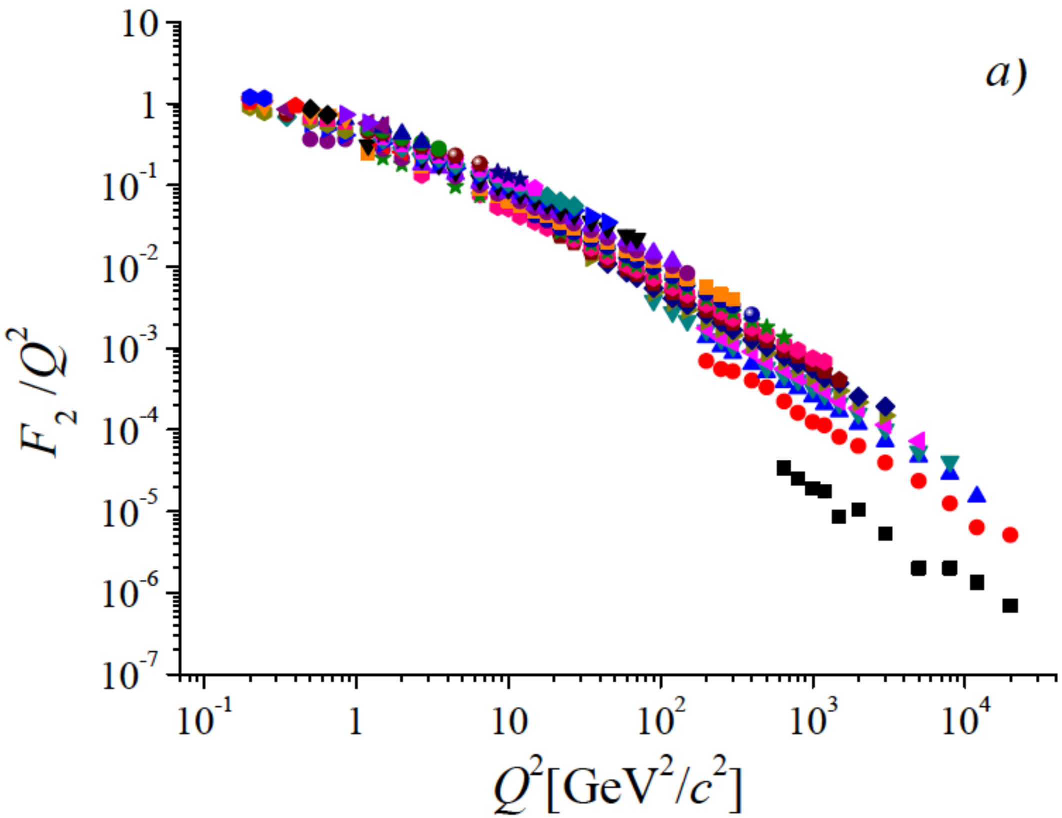


Saturation scale: energy and x dependence

$$Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$

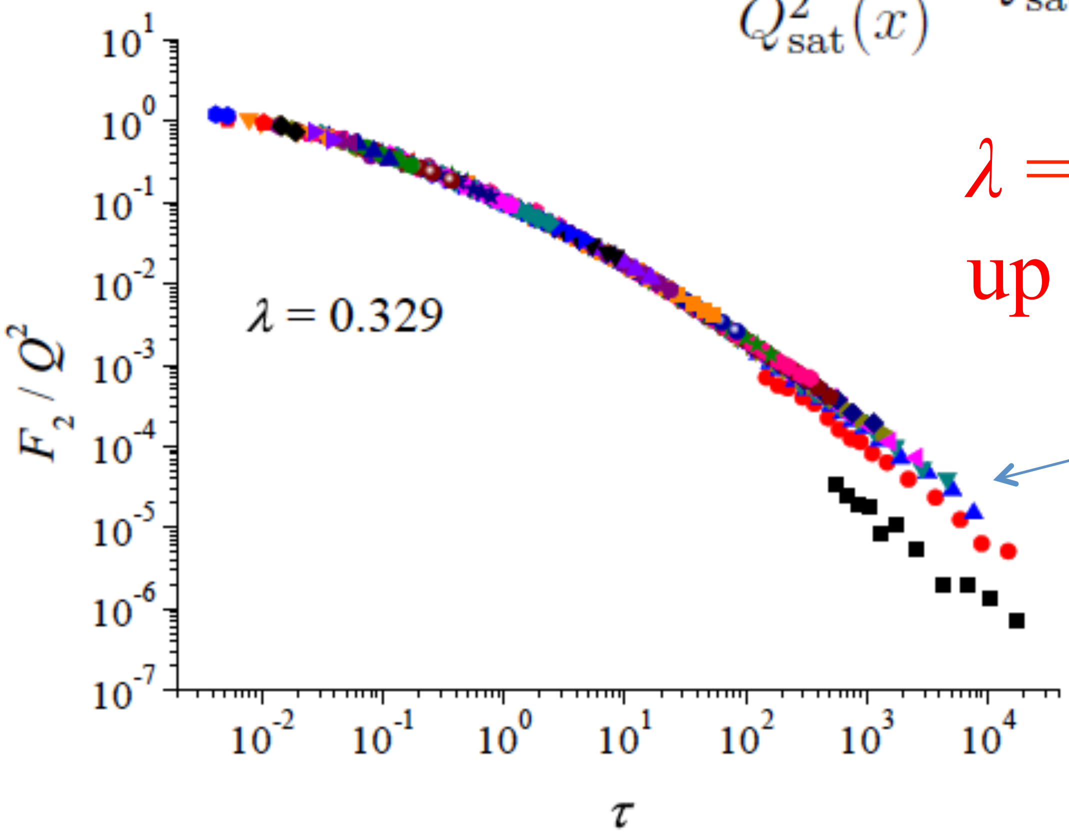


A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski
PRL 86 (2001) 596-599

M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]

Saturation scale: energy and x dependence

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$

up to $x = 0.08$ (!)

large x

Saturation at the LHC

Bjorken x 's of colliding partons: $$x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}$$

at mid rapidity for GeV transverse momenta

$$x_{1,2} \sim 10^{-2} - 10^{-3}$$

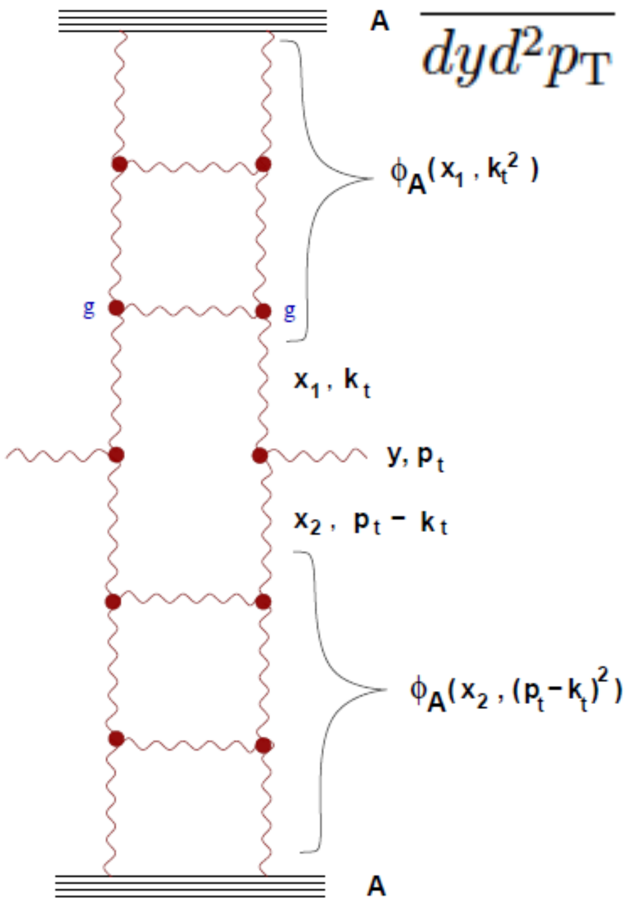
we are in the saturation regime characterized by the saturation scale

$$Q_s(x) \sim \text{GeV}$$

proton-proton @ LHC

Basics of geometrical scaling

Gribov, Levin Ryskin, *High p_T Hadrons In The Pionization Region In QCD*. Phys.Lett.B100:173-176,1981.



$$\frac{d\sigma}{dy d^2p_T} = \frac{3\pi\alpha_s}{2p_T^2}\int d^2\vec{k}_T\, \varphi_1(x_1,\vec{k}_T^2)\varphi_2(x_2,(\vec{k}-\vec{p})_T^2)$$

$$x_{1,2} = \frac{p_T}{\sqrt{s}}e^{\pm y}$$

gluon distribution   unintegrated glue

$$xG(x,Q^2) = \int^{Q^2} dk_T^2\, \varphi(x,k_T^2)$$

Kharzeev, Levin
Phys.Lett.B523:79-87,2001.

Basics of geometrical scaling

gluon distribution — unintegrated glue

$$xG(x,Q^2)=\int^{Q^2} dk_T^2\,\varphi(x,k_T^2)$$

Golec-Biernat – Wuesthoff (DIS)

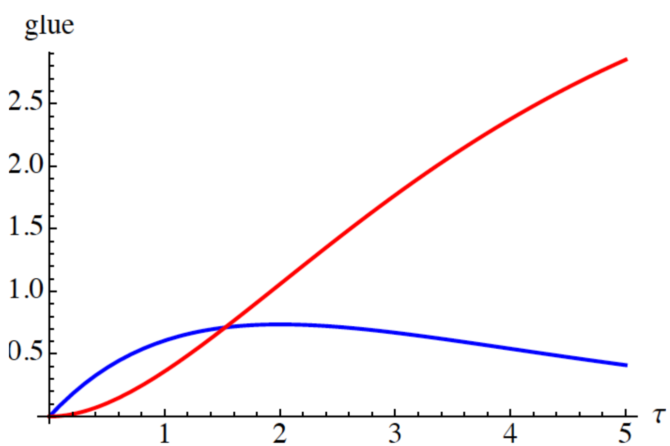
$$\varphi(x,k_T^2)=S_\perp\frac{3}{4\pi^2}\frac{k_T^2}{Q_s(x)^2}\exp\left(-k_T^2/Q_s(x)^2\right)$$

$$S_\perp=\sigma_0$$

Kharzeev – Levin (AA)

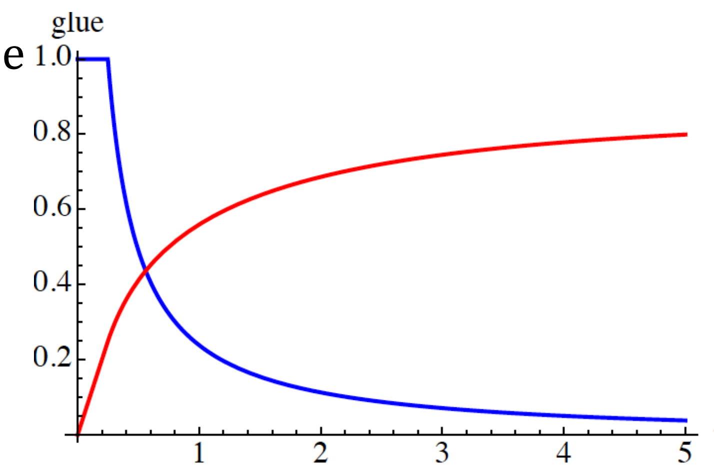
$$\varphi(x,k_T^2)=S_\perp\begin{cases}1 & \text{for } k_T^2<Q_s(x)^2\\ Q_s(x)^2/k_T^2 & \text{for } Q_s(x)^2<k_T^2\end{cases}$$

$S_\perp$ is the transverse size given by geometry



scaling variable

$$\tau=\frac{p_T^2}{Q_s^2(x)}$$



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dyd^2p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2}\frac{Q_s^2(x)}{p_{\mathrm{T}}^2}\int \frac{d^2\vec{k}_{\mathrm{T}}}{Q_s^2(x)}\varphi_1\left(\vec{k}_{\mathrm{T}}^2/Q_s^2(x)\right)\varphi_2\left((\vec{k}-\vec{p})_{\mathrm{T}}^2/Q_s^2(x)\right)$$

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$$\frac{d\sigma}{dyd^2p_{\mathrm{T}}} = S_\perp^2\mathcal{F}(\tau) \qquad \tau = \frac{p_{\mathrm{T}}^2}{Q_s^2(x)}$$

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_{\rm T}} = \frac{3\pi\alpha_{\rm s}}{2} \frac{Q_s^2(x)}{p_{\rm T}^2} \int \frac{d^2\vec{k}_{\rm T}}{Q_s^2(x)} \varphi_1\left(\vec{k}_{\rm T}^2/Q_s^2(x)\right) \varphi_2\left((\vec{k}-\vec{p})_{\rm T}^2/Q_s^2(x)\right)$$

$$\frac{d\sigma}{dy d^2 p_{\rm T}} = S_\perp^2 \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_s^2(x)}$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_{\rm T} = S_\perp^2 Q_s^2(x) \int \mathcal{F}(\tau) d\tau = \frac{1}{\kappa} S_\perp^2 Q_s^2(x)$$

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2}\frac{Q_s^2(x)}{p_T^2}\int \frac{d^2\vec{k}_T}{Q_s^2(x)}\varphi_1\left(\vec{k}_T^2/Q_s^2(x)\right)\varphi_2\left((\vec{k}-\vec{p})_T^2/Q_s^2(x)\right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \qquad \tau = \frac{p_T^2}{Q_s^2(x)}$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_T = S_\perp^2 Q_s^2(x) \int \mathcal{F}(\tau) d\tau = \frac{1}{\kappa} S_\perp^2 Q_s^2(x)$$

$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{S_\perp^2}{\kappa} Q_s^2(x) \rightarrow Q_s^2(x) = \frac{\kappa}{S_\perp}\frac{dN}{dy}$$

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

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$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{S_\perp^2}{\kappa} Q_s^2(x) \rightarrow \boxed{Q_s^2(x) = \frac{\kappa}{S_\perp} \frac{dN}{dy}}$$

saturation scale = gluon density per transverse area

Geometrical scaling of p_T distribution

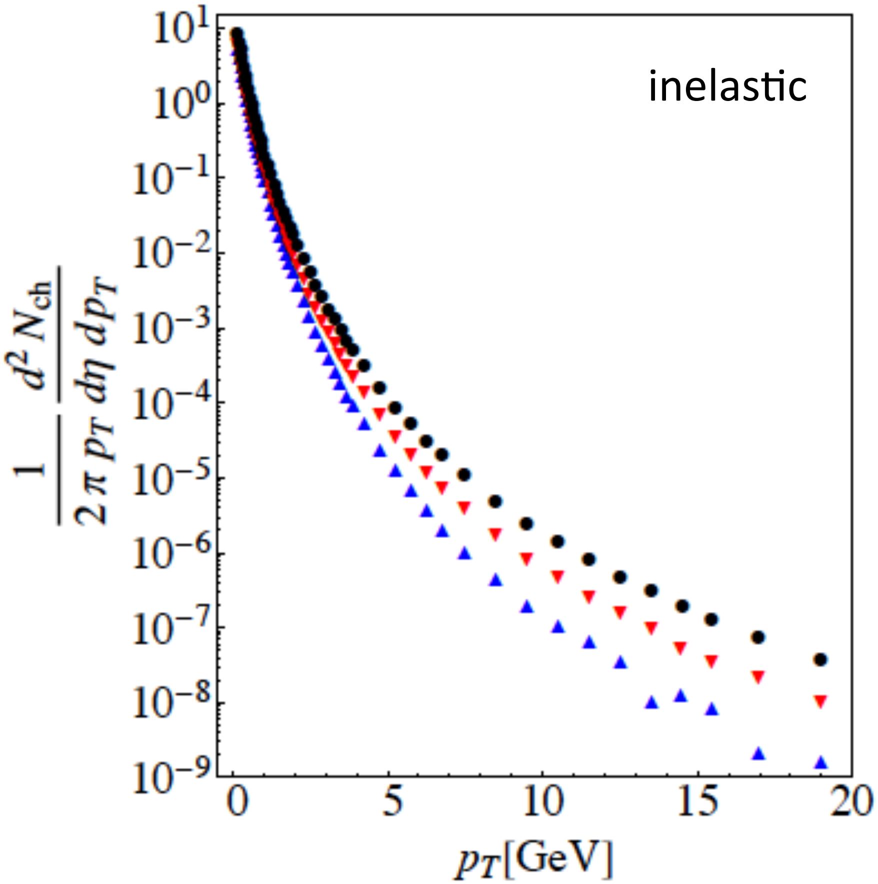
L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566
Phys.Rev. D87 (2013) 071502(R)

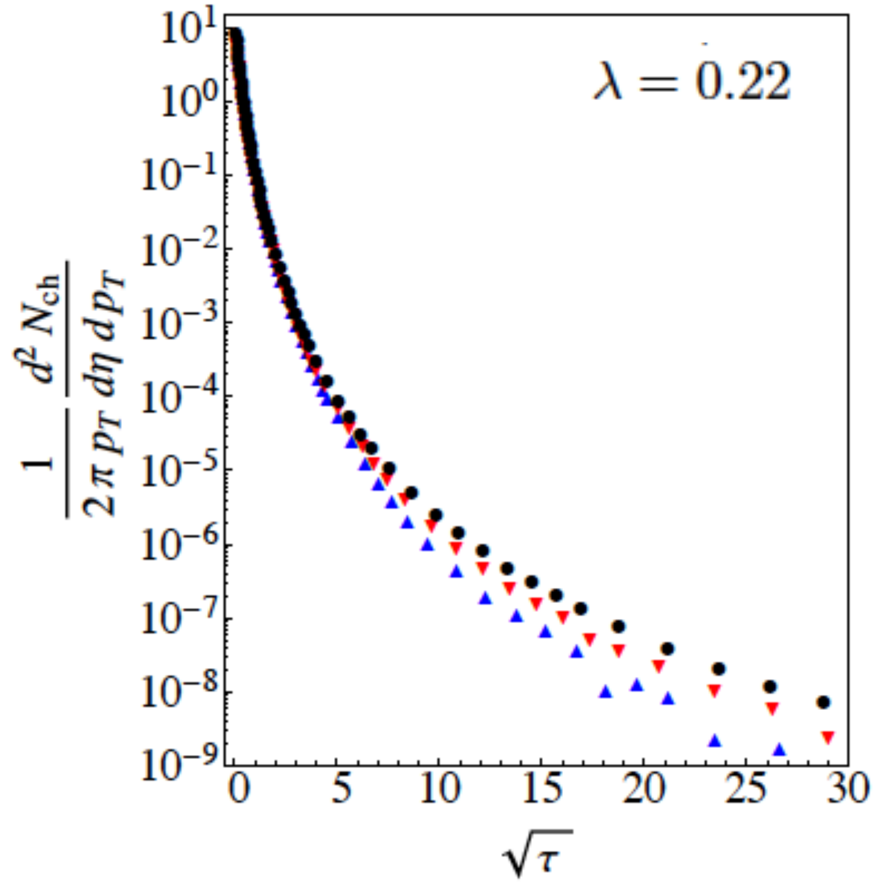
$$\tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

Determination of lambda

$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662





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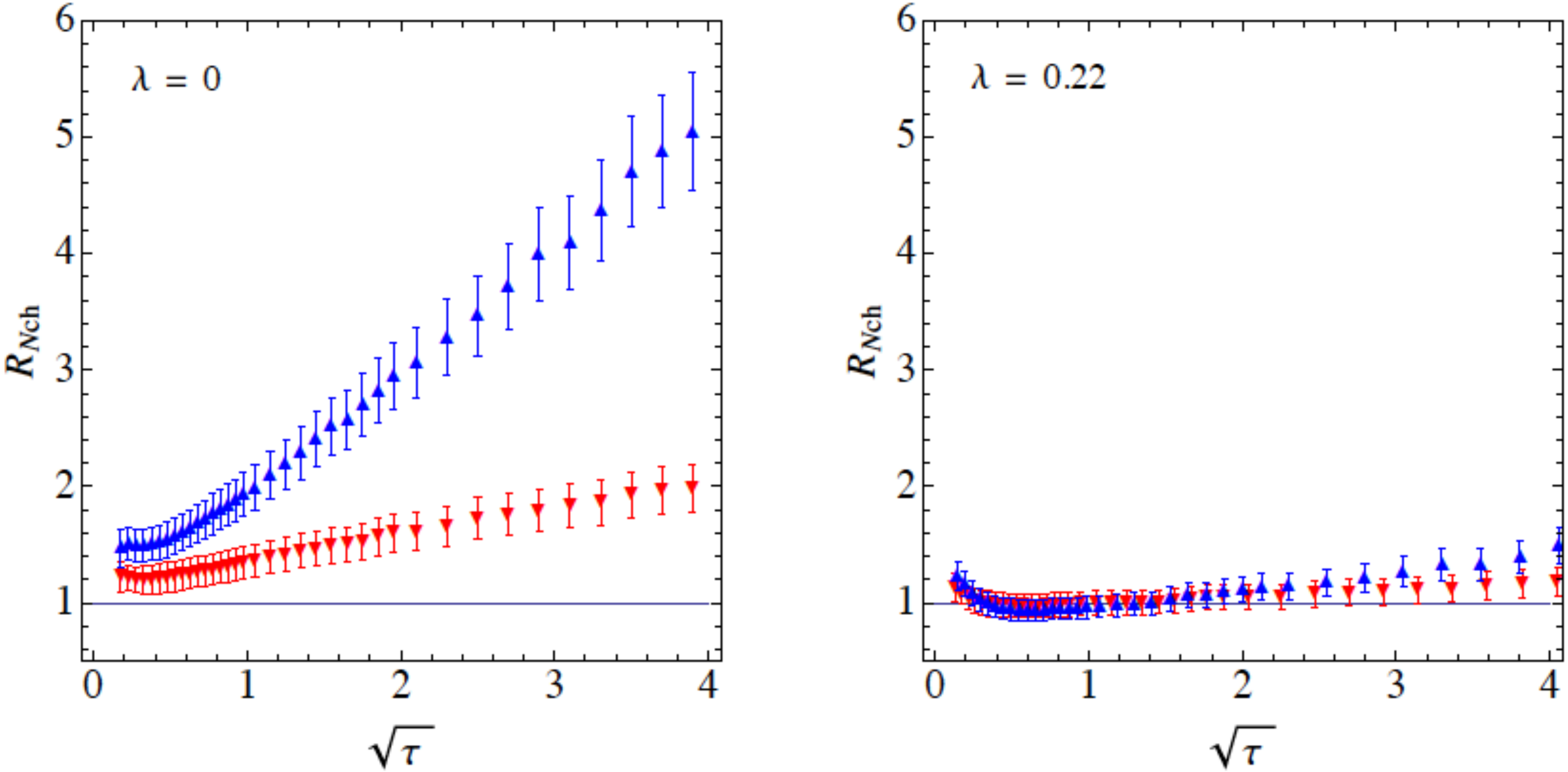
$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_{\perp} \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662



Consequences of GS

$$\frac{dN_{\mathrm{ch}}}{dy\,dp_{\mathrm{T}}^2} = \frac{1}{Q_0^2} F(\tau) \quad \Rightarrow \quad \frac{dN_{\mathrm{ch}}}{dy} = \int \frac{dp_{\mathrm{T}}^2}{Q_0^2} F(\tau)$$

$$\tau = \frac{p_{\mathrm{T}}^2}{Q_0^2}\left(\frac{p_{\mathrm{T}}}{W}\right)^{\lambda/2}$$

$$W \sim \sqrt{s}$$

Consequences of GS

$$\frac{dN_{\rm ch}}{dy dp_{\rm T}^2} = \frac{1}{Q_0^2} F(\tau) \quad \Rightarrow \quad \frac{dN_{\rm ch}}{dy} = \int \frac{dp_{\rm T}^2}{Q_0^2} F(\tau)$$

$$\tau = \frac{p_{\rm T}^2}{Q_0^2}\left(\frac{p_{\rm T}}{W}\right)^{\lambda/2}$$

integral over $d\tau$ is energy independent

$$W \sim \sqrt{s}$$

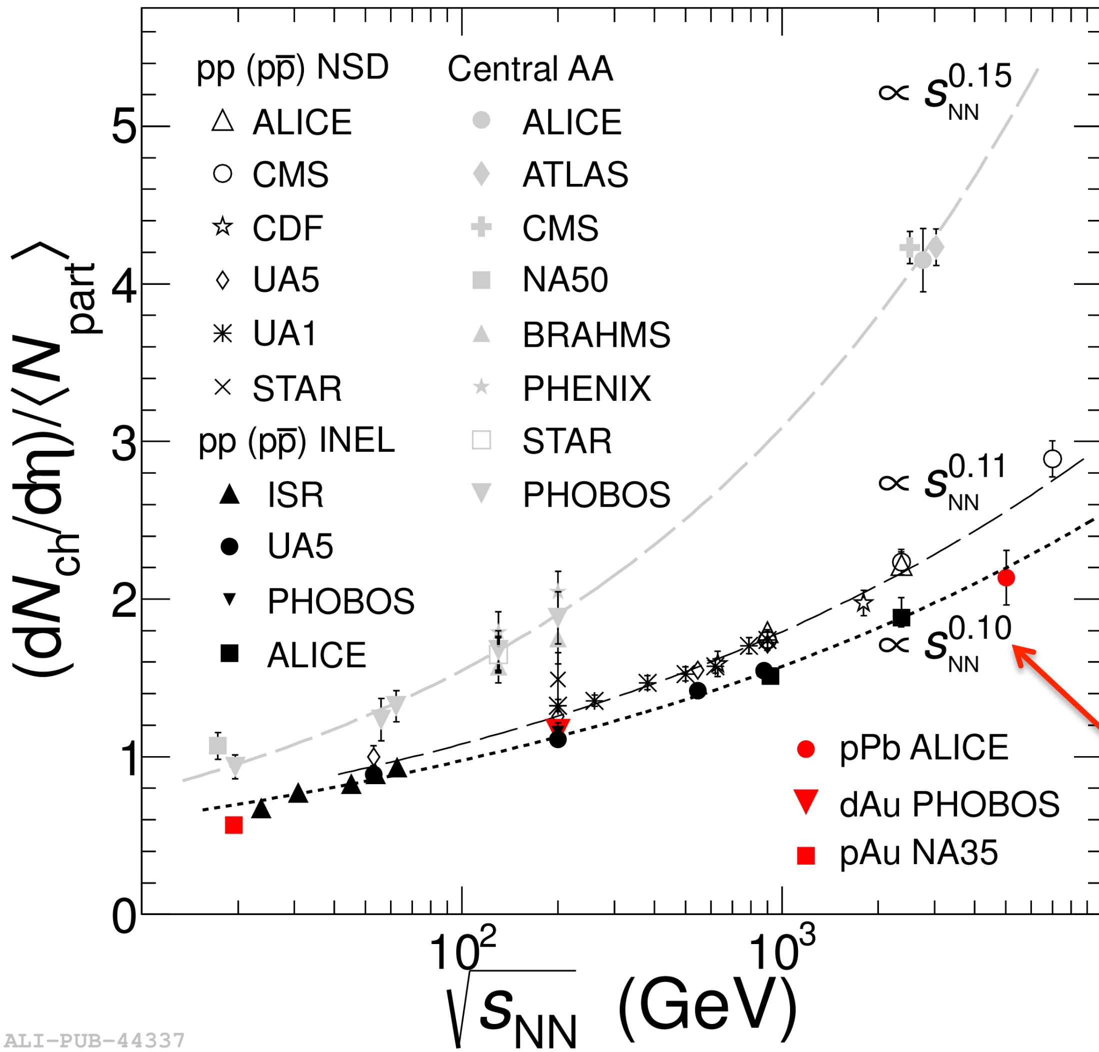
$$\frac{dp_{\rm T}^2}{Q_0^2} = \frac{2}{2+\lambda}\left(\frac{W}{Q_0}\right)^{\frac{2\lambda}{2+\lambda}} \tau^{-\frac{\lambda}{2+\lambda}} d\tau$$

effective growth of multiplicity is slower than λ

$$\lambda_{\rm eff} = \frac{2\lambda}{2+\lambda} < \lambda \quad = 0.23$$

Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf



plot: P. Braun-Munzinger, 54 Cracow School of Theoretical Physics (from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

transverse area is energy indpendent

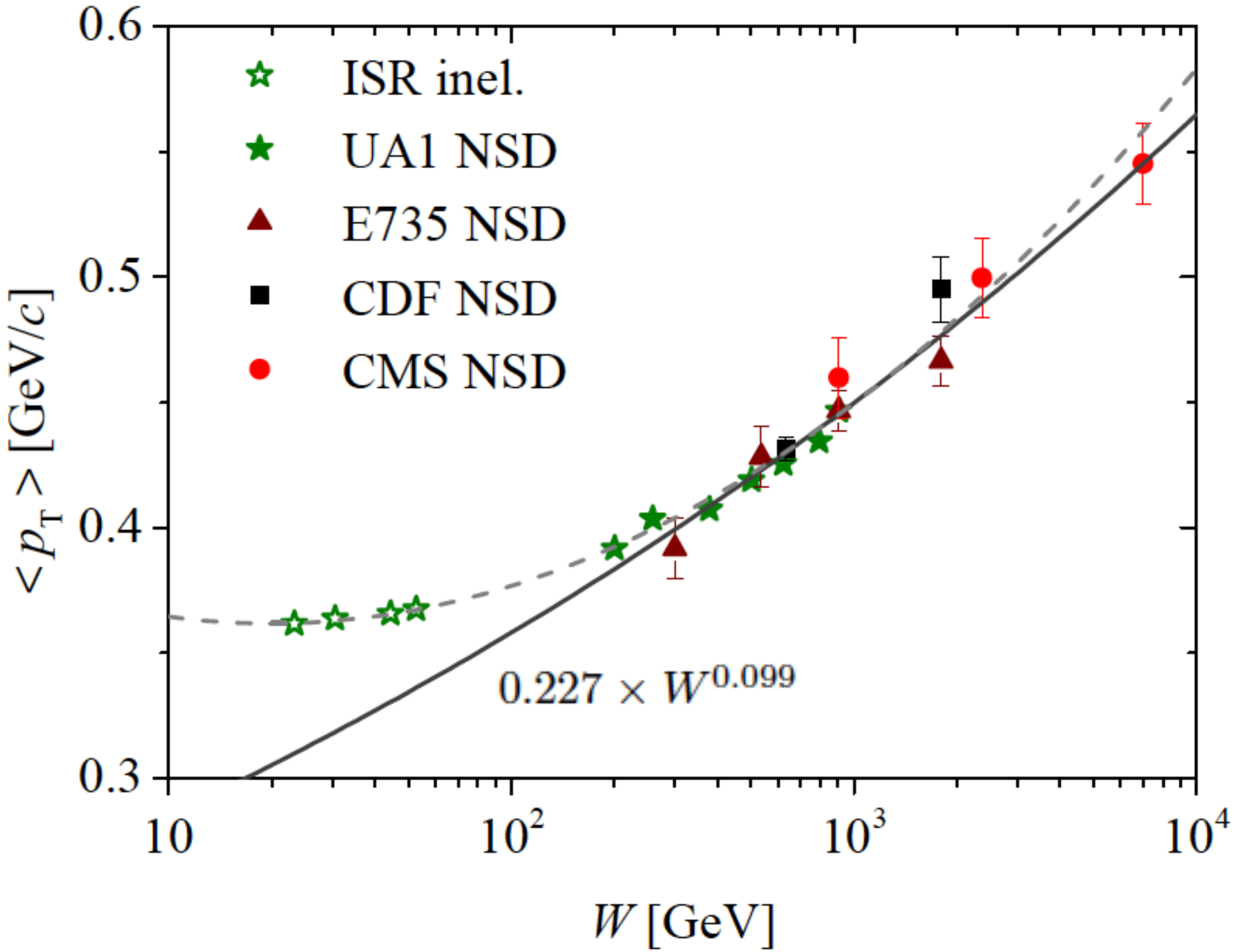
$$\lambda/(2+\lambda) \simeq 0.099$$

Average transverse momentum

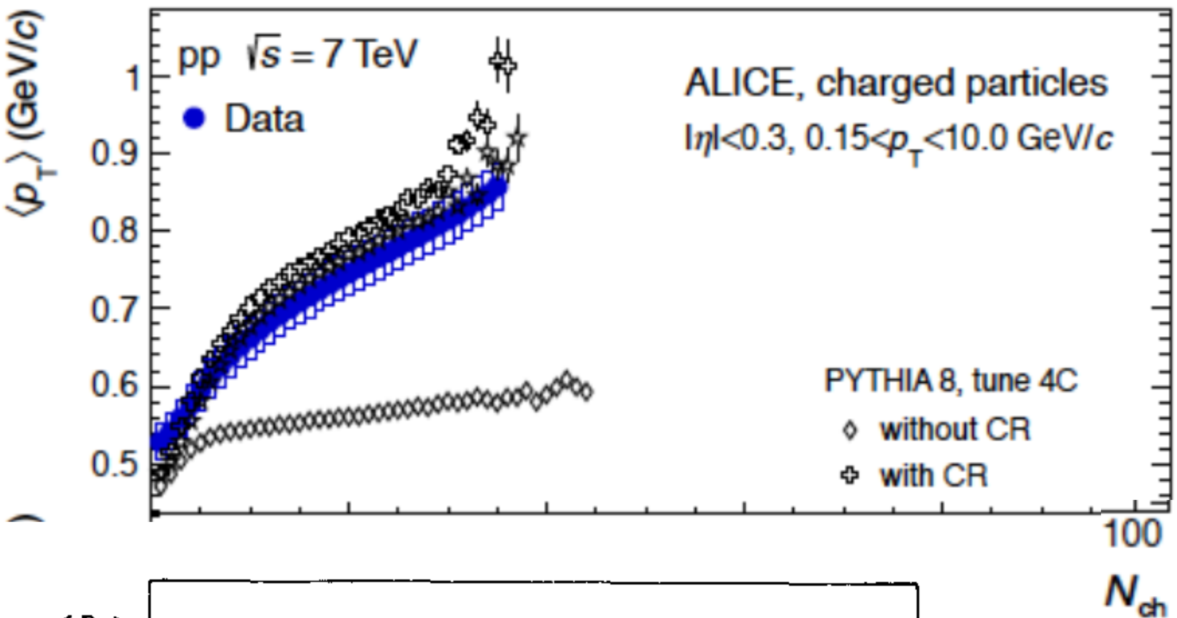
$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau)$$

$$\langle p_{\rm T} \rangle = \frac{\int p_T \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T}{\int \frac{dN_{\rm g}}{dy d^2 p_{\rm T}} d^2 p_T} \sim \bar{Q}_{\rm s}(W) \sim Q_0 \left(\frac{W}{Q_0} \right)^{\lambda/(2+\lambda)}$$

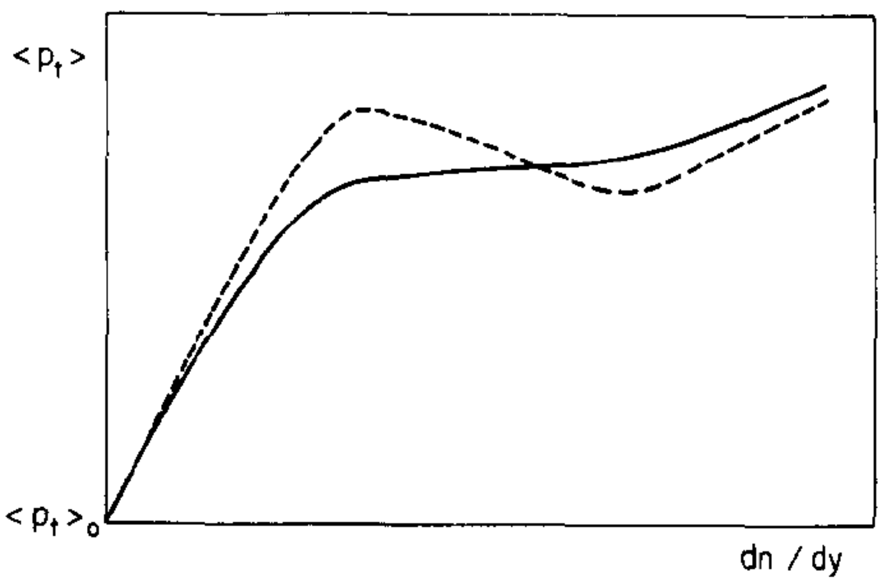
Average transverse momentum



Mean p_T as a function of N_{ch}



- $\langle p_T \rangle(N_{ch})$ – correlations are sensitive to the fine details of dynamics
- difficult to describe by untuned MonteCarlos
- possible sign of phase transition



MULTIPLICITY DEPENDENCE OF p_t SPECTRUM AS A POSSIBLE SIGNAL FOR A PHASE TRANSITION IN HADRONIC COLLISIONS

L. VAN HOVE
CERN, Geneva, Switzerland

Received 25 August 1982

Mean p_T as a function of N_ch

$$\langle p_\mathrm{T} \rangle \sim \bar{Q}_\mathrm{s}(W)$$

Mean p_T as a function of N_{ch}

$$\langle p_{\rm T}\rangle \sim \bar{Q}_{\rm s}(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

interaction radius

Mean p_T as a function of N_{ch}

$$\langle p_T \rangle \sim \bar{Q}_s(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

interaction radius

phenomenological formula:

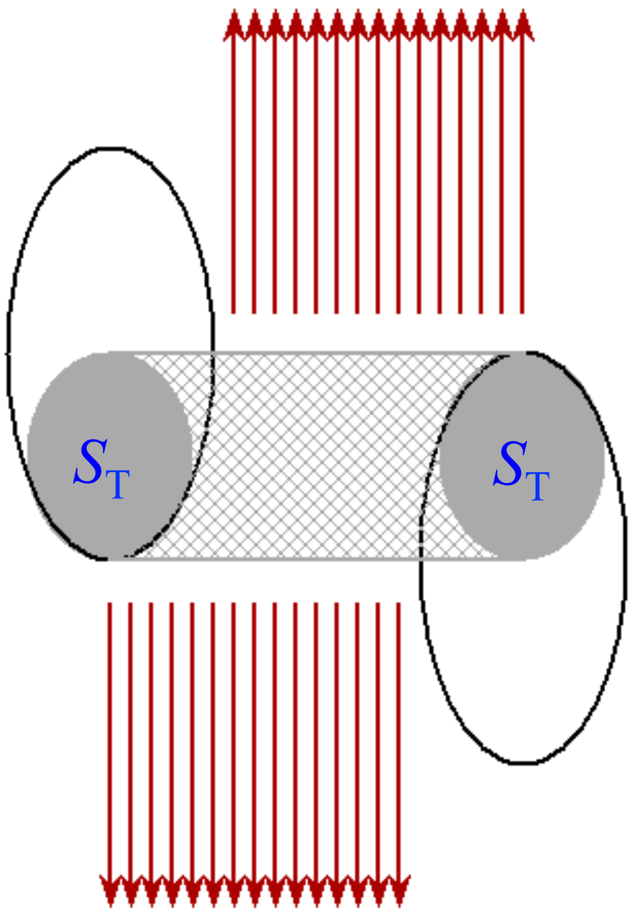
$$\langle p_T \rangle = \alpha + \beta \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

nonperturbaitive
coefficient

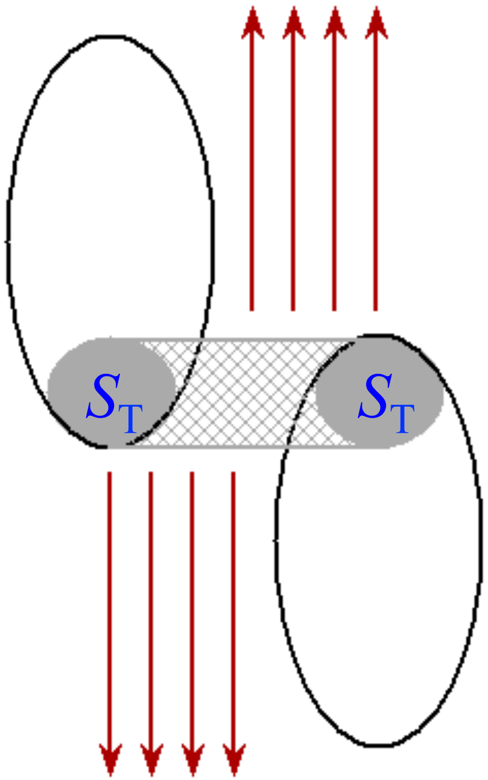
α, β do not depend on energy, do depend on particle species

Interaction radius

Transverse size and expansion time (longitudinal size) are proportional for fixed multiplicity



large multiplicity



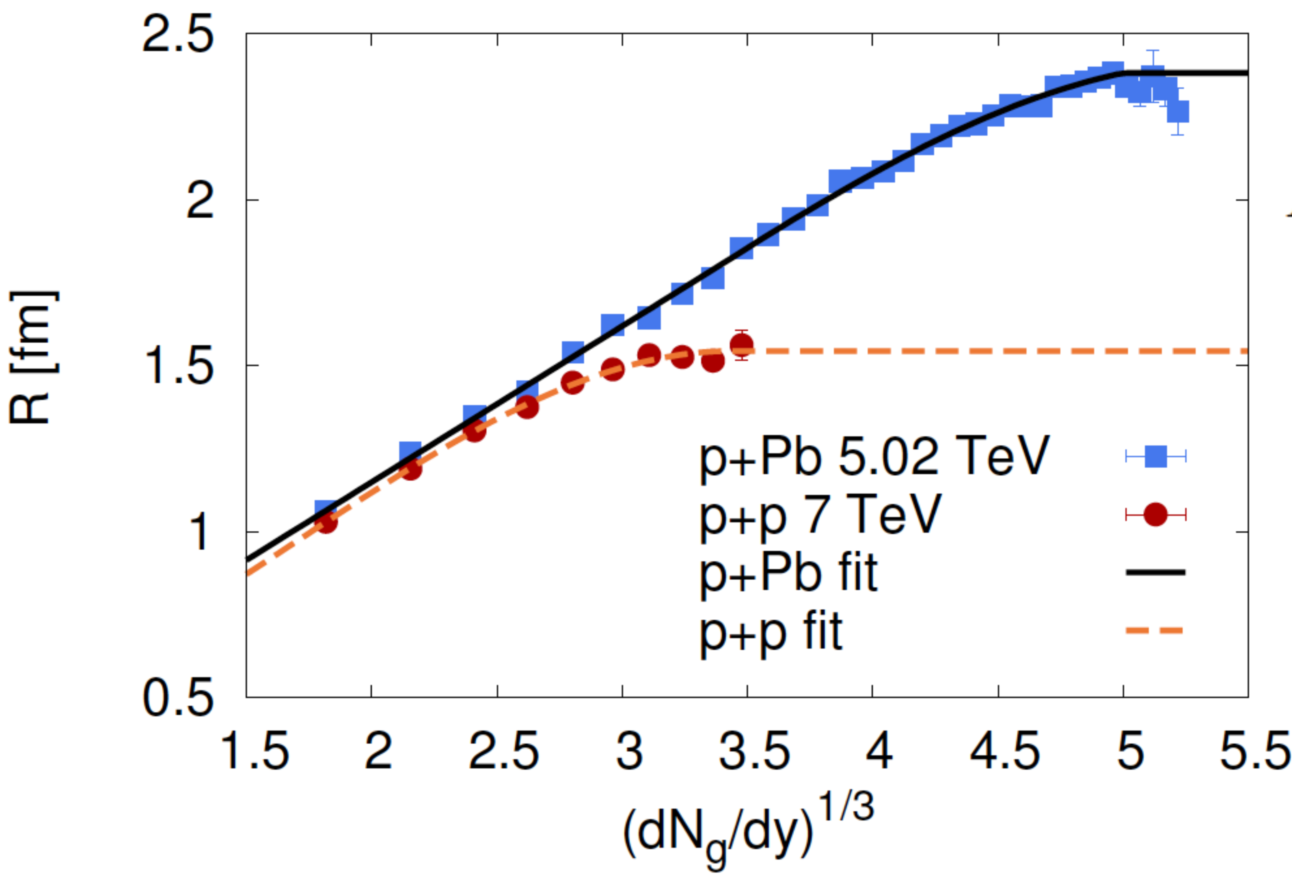
small multiplicity

similar effect in multipomeron model, where string tension is growing with multiplicity

M. A. Braun, C. Pajares
Phys. Lett. B 287, 154(1992)
Nucl. Phys. B 390 , 542, 559 (1993)
N. Armesto, D.A. Derkach, G.A. Feofilov
Phys. of At. Nuclei 71, 2087 (2008)

Interaction radius

A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan,
Initial state geometry and the role of hydrodynamics in proton-proton, proton-nucleus and deuteron-nucleus collisions,
Phys. Rev. C 87 (2013) 064906, [arXiv:1304.3403 [nucl-th]].



$$N_{\rm ch} = \frac{1}{\gamma \Delta y} \int_{\Delta y} \frac{dN_{\rm g}}{dy} dy$$

Scaling of mean p_T

$$\langle p_T \rangle = \alpha + \beta \frac{\sqrt{N_{ch}}}{R(\gamma N_{ch})}$$

parton-hadron duality ↑

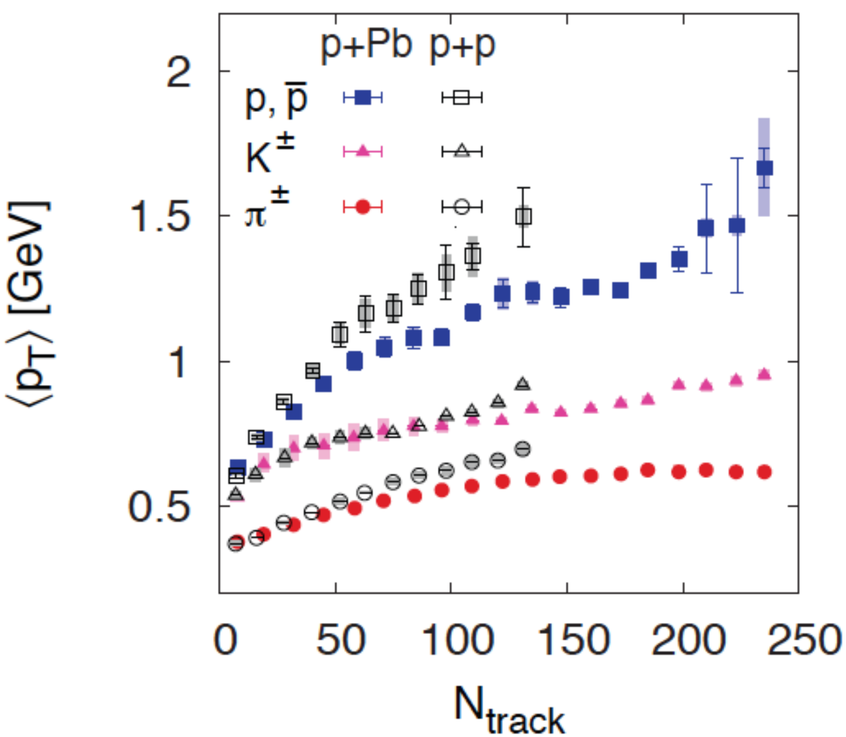
Scaling of mean p_T

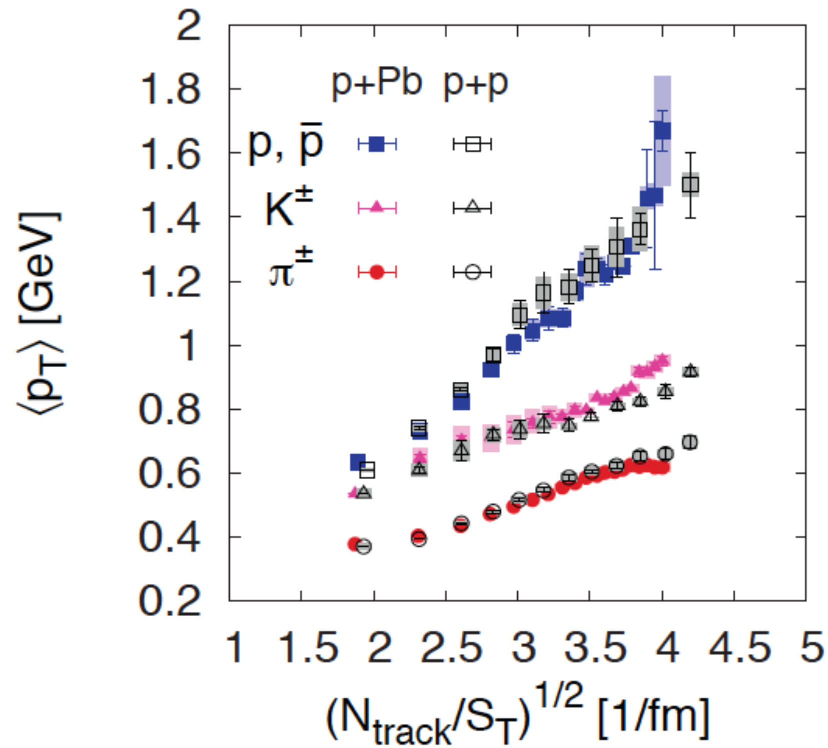
$$\langle p_T \rangle = \alpha + \beta \frac{\sqrt{N_{ch}}}{R(\gamma N_{ch})}$$

scaling variable

parton-hadron duality

CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847





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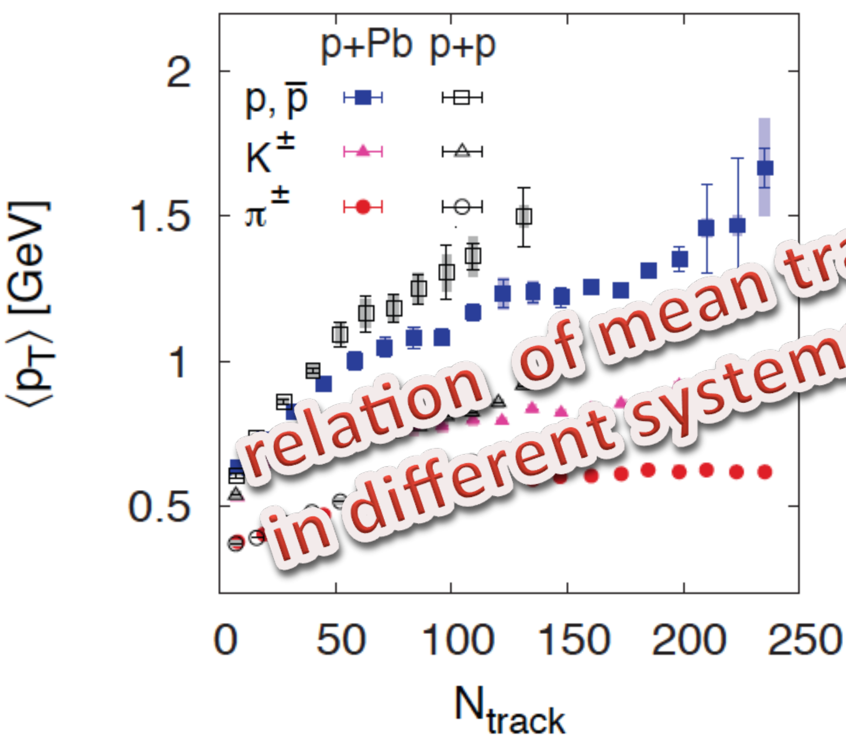
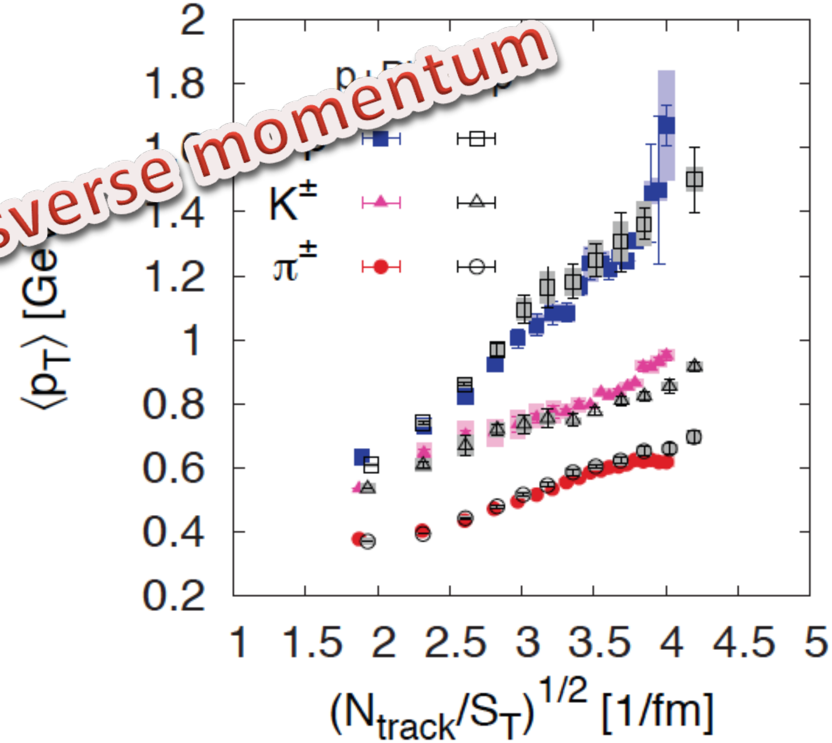
parton-hadron duality

CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847





relation of mean transverse momentum in different systems

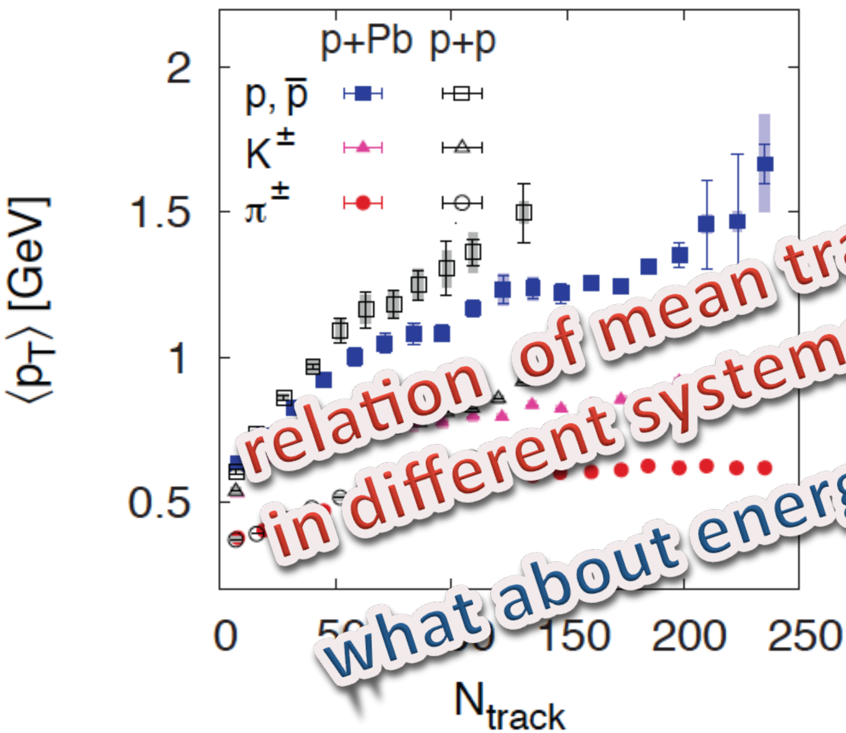
Scaling of mean p_T

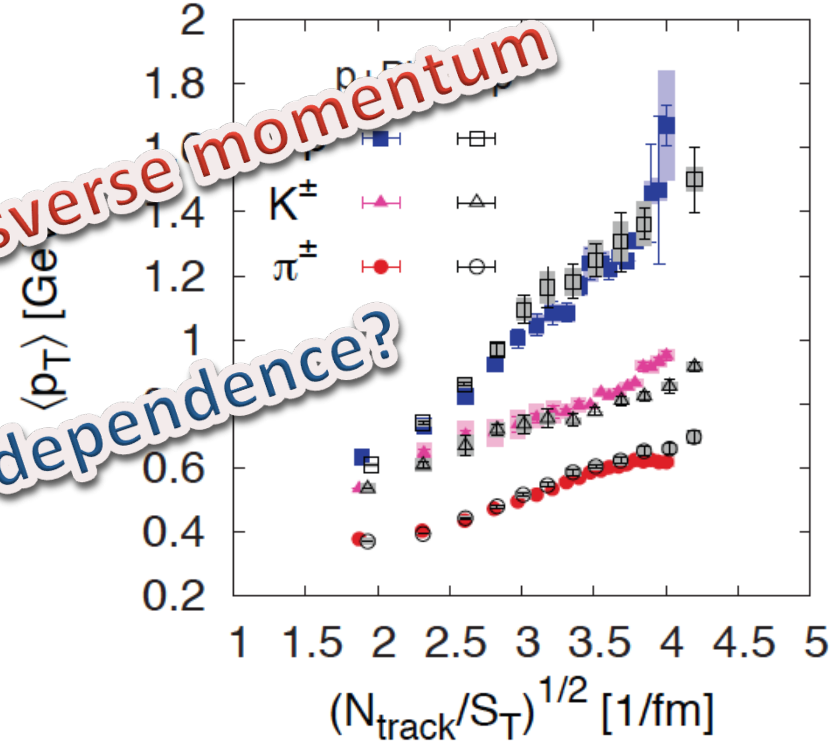
$$\langle p_T \rangle = \alpha + \beta \frac{\sqrt{N_{ch}}}{R(\gamma N_{ch})}$$

scaling variable

parton-hadron duality

CMS Collaboration, Eur. Phys. J. C72 (2013) 2164, C74 (2014) 2847





relation of mean transverse momentum
in different systems

what about energy dependence?

Energy dependence of mean p_{T} - apparent paradox?

$$\frac{dN_{\rm ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

transverse area is
energy indpendent

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$$\langle p_{\rm T} \rangle \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

If one *fixes* multiplicity and *then* changes energy, transverse area has to change accordingly

$$\langle p_{\rm T} \rangle \sim \bar{Q}_{\rm s}(W)$$

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$$\langle p_{\rm T}\rangle \sim \bar{Q}_{\rm s}(W)$$

$$\langle p_{\rm T}\rangle|_W = \alpha + \beta \frac{\sqrt{N_{\rm ch}}}{R(\sqrt[3]{\gamma N_{\rm ch}})|_W}$$

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If one *fixes* multiplicity and *then* changes energy, transverse area has to change accordingly

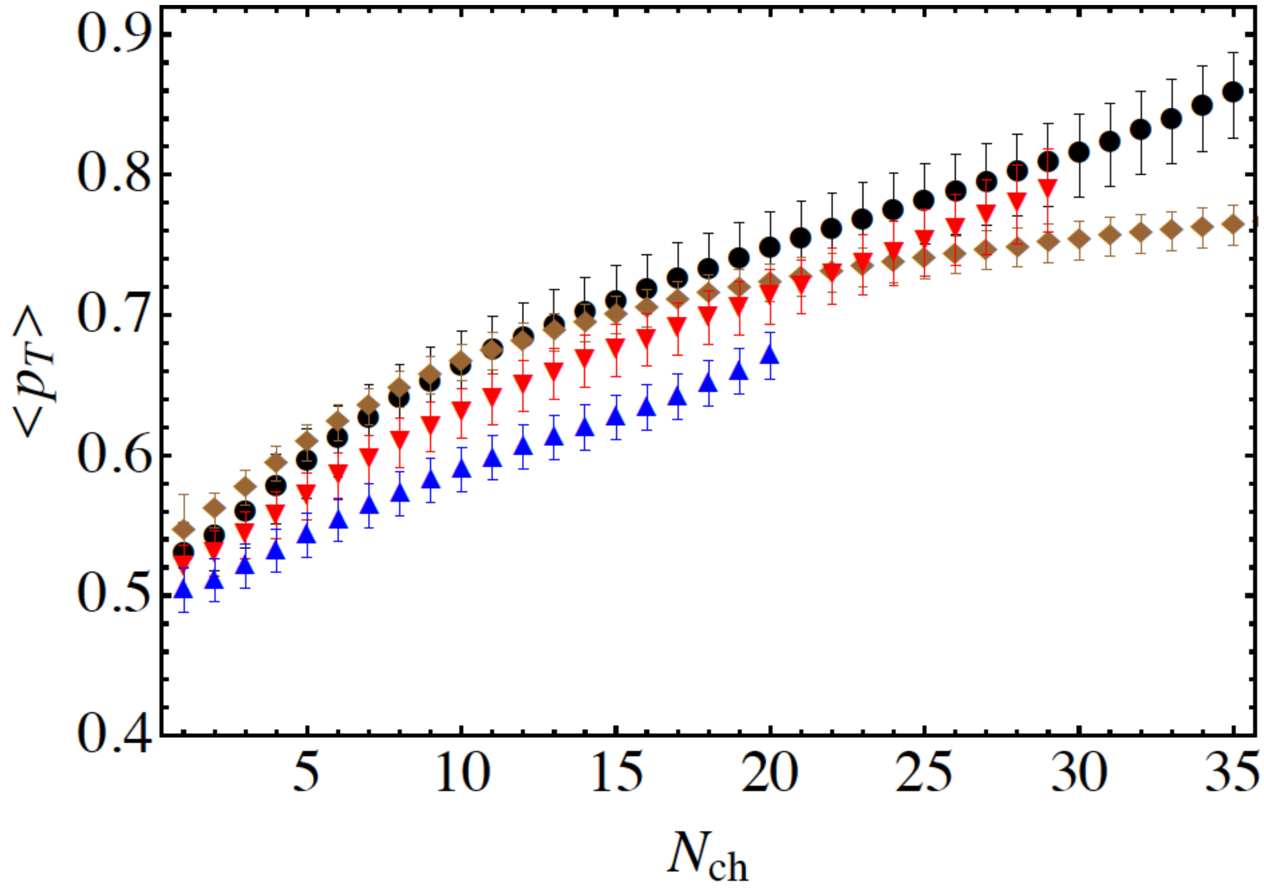
$$\langle p_T \rangle \sim \bar{Q}_s(W)$$

new scaling variable

$$\langle p_T \rangle|_W = \alpha + \beta \frac{\sqrt{N_{\rm ch}}}{R(\sqrt[3]{\gamma N_{\rm ch}})|_W} = \alpha + \beta \left(\frac{W}{W_0}\right)^{\lambda/(2+\lambda)} \frac{\sqrt{N_{\rm ch}}}{R(\sqrt[3]{\gamma N_{\rm ch}})|_{W_0}}$$

Mean p_T scaling

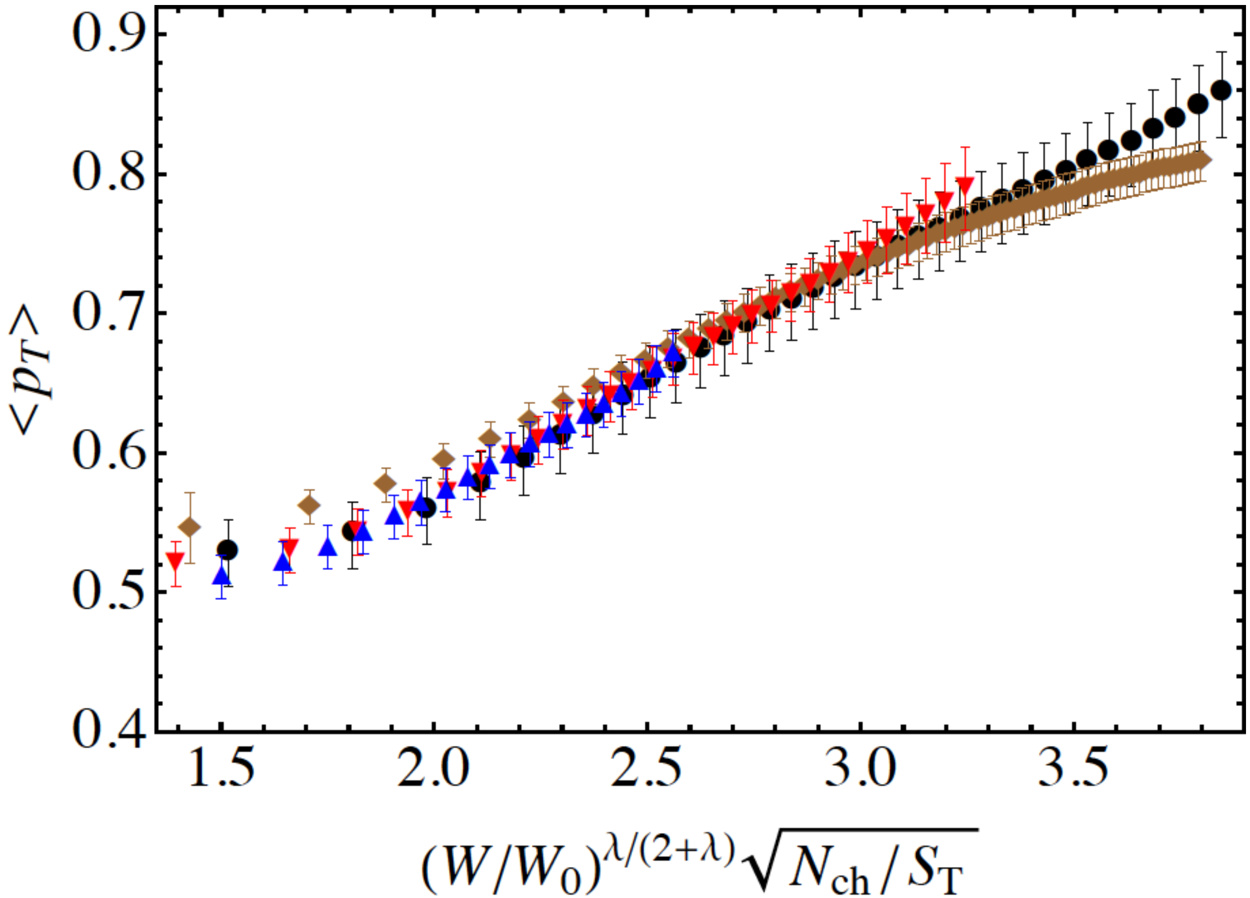
ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]



pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV

Mean p_T scaling

ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]



pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV

Violation
of Geometrical Scaling

Violation of GS for $y \neq 0$

M. P. Phys.Rev. D87 (2013) 071502(R)

$$x_{1,2} = \frac{p_{\mathrm{T}}}{\sqrt{s}} e^{\pm y}$$

x_1 can be large, so that gluons in 1 are dilute
x_2 is then small in the dense region

Kinematical range of GS in pp

$$x_1 < x_{\max}$$

⇓

$$p_{\mathrm{T}\max}(W, y) < x_{\max} W \, e^{-y}$$

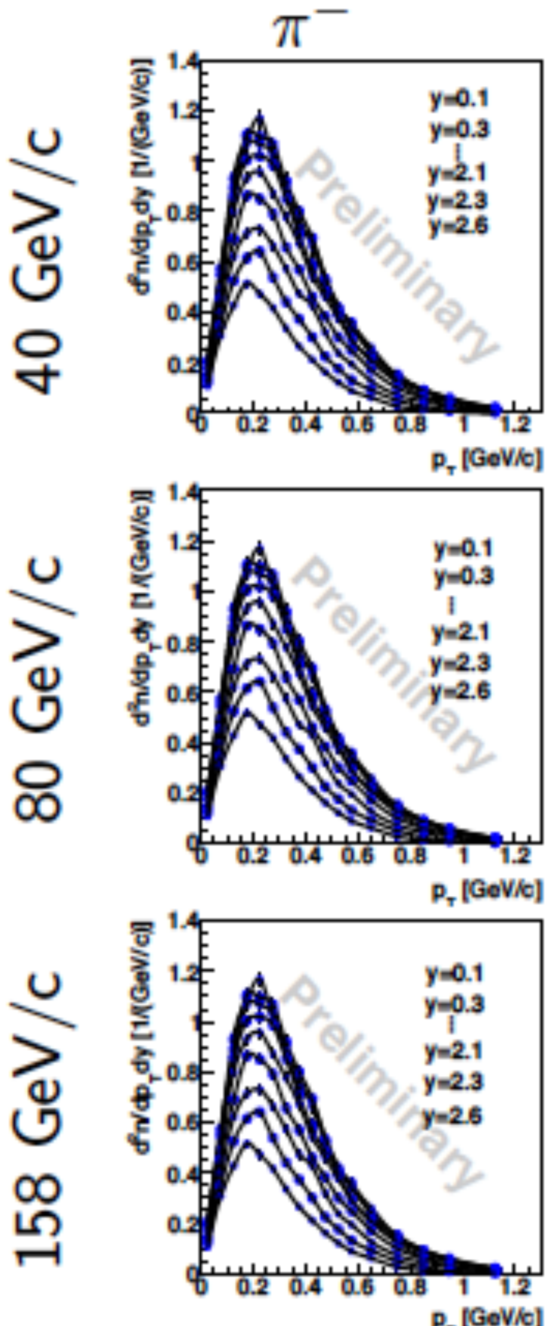
Kinematical range of GS in pp

$$x_1 < x_{\max}$$

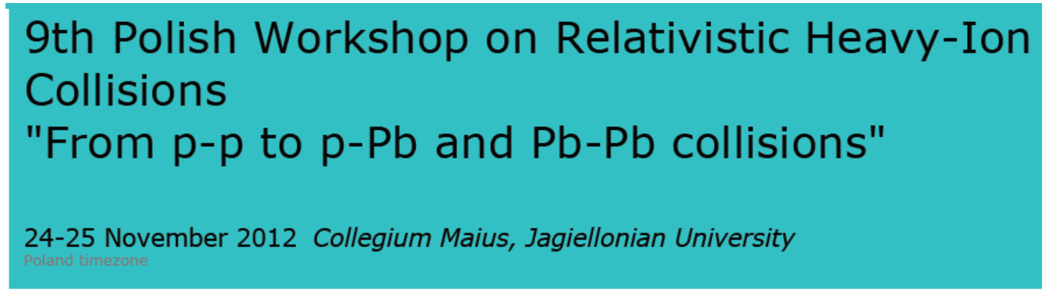
$$p_{\mathrm{T}\max}(W, y) < x_{\max} W \, e^{-y}$$

transverse momentum schould be larger than some nonperturbative scale Λ

$$p_{\mathrm{T}\min} > \Lambda$$



NA61 Shine data



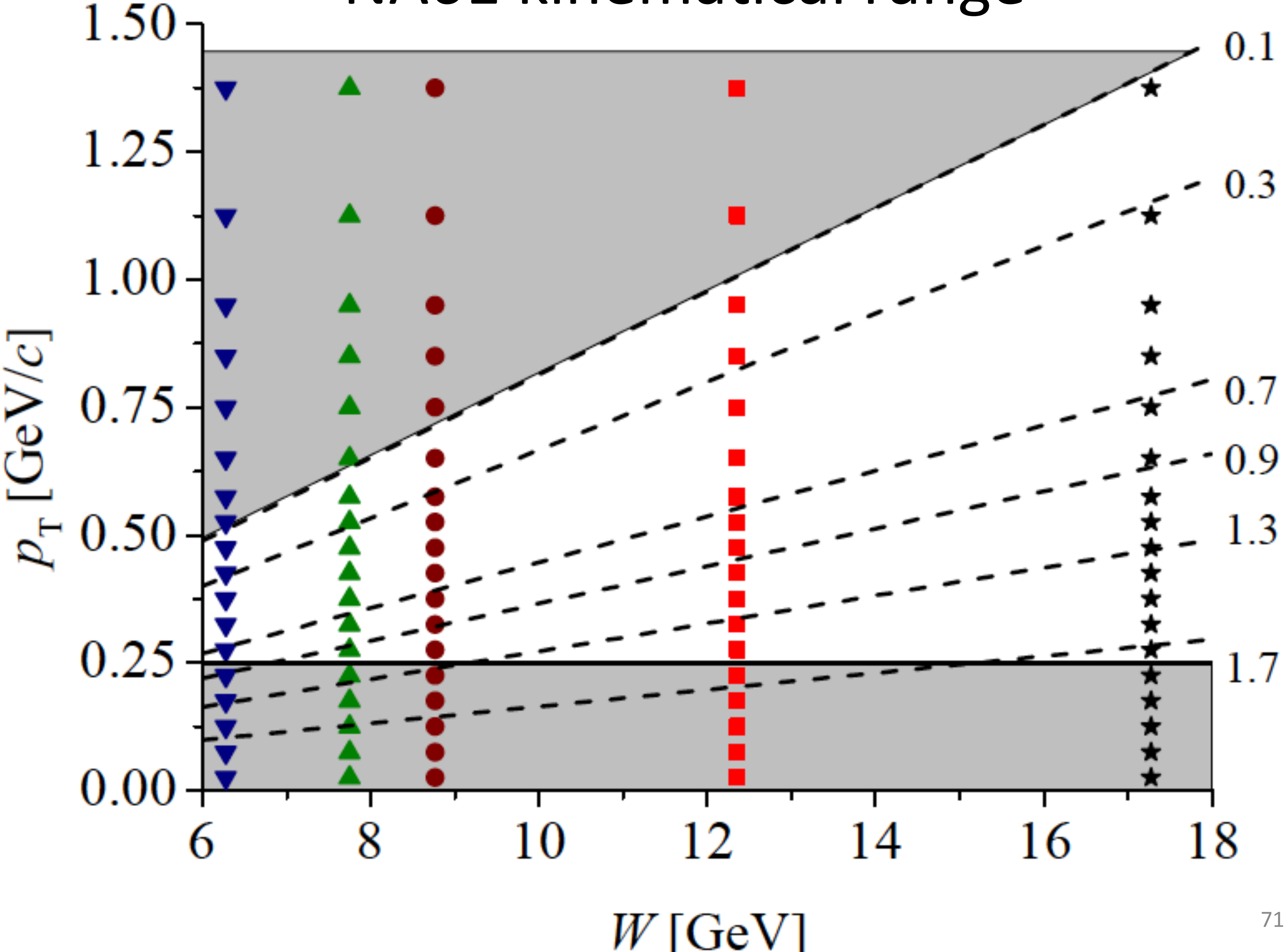
Hadron spectra: p+p vs. Pb+Pb at the SPS energies

Szymon Puławski
for NA61/SHINE Collaboration

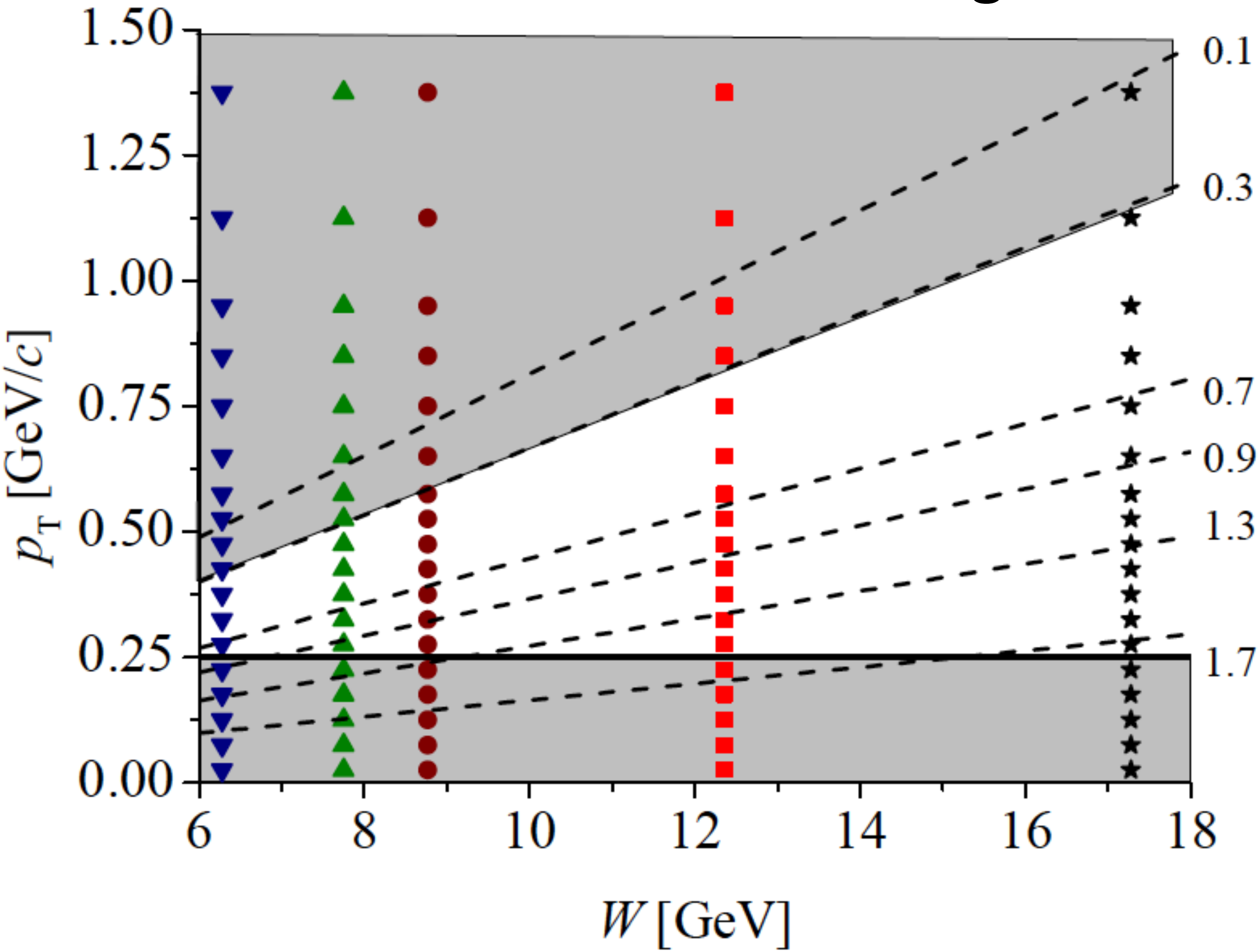
University of Silesia, Katowice

- Data analyzed:
  - p+p @ 20 GeV/c ($\sqrt{s}$ = 6.2 GeV): $1.3{\cdot}10^6$ events
  - p+p @ 31 GeV/c ($\sqrt{s}$ = 7.7 GeV): $3.1{\cdot}10^6$ events
  - p+p @ 40 GeV/c ($\sqrt{s}$ = 8.8 GeV): $5.2{\cdot}10^6$ events
  - p+p @ 80 GeV/c ($\sqrt{s}$ = 12.3 GeV): $4.3{\cdot}10^6$ events
  - p+p @ 158 GeV/c ($\sqrt{s}$ = 17.3 GeV): $3.5{\cdot}10^6$ events

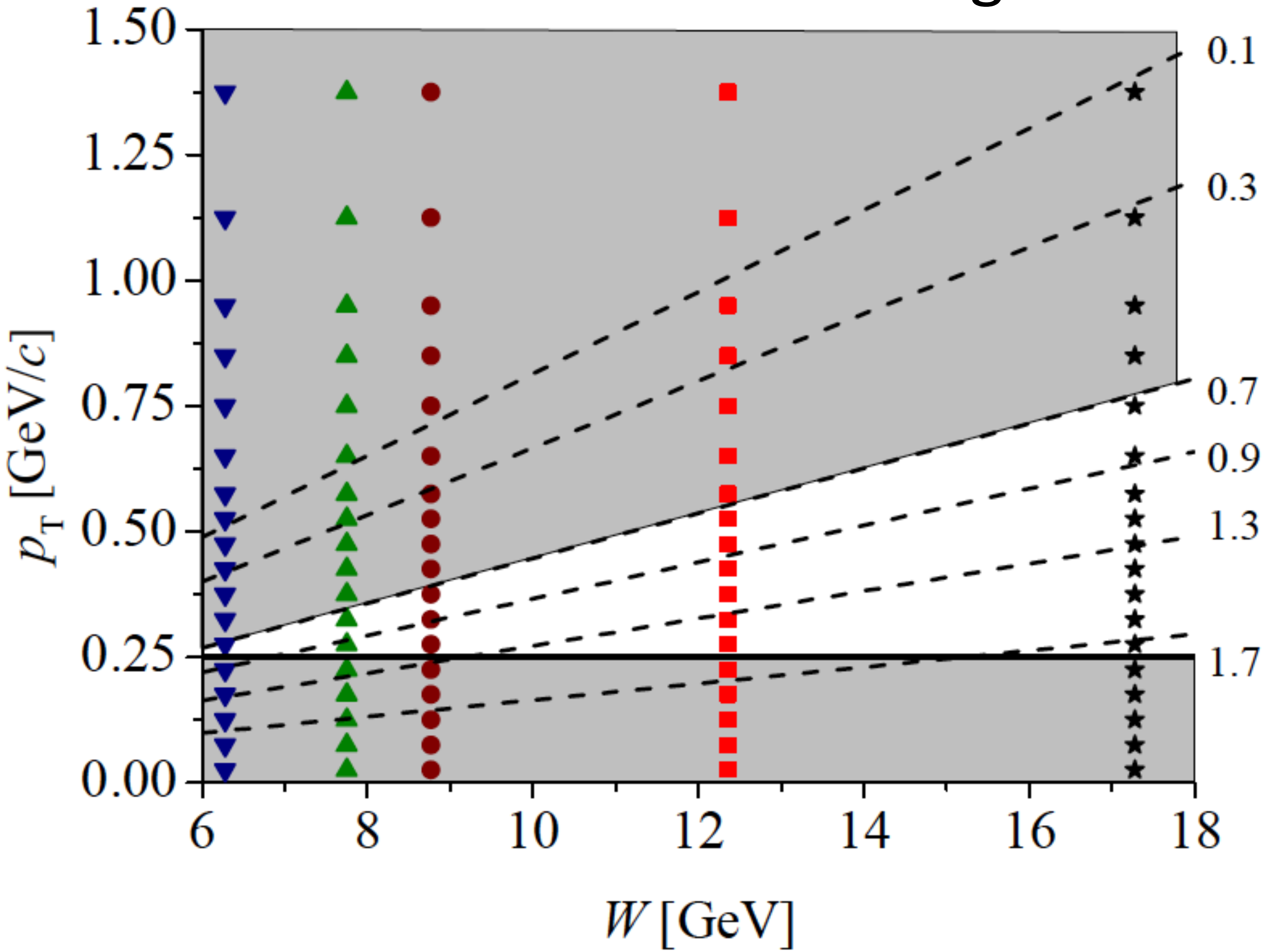
NA61 kinematical range



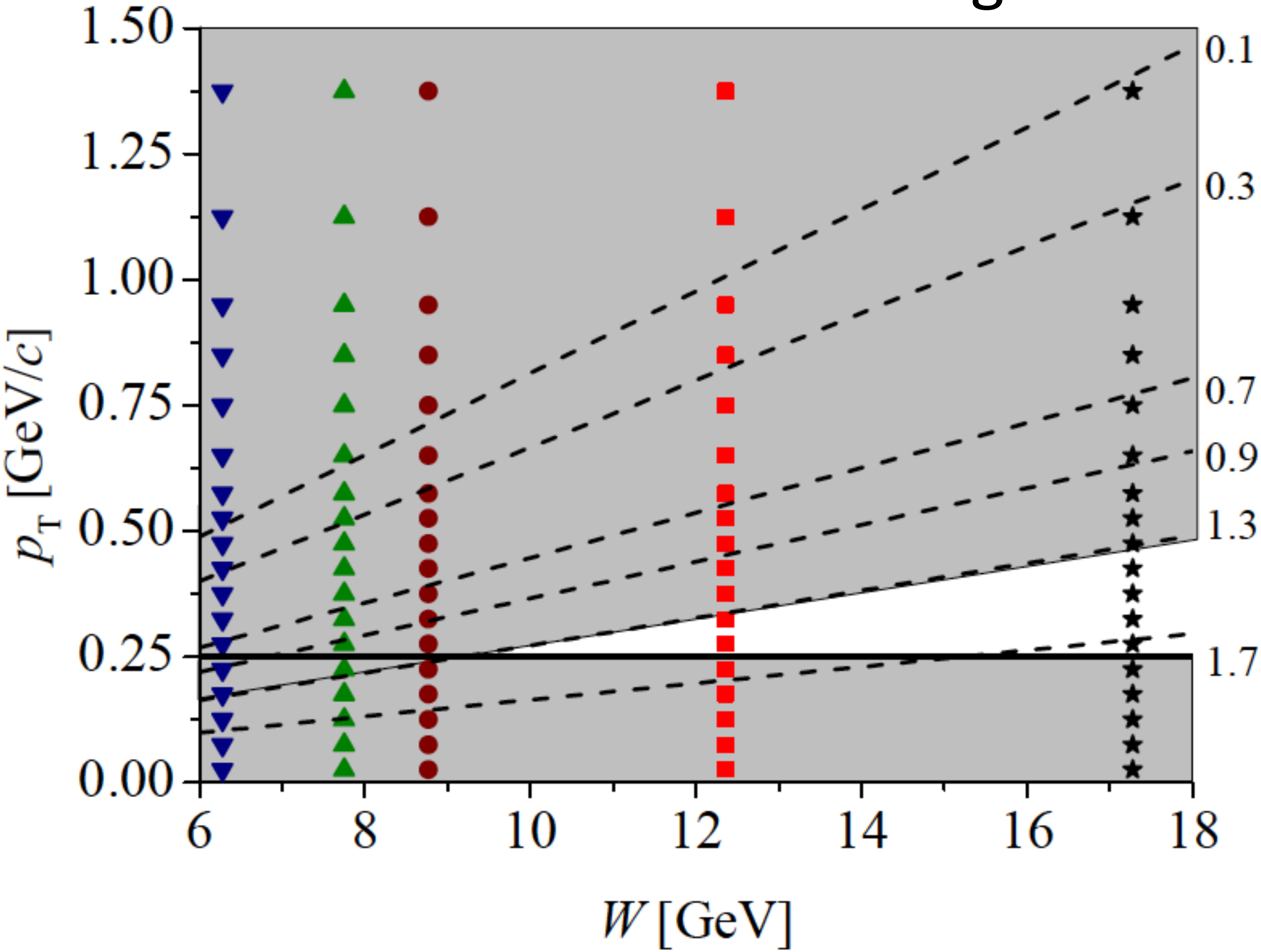
NA61 kinematical range

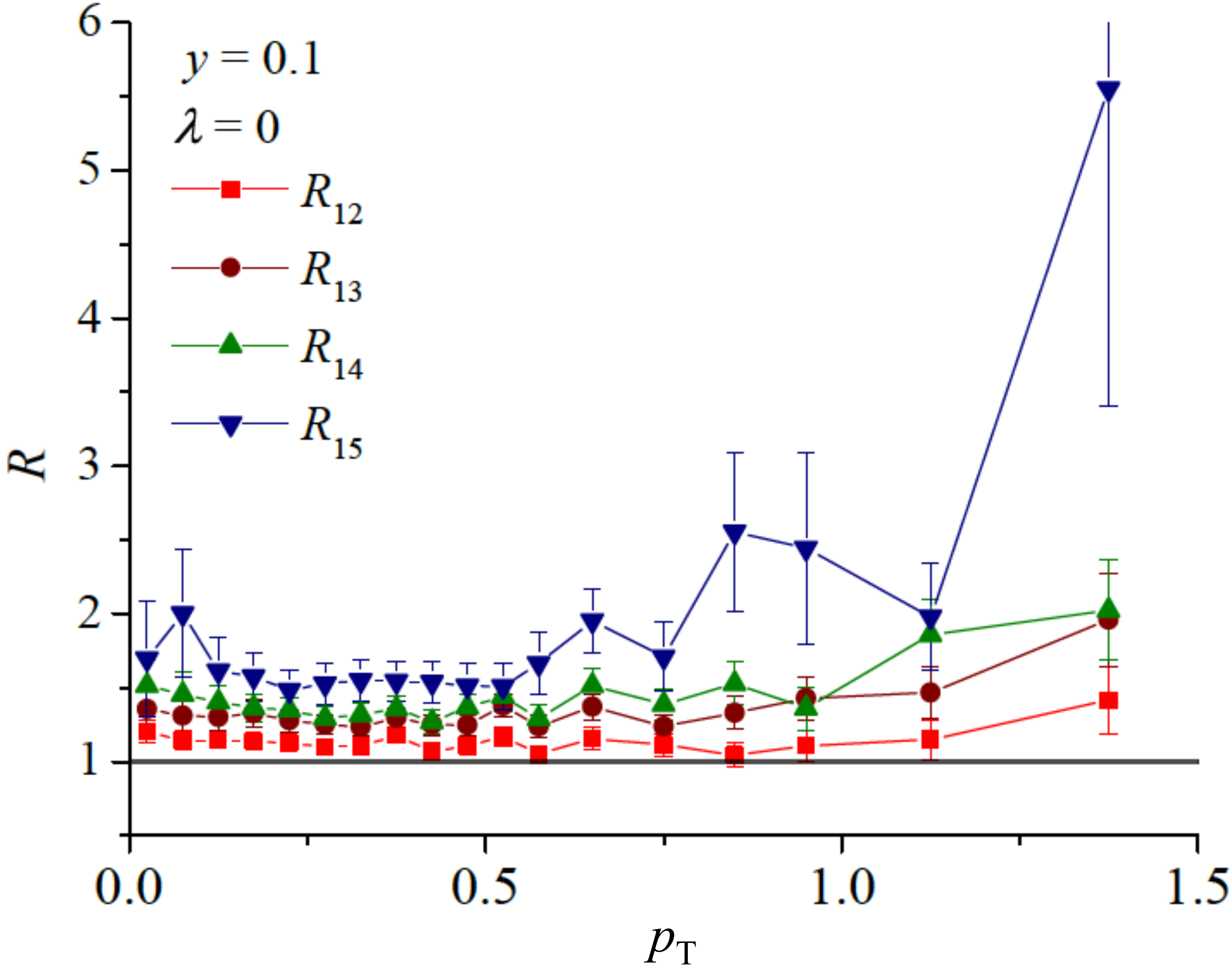


NA61 kinematical range

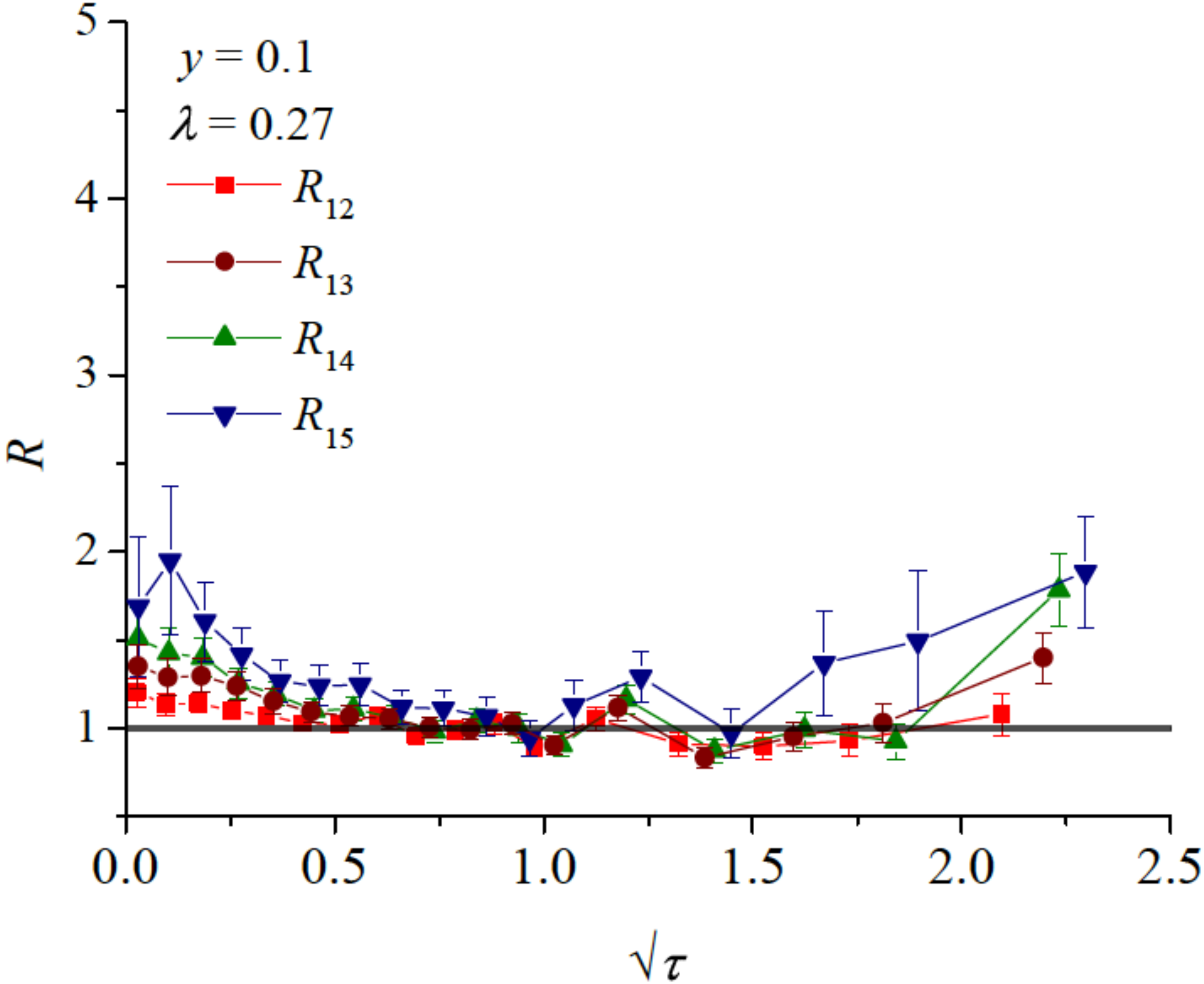


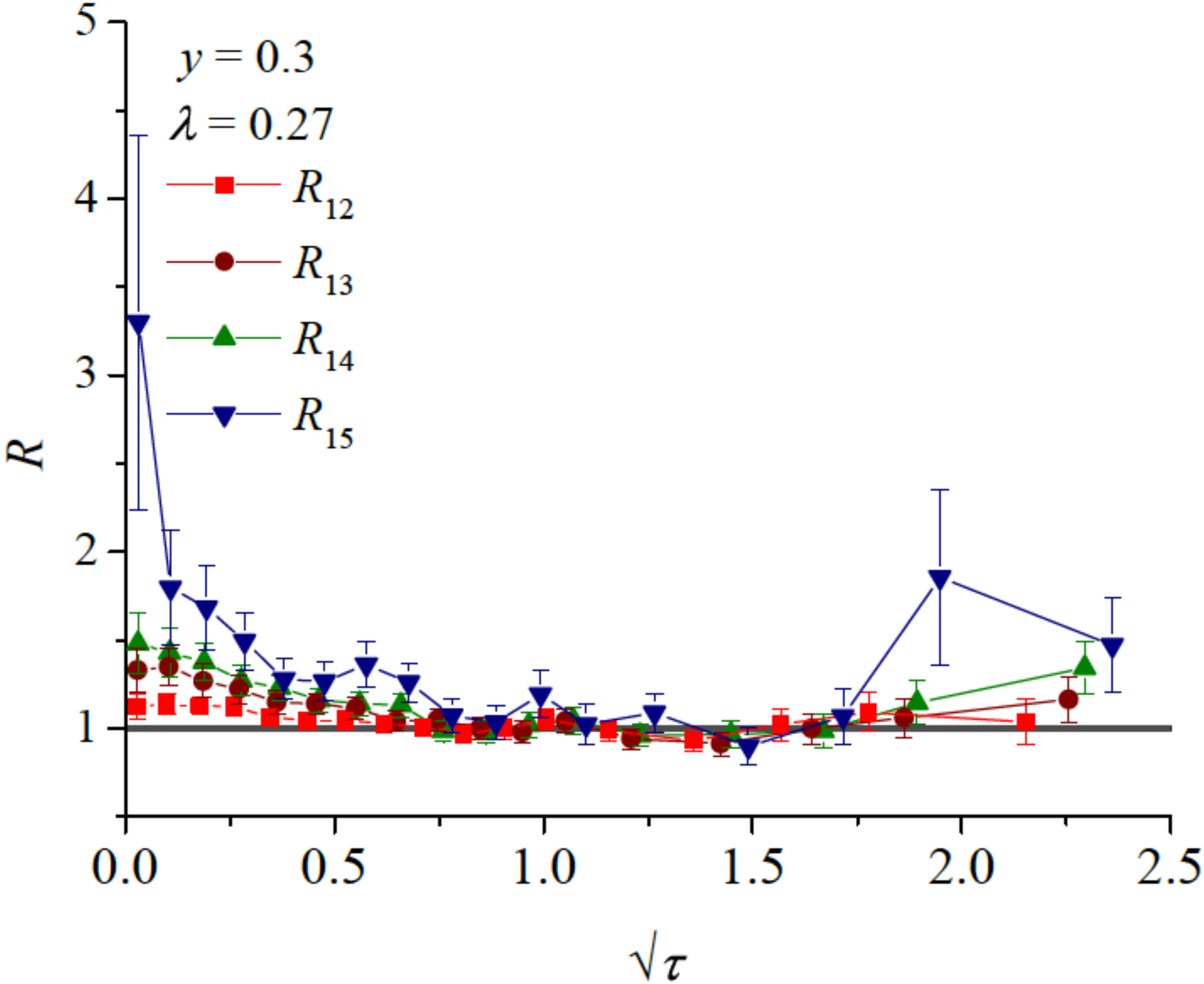
NA61 kinematical range



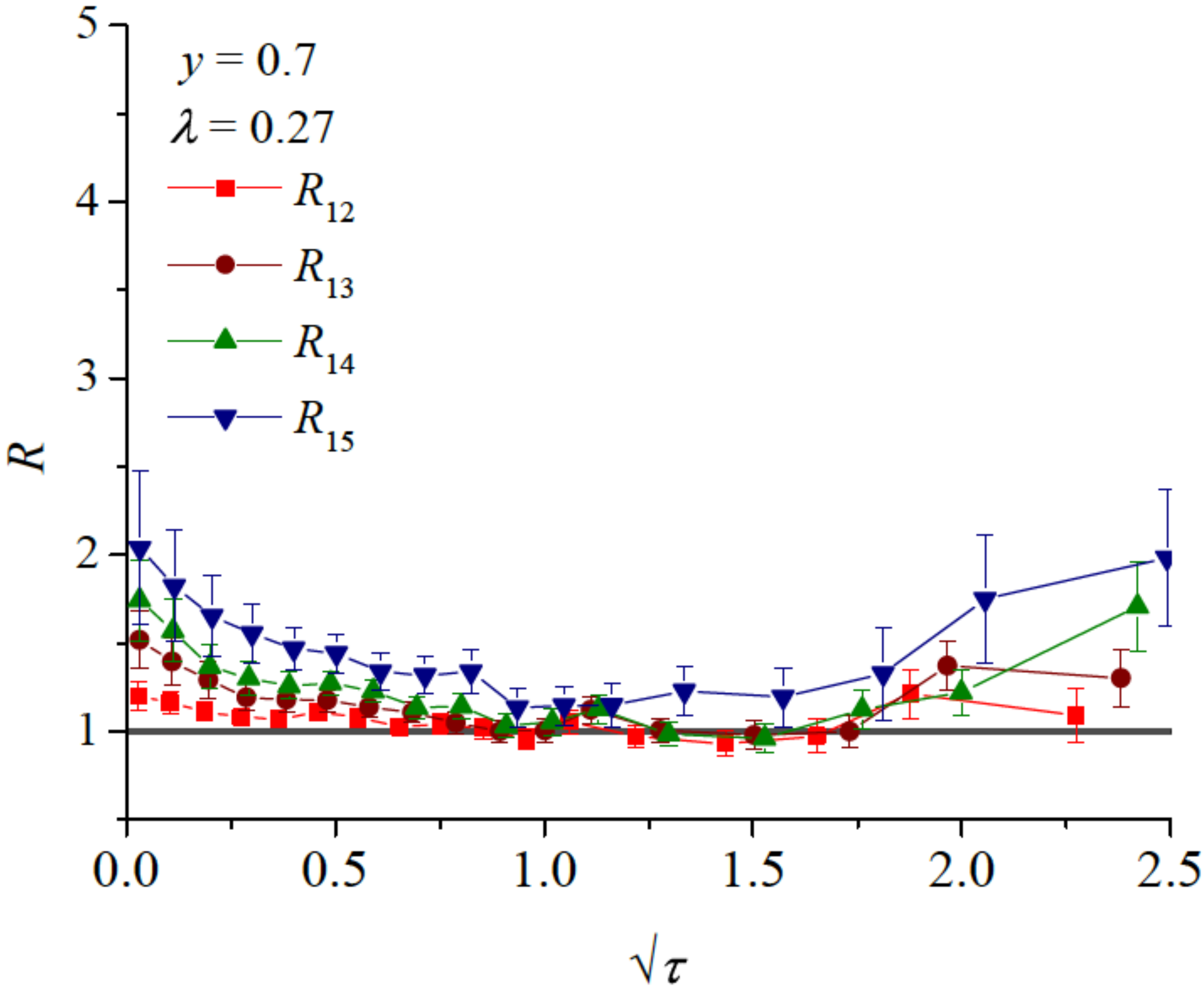


y = 0.1
λ = 0
R12
R13
R14
R15
R
6
5
4
3
2
1
0.0
0.5
1.0
1.5
pT

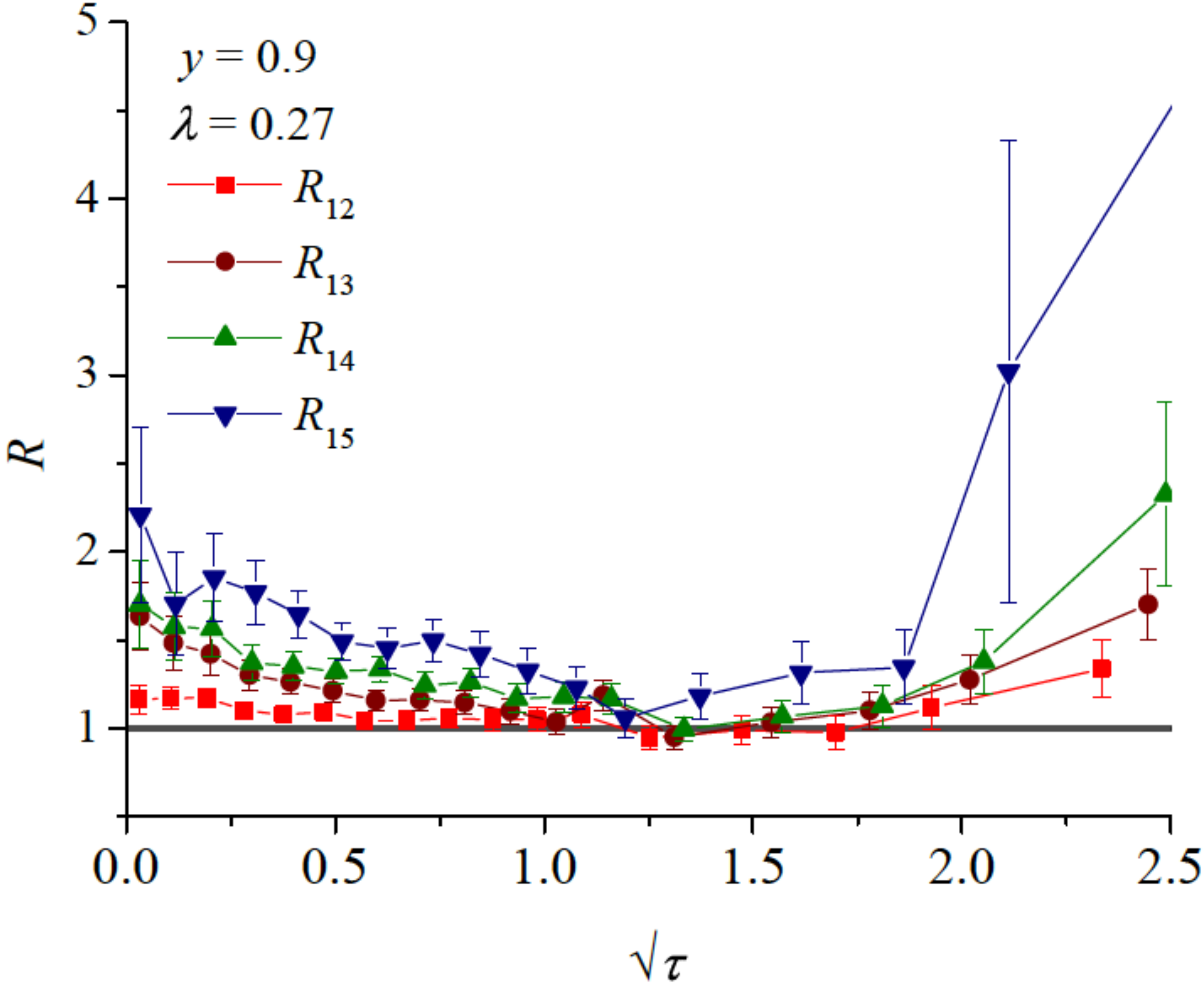

$y = 0.1$
$\lambda = 0.27$
R_{12}
R_{13}
R_{14}
R_{15}
R
$\sqrt{\tau}$
0.0
0.5
1.0
1.5
2.0
2.5
1
2
3
4
5



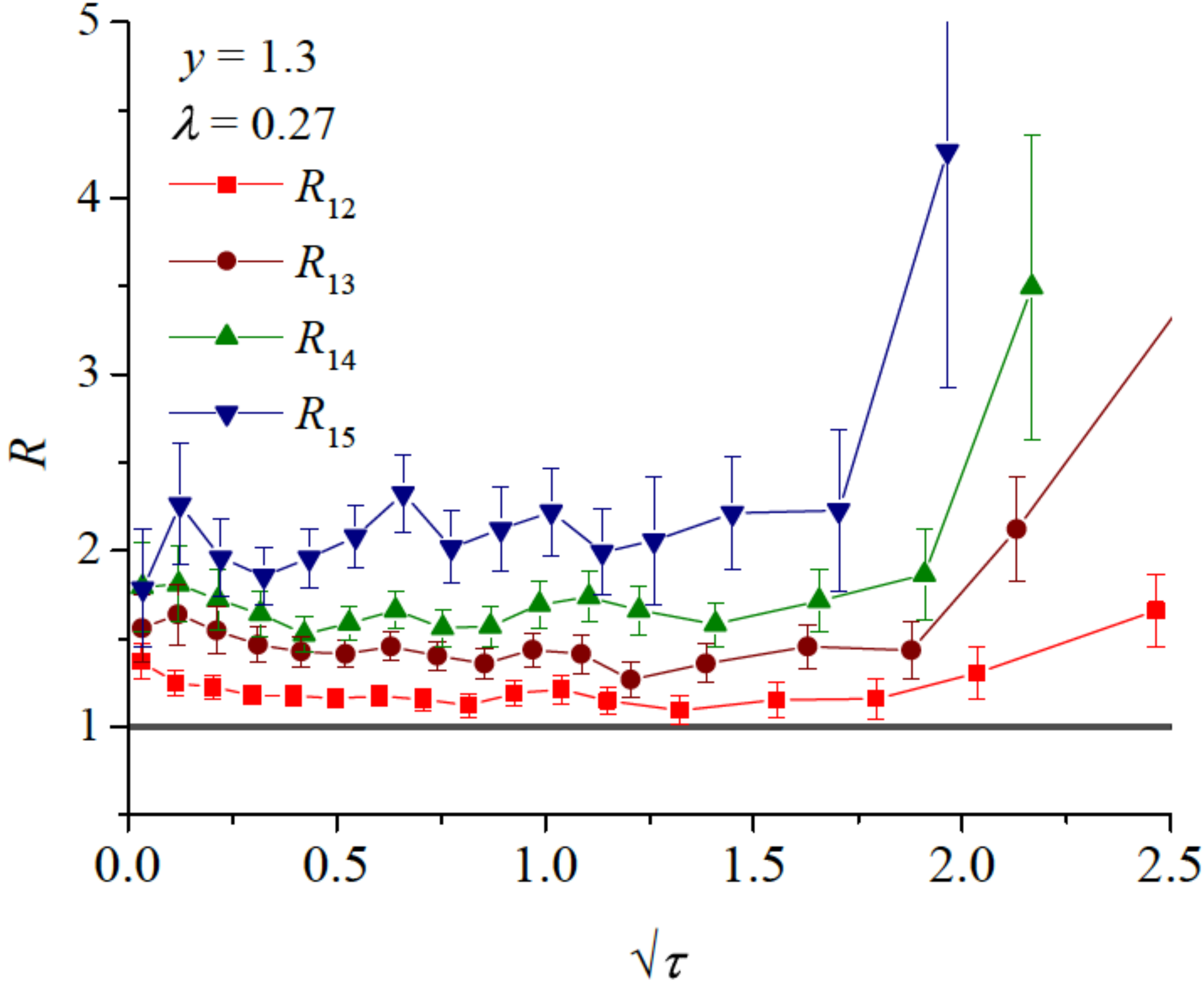
$y = 0.3$
$\lambda = 0.27$
R_{12}
R_{13}
R_{14}
R_{15}
R
$\sqrt{\tau}$
0.0
0.5
1.0
1.5
2.0
2.5
1
2
3
4
5



y = 0.7
λ = 0.27
R12
R13
R14
R15
R
√τ
5
4
3
2
1
0.0
0.5
1.0
1.5
2.0
2.5



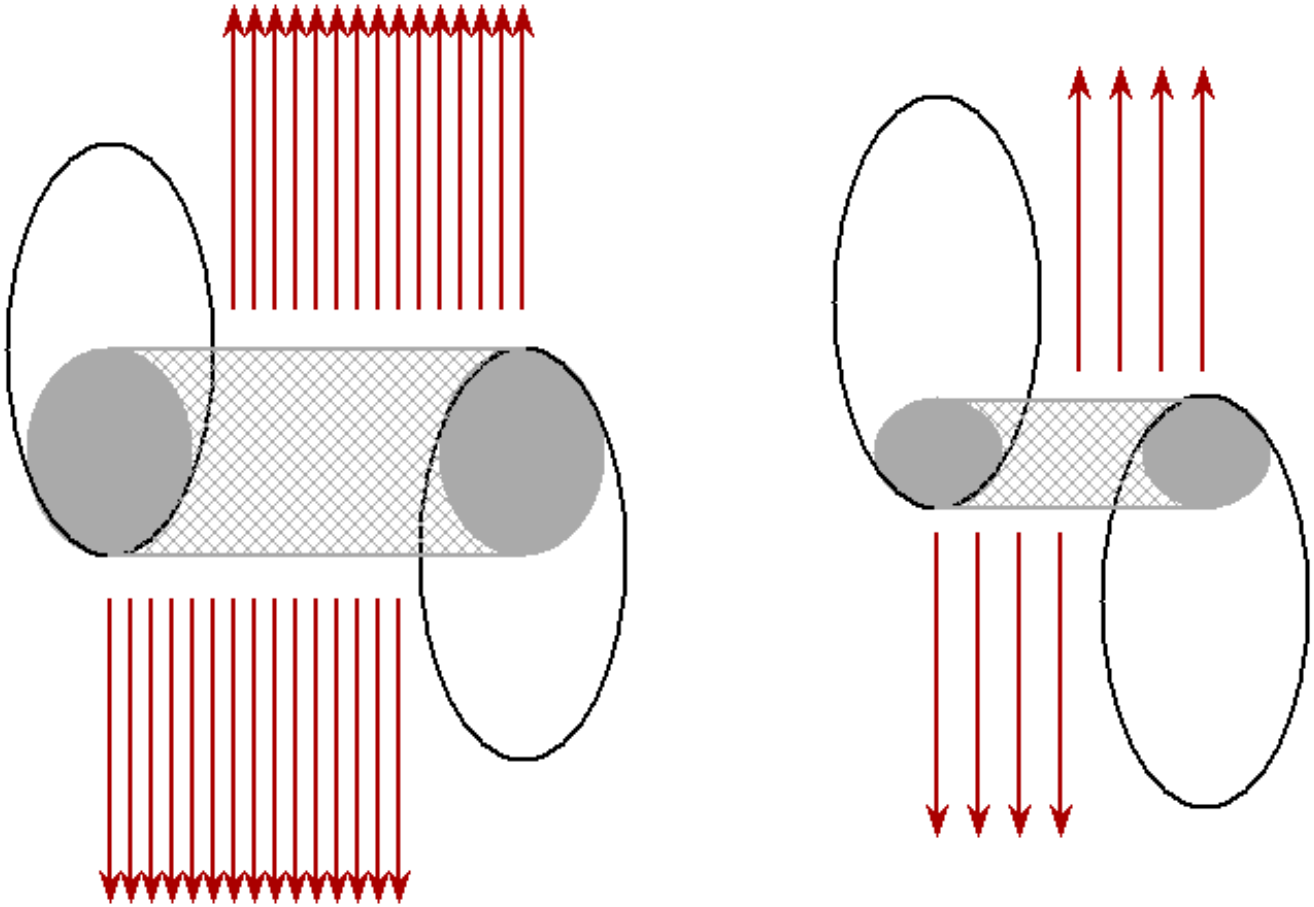
$y = 0.9$
$\lambda = 0.27$
R_{12}
R_{13}
R_{14}
R_{15}
R
1
2
3
4
5
0.0
0.5
1.0
1.5
2.0
2.5
$\sqrt{\tau}$



y = 1.3
λ = 0.27
R12
R13
R14
R15
R
√τ
5
4
3
2
1
0.0
0.5
1.0
1.5
2.0
2.5

Geometrical Scaling in Heavy Ion Collisions

GS in HI: centrality



small centrality

large centrality

number of participants

$$N_{\mathrm{part}} \sim V$$

GS in HI: centrality dependence

$$S_\perp \sim N_{\rm part}^{2/3}$$

$$\frac{dN}{dy} \sim N_{\rm part}$$

Geometrical Scaling of Direct-Photon Production
in Hadron Collisions from RHIC to the LHC
Christian Klein-Bösing, Larry McLerran arXiv:1403.1174

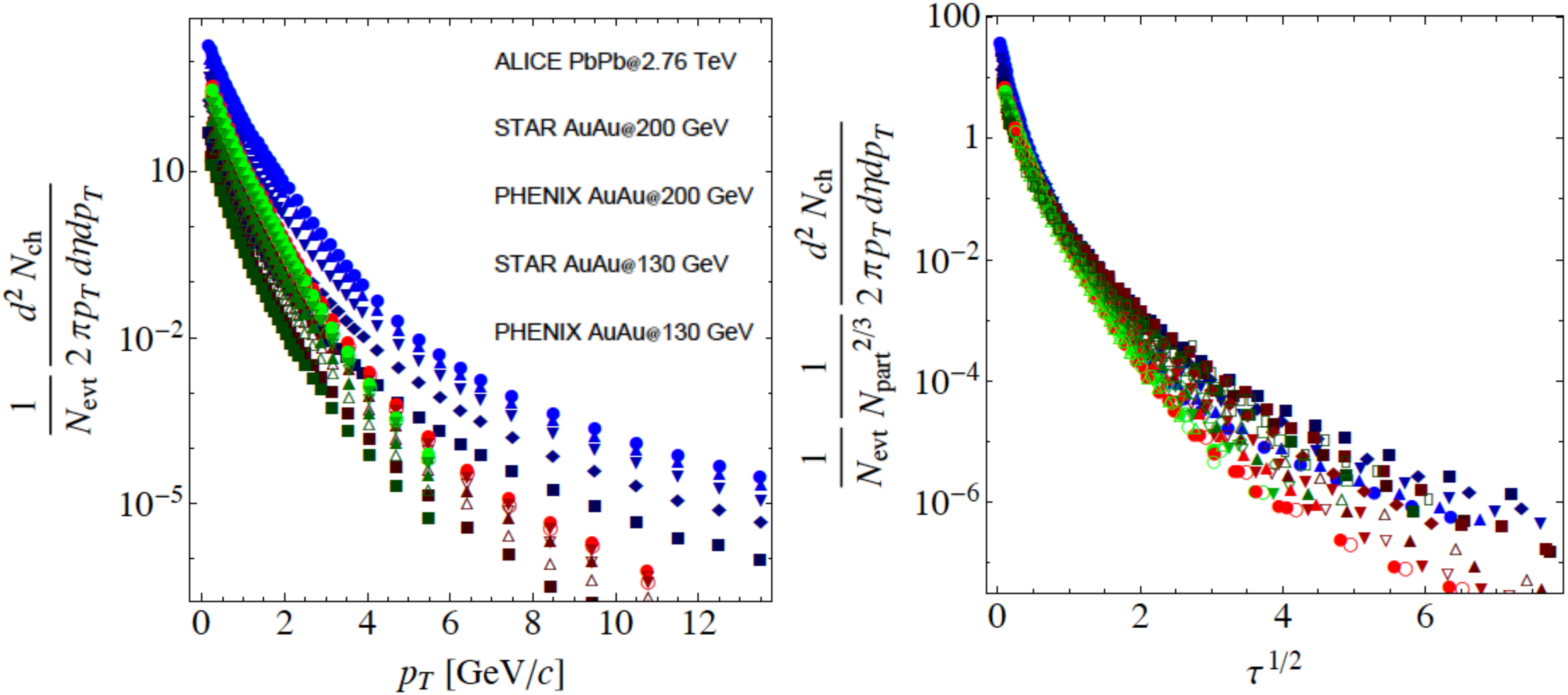
Scaling of the saturation scale:

$$Q_s^2(x) = \frac{\kappa}{S_\perp}\frac{dN}{dy} \sim N_{\rm part}^{1/3}\left(\frac{\sqrt{s}}{p_{\rm T}}\right)^\lambda$$

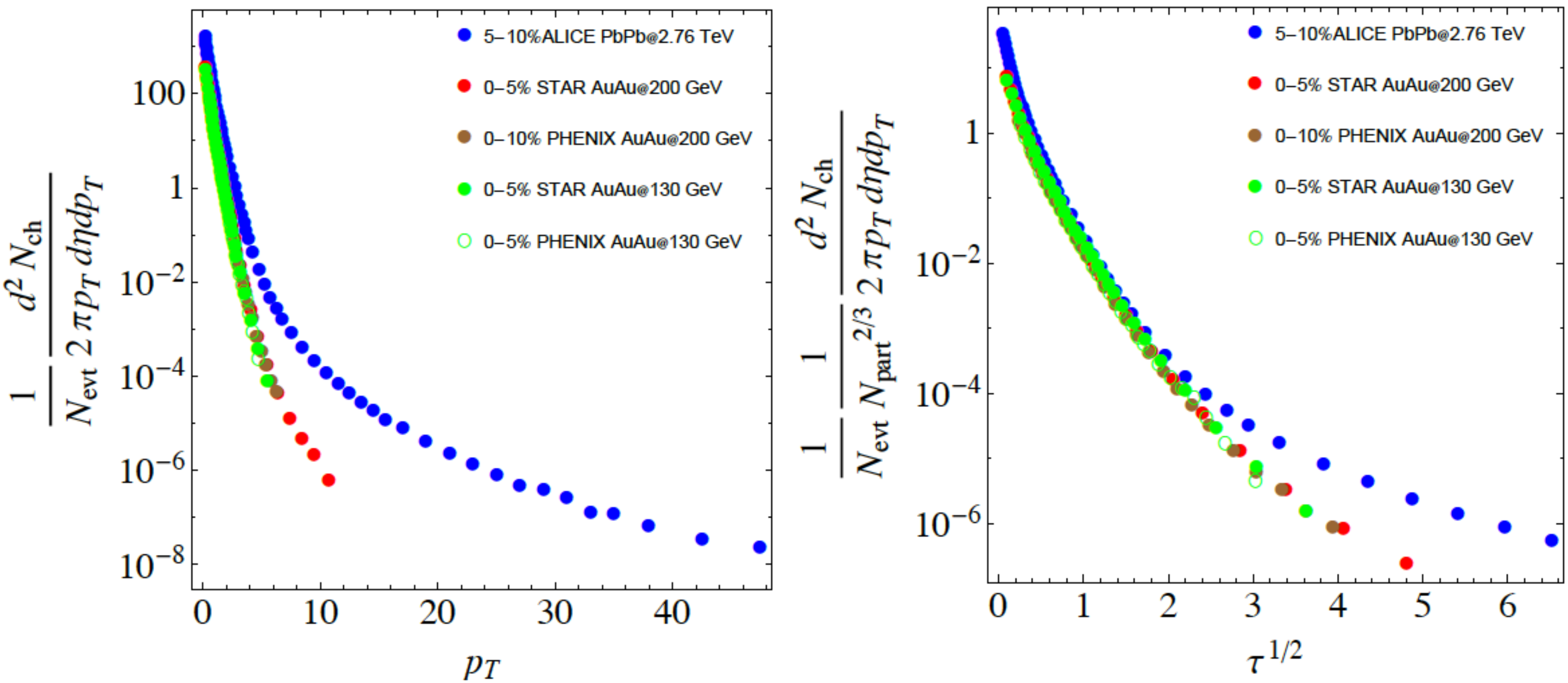
$$\frac{Q_0^2}{N_{\rm part}^{2/3}}\frac{dN_{\rm ch}}{2\pi p_{\rm T}\, d\eta dp_{\rm T}} = F(\tau)$$

$$\tau = \frac{1}{N_{\rm part}^{1/3}}\frac{p_{\rm T}^2}{Q_0^2}\left(\frac{p_{\rm T}}{W}\right)^\lambda$$

GS in HI

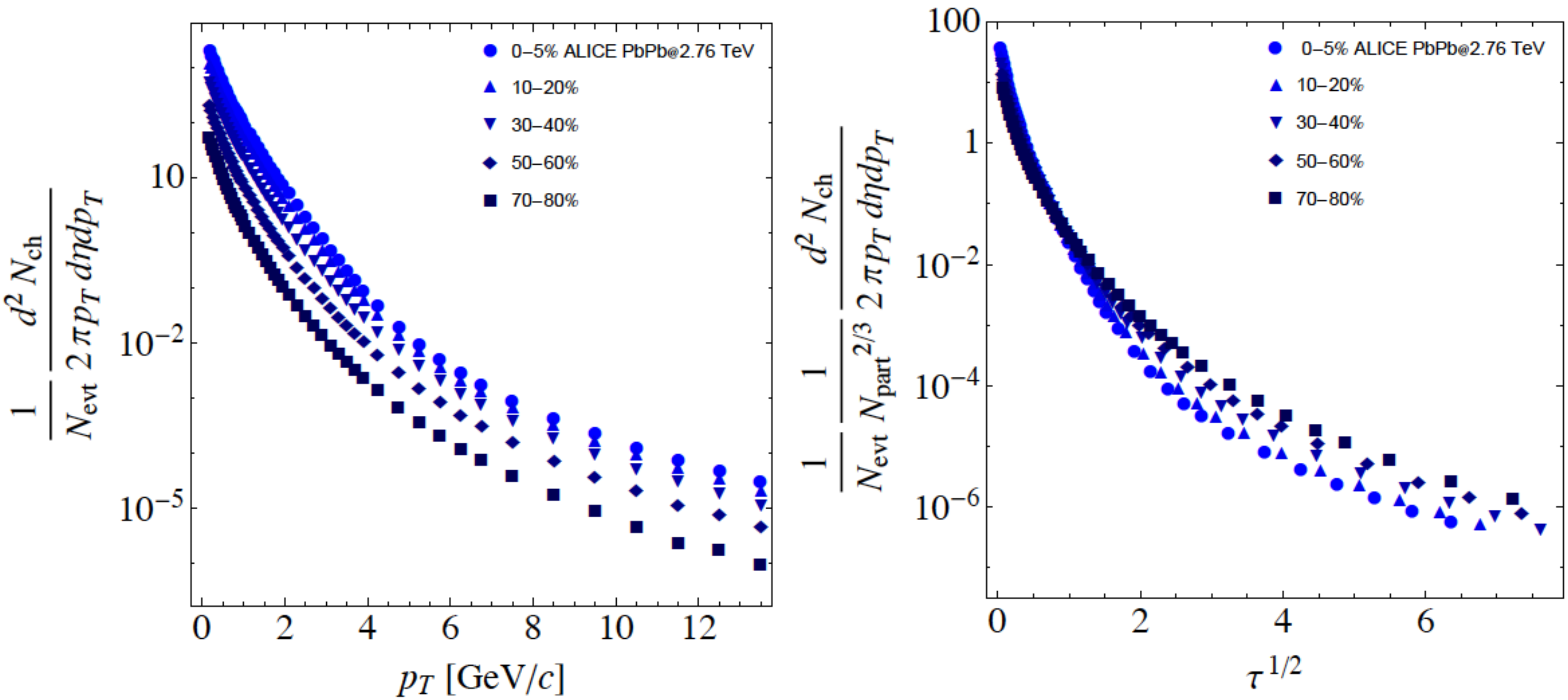

ALICE PbPb@2.76 TeV
STAR AuAu@200 GeV
PHENIX AuAu@200 GeV
STAR AuAu@130 GeV
PHENIX AuAu@130 GeV
$\frac{1}{N_{evt}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
p_T [GeV/c]
$\frac{1}{N_{evt} N_{part}^{2/3}} \frac{d^2 N_{ch}}{2\pi p_T d\eta dp_T}$
$\tau^{1/2}$

Energy Scaling in HI



energy scaling works quite well, why?

Centrality Scaling in HI



Summary

- QCD evolution equations lead to overabundance of gluons
- Nonlinear evolution introduces new scale: *saturation momentum*
- GS should emerge if no other scales are present
- GS in DIS works for rather high Bjorken x
- GS works also for charged particles in pp
- GS for mean p_T and for $\langle p_T \rangle (N_{ch})$
- GSV is found for $y \neq 0$ in agreement with expectations
- Energy and centrality dependence of GI in HI

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- GS for mean p_{T} and for $\langle p_{\mathrm{T}} \rangle (N_{\mathrm{ch}})$
- GSV is found for $y \neq 0$ in agreement with expectations
- Energy and centrality dependence of GS in HI
- Is GS a real sign of saturation?
- Why in pp GS is not washed out by FSI?
- Why in HI hydro preserves (at least partially) GS?
- Nonuniversality: different values of λ