Influence of interactions on particle production induced by time-varying mass terms

[*JHEP* **1503** (2015) 113 (arXiv:1412.7442 [hep-ph])]

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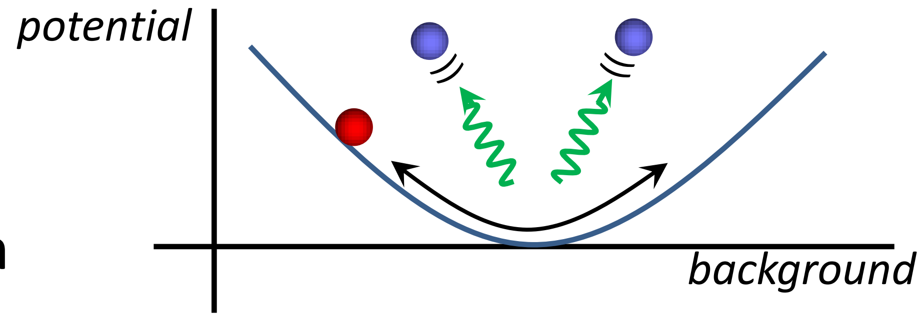
Collaborators : Olga Fuksińska and Zygmunt Lalak (UW)

Outline

1. Introduction
2. Bogoliubov transformation law with interaction terms
3. Application to our model
4. Summary

1. Introduction



- Particle production from vacuum
  - It is known that a varying background causes production of particles
    - Oscillating Electric field → pair production of electrons

      [E. Brezin and C. Itzykson, *Phys. Rev.* **D 2**,1191 (1970)]

    - Changing metric → gravitational particle production

      [L. Parker, *Phys. Rev.* **183**, 1057 (1969)]

      [L. H. Ford, *Phys. Rev.* **D 35**, 2955 (1987)]

    - Oscillating inflaton → (p)reheating

      [L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994)]

      [L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev.* **D 56**, 3258 (1997)]

Example of scalar particle production (overview)

[L.Kofman, A.Linde, X.Liu, A.Maloney, L.McAllister, E.Silverstein, JHEP **0405**, 030 (2004)]

- Let us consider : $\mathcal{L}_{int} = -\frac{1}{2} g^2 |\phi|^2 \chi^2$

χ : real scalar particle (*quantum*)

g : coupling

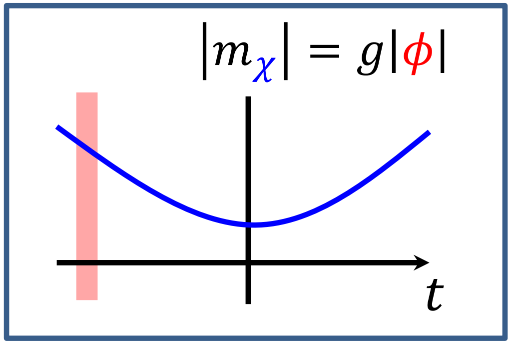
ϕ : complex scalar field (*classical*)

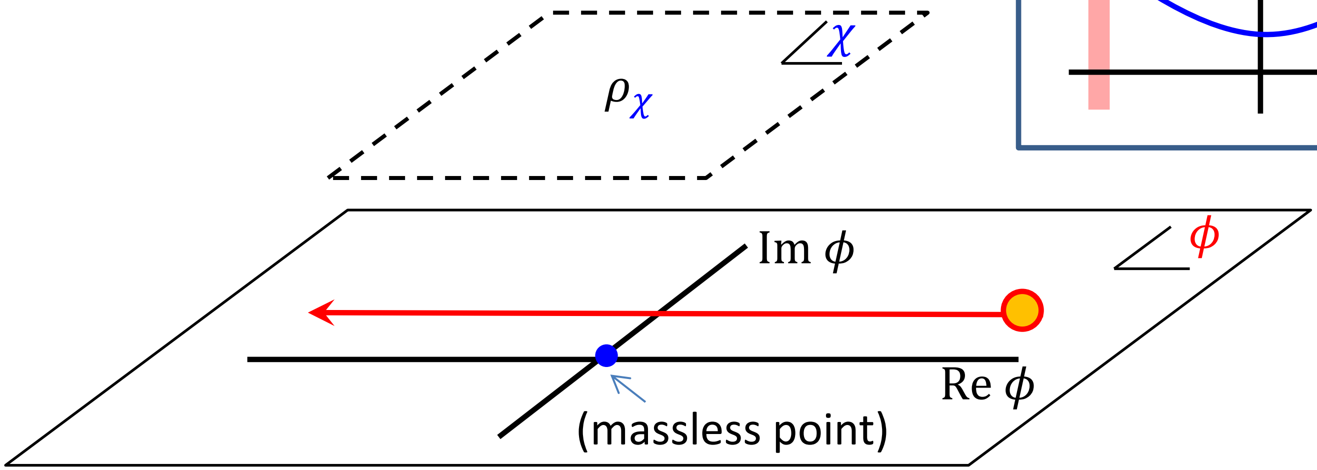
- If ϕ goes near the origin…

→ mass of χ ($m_\chi = g\phi$) becomes small around $|\phi| = 0$

→ kinetic energy of ϕ converts to χ

★ χ particles are produced !!





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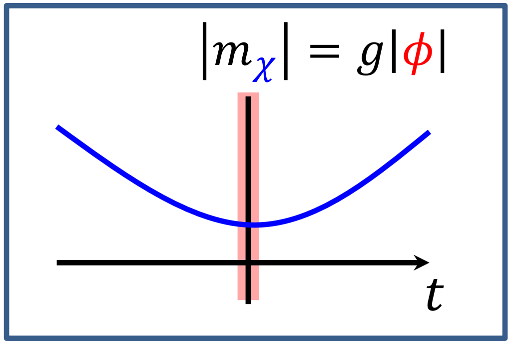

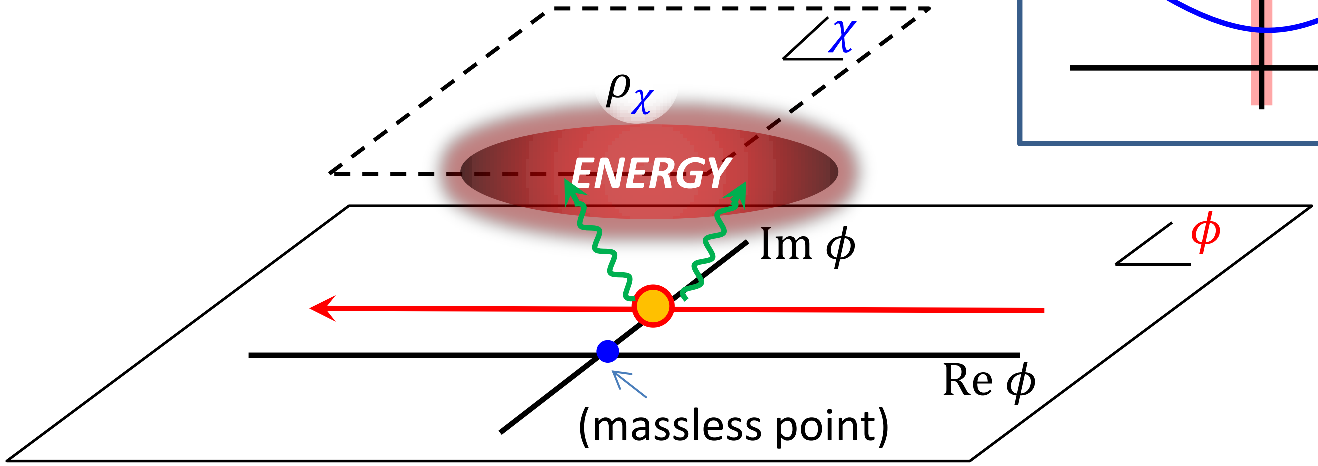

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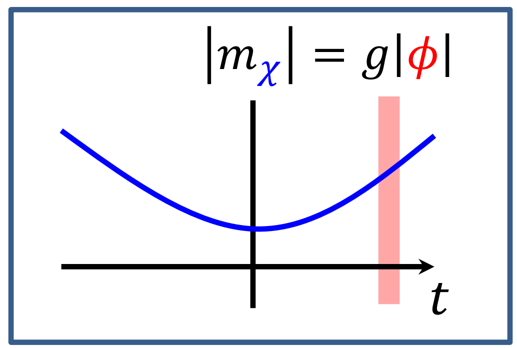

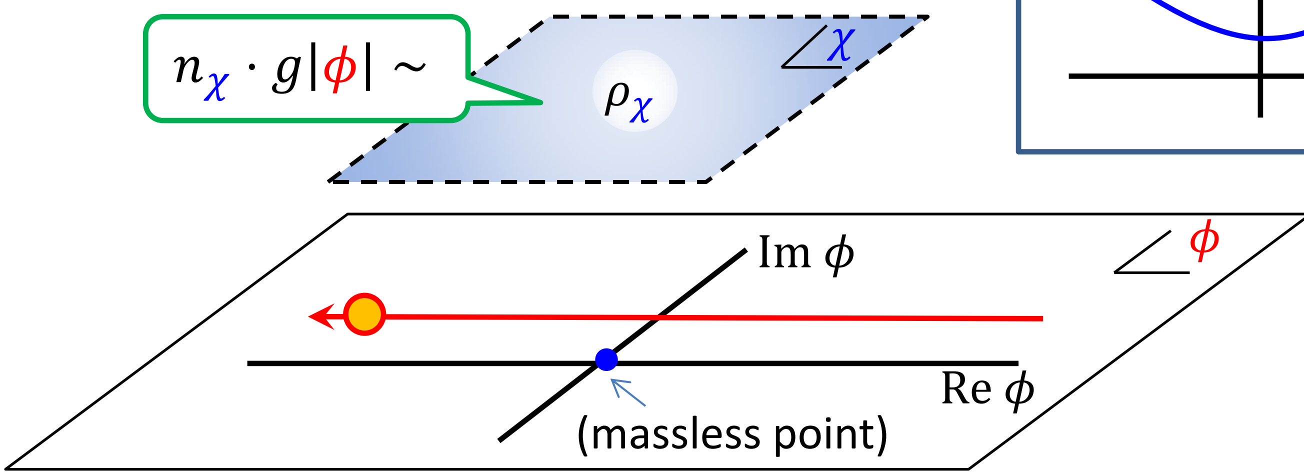

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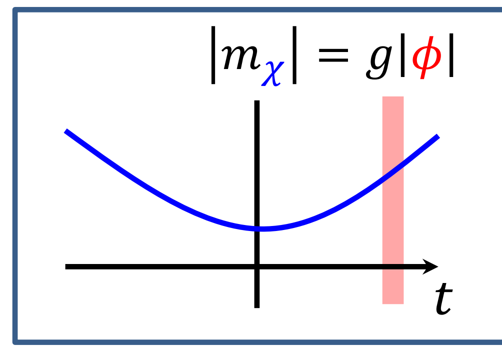
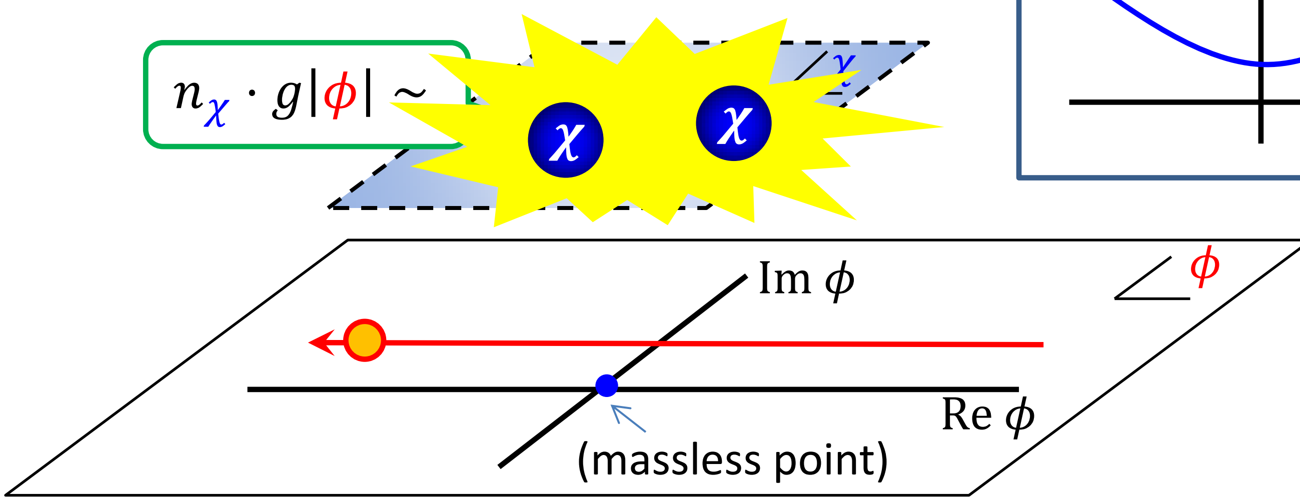




Equations of motion for χ:

- $0 = (\partial^2 + g^2|\phi|^2)\chi$

*Assumption : $\phi = \phi(t)$

plane wave expansion: $\chi = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left(\chi_k(t)a_{\mathbf{k}} + \chi_k^*(t)a_{-\mathbf{k}}^\dagger\right)$

plane wave

creation/annihilation op.

time-dependent wave func.

$$0 = \partial_0^2\chi_k + \omega_k^2(t)\chi_k \quad , \quad (\chi_k, \chi_k) = 1$$

$$\left(\omega_k \equiv \sqrt{k^2 + g^2|\phi|^2}, \qquad (A, B) \equiv i\left(A^\dagger\dot{B} - \dot{A}^\dagger B\right) \right)$$

- If $\chi_k^{\rm in}$ is a solution, a linear combination $\chi_k^{\rm in}$ and $\chi_k^{\rm in*}$ is also a solution

$$\chi_k^{\rm out} = \alpha_k^*\chi_k^{\rm in} - \beta_k^*\chi_k^{\rm in*}$$
$$(|\alpha_k|^2 - |\beta_k|^2 = 1)$$

$$\left(\begin{array}{lll} \chi_k^{\rm in} & : \; i\partial_0\chi_k^{\rm in} \sim +\omega_k\chi_k^{\rm in} & @\, t = -\infty \\ \chi_k^{\rm out} & : \; i\partial_0\chi_k^{\rm out} \sim +\omega_k\chi_k^{\rm out} & @\, t = +\infty \end{array} \right)$$

Transformation law for operators

$$\chi_k^{\mathrm{out}} = \alpha_k^* \chi_k^{\mathrm{in}} - \beta_k^* \chi_k^{\mathrm{in}*}$$

- $$\chi = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left(\chi_k^{\mathrm{out}} a_{\mathbf{k}}^{\mathrm{out}} + \chi_k^{\mathrm{out}*} a_{-\mathbf{k}}^{\mathrm{out}\dagger} \right)$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left(\chi_k^{\mathrm{in}} \left(\alpha_k^* a_{\mathbf{k}}^{\mathrm{out}} - \beta_k a_{-\mathbf{k}}^{\mathrm{out}\dagger} \right) + \chi_k^{\mathrm{in}*} \left(-\beta_k^* a_{\mathbf{k}}^{\mathrm{out}} + \alpha_k a_{-\mathbf{k}}^{\mathrm{out}\dagger} \right) \right)$$

$= a_{\mathbf{k}}^{\mathrm{in}}$ $= a_{-\mathbf{k}}^{\mathrm{in}\dagger}$

- Transformation law (***Bogoliubov transformation***)

$$\chi_k^{\mathrm{out}} = \alpha_k^* \chi_k^{\mathrm{in}} - \beta_k^* \chi_k^{\mathrm{in}*}, \qquad a_{\mathbf{k}}^{\mathrm{out}} = \alpha_k a_{\mathbf{k}}^{\mathrm{in}} + \beta_k a_{-\mathbf{k}}^{\mathrm{in}\dagger}$$

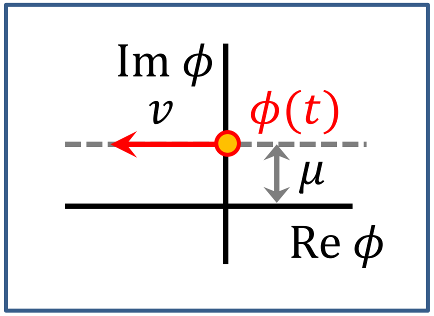
$$\left(|\alpha_k|^2 - |\beta_k|^2 = 1 \right)$$

Produced number (occupation number)

$$n_k \equiv \langle 0^{\mathrm{in}} | a_{\mathbf{k}}^{\mathrm{out}\dagger} a_{\mathbf{k}}^{\mathrm{out}} | 0^{\mathrm{in}} \rangle = V \cdot |\beta_k|^2$$

- With WKB method

$$|\beta_k|^2 \sim \exp\left[2\, \mathrm{Im} \int dt\, \omega(t) \right] = \exp\left[-\pi \frac{k^2 + g^2\mu^2}{gv} \right]$$



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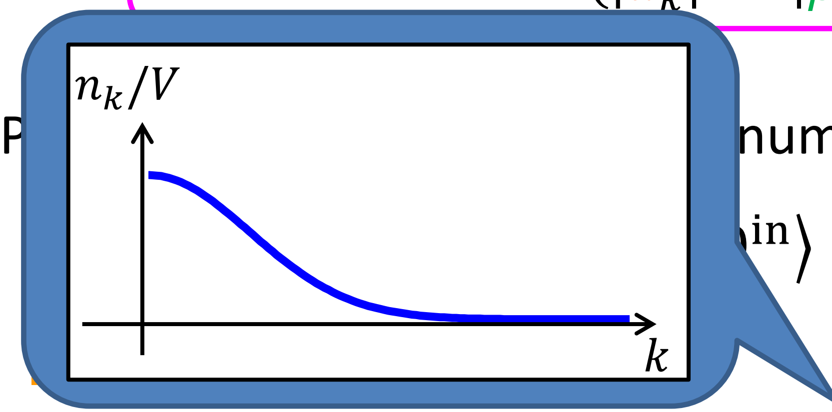
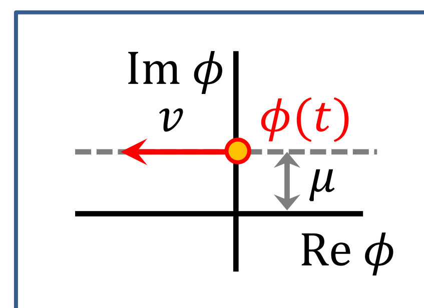


P … number)

$\text{in}\rangle = V \cdot |\beta_k|^2$



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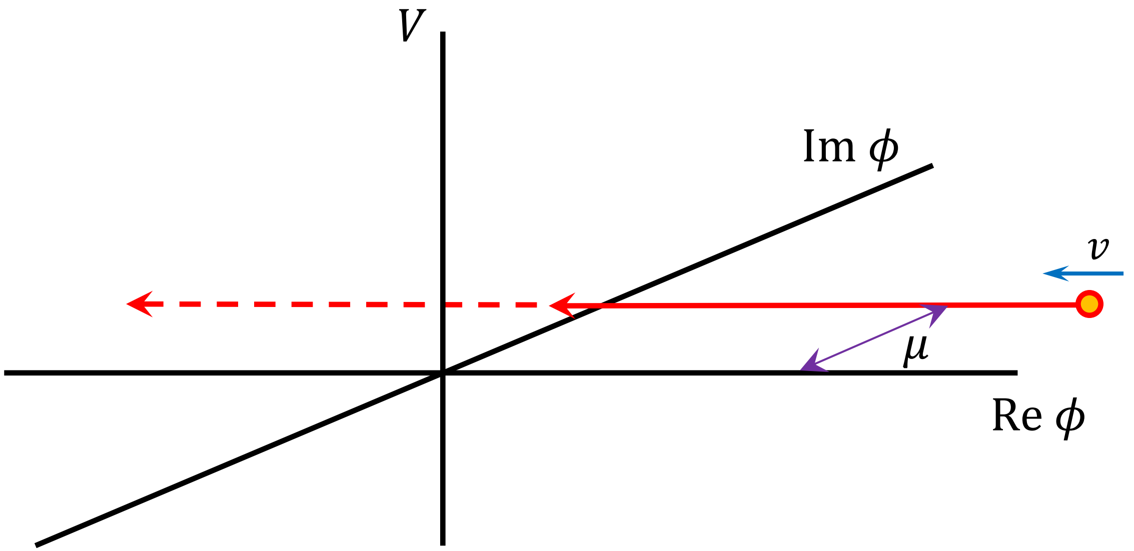
After non-perturbative particle production

$$\mathcal{H} = |\dot{\phi}|^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\nabla\chi)^2 + \frac{1}{2}g^2|\phi|^2\chi^2$$

$$= \rho_\chi \sim n_\chi \cdot g|\phi|$$

Linear potential is established for ϕ!



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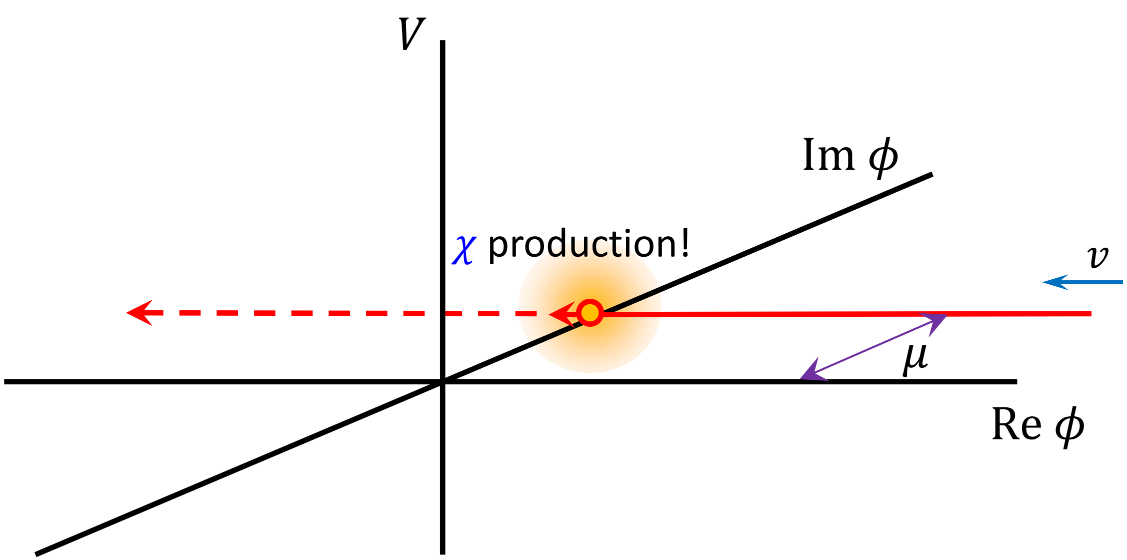

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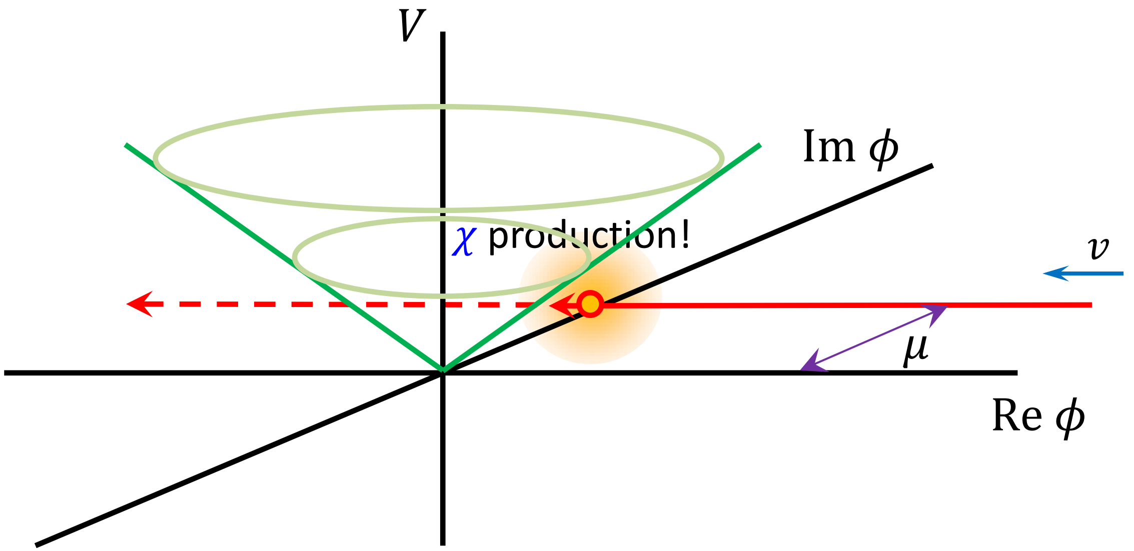

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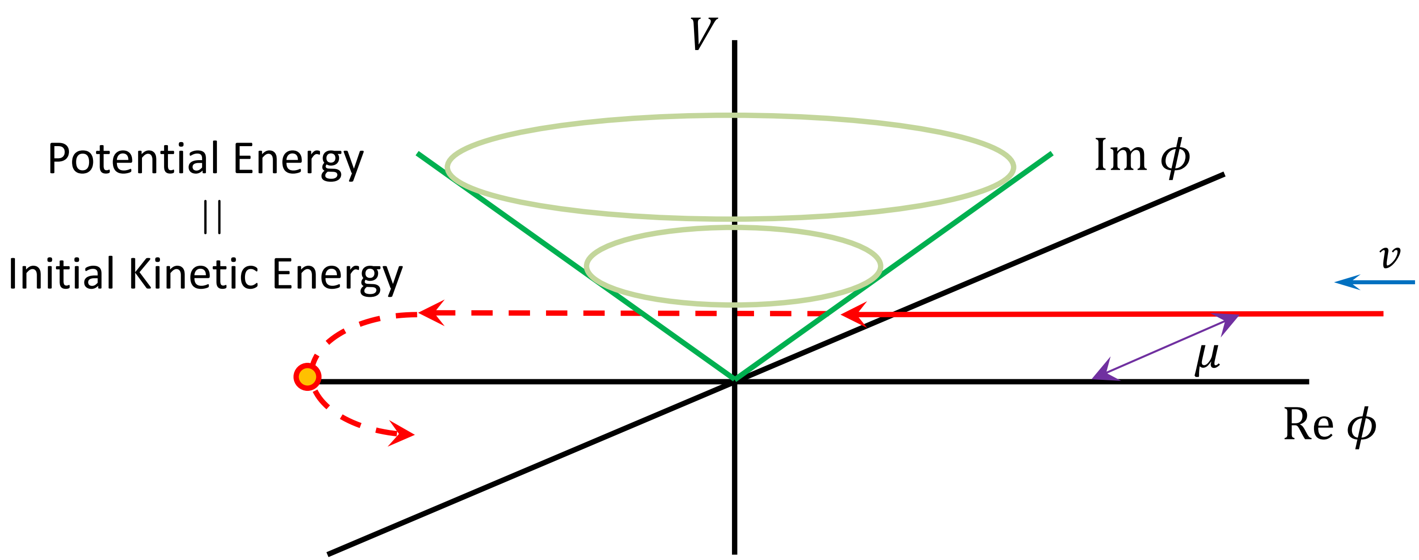

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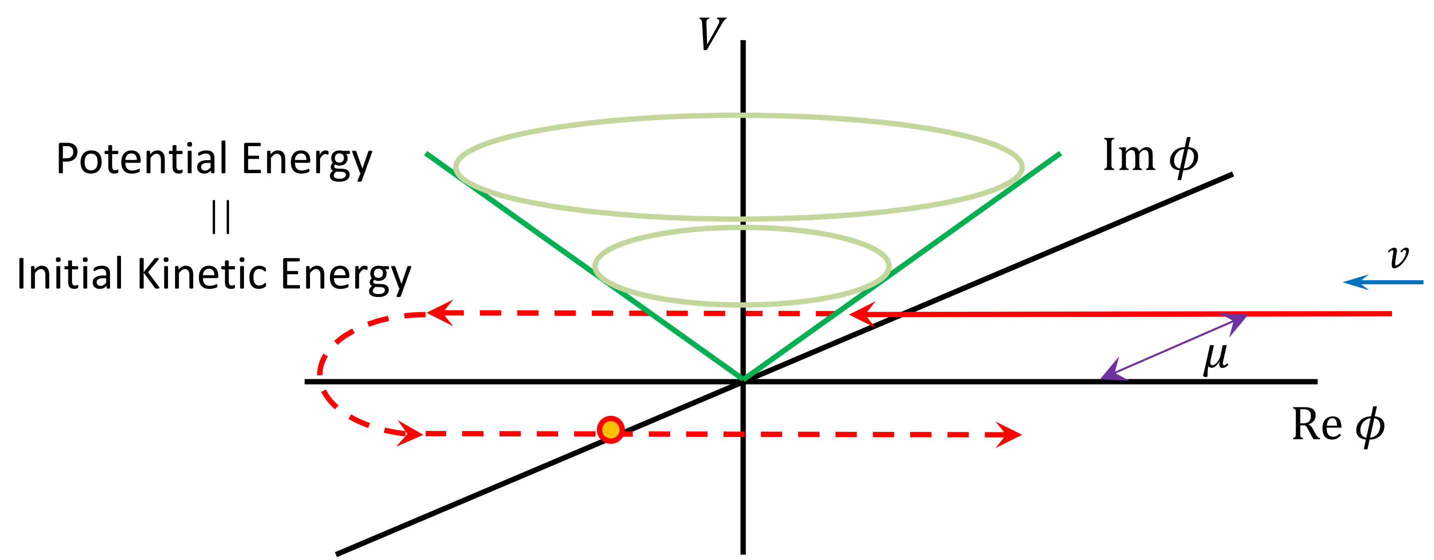

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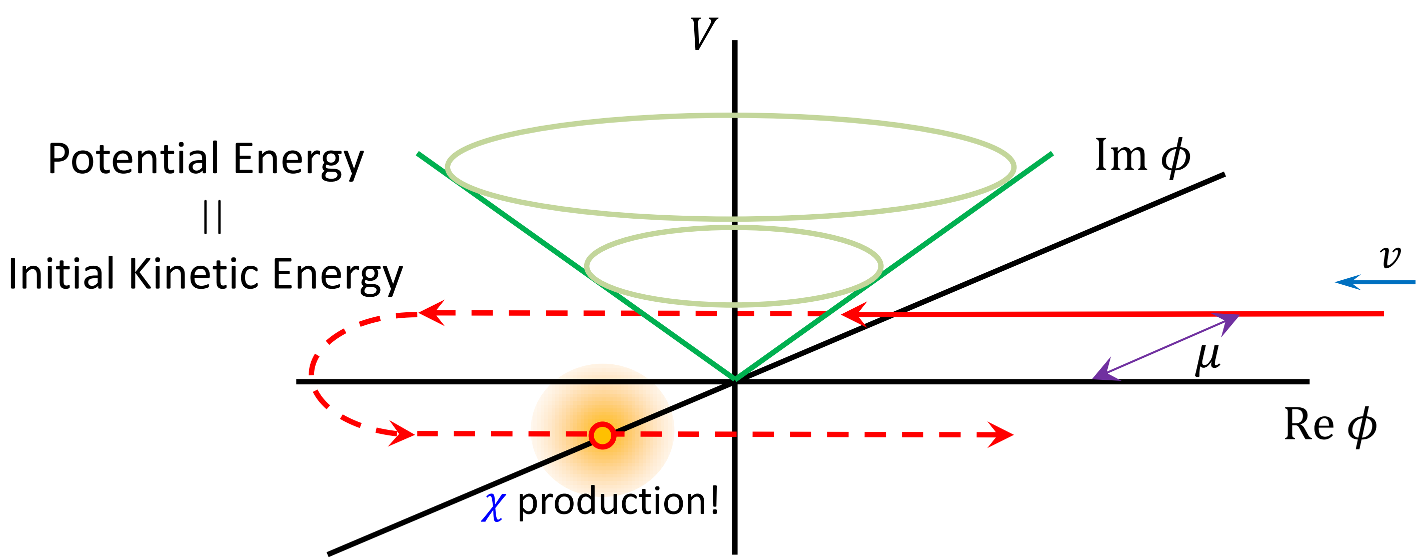

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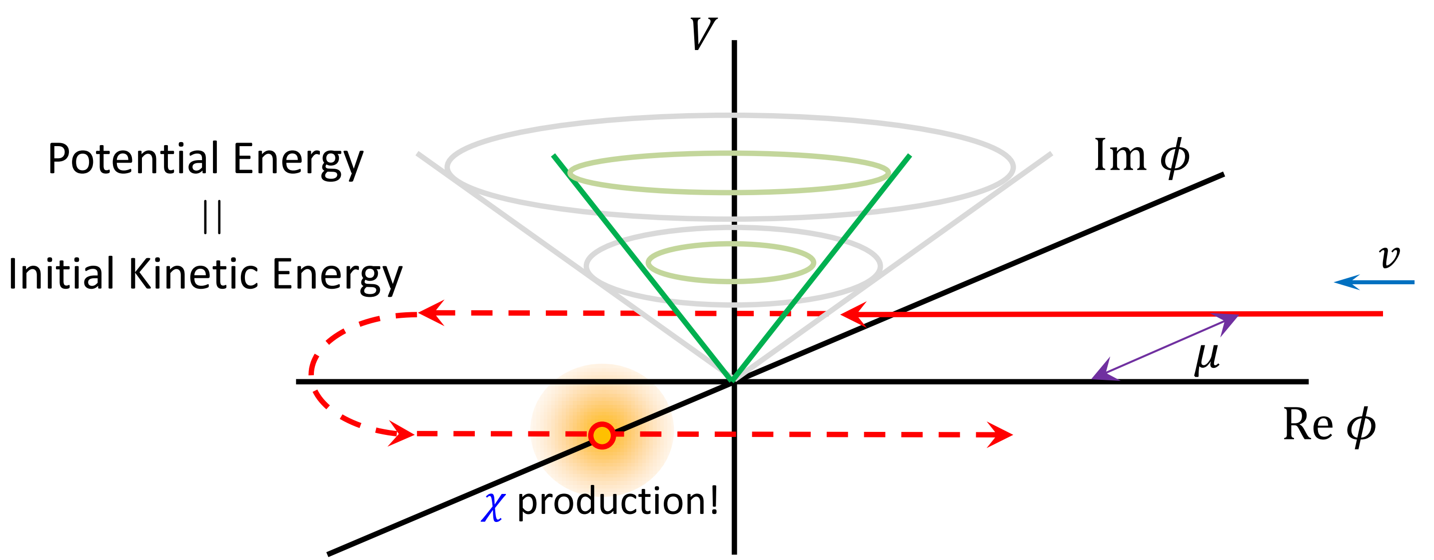

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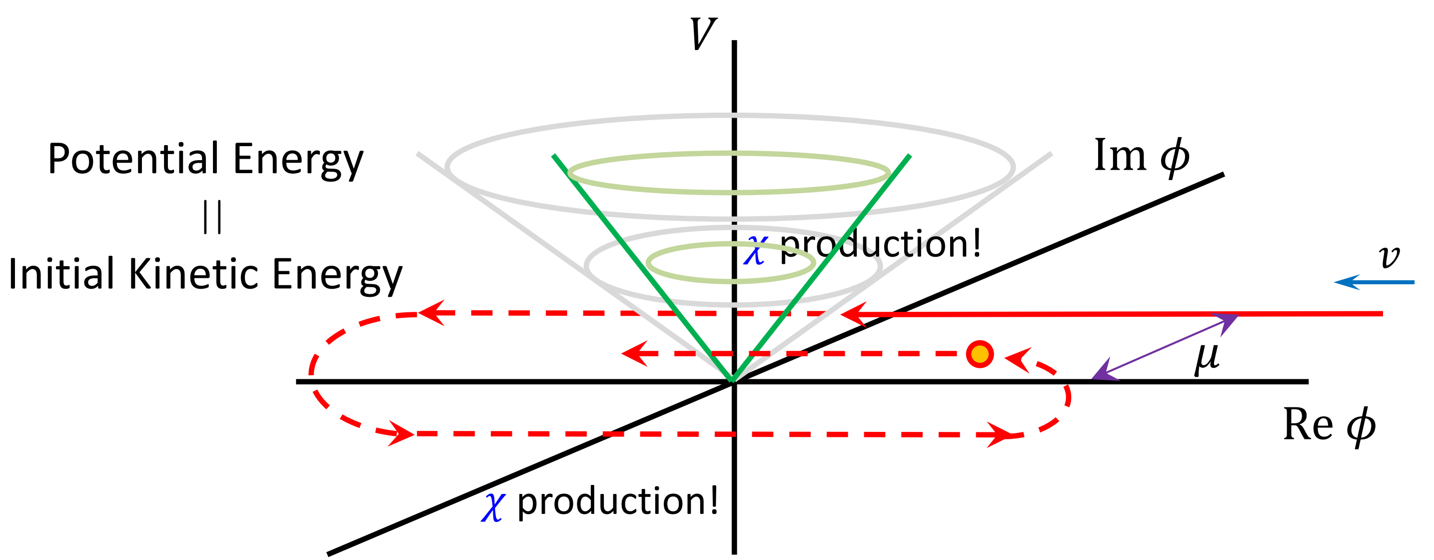

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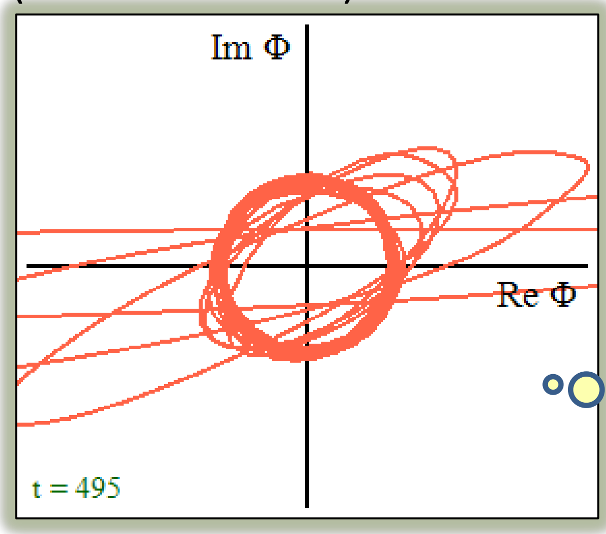
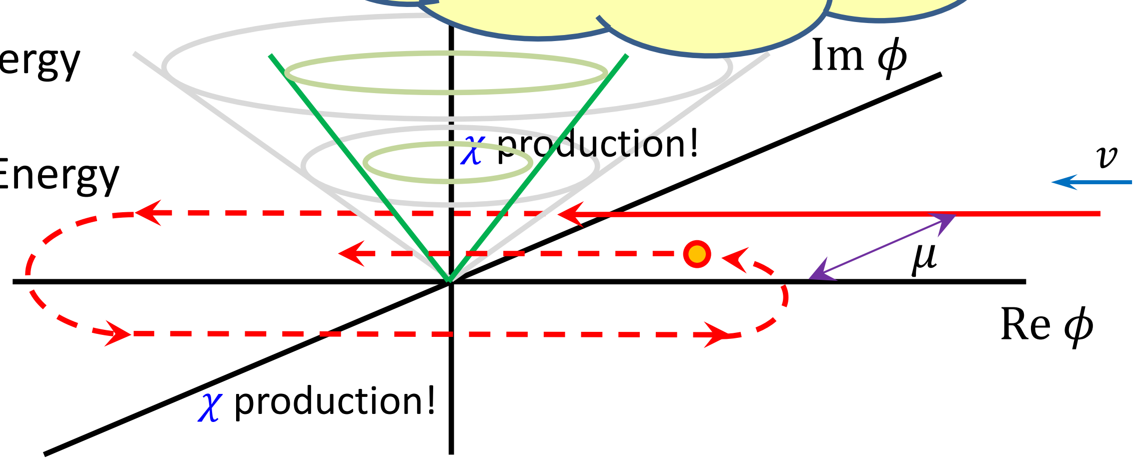
(numerical result)



➡ Linear potential is established for ϕ!

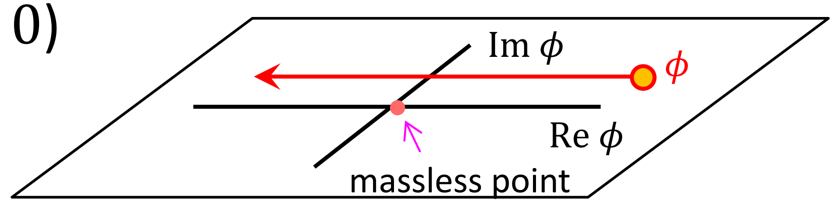
ϕ is trapped around the massless point.

Potential Energy
||
Initial Kinetic Energy



★ Brief summary of introduction

1. Variable mass (approaching to $\phi \sim 0$)



2. Particle production (χ)

- produced (occupation) number :

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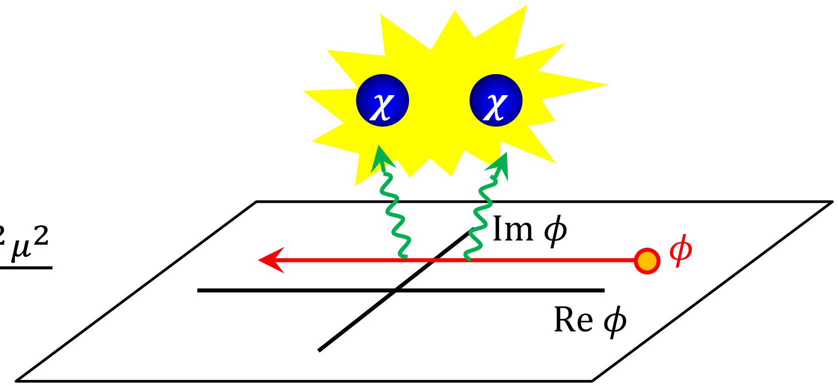
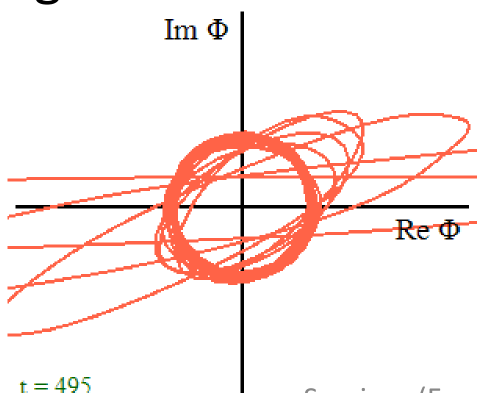
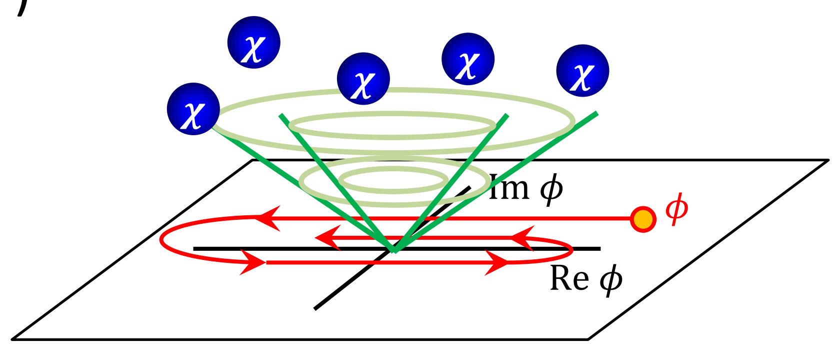
$$= V \cdot |\beta_k|^2 \sim V \cdot e^{-\pi \frac{k^2 + g^2 \mu^2}{g v}}$$



3. Trapping around massless point (ϕ)





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- How do (quantum) interaction terms affect particle production?
  - Usually production rates are calculated in the purely classical background

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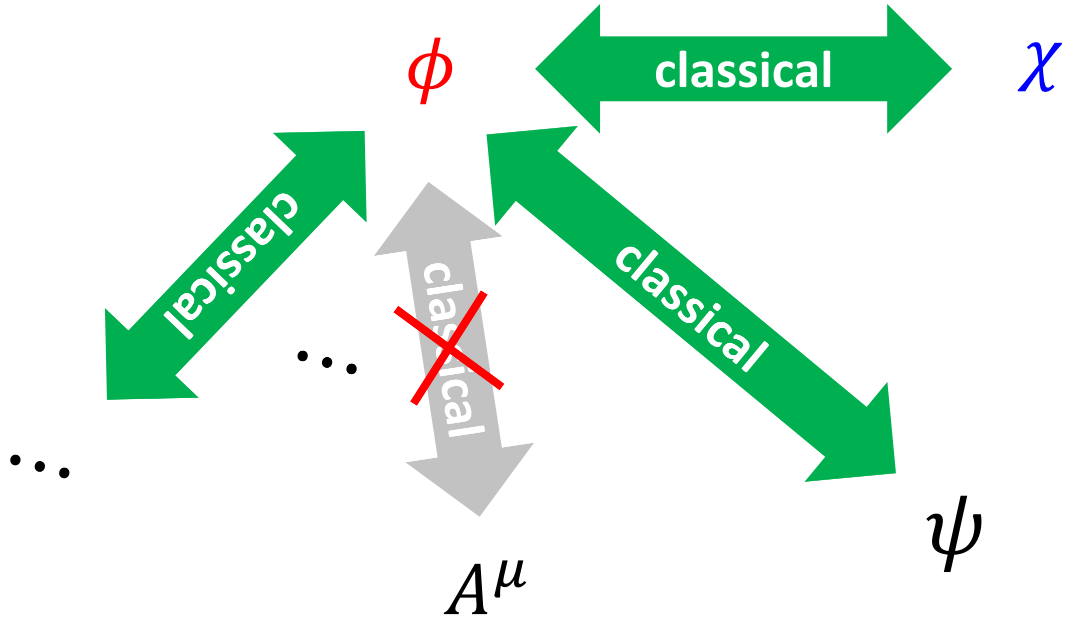

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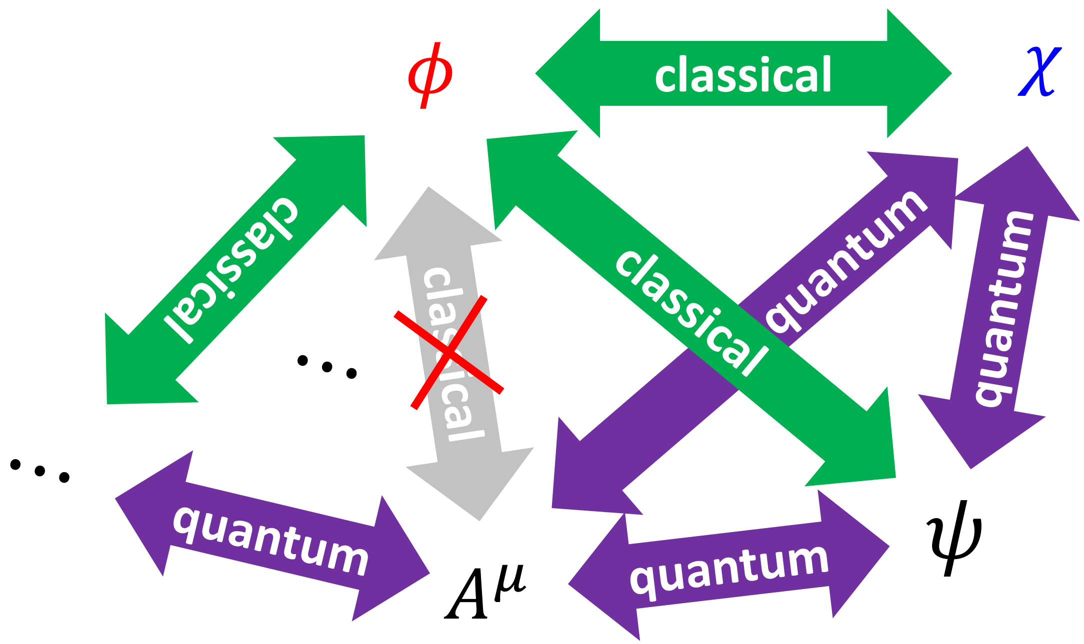


In this talk we consider a simple SUSY model

Superpotential :

$$W = \frac{1}{2} g \Phi X^2$$

$\Phi = \phi + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi$

$X = \chi + \sqrt{2}\theta\psi_\chi + \theta^2 F_\chi$

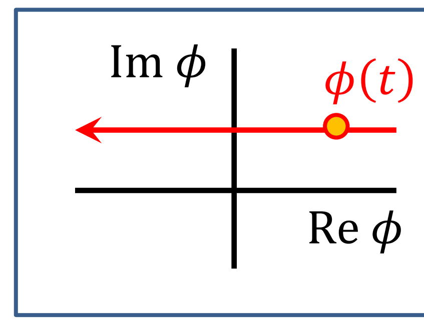
g : coupling

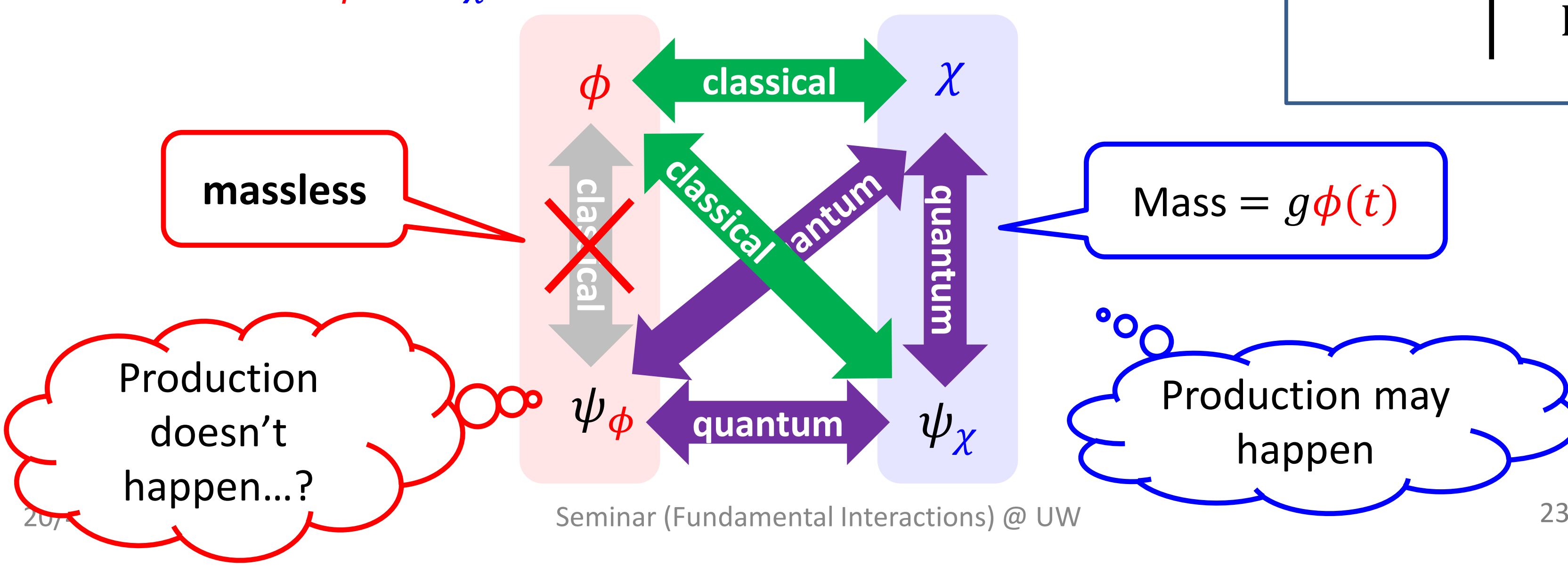
→ Interaction terms in components :

$$\mathcal{L}_{int} = -g^2|\phi|^2|\chi|^2 - \frac{1}{4}g^2|\chi|^4 - g\left(\frac{1}{2}\phi\psi_\chi\psi_\chi + \psi_\phi\psi_\chi\chi + (h.c.)\right)$$

Stationary point :

- $\chi = \psi_\phi = \psi_\chi = 0$, but ϕ can have ***any*** value





2. Bogoliubov transformation law with interaction terms

Our aim:

$\phi \quad \psi_\phi \quad \chi \quad \psi_\chi$

Evaluation of produced particle number for all species

$$n_k(t^{\text{out}}) \equiv \langle 0^{\text{in}} | a_{\mathbf{k}}^{\text{out}\dagger} a_{\mathbf{k}}^{\text{out}} | 0^{\text{in}} \rangle$$

→ To find the Bogoliubov transformation law in the interacting theory

$$a_{\mathbf{k}}^{\text{out}} \longleftrightarrow a_{\mathbf{k}}^{\text{in}}$$

Method

- Schwinger-Keldysh formalism
- **Yang-Feldman formalism** ⬅
- Others (but I don't know…)

An example with a real scalar field Ψ (1/4)

mass, c-number

source, operator

- Operator field equation : $0 = (\partial^2 + M^2(x))\Psi(x) + J(x)$
- Commutation relation : $[\Psi(\mathbf{x}), \dot{\Psi}(\mathbf{y})] = i\delta^3(\mathbf{x}-\mathbf{y})$

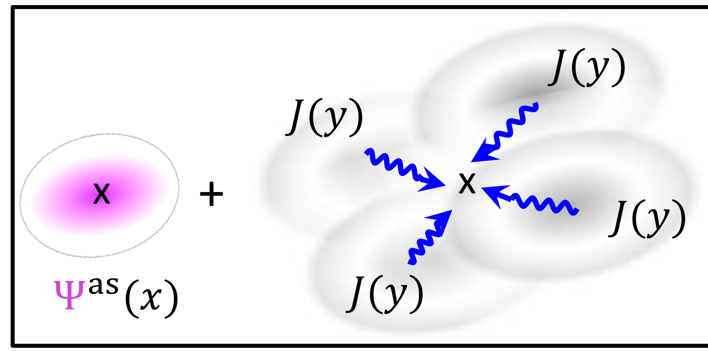
→ Formal solution (***Yang-Feldman equations***)

$$\Psi(x) = \sqrt{Z}\Psi^{\mathrm{as}}(x) - iZ\int_{y^0=t^{\mathrm{as}}}^{y^0=x^0} d^4y\,[\Psi^{\mathrm{as}}(x), \Psi^{\mathrm{as}}(y)]\,J(y)$$

Z : const.

Ψ^{as} : asymptotic field
$0 = (\partial^2 + M^2)\Psi^{\mathrm{as}}$

- $x^0 = t^{\mathrm{as}} \implies \Psi(x^{\mathrm{as}}) = \sqrt{Z}\Psi^{\mathrm{as}}(x^{\mathrm{as}})$



- If we take $t^{\mathrm{as}} = t^{\mathrm{in}} = -\infty$ or $t^{\mathrm{as}} = t^{\mathrm{out}} = +\infty$,

$$\Psi^{\mathrm{out}}(x^{\mathrm{out}}) = \Psi^{\mathrm{in}}(x^{\mathrm{out}}) - i\sqrt{Z}\int d^4y\,[\Psi^{\mathrm{in}}(x^{\mathrm{out}}), \Psi^{\mathrm{in}}(y)]\,J(y)$$

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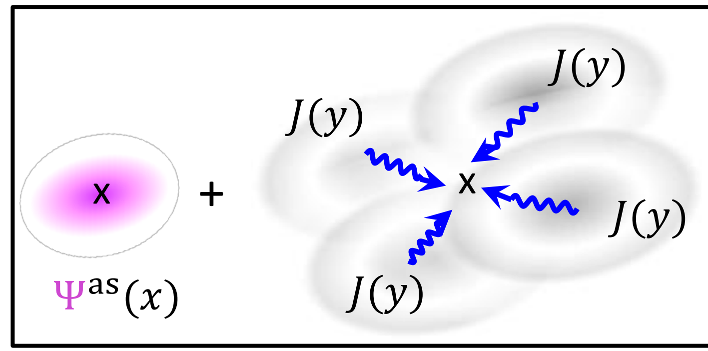
$\Psi^{\rm as}$: asymptotic field

$0 = (\partial^2 \ldots \Psi^{\rm as}$

$a_{\mathbf{k}}^{\rm out}$

$a_{\mathbf{k}}^{\rm in}$



- If we take $t^{\rm as} = t^{\rm in} = -\infty$ or $t^{\rm as} = t^{\rm out} = +\infty$,

$$\Psi^{\rm out}(x^{\rm out}) = \Psi^{\rm in}(x^{\rm out}) - i\sqrt{Z}\int d^4y\left[\Psi^{\rm in}(x^{\rm out}), \Psi^{\rm in}(y)\right]J(y)$$

An example with a real scalar field Ψ (2/4)

- Ψ^{as} is free particle, so we can expand with plane waves as

$$\Psi^{\text{as}}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\Psi_k^{\text{as}}(x^0) a_{\mathbf{k}}^{\text{as}} + \Psi_k^{\text{as}*}(x^0) a_{-\mathbf{k}}^{\text{as}\dagger} \right)$$

plane wave

wave func.
$0 = \partial_0^2 \Psi_k^{\text{as}} + (\mathbf{k}^2 + M^2) \Psi_k^{\text{as}}$

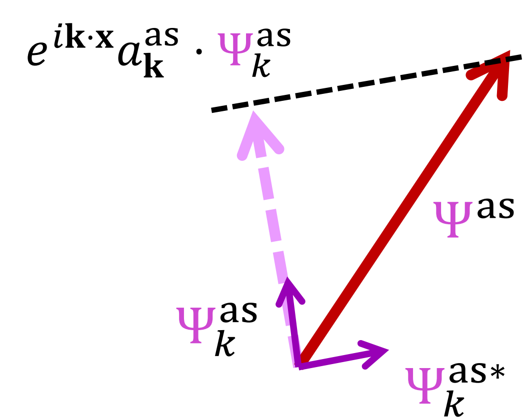
creation/annihilation op.

- inner product relation : $(\Psi_k^{\text{as}}, \Psi_k^{\text{as}}) = 1/Z$

which comes from conditions

$$[a_{\mathbf{k}}^{\text{as}},\ a_{\mathbf{k}'}^{\text{as}\dagger}] = (2\pi)^3 \delta^3(\mathbf{k}-\mathbf{k}') \ ,\ Z\left[\Psi^{\text{as}}(\mathbf{x}), \dot{\Psi}^{\text{as}}(\mathbf{y})\right]_{t\to t^{\text{as}}} = i\hbar\delta^3(\mathbf{x}-\mathbf{y})$$

$$a_{\mathbf{k}}^{\text{as}} = Z \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\Psi_k^{\text{as}}, \Psi^{\text{as}} \right)$$



- a projection from Ψ^{as} to Ψ_k^{as} direction

An example with a real scalar field Ψ (3/4)

Relation between $a_{\mathbf{k}}^{\mathrm{in}}$ and $a_{\mathbf{k}}^{\mathrm{out}}$

$$a_{\mathbf{k}}^{\mathrm{out}} = Z \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\Psi_k^{\mathrm{out}}, \Psi^{\mathrm{out}}\right)$$

$$\Psi^{\mathrm{out}}(x^{\mathrm{out}}) = \Psi^{\mathrm{in}}(x^{\mathrm{out}}) - i\sqrt{Z} \int d^4y \left[\Psi^{\mathrm{in}}(x^{\mathrm{out}}), \Psi^{\mathrm{in},*}(y)\right] J(y)$$

$$a_{\mathbf{k}}^{\mathrm{out}} = \alpha_k a_{\mathbf{k}}^{\mathrm{in}} + \beta_k a_{-\mathbf{k}}^{\mathrm{in}\dagger} - i\sqrt{Z} \int d^4x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\alpha_k \Psi_k^{\mathrm{in}*} - \beta_k \Psi_k^{\mathrm{in}}\right) J(y)$$

(usual) Bogoliubov tfn law

Interaction effects

$$\alpha_k \equiv Z\left(\Psi_k^{\mathrm{out}}, \Psi_k^{\mathrm{in}}\right)$$

$$\beta_k \equiv Z\left(\Psi_k^{\mathrm{out}}, \Psi_k^{\mathrm{in}*}\right)$$

$\Rightarrow$

$$\Psi_k^{\mathrm{in}} = \alpha_k \Psi_k^{\mathrm{out}} + \beta_k \Psi_k^{\mathrm{out}*}$$

$$\Psi_k^{\mathrm{out}} = \alpha_k^* \Psi_k^{\mathrm{in}} - \beta_k^* \Psi_k^{\mathrm{in}*}$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

An example with a real scalar field Ψ (4/4)

Produced (occupation) number :

$$n_k = \langle 0^{\rm in} | a_{\mathbf{k}}^{{\rm out}\dagger} a_{\mathbf{k}}^{\rm out} | 0^{\rm in} \rangle$$

$$= \left| \left(\beta_k a_{-\mathbf{k}}^{{\rm in}\dagger} - i\sqrt{Z} \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\alpha_k \Psi_k^{{\rm in}*} - \beta_k \Psi_k^{\rm in} \right) J \right) | 0^{\rm in} \rangle \right|^2$$

$$= \begin{cases} V \cdot |\beta_k|^2 \; + \; \cdots & (\beta_k \neq 0) \\ 0 \; + Z \left| \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \, \Psi_k^{{\rm in}*} J \, | 0^{\rm in} \rangle \right|^2 & (\beta_k = 0) \end{cases}$$

→ Particles can be produced <u>even if $\beta_k = 0$!</u>

In case of a fermionic field

Inner product becomes

$$(A, B) \equiv i\left(A^\dagger \dot{B} - \dot{A}^\dagger B\right) \quad \Longrightarrow \quad (A, B)_F \equiv \bar{A}\gamma^0 B$$ (A, B : 4-component)

In case of Majorana fermions, the formulae are more complicated

★ Brief summary of this section

1. Bogoliubov transformation law with interaction effects

$$a_{\mathbf{k}}^{\text{out}} = \alpha_k a_{\mathbf{k}}^{\text{in}} + \beta_k a_{-\mathbf{k}}^{\text{in}\dagger} - i\sqrt{Z}\int d^4x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\left(\alpha_k \Psi_k^{\text{in}*} - \beta_k \Psi_k^{\text{in}}\right) J(y)$$

(usual) Bogoliubov tfn law

Interaction effects

- Wave functions' law keeps ordinary form

$$\Psi_k^{\text{out}} = \alpha_k^* \Psi_k^{\text{in}} - \beta_k^* \Psi_k^{\text{in}*}$$

2. The particle production can happen even if $\beta_k = 0$

3. Application to our model

Model (again)

$$\mathcal{L}_{int} = -g^2|\phi|^2|\chi|^2 - \frac{1}{4}g^2|\chi|^4 - g\left(\frac{1}{2}\phi\psi_\chi\psi_\chi + \psi_\phi\psi_\chi\chi + (h.c.)\right)$$

Equation of Motion :

$$\phi \ : \quad 0 = (\partial^2 + \underline{g^2|\chi|^2})\phi + \underline{\frac{1}{2}g\psi_\chi^\dagger\psi_\chi^\dagger}$$

$$\chi \ : \quad 0 = \left(\partial^2 + g^2|\phi|^2 + \underline{\frac{1}{2}g^2|\chi|^2}\right)\chi + \underline{g\psi_\phi^\dagger\psi_\chi^\dagger}$$

$$\psi_\phi : \quad 0 = \bar{\sigma}^\mu\partial_\mu\psi_\phi + \underline{ig\chi^\dagger\psi_\chi^\dagger}$$

$$\psi_\chi : \quad 0 = \bar{\sigma}^\mu\partial_\mu\psi_\chi + ig\phi^\dagger\psi_\chi^\dagger + \underline{ig\chi^\dagger\psi_\phi^\dagger}$$

ϕ has a backgound

χ , ψ_ϕ, ψ_χ : no background

$\underline{***}$ ⇨ source terms

EOM for asymptotic fields (as = in, out):

$$\phi^{\text{as}} \ : \ 0 = \partial^2\phi^{\text{as}}$$

$$\chi^{\text{as}} \ : \ 0 = (\partial^2 + g^2|\langle\phi\rangle|^2)\chi^{\text{as}}$$

$$\psi_\phi^{\text{as}} \ : \ 0 = \bar{\sigma}^\mu\partial_\mu\psi_\phi^{\text{as}}$$

$$\psi_\chi^{\text{as}} \ : \ 0 = \bar{\sigma}^\mu\partial_\mu\psi_\chi^{\text{as}} + ig\langle\phi^\dagger\rangle\psi_\chi^{\text{as}\dagger}$$

$$\left[\ \langle\phi\rangle \equiv \langle 0^{\text{in}}|\phi|0^{\text{in}}\rangle\ \right]$$

Definition of wave functions (Assuming $\langle\phi\rangle = \langle\phi(t)\rangle$)

- $$\phi^{\rm as} = \langle\phi^{\rm as}\rangle + \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\phi_k^{\rm as} a_{\phi\mathbf{k}}^{+,\rm as} + \phi_k^{\rm as*} a_{\phi-\mathbf{k}}^{-,\rm as\dagger}\right)$$

$$0 = \ddot{\phi}_k^{\rm as} + \mathbf{k}^2 \phi_k^{\rm as} \rightarrow \phi_k^{\rm in} = \phi_k^{\rm out} \propto e^{-ikt} \rightarrow \left|\beta_{\phi k}\right|^2 = 0$$

- $$\chi^{\rm as} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\chi_k^{\rm as} a_{\chi\mathbf{k}}^{\rm as} + \chi_k^{\rm as*} b_{\chi-\mathbf{k}}^{\rm as\dagger}\right)$$

$$0 = \ddot{\chi}_k^{\rm as} + (\mathbf{k}^2 + g^2|\langle\phi\rangle|^2)\chi_k^{\rm as} \rightarrow \left|\beta_{\chi k}\right|^2 \equiv \left|Z_\chi\left(\chi_k^{\rm out}, \chi_k^{\rm in*}\right)\right|^2 \neq 0$$

- $$\psi_\phi^{\rm as} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(e_\mathbf{k}^+ \psi_{\phi k}^{+,\rm as} a_{\psi_\phi\mathbf{k}}^{+\rm as} + e_\mathbf{k}^- \psi_{\phi k}^{-,\rm as*} a_{\psi_\phi-\mathbf{k}}^{-\rm as\dagger}\right)$$

↑ (* eigen vector for helicity)

$$0 = \dot{\psi}_{\phi k}^{\rm as} + i|\mathbf{k}|\psi_{\phi k}^{\rm as} \rightarrow \psi_{\phi k}^{\rm in} = \psi_{\phi k}^{\rm out} \propto e^{-ikt} \rightarrow \left|\beta_{\psi_\phi k}\right|^2 = 0$$

↓ (* phase)

- $$\psi_\chi^{\rm as} = \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e_\mathbf{k}^s \left(\psi_{\chi k}^{(+)s,\rm as} a_{\psi_\chi\mathbf{k}}^s - s e^{-i\theta_\mathbf{k}} \psi_{\chi k}^{(-)s,\rm as*} a_{\psi_\chi-\mathbf{k}}^{s\dagger}\right)$$

$$0 = \dot{\psi}_{\chi k}^{(+)s,\rm as} + is|\mathbf{k}|\psi_{\chi k}^{(+)s,\rm as} + ig\langle\phi^\dagger\rangle\psi_{\chi k}^{(-)s,\rm as}$$

$$0 = \dot{\psi}_{\chi k}^{(-)s,\rm as} - is|\mathbf{k}|\psi_{\chi k}^{(-)s,\rm as} + ig\langle\phi\rangle\psi_{\chi k}^{(+)s,\rm as}$$

$$\rightarrow \left|\beta_{\psi_\chi\mathbf{k}}^s\right|^2 \equiv \left|Z_{\psi_\chi}\left(\psi_{\chi k}^{(+)s,\rm out}\psi_{\chi k}^{(-)s,\rm in} - \psi_{\chi k}^{(-)s,\rm out}\psi_{\chi k}^{(+)s,\rm in}\right)\right|^2 \neq 0$$

Analytical results (massive particles χ, ψ_χ; $\beta_k \neq 0$)

- $n_{\chi k}/V \equiv \left\langle 0^{\text{in}} \middle| a^{+,\text{out}\dagger}_{\chi\mathbf{k}} a^{+,\text{out}}_{\chi\mathbf{k}} \middle| 0^{\text{in}} \right\rangle / V + \left\langle 0^{\text{in}} \middle| a^{-,\text{out}\dagger}_{\chi\mathbf{k}} a^{-,\text{out}}_{\chi\mathbf{k}} \middle| 0^{\text{in}} \right\rangle / V$

$$= 2\left|\beta_{\chi k}\right|^2 + \cdots$$

$$= 2\exp\left[2\,\mathrm{Im}\int dt\,\sqrt{k^2 + g^2|\langle\phi\rangle|^2}\right] + \cdots$$

$$= 2\exp\left[-\pi\frac{k^2+g^2\mu^2}{gv}\right] + \cdots$$

- $n_{\psi_\chi k}/V \equiv \sum_s \left\langle 0^{\text{in}} \middle| a^{s,\text{out}\dagger}_{\psi_\chi\mathbf{k}} a^{s,\text{out}}_{\psi_\chi\mathbf{k}} \middle| 0^{\text{in}} \right\rangle / V$

$$= \sum_s \left|\beta^s_{\psi_\chi k}\right|^2 + \cdots$$

$$= 2\exp\left[-\pi\frac{k^2+g^2\mu^2}{gv}\right] + \cdots$$

Analytical results (massless particle $\tilde{\phi}$; $\beta_{\phi k} = 0$)

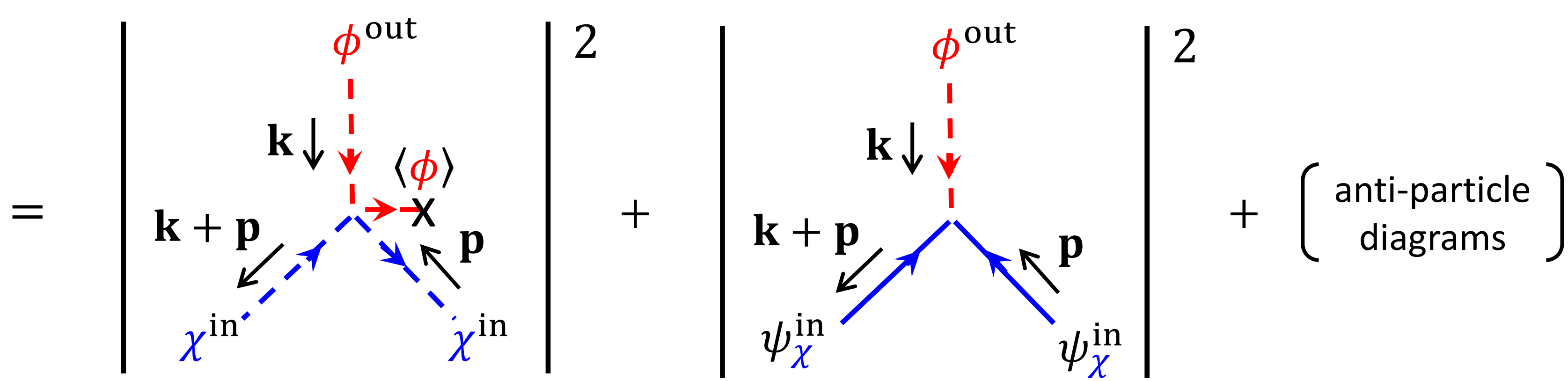
$$n_{\phi k}/V \equiv \left\langle 0^{\rm in} \middle| \tilde{a}^{+,{\rm out}\dagger}_{\phi\mathbf{k}} \tilde{a}^{+,{\rm out}}_{\phi\mathbf{k}} \middle| 0^{\rm in} \right\rangle / V + \left\langle 0^{\rm in} \middle| \tilde{a}^{-,{\rm out}\dagger}_{\phi\mathbf{k}} \tilde{a}^{-,{\rm out}}_{\phi\mathbf{k}} \middle| 0^{\rm in} \right\rangle / V$$

$$\left(\ \tilde{a} \equiv a - \langle 0^{\rm in} | a | 0^{\rm in} \rangle \ \right)$$

$$\vdots$$

$$= g^2 \int \frac{d^3p}{(2\pi)^3} \Bigg[Z_\phi Z_\chi^2 \left(\left| \int dt \, \phi_k^{\rm out} \, \chi_{|\mathbf{k}+\mathbf{p}|}^{\rm in} \, \chi_p^{\rm in} \cdot g \langle \phi \rangle \right|^2 + \left| \int dt \, \phi_k^{\rm out} \, \chi_{|\mathbf{k}+\mathbf{p}|}^{\rm in} \, \chi_p^{\rm in} \cdot g \langle \phi^\dagger \rangle \right|^2 \right)$$

$$+ \frac{1}{4} Z_\phi Z_{\psi_\chi}^2 \sum_{s,r,q} \left(1 + sr \frac{\mathbf{p}\cdot(\mathbf{k}+\mathbf{p})}{p|\mathbf{k}+\mathbf{p}|} \right) \left| \int dt \, \phi_k^{\rm out} \, \psi_{\chi|\mathbf{k}+\mathbf{p}|}^{(q)s,{\rm in}} \, \psi_{\chi p}^{(q)r,{\rm in}} \right|^2 \Bigg] + \cdots$$



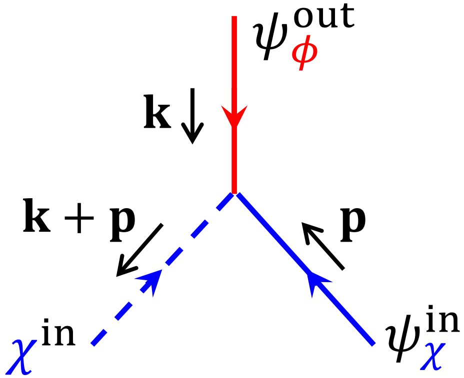
Analytical results (massless particle ψ_ϕ; $\beta_{\psi_\phi k} = 0$)

$$n_{\psi_\phi k}/V \equiv \sum_s \left\langle 0^{\text{in}} \middle| a^{s,\text{out}\dagger}_{\psi_\phi \mathbf{k}} a^{s,\text{out}}_{\psi_\phi \mathbf{k}} \middle| 0^{\text{in}} \right\rangle / V$$

$$n_k \sim Z \left| \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \, \Psi^{\text{in}*}_k \, J \, |0^{\text{in}}\rangle \right|^2$$

$$\vdots$$

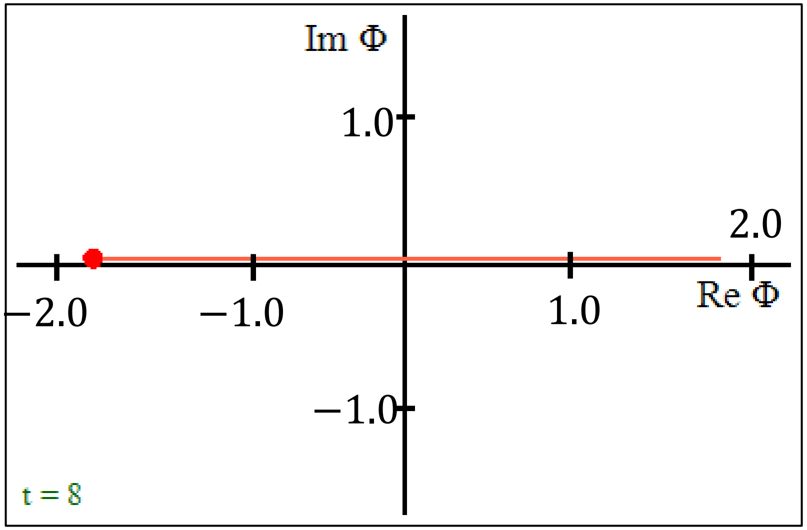
$$= g^2 \int \frac{d^3p}{(2\pi)^3} Z_\chi Z_{\psi_\phi} Z_{\psi_\chi} \sum_{s,r} \frac{1}{2}\left(1 - sr\frac{\mathbf{p}\cdot\mathbf{k}}{pk}\right) \times \left| \int dt \, \psi^{\text{out}}_{\phi k} \, \chi^{\text{in}}_{|\mathbf{k}+\mathbf{p}|} \, \psi^{(s)r,\text{in}}_{\chi p} \right|^2 + \cdots$$

$\sim \Big|$  $\Big|^2$ $+$ (anti-particle diagram)

★ The main channel of massless particles production is "inverse decay"

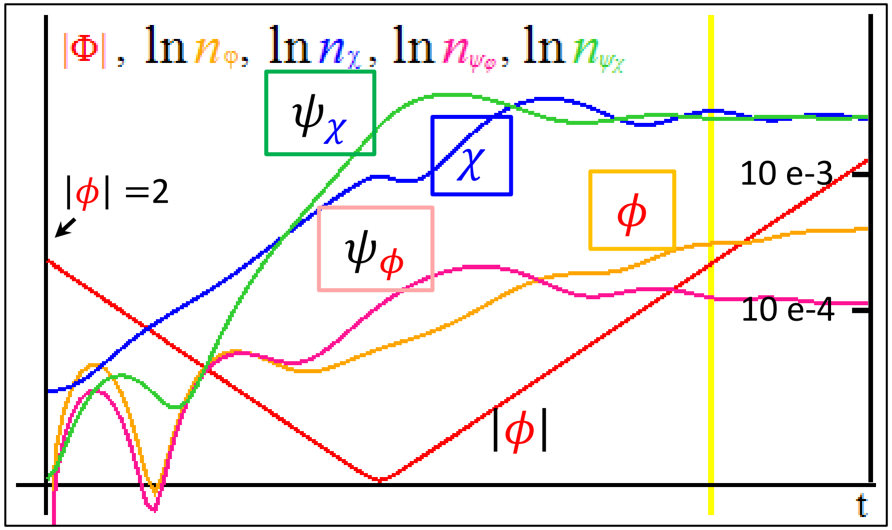
Numerical results : case 1 ($g = 1,\ v = 0.5,\ \mu = 0.05$)

Trajectory:

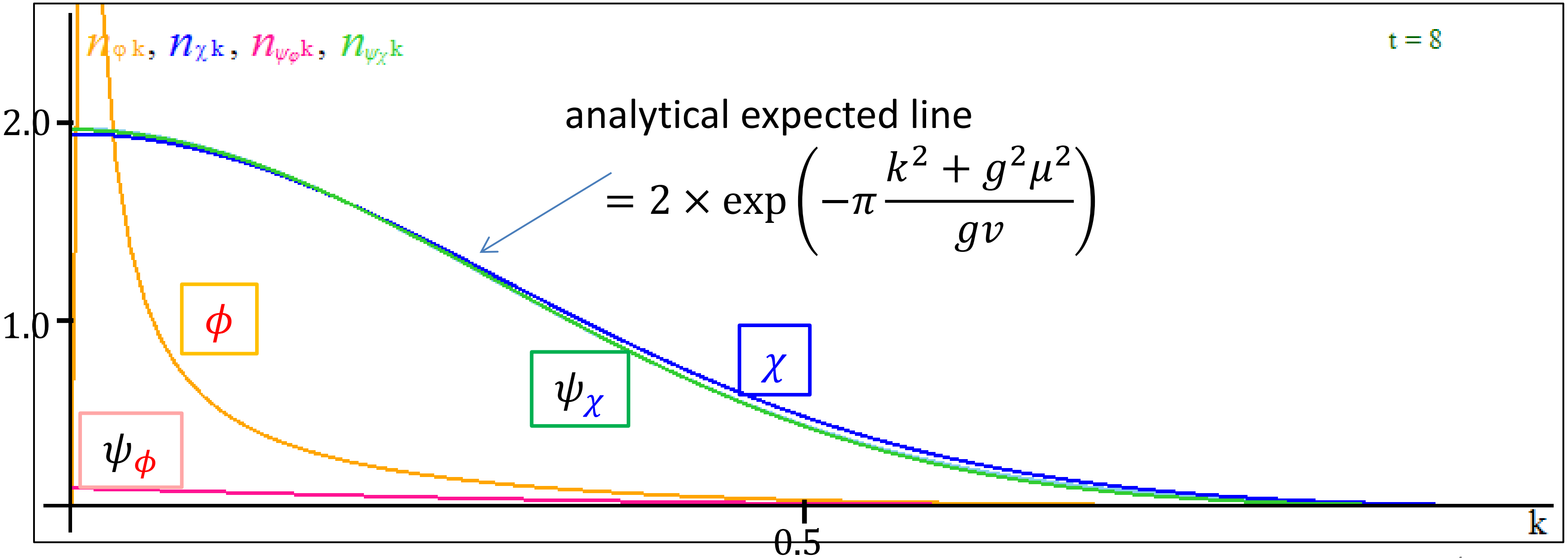


Background

Number densities



Distributions

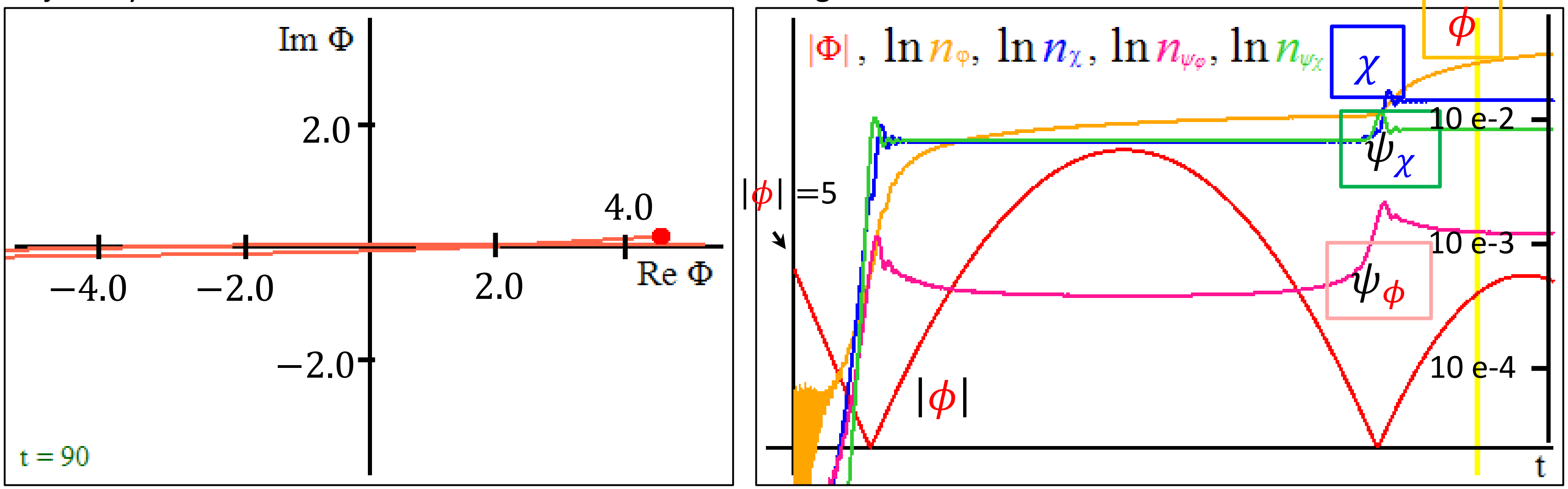


Numerical results : case 2 ($g = 2,\ v = 0.5,\ \mu = 0.05$)

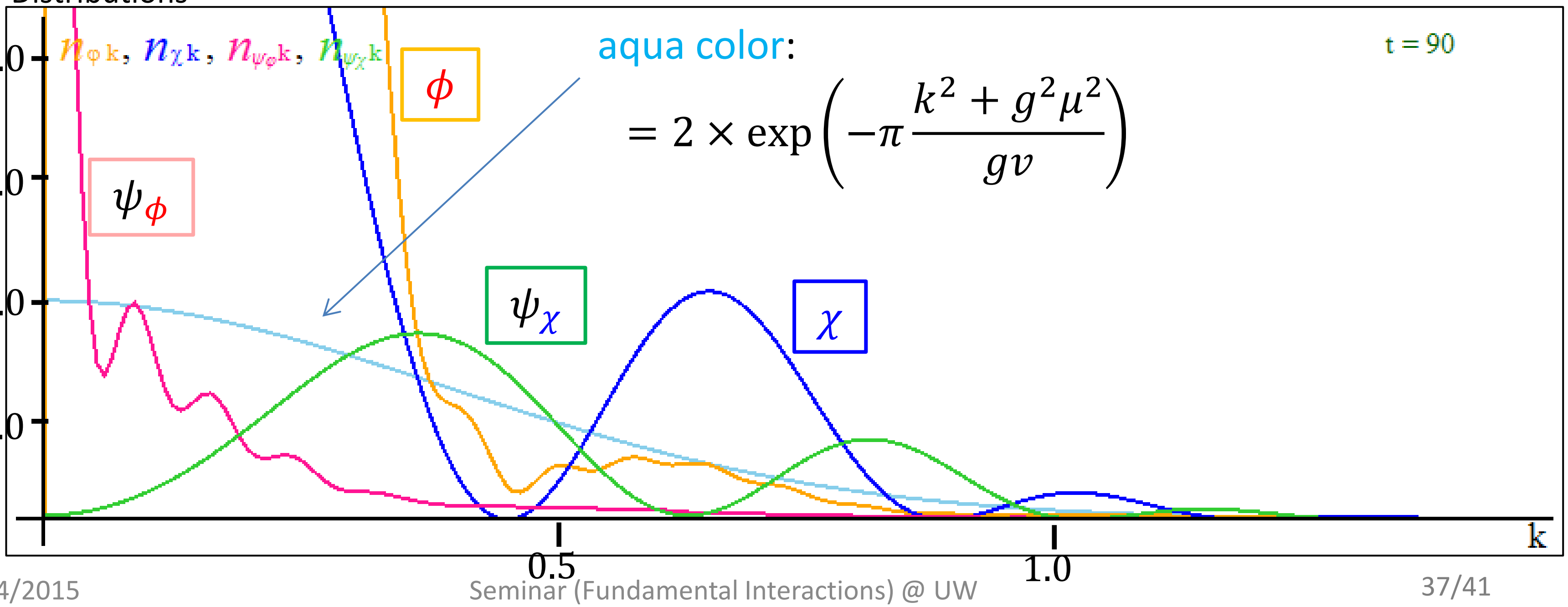
Trajectory:

Background

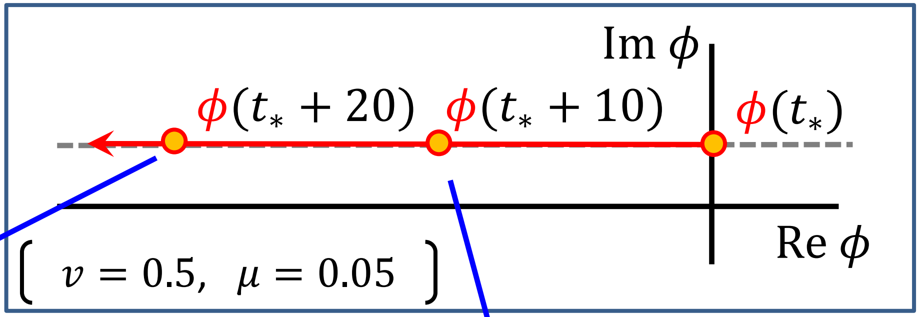
Number densities

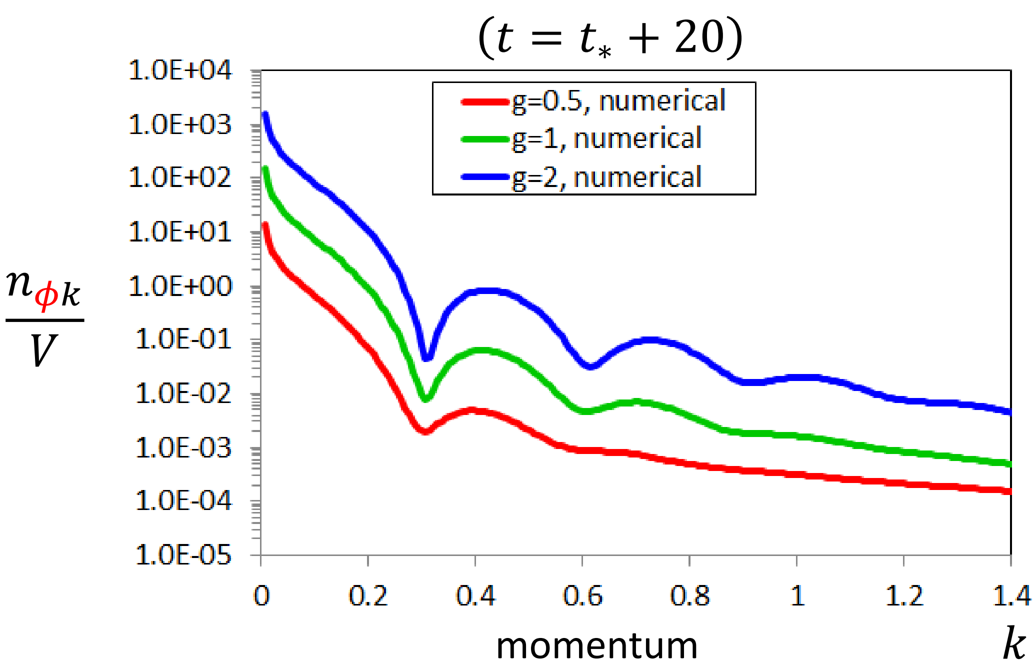


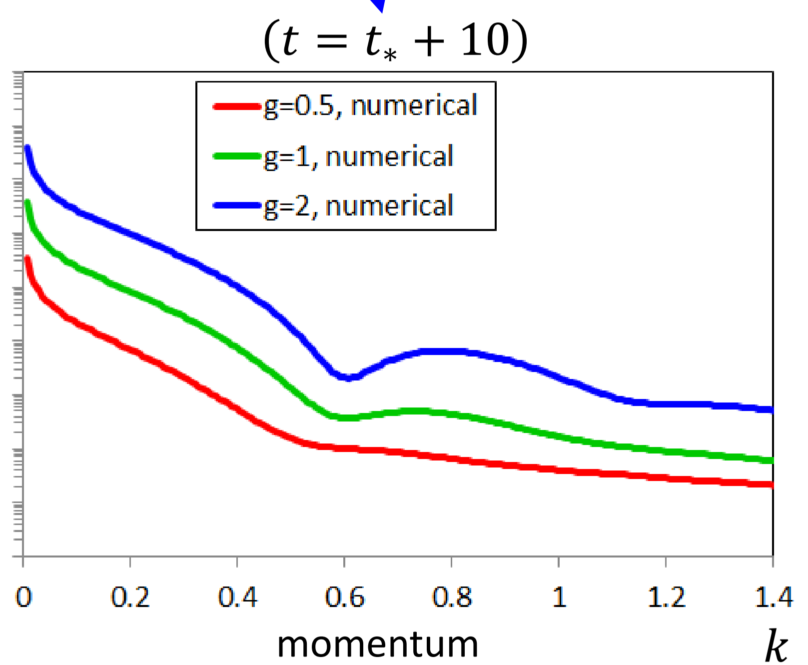
Distributions



Fitting function for distributions (massless boson ϕ)

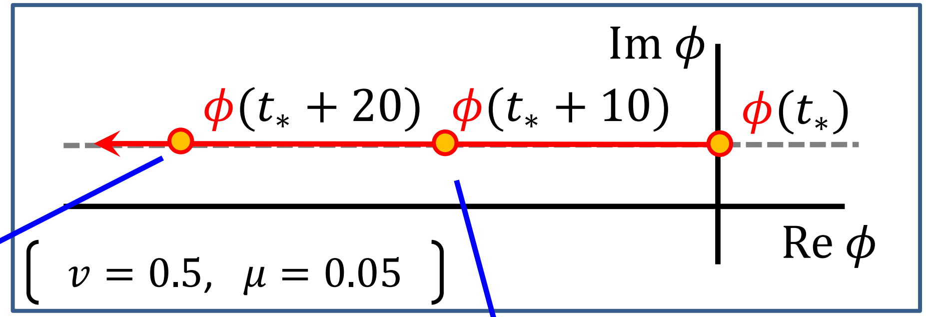


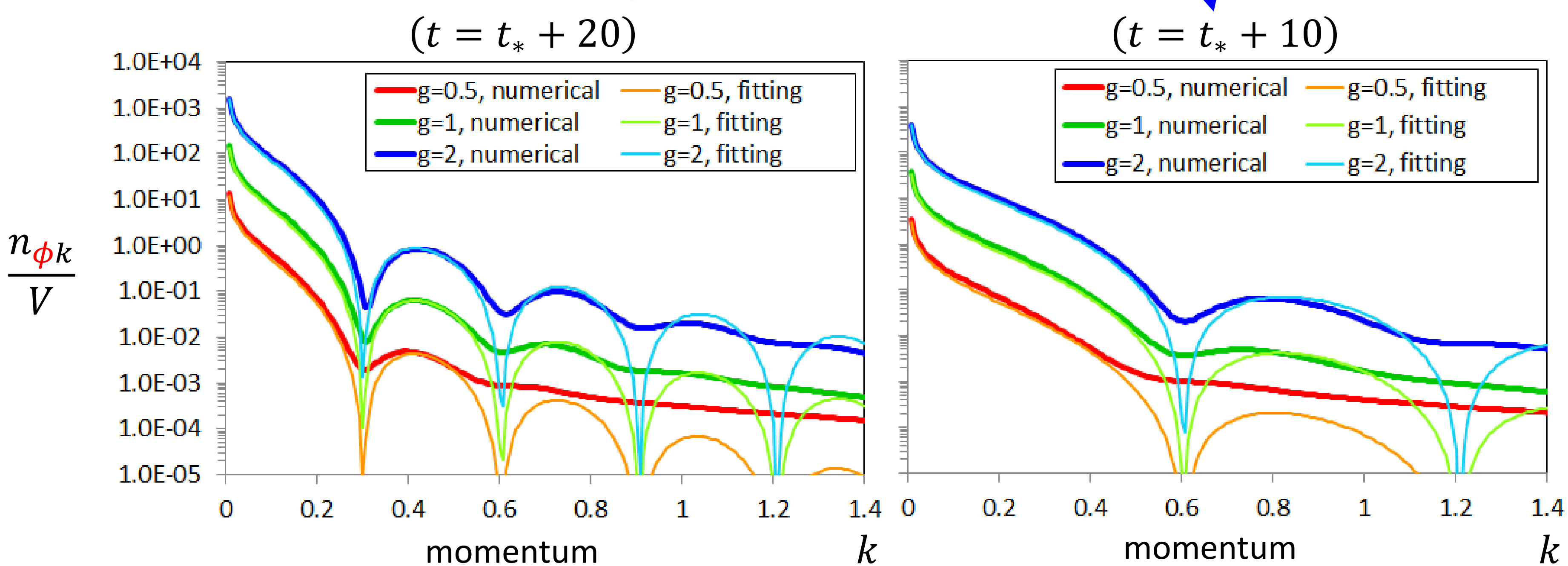




Fitting function for distributions

(massless boson ϕ)





$$\frac{n_{\phi k}}{V} \sim 0.16 \frac{g^2}{4\pi} \cdot \frac{1}{\exp\left[\sqrt{\pi k^2 / g|v|}\right] - 1} \cdot g|v|(t - t_*)^2 \left[\frac{\sin 0.52k(t - t_*)}{0.52k(t - t_*)}\right]^2$$

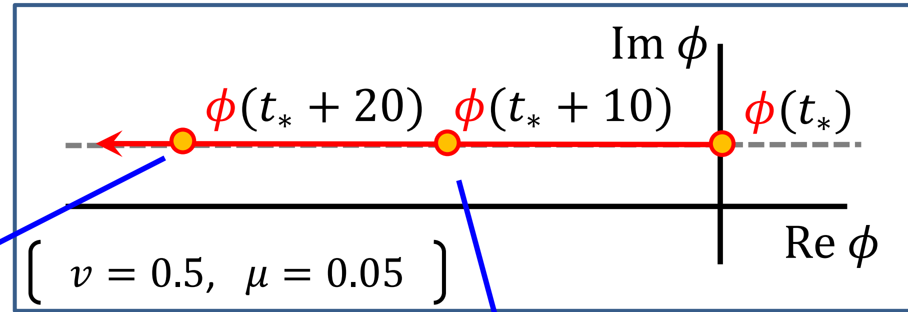
Perturbative suppression factor

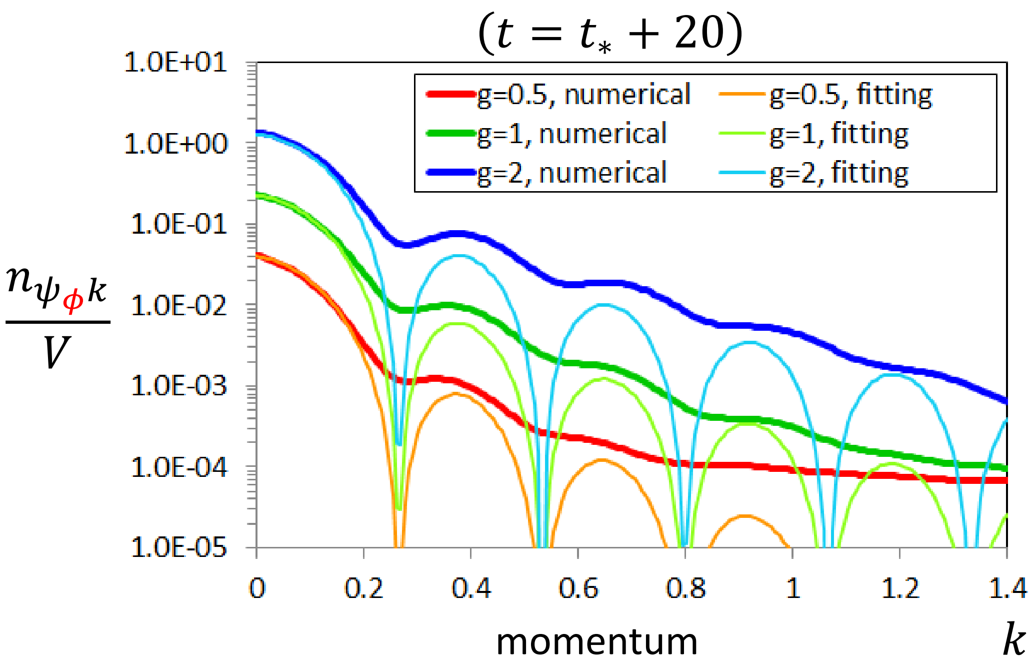
Equilibrium distribution

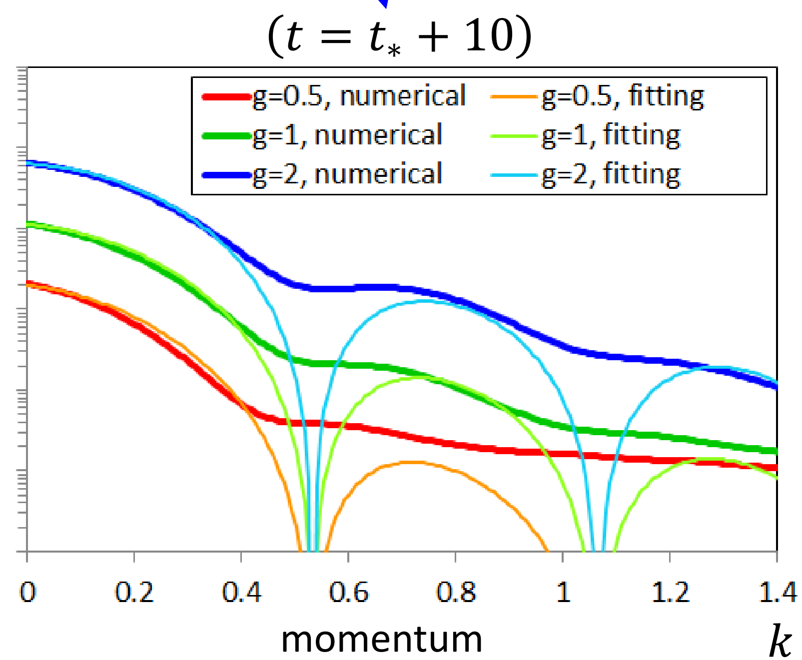
Time evolution part

Fitting function for distributions

(massless fermion ψ_ϕ)







$$\frac{n_{\phi k}}{V} \sim 0.40 \frac{g^2}{4\pi} \cdot \frac{1}{\exp\left[\sqrt{\pi k^2 / g|v|}\right] + 1} \cdot \sqrt{g|v|(t-t_*)^2} \left[\frac{\sin 0.59 k(t-t_*)}{0.59 k (t-t_*)}\right]^2$$

Perturbative suppression factor

Equilibrium distribution

Time evolution part

4. Summary

1. We have found the Bogoliubov transformation law in the interacting theory using the Yang-Feldman formalism
   - In general, particle production is possible even if $\beta_k = 0$

2. We calculated produced particle's (occupation) number
   - $\frac{n_{\chi k}}{V} = \frac{n_{\psi_\chi k}}{V} \sim 2 \exp\left[-\pi \frac{k^2+g^2\mu^2}{gv}\right]$ (analytical results)
   - $\frac{n_{\phi k}}{V}, \frac{n_{\psi_\phi k}}{V} \propto \frac{g^2}{4\pi} \times$ (Eq. distribution) (fitting)
     - Massive and massless particles are produced at the same time
     - The main channel of massless particle production is inverse decay
     - Although the produced number of massless particles is suppressed by $g^2/4\pi$, it is possible to create a sizable amount of particles if the coupling is reasonably strong