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Vacuum Stability and Landau Poles of SU(3) Scalars

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2 Motivation

- Sommerfeld enhancement for dark matter from higher multiplets

  El Hedri, Kaminska & de Vries, 1612.02825

- CP-violation with an unbroken CP-transformation

  Ratz & Trautner, 1612.08984

- Different confinement scales for different multiplets

  Kubo, Lim & Lindner, 1403.4262

3 Model(s)

- Standard Model & one scalar S in multiplet $\mathbf{R}$ of colour
- Choices are 3, 8, 10, 15, 15′, 21, … of $SU(3)_c$
- To be asymptotically free,

$$N_s T(\mathbf{R}_s) < 33 - 2N_f = 21$$

- The largest multiplet to consider is 15′

4 Model(s)

The Lagrangian is given by

$$\mathrm{L} = \mathrm{L}_{\mathrm{SM}}^{\mathrm{gauge,Yukawa}} + |D_\mu H|^2 + |D_\mu S|^2 - \mu_H^2 |H|^2 - m_S^2 |S|^2 - V_{\mathrm{quartic}},$$

where

$$V_{\mathrm{quartic}} = \lambda_H |H|^2 + \lambda_{SH} |S|^2 |H|^2 + V_{\mathbf{R}}(S)$$

5 Vacuum Stability Conditions

- The self-coupling potential of S in the representation **R** can be written as

$$V_{\mathbf{R}}(S) = (\lambda_S + \lambda_{Si}\rho_i)|S|^4,$$

where ρ_i are *orbit space parameters*

- The full potential

$$V_{\text{quartic}} = \lambda_H|H|^2 + \lambda_{SH}|S|^2|H|^2 + V_{\mathbf{R}}(S)$$

is bounded from below if

$$\lambda_H > 0, \quad \lambda_S + \lambda_{Si}\rho_i > 0,$$
$$\lambda_{SH} > -2\sqrt{\lambda_H\left(\lambda_S + \lambda_{Si}\rho_i\right)}$$

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6 Potential of 3

- 3 of $SU(3)$ is a vector S^i with $(S^i)^\dagger \equiv S_i$
- The scalar potential of 3 is

$$V_3(S) = \lambda_S (S_i S^i)^2$$

- Bounded from below if $\lambda_S > 0$

7 Potential of 8

- 8 of $SU(3)$ is a Hermitian traceless matrix S^i_j
- Cayley-Hamilton theorem: $\mathrm{tr}\, S^4 \propto (\mathrm{tr}\, S^2)^2$
- The scalar potential of 8 is

$$V_8(S) = \lambda_S (\mathrm{tr}\, S^2)^2$$

- Bounded from below if $\lambda_S > 0$

8 Potential of 6

- 6 of $SU(3)$ is a complex symmetric matrix S^{ij}
- The scalar potential of 6 is

$$
\begin{aligned}
V_6(S) &= \lambda_S(\operatorname{tr} S^\dagger S)^2 + \lambda_{S\,I} \operatorname{tr} S^\dagger S S^\dagger S \\
&= (\lambda_S + \lambda_{S\,I}\rho)|S|^4,
\end{aligned}
$$

where $|S|^2 = \operatorname{tr} S^\dagger S$ and $\rho = \dfrac{\operatorname{tr} S^\dagger S S^\dagger S}{(\operatorname{tr} S^\dagger S)^2}$

- Minimise $\lambda_S + \lambda_{S\,I}\rho$ over ρ: need ρ_{min} and ρ_{max}

9 Potential of 6

- The scalar potential of 6 is

$$V_6(S) = \lambda_S(\mathrm{tr}\, S^\dagger S)^2 + \lambda_{S\,\mathrm{I}}\, \mathrm{tr}\, S^\dagger S S^\dagger S$$
$$= \lambda_S(\mathrm{tr}\, M)^2 + \lambda_{S\,\mathrm{I}}\, \mathrm{tr}\, M^2$$
$$= (\lambda_S + \lambda_{S\,\mathrm{I}}\rho)|S|^4,$$

where $M_i^j = S_{ik}S^{jk}$ and $\mathrm{tr}\, M = \mathrm{tr}\, S^\dagger S = |S|^2$

9 Potential of 6

- The scalar potential of 6 is

$$\begin{aligned} V_6(S) &= \lambda_S (\operatorname{tr} S^\dagger S)^2 + \lambda_{S1} \operatorname{tr} S^\dagger S S^\dagger S \\ &= \lambda_S (\operatorname{tr} M)^2 + \lambda_{S1} \operatorname{tr} M^2 \\ &= (\lambda_S + \lambda_{S1}\rho)|S|^4, \end{aligned}$$

where $M_i^j = S_{ik} S^{jk}$ and $\operatorname{tr} M = \operatorname{tr} S^\dagger S = |S|^2$

- Then the orbit parameter is

$$\rho = \frac{\operatorname{tr} S^\dagger S S^\dagger S}{(\operatorname{tr} S^\dagger S)^2} = \frac{\operatorname{tr} M^2}{(\operatorname{tr} M)^2} = \frac{\sum_i d_i^2}{(\sum_i d_i)^2}$$

- $\rho_{min} = \frac{1}{3}$ and $\rho_{max} = 1$

10 Vacuum Stability of 6

- The potential is bounded from below if

$$\lambda_S + \lambda_{S1}\rho > 0$$

- The vacuum stability conditions are thus

$$\lambda_S + \frac{1}{3}\lambda_{S1} > 0, \qquad \lambda_S + \lambda_{S1} > 0$$

- Non-zero ρ_{min} allows for $\lambda_S < 0$
- Can play a rôle for asymptotic safety

Giudice, Isidori, Salvio & Strumia, 1412.2769

Potential of 10

- 10 of $SU(3)$ is the symmetric tensor S^{ijk}
- The scalar potential of 10 is

$$\begin{aligned} V_{10}(S) &= \lambda_S (S_{ijk}S^{ijk})^2 + \lambda_{S1} S_{ijm}S^{ijn}S_{kln}S^{klm} \\ &= \lambda_S (\mathrm{tr}\, M)^2 + \lambda_{S1}\, \mathrm{tr}\, M^2, \end{aligned}$$

where $M_i^j = S_{ikl}S^{jkl}$

- Again,

$$\rho = \frac{\mathrm{tr}\, M^2}{(\mathrm{tr}\, M)^2}$$

and $\rho_{\mathrm{min}} = \dfrac{1}{3}$ and $\rho_{\mathrm{max}} = 1$

12 Potential of 15′

- 15′ of $SU(3)$ is a completely symmetric tensor S^{ijkl}
- The scalar potential of 15′ is

$$
\begin{aligned}
V_{15'}(S) &= \lambda_S (S_{ijkl} S^{ijkl})^2 + \lambda_{S1} S_{ijkp} S^{ijkq} S_{lmnq} S^{lmnp} \\
&\quad + \lambda_{S2} S_{ijmn} S^{ijpq} S_{klpq} S^{klmn} \\
&= (\lambda_S + \lambda_{S1}\rho_1 + \lambda_{S2}\rho_2)|S|^4
\end{aligned}
$$

- Vacuum is stable if

$$\lambda_S + \lambda_{S1}\rho_1 + \lambda_{S2}\rho_2 > 0$$

for all allowed ρ_1 and ρ_2

13 Vertices of Orbit Space

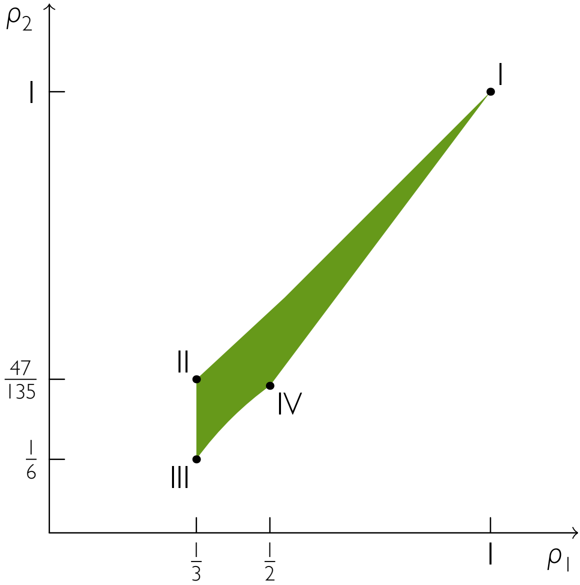
At a vertex V,

$$\frac{\partial \rho_i}{\partial S_a^V} = 0$$

- $S_a^V + \delta S_a$ yields $\rho_i^V + \delta\rho_i$
- The deviation cannot move out of the orbit space, so $\delta\rho_i = 0$

Kim, Nucl.Phys. B197 (1982) 174

14 Orbit Space of 15′



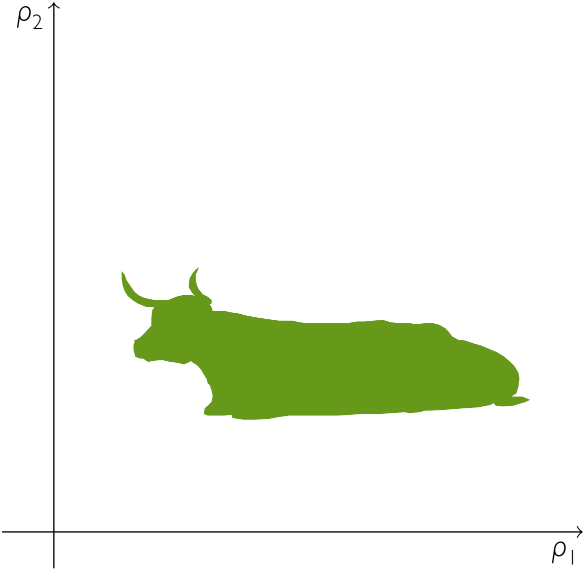
15 Vacuum Stability

Vacuum stability conditions are yielded by the *convex hull* of the orbit space:

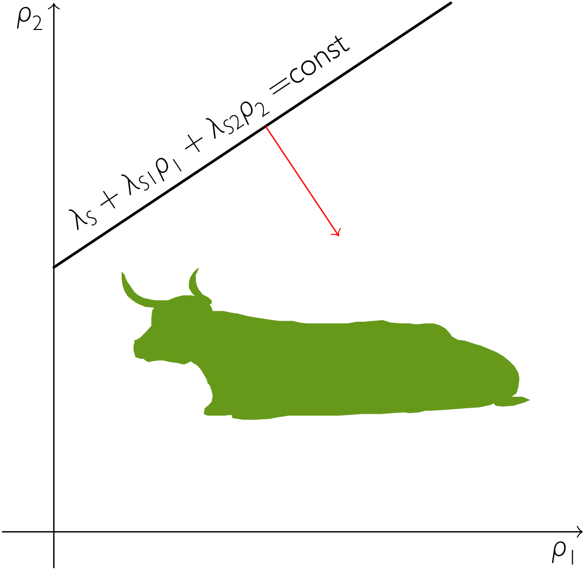
$$\lambda_i \rho_i^A > 0, \;\; \lambda_i \rho_i^B > 0 \quad \Longrightarrow \quad \lambda_i \left[\eta \rho_i^A + (1 - \eta) \rho_i^B \right] > 0$$

with $0 \leqslant \eta \leqslant 1$

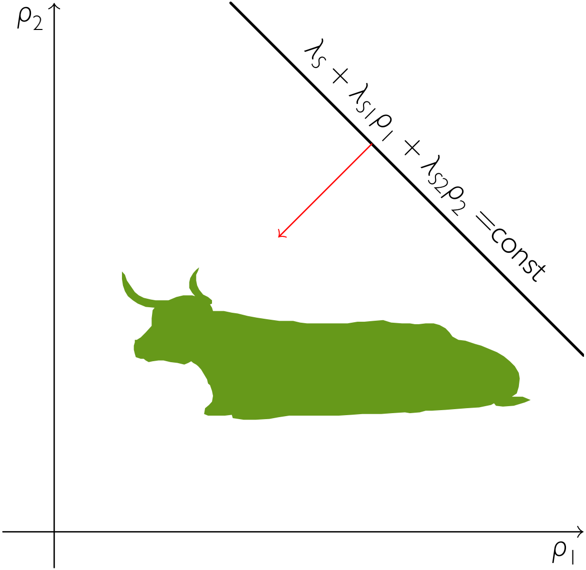
16 Vacuum Stability



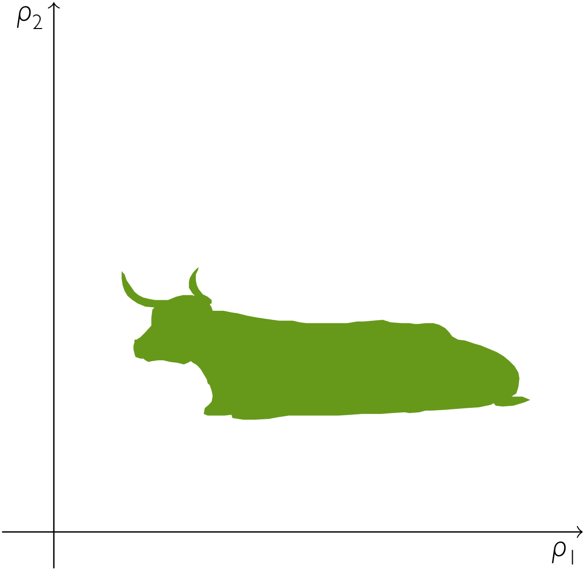
17 Vacuum Stability



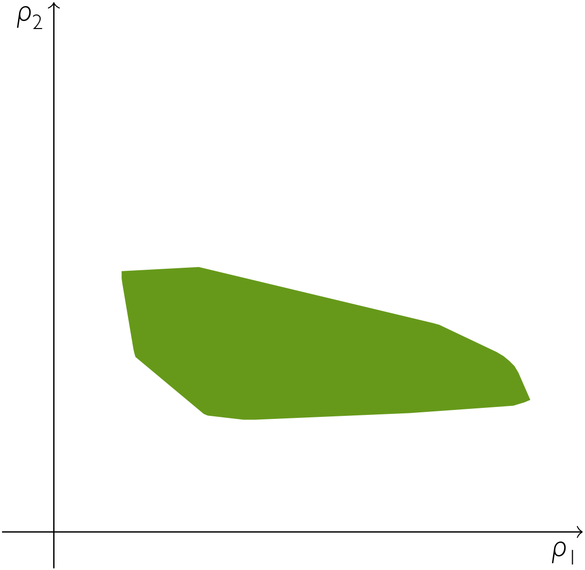
18 Vacuum Stability



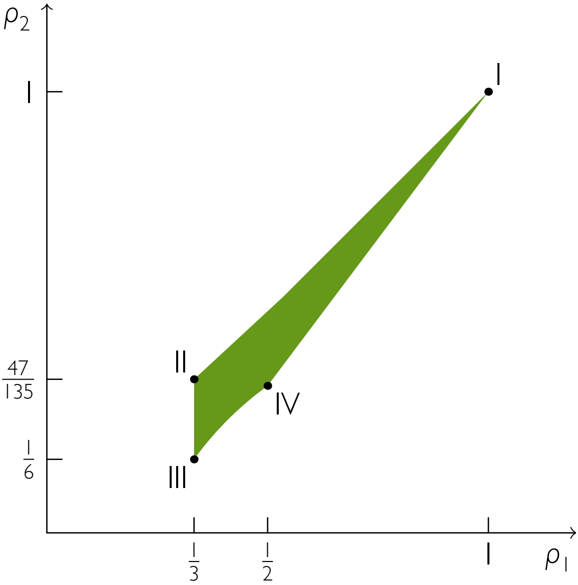
19 Vacuum Stability



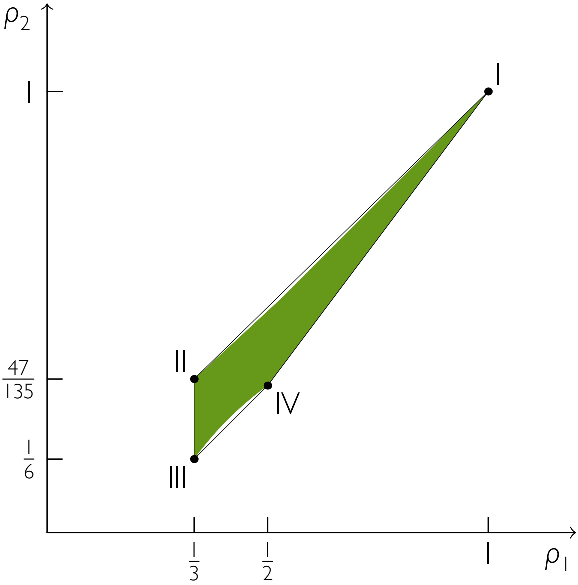
20 Vacuum Stability



21 Orbit Space of 15′



22 Orbit Space of 15′



23 Vacuum Stability of $15'$

- Vertices of the orbit space are

$$\vec{\rho}_{\rm I} = (1, 1), \quad \vec{\rho}_{\rm II} = (\tfrac{1}{3}, \tfrac{47}{135}), \quad \vec{\rho}_{\rm III} = (\tfrac{1}{3}, \tfrac{1}{6}) \quad \text{and} \quad \vec{\rho}_{\rm IV} = (\tfrac{1}{2}, \tfrac{1}{3})$$

- Vacuum stability conditions are given by

$$\begin{aligned}
\text{I} &: \quad \lambda_S + \lambda_{S1} + \lambda_{S2} > 0, \\
\text{II} &: \quad \lambda_S + \frac{1}{3}\lambda_{S1} + \frac{47}{135}\lambda_{S2} > 0, \\
\text{III} &: \quad \lambda_S + \frac{1}{3}\lambda_{S1} + \frac{1}{6}\lambda_{S2} > 0, \\
\text{IV} &: \quad \lambda_S + \frac{1}{2}\lambda_{S1} + \frac{1}{3}\lambda_{S2} > 0
\end{aligned}$$

24 Potential of 15

- 15 of $SU(3)$ is a tensor S^{ij}_k that is symmetric in the upper indices and traceless
- The scalar potential of 15 is

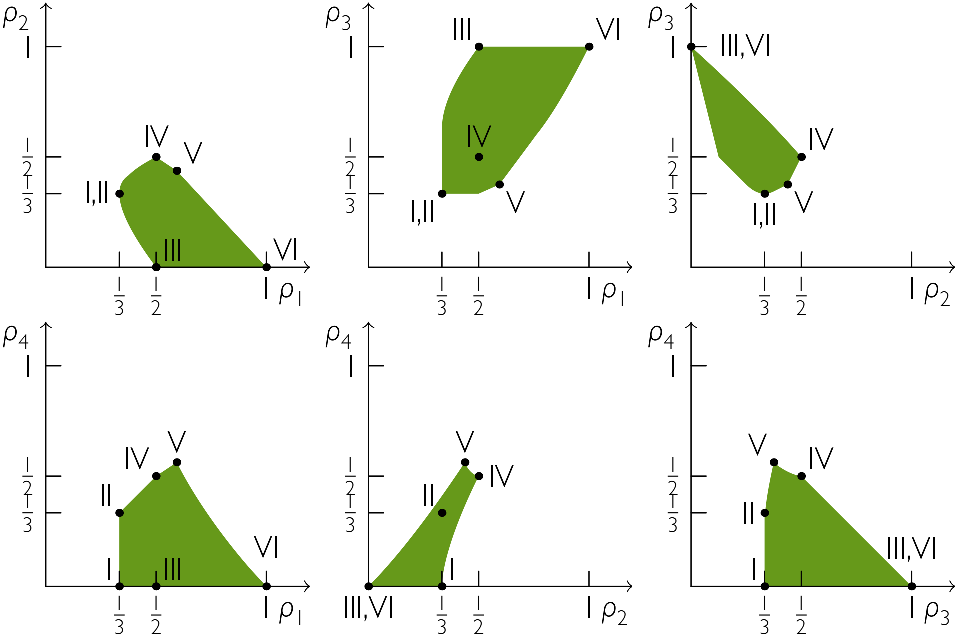
$$
\begin{aligned}
V_{15}(S) &= \lambda_S (S^k_{ij} S^{ij}_k)^2 + \lambda_{S1} S^i_{jm} S^{jn}_i S^k_{ln} S^{lm}_k + \lambda_{S2} S^i_{jm} S^{jn}_i S^m_{kl} S^{kl}_n \\
&\quad + \lambda_{S3} S^m_{ij} S^{ij}_n S^n_{kl} S^{kl}_m + \lambda_{S4} S^i_{jm} S^{km}_l S^j_{in} S^{ln}_k \\
&= (\lambda_S + \lambda_{S1}\rho_1 + \lambda_{S2}\rho_2 + \lambda_{S3}\rho_3 + \lambda_{S4}\rho_4)\,|S|^4
\end{aligned}
$$

25 Orbit Space of 15

The orbit space lies within the 4-box

$$\frac{1}{3} \leqslant \rho_1 \leqslant 1,$$
$$0 \leqslant \rho_2 \leqslant \frac{1}{2},$$
$$\frac{1}{3} \leqslant \rho_3 \leqslant 1,$$
$$0 \leqslant \rho_4 \leqslant \frac{9}{16}$$

2D Projections of Orbit Space of 15



27 Orbit Space of 15

- The full analytical shape of the orbit space of 15 would be rather hard to find
- For vacuum stability conditions, we need its convex hull
- We use the QuickHull algorithm to find its four-dimensional convex hull

  Loren Petrich's Mathematica code **http://lpetrich.org/Science/#CHDV**

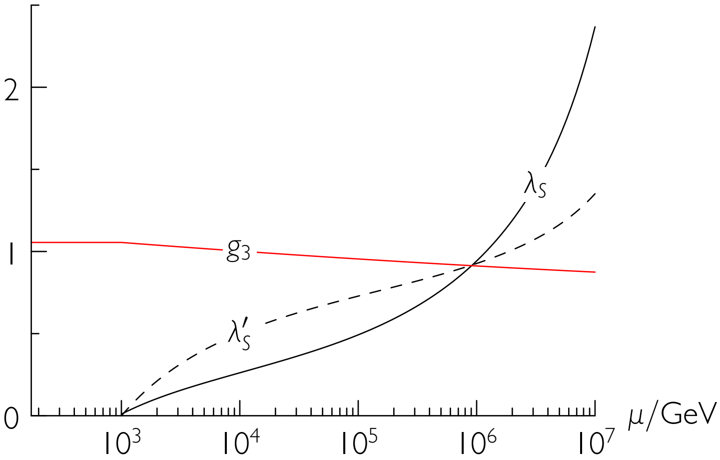
28 RGE Running of Higher Multiplets

- We use PyR@TE 2 to calculate the RGEs
  Lyonnet, Schienbein, Staub and Wingerter, 1309.7030;
  Lyonnet, Schienbein, 1608.07274
- The RGE for a single self-coupling has the form

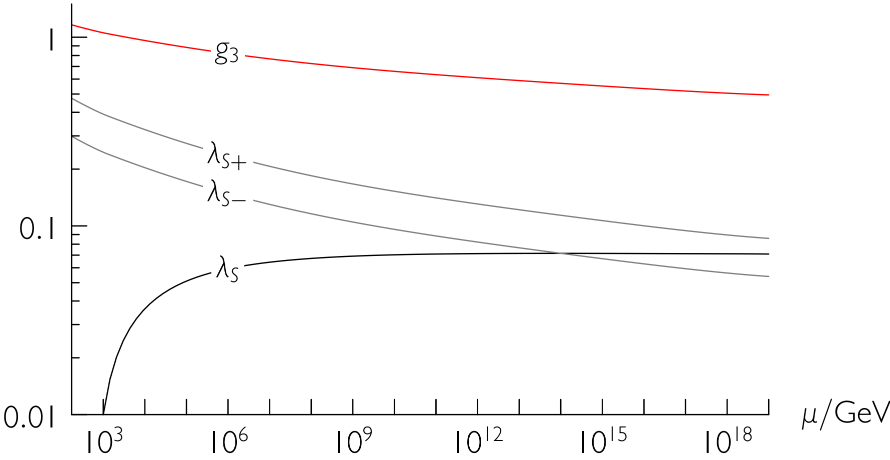
$$\frac{d\lambda_S}{d(\ln\mu)} = b_\lambda \lambda_S^2 - b_{\lambda g}\, g_3^2 \lambda_S + b_{\lambda gg}\, g_3^4 + 2\lambda_{SH}^2$$

- The strong coupling g_3 always generates λ_S

29 Running Self-Couplings of 10

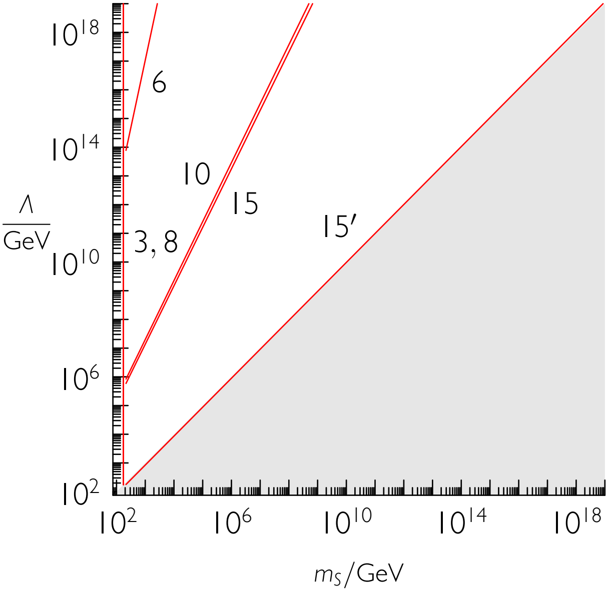


"Walking" Self-Coupling of 3



- β_{λ_S} must have real roots $\lambda_{S\pm}$, so $b_{\lambda g}^2 > 4 b_\lambda b_{\lambda gg}$

Landau Pole *vs.* Mass of *S*



32 Conclusions

- If the potential depends on the orbit space parameters linearly, we need only the convex hull of the orbit space
- Landau poles are low for most higher multiplets of $SU(3)_c$ due to large g_3
- Self-couplings of 3 and 8 *walk*, rather than run