Preheating and light fields

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in collaboration with
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based on:
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Motivation

Preheating - exponentially and non-perturbatively produced states typically correspond to the fields directly interacting with the inflaton, they do affect the mass term of the inflaton through back-reaction effects

...
L. Kofman, A. Linde, A. Starobinsky: 9405187
L. Kofman, A. Linde, A. Starobinsky: 9704452
J. H. Traschen, R. H. Brandenberger: PRD42 (1990)
A. D. Dolgov, D. P. Kirilova, Sov. J. Nucl. Phys. 51 (1990)
R. Allahverdi et al.: 1001.2600
M. A. Amin et al.: 1410.3808
...

In our previous study S. Enomoto, O. Fuksińska, Z. Lalak: 1412.7442 we showed that light fields which are not coupled directly to the background can be produced due to quantum corrections and their abundance can be sizeable, even for the massless case.

Models

I) Two scalar system:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}g^2\phi^2\chi^2$$

ϕ - inflaton, $\langle 0^{\rm in}|\,\phi\,|0^{\rm in}\rangle = \langle\phi(t)\rangle$
χ - another scalar field coupled directly to ϕ, $m_\phi \gg m_\chi$, $\langle\chi\rangle = 0$

II) System with the additional light sector:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}(\partial\chi)^2 - \tfrac{1}{2}m_\phi^2\phi^2 - \tfrac{1}{2}m_\chi^2\chi^2 - \tfrac{1}{4}g^2\phi^2\chi^2 \\ + \textstyle\sum_n \tfrac{1}{2}(\partial\xi_n)^2 - \sum_n \tfrac{1}{2}m_\xi^2\xi_n^2 - \sum_n \tfrac{1}{4}y^2\chi^2\xi_n^2$$

ξ_n - N light or massless fields not coupled to ϕ, $m_\phi \gg m_\xi$, $\langle\xi_n\rangle = 0$
χ particles are produced resonantly and ξ_n through the interactions with χ

Instant preheating

"We describe a new efficient mechanism of reheating. Immediately after rolling down the rapidly moving inflaton field ϕ produces particles χ, which may be either bosons or fermions. This is a nonperturbative process which occurs almost instantly; no oscillations or parametric resonance is required. (...) When the particles χ become sufficiently heavy, they rapidly decay to other, lighter particles. (...)"

G. Felder, L. Kofman, A. Linde: 9812289

- three fields - background ϕ, χ interacting with ϕ and some other field ψ not coupled to ϕ
- χ particles produced within one-time oscillation of ϕ decay immediately to ψ before the next oscillation of ϕ
- ψ states can be also produced even though there is no direct interaction between ϕ and ψ

	our work	instant preheating
mechanism of production	the quantum corrections	decay
origin of the quenching	backreaction	rapid decay

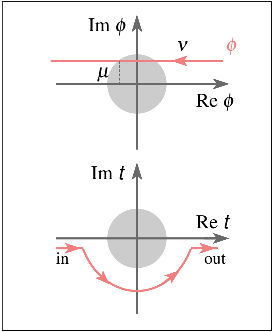
Nature of the oscillations L. v Kofman et al.: 0403001

$$V = \frac{1}{2}g^2|\phi|^2\chi^2$$

- asymptotically: $\phi(t) = vt + i\mu$, $\langle\chi\rangle = 0$
- non-adiabatic region: $|\phi| \lesssim \sqrt{v/g}$
- background field in **non-adiabatic region**: χ particles are produced $\left(\Delta t \sim (gv)^{-1/2}\right)$
- produced particles induce a new **linear potential**

$$\rho_\chi = \int \frac{d^3k}{(2\pi)^3} n_k \sqrt{k^2 + g^2|\phi(t)|^2} \approx g|\phi(t)|n_\chi$$

and an attractive force ("oscillations")



Numerical results for multi-scalar systems

We are interested in time-evolution of particle number density for each species:

$$n(t) = \int \frac{d^3k}{(2\pi)^3} \frac{\langle N_{\mathbf{k}} \rangle}{V}$$

$$N_k(t) = \frac{1}{2\omega_k} \left(\dot{\hat{\phi}}^\dagger_{\mathbf{k}} \dot{\hat{\phi}}_{\mathbf{k}} + \omega_k^2 \hat{\phi}^\dagger_{\mathbf{k}} \hat{\phi}_{\mathbf{k}} \right) + \frac{i}{2} \left(\hat{\phi}^\dagger_{\mathbf{k}} \dot{\hat{\phi}}_{\mathbf{k}} + \dot{\hat{\phi}}^\dagger_{\mathbf{k}} \hat{\phi}_{\mathbf{k}} \right)$$

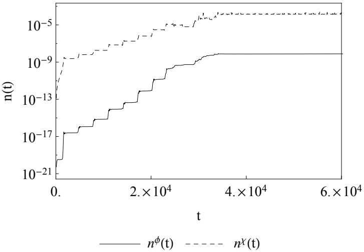
- solve eoms for all the species for t_{ini} and calculate their number density
- do the same for $t_{ini} + \Delta t$ taking into account the back-reaction of previously produced states on the evolution of the background (given by the induced potential coming from non-zero energy density)
- repeat it till you reach t_{fin}

Two scalar system

According to L. Kofman et al.: 0403001 the first production of χ particles results in the number density

$$n_\chi^{(1)} \sim \frac{\left[g m_\phi \langle\phi(0)\rangle\right]^{3/2}}{(2\pi)^3} \sim 4 \cdot 10^{-9}$$

we are in agreement!



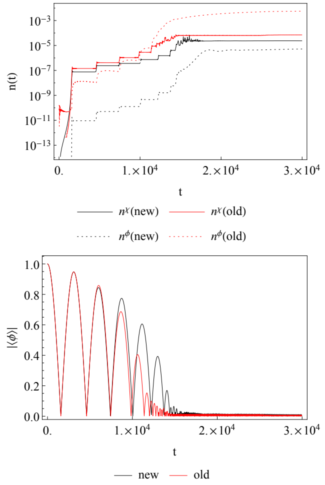
$g = 0.1$, $m_\phi = 0.001$
($m_\phi \sim 5 \times 10^{14}$ GeV)

In general it is difficult to obtain the analytical results for indirect production products, like $\tilde{\phi}$. In this system it is a safe approximation to neglect its production but it depends on the target precision.

The new method vs the old one

old S. Enomoto, O. Fuksińska, Z. Lalak: 1412.7442	new O. Czerwińska, S. Enomoto, Z. Lalak: 1701.00015
massless background	massive background
asymptotic approximation	interacting field theory
artificial infinite growth	no secularity
for massless states	
(secularity)	

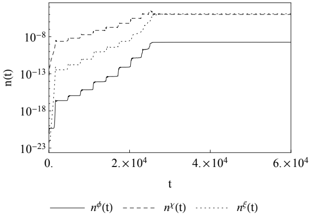
However:
the old results with secularity are still applicable
at the early stages of particle production process.



$g = 1, m_\phi = 0.001$

System with the additional light sector

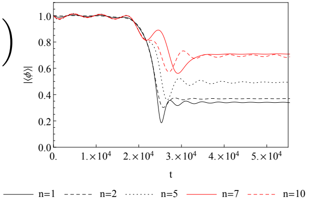
- all the states are produced abundantly
- for $n_\xi \sim n_\chi$: quenching of the preheating (due to enhancement of the back-reaction effects: $n_\xi \uparrow$)
- expectation: most of the energy would be transferred to ξ_n fields as they are very light but: $N \uparrow \Leftrightarrow |\langle\phi\rangle|^{final} \uparrow$, energy transfer to $\xi \downarrow$



$g = 0.1, y = 1, n = 1, m_\phi = 0.001$

The physical mass of χ

$$M_\chi^2 = m_\chi^2 + \frac{1}{2}g^2\langle\phi\rangle^2 + \frac{1}{2}g^2\int\frac{d^3p}{(2\pi)^3}\left(\frac{1}{V}\langle\hat{\phi}^\dagger_{\mathbf{p}}\hat{\phi}_{\mathbf{p}}\rangle - \frac{1}{2\omega_{\phi p}}\right)$$
$$+\frac{1}{2}y^2\sum_n\left(\frac{1}{V}\langle\hat{\xi}^\dagger_{n\mathbf{p}}\hat{\xi}_{n\mathbf{p}}\rangle - \frac{1}{2\omega_{\xi p}}\right) + \mathcal{O}(y^4, y^2g^2, g^4)$$



$g = 0.1, y = 1, m_\phi = 0.001$

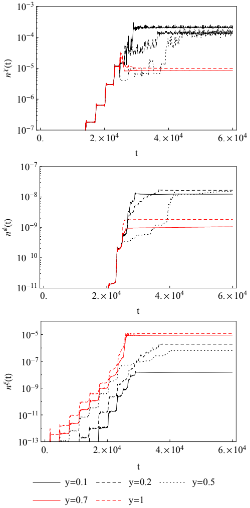
Once ϕ or ξ_n are produced they also generate χ's effective mass which results in particle production area becoming narrower: $n_\chi \downarrow$.

Varying y with fixed g

$\frac{1}{4}g^2\phi^2\chi^2 \quad \sum_n \frac{1}{4}y^2\chi^2\xi_n^2$

produced states	effect of varied y
χ, ϕ	does not influence the initial stage of preheating influences the final n_χ and n_ϕ: $y\uparrow \quad \Leftrightarrow \quad n_\chi^{final}\downarrow, n_\phi^{final}\downarrow$
ξ_n	both initial and final stages are strongly influenced $y\uparrow \quad \Leftrightarrow \quad n_\xi^{final}\uparrow$ $y\downarrow\Leftrightarrow$ energy transfer to $\langle\phi\rangle\uparrow$

For $n_\xi^{final} \sim n_\chi^{final}$: quenching of the preheating ($y = 0.7$ and $y = 1$)



$g = 0.1, n = 1, m_\phi = 0.001$

Varying y with fixed g

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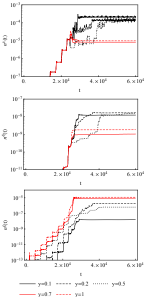
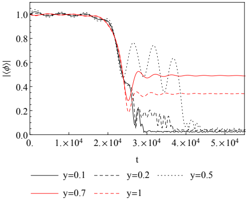




$g = 0.1, n = 1, m_\phi = 0.001$

Thank you for your attention.

Back-up slides

EOMs

$$
\begin{aligned}
\langle\hat{\phi}_{\mathbf{k}}^\dagger\hat{\phi}_{\mathbf{k}}\rangle^\cdot &= \langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\hat{\phi}_{\mathbf{k}}\rangle + \langle\hat{\phi}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle \\
\langle\hat{\phi}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle^\cdot &= \langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle + \langle\hat{\phi}_{\mathbf{k}}^\dagger\ddot{\hat{\phi}}_{\mathbf{k}}\rangle \\
&= \langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle - \omega_k^2\langle\hat{\phi}_{\mathbf{k}}^\dagger\hat{\phi}_{\mathbf{k}}\rangle - \langle\hat{\phi}_{\mathbf{k}}^\dagger\hat{J}_{\mathbf{k}}\rangle \\
\langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle^\cdot &= \langle\ddot{\hat{\phi}}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle + \langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\ddot{\hat{\phi}}_{\mathbf{k}}\rangle \\
&= -\omega_k^2(\langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\hat{\phi}_{\mathbf{k}}\rangle + \langle\hat{\phi}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle) - \langle\dot{\hat{\phi}}_{\mathbf{k}}^\dagger\hat{J}_{\mathbf{k}}\rangle - \langle\hat{J}_{\mathbf{k}}^\dagger\dot{\hat{\phi}}_{\mathbf{k}}\rangle
\end{aligned}
$$

where

$$
\hat{J}_{\mathbf{k}} \equiv \int d^3x e^{-\mathbf{k}\cdot\mathbf{x}} J(t,\mathbf{x}).
$$

Physical mass of ϕ is determined by the relation:

$$
\begin{gathered}
0 = \langle\hat{\phi}_{\mathbf{k}}^\dagger\hat{J}_{\mathbf{k}}\rangle = (m^2 - M^2)\langle\hat{\phi}_{\mathbf{k}}^\dagger\hat{\phi}_{\mathbf{k}}\rangle \\
+ \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \left\langle \hat{\phi}_{\mathbf{k}}^\dagger \frac{dV(x)}{d\phi} \right\rangle
\end{gathered}
$$

to remove the infinite part of the mass correction.

Expansion of the universe

we neglect the expansion of the universe $\Leftrightarrow$ we assume that the mean time the trajectory spends in the non-adiabatic region is smaller than the Hubble time

$$\frac{1}{\sqrt{gv}} < \frac{2}{3H(w+1)}$$

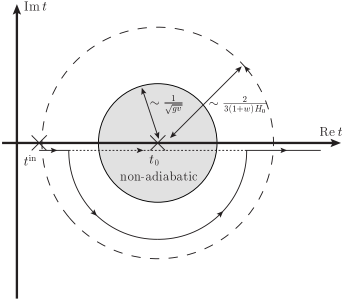
Following K. Enqvist, M. Sloth: 0109214 the number density of produced particles in the expanding universe is

$$n_\chi^{(j)} \sim n_\chi^{(1)} \cdot 3^{j-1} \left(\frac{5}{2}\right)^{3/2} \frac{1}{j^{5/2}}$$

j - the number of oscillations.

For $j \sim 10$ and $n_\chi^{(10)} \sim 1 \times 10^{-6}$, the oscillation phase finishes when $\frac{1}{2} m_\phi^2 \langle \phi_j \rangle^2 \sim \rho_\chi^{(j)} \sim g \langle \phi_j \rangle n_\chi^{(j)}$:

we are in agreement!



Bogoliubov transformation (L.E. Parker & D.J. Toms, N. D. Birrell & P. C. W. Davies, ...)

These two sets of operators act in the same Hilbert space so we can express one using another

$$a_k^{\text{out}} = \alpha_k a_k^{\text{in}} + \beta_k a_k^{\text{in}\,\dagger}$$
$$a_k^{\text{out}\,\dagger} = \alpha_k^* a_k^{\text{in}\,\dagger} + \beta_k^* a_k^{\text{in}}$$

and calculate commutation relation in the new basis

$$[a_k^{\text{out}}, a_k^{\text{out}\,\dagger}] = [\alpha_k a_k^{\text{in}} + \beta_k a_k^{\text{in}\,\dagger}, \alpha_k^* a_k^{\text{in}\,\dagger} + \beta_k^* a_k^{\text{in}}] = \ldots = \left(|\alpha_k|^2 - |\beta_k|^2\right)[a_k^{\text{in}}, a_k^{\text{in}\,\dagger}]$$

Commutation relation is fixed so we obtain the **normalization condition** for **Bogoliubov coefficients** in case of the scalar field

$$|\alpha_k|^2 - |\beta_k|^2 = 1.$$

For fermions: $|\alpha_k|^2 + |\beta_k|^2 = 1$ because of the different form of commutation relation.

Occupation number of produced particles

$$n_k \equiv \langle 0^{\text{in}}|N_k|0^{\text{in}}\rangle = \langle 0^{\text{in}}|a_{\vec{k}}^{\text{out}\,\dagger} a_{\vec{k}}^{\text{out}}|0^{\text{in}}\rangle = V|\beta_k|^2.$$

It seems that if $\beta_k = 0$ particles are not produced.

Effect of interactions

A scalar field Ψ has equation of motion of the form

$$\left(\partial^2 + M^2(x)\right)\Psi(x) + J(x) = 0,$$

where $J(x)$ describes possible interactions (**source term**).

Its solution (for weak coupling) is called **Yang-Feldman equation** (analogy with retarded potential in electrodynamics) (C. N. Yang and D. Feldman,Phys. Rev. 79, 972 (1950))

$$\Psi(x) = \sqrt{Z}\Psi^{as}(x) - iZ\int_{t_{as}}^{x^0} dy^0 \int d^3y [\Psi^{as}(x), \Psi^{as}(y)] J(y),$$

where

$$\Psi(t^{as}, \vec{x}) = \sqrt{Z}\Psi^{as}(t^{as}, \vec{x})$$

is a free **asymptotic field**.

Effect of interactions

Expanding our asymptotic field into **mode functions** (it's a free field: $J(x) = 0$)

$$\Psi^{as}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left(\Psi_k^{as}(x^0) a_{\vec{k}}^{as} + \Psi_k^{as\,*}(x^0) a_{-\vec{k}}^{as\,\dagger} \right)$$

and taking time-dependent **inner product relation** that comes from canonical commutation relations

$$\dot{\Psi}_k^{as\,*} \Psi_k^{as} - \Psi_k^{as\,*} \dot{\Psi}_k^{as} = i/Z$$

allows us to get

$$a_{\vec{k}}^{as} = -iZ \int d^3x e^{-i\vec{k}\cdot\vec{x}} \left(\dot{\Psi}_k^{as\,*} \Psi^{as} - \Psi_k^{as\,*} \dot{\Psi}^{as} \right)$$

which means

$$a_{\vec{k}}^{out} = \alpha_k a_{\vec{k}}^{in} + \beta_k a_{-\vec{k}}^{in\,\dagger} - i\sqrt{Z} \int d^4x e^{-i\vec{k}\cdot\vec{x}} \left(-\beta_k \Psi_k^{in}(x^0) + \alpha_k \Psi_k^{in\,*}(x^0) \right) J(x).$$

That establishes the **generalized Bogoliubov transformation** with coefficients defined as some combination of Z, Ψ_k^{in} and Ψ_k^{out} with usual normalization.

Effect of interactions

Occupation number is now

$$n_k = \begin{cases} V|\beta_k|^2 + \ \dots & (\beta_k \neq 0) \\ 0 + Z|\int d^4x e^{-i\vec{k}\cdot\vec{x}} \Psi_k^{\text{in}\,*} J|0^{\text{in}}\rangle|^2 & (\beta_k = 0) \end{cases}$$

Particles are produced even if $\beta_k = 0$.

How big is that effect?

Vacuum in curved spacetime

In Minkowski space vacuum is the lowest energy-eigenstate of the Hamiltonian (one and only). All inertial observers will always measure the same vacuum.

In curved spacetime Hamiltonian is explicitly time-dependent and no set of mode functions is distinguished. There is some freedom in choosing the set of modes that serves as vacuum.

Examples:

- **instantenous vacuum** - minimizing instantenous Hamiltonian
- **adiabatic vacuum** - based on WKB approximation

But: for asymptotically flat geometry we can describe our vacuum in analogy with Minkowski case and we can well define "in" and "out" states then

$$a(t) \sim \begin{cases} a_1 \text{ for } t \to -\infty \\ a_2 \text{ for } t \to +\infty \end{cases}$$

Quantization of the scalar field in curved spacetime

The simplest action for a free scalar field Ψ minimally coupled to gravity in FRW flat metric $\left(ds^2 = dt^2 - a^2(t)d^2\vec{x}\right)$:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial^{\mu}\Psi\partial_{\mu}\Psi - \frac{1}{2}m^2\Psi^2\right).$$

Decomposition of the scalar field into **modes** $v_{\vec{k}}$:

$$\Psi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left(v_{\vec{k}}(t,\vec{x})a_{\vec{k}}(t,\vec{x}) + v^*_{\vec{k}}(t,\vec{x})a^{\dagger}_{\vec{k}}(t,\vec{x})\right)$$

in our notation should asymptotically correspond to positive-energy Minkowski solution:

$$w_{\vec{k}} = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\tilde{\omega}_k}}e^{i(\vec{k}\vec{x}-\tilde{\omega}_k t)}$$

for $\tilde{\omega}_k^2 = |\vec{k}|^2 + m^2$. So we choose:

$$v_{\vec{k}} = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{a^3(t)}}\frac{1}{\sqrt{2\omega_k}}e^{i(\vec{k}\vec{x}-\int_0^t \omega_k(t')dt')}$$

with $\omega_k^2 = \left(\frac{k}{a(t)}\right)^2 + m^2$.

Quantization of the scalar field in curved spacetime

Solving **Euler-Lagrange equation** gives the equation of motion for field Ψ

$$\ddot{\Psi} + 3\frac{\dot{a}}{a}\dot{\Psi} - \frac{\nabla^2}{a^2}\Psi + m^2\Psi = 0$$

which means the following equation of motion for modes:

$$\ddot{v}_{\vec{k}} + \omega^2_{\vec{k}}(t) v_{\vec{k}} = 0$$

that is simply the harmonic oscillator equation with time-dependent frequency. Solving it means finding some particular set of modes.

Vacuum state is defined by: $\forall_{\vec{k}} \quad a_{\vec{k}}|0\rangle = 0$.

Commutation relations:

$$[a_{\vec{k}}, a^{\dagger}_{\vec{k}'}] = \delta(\vec{k} - \vec{k}')$$
$$[a_{\vec{k}}, a_{\vec{k}'}] = 0$$

Our notation

- annihilation and creation operators depend only on time (no space dependence), it is connected with the definition of the asymptotic field: $a = a(t^{as})$
- it does not describe the time evolution of the system but it means that if field $\phi(t, \vec{x})$ evolve from t^{as} to t, operator a_k keeps $a_k(t^{as})$
- time evolution of the system is described by the wave function
- the definition of the number operators is tuned at each moment of time so that they can lead to correct quantities (instantenous approach)

Normalization of modes

Mode functions $v_{\vec{k}}$ are orthonormal in a sense:

$$\left(v_{\vec{k}}, v_{\vec{k'}}\right) = \delta(\vec{k} - \vec{k'})$$

$$\left(v_{\vec{k}}, v^*_{\vec{k'}}\right) = 0$$

with scalar product

$$\left(f_1, f_2\right) = i \int d^3x |g|^{1/2} \left[f_1^*(\vec{x}, t) \cdot \partial_0 f_2(\vec{x}, t) - \partial_0 f_1^*(\vec{x}, t) \cdot f_2(\vec{x}, t) \right].$$

Particle production after inflation

- preheating - parametric resonance (forced oscillator)
  - classical inflaton background ϕ induces the quantum production of χ particles
  - solving Matthieu equation: $\chi_k'' + (A_k - 2q\cos(2z))\chi_k = 0$,
    where $A_k = \frac{k^2}{m^2} + 2q$ and $q = \frac{g^2\Phi^2}{4m^2} = \text{const}$
- slow reheating and thermalization - oscillations around the minimum of the potential, perturbative production

D. Baumann "The Physics of Inflation"