Anatomy of single field inflationary models for primordial black holes

Scalars 2023

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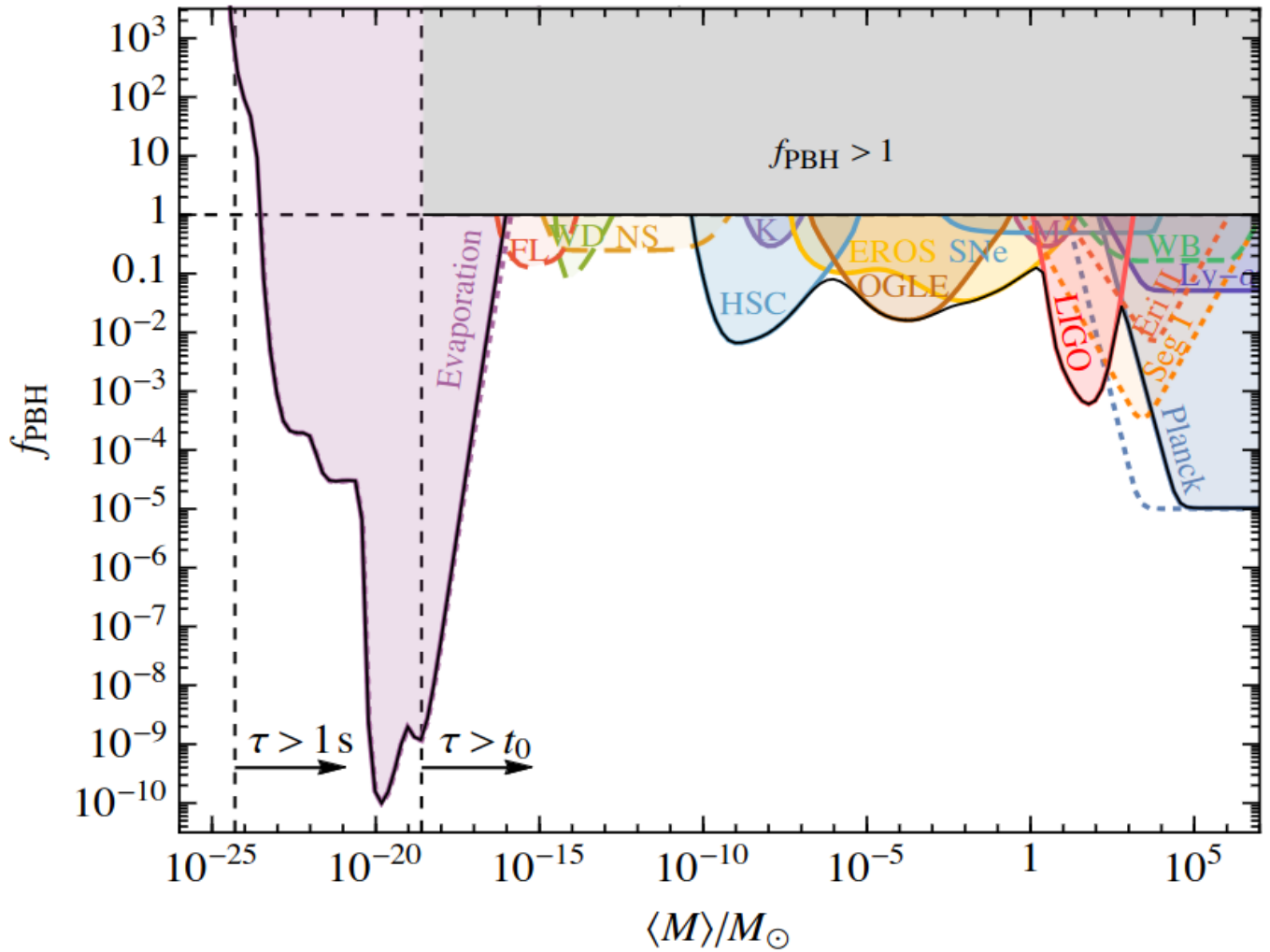
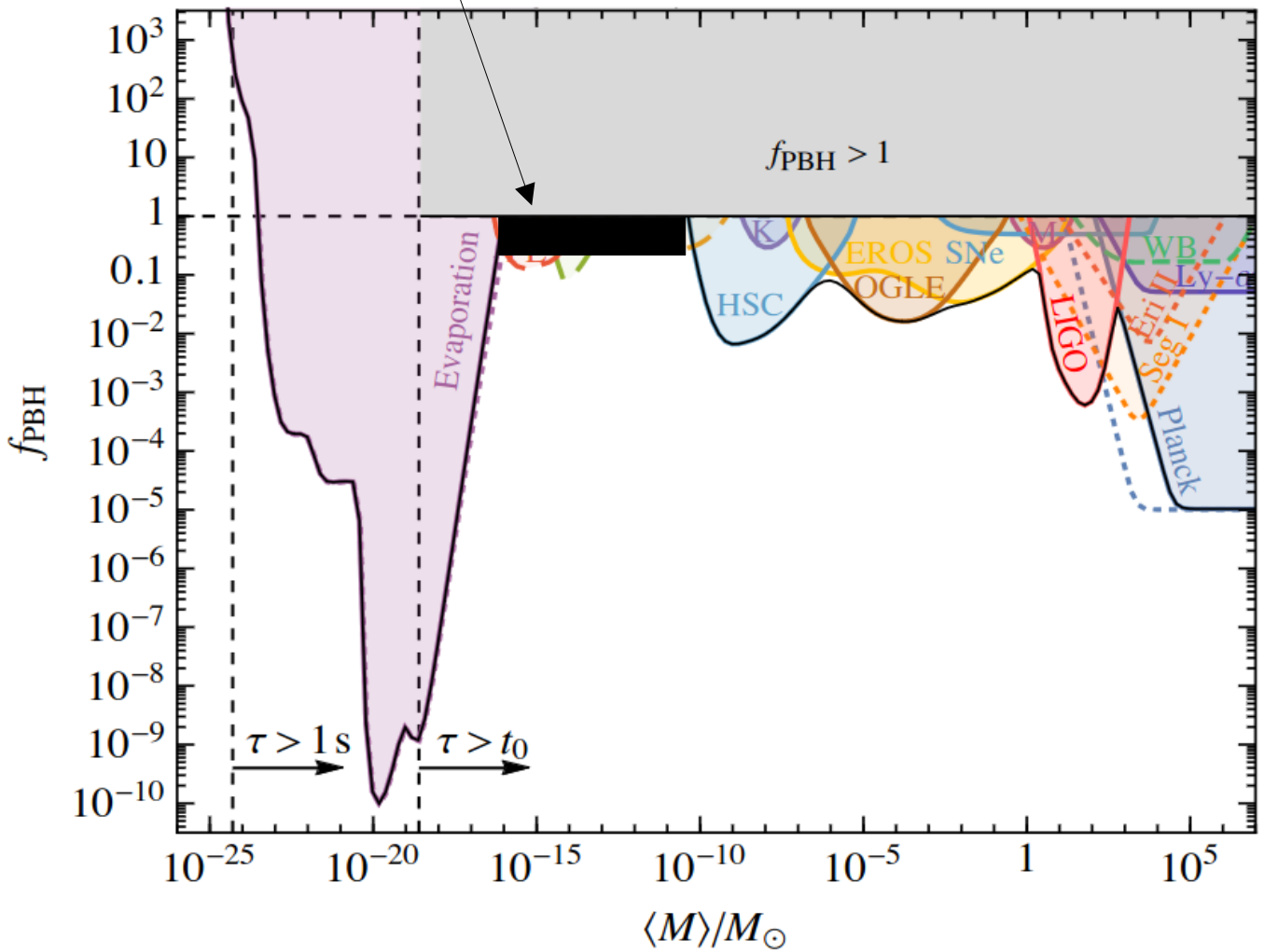
A. Karam, NK, E. Tomberg, V. Vaskonen, H. Veermäe 2205.13540

A. Karam, NK, E. Tomberg, A. Racioppi, H. Veermäe 2305.09630


$f_{\rm PBH} > 1$
Evaporation
FL
WD
NS
HSC
EROS
SNe
OGLE
LIGO
WB
Ly-α
Planck
$\tau > 1\,{\rm s}$
$\tau > t_0$
$f_{\rm PBH}$
$\langle M \rangle / M_\odot$
10^{3}
10^{2}
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-25}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}

PBH as all dark matter
$M_{\rm PBH} = 10^{17} - 10^{22}{\rm g}$



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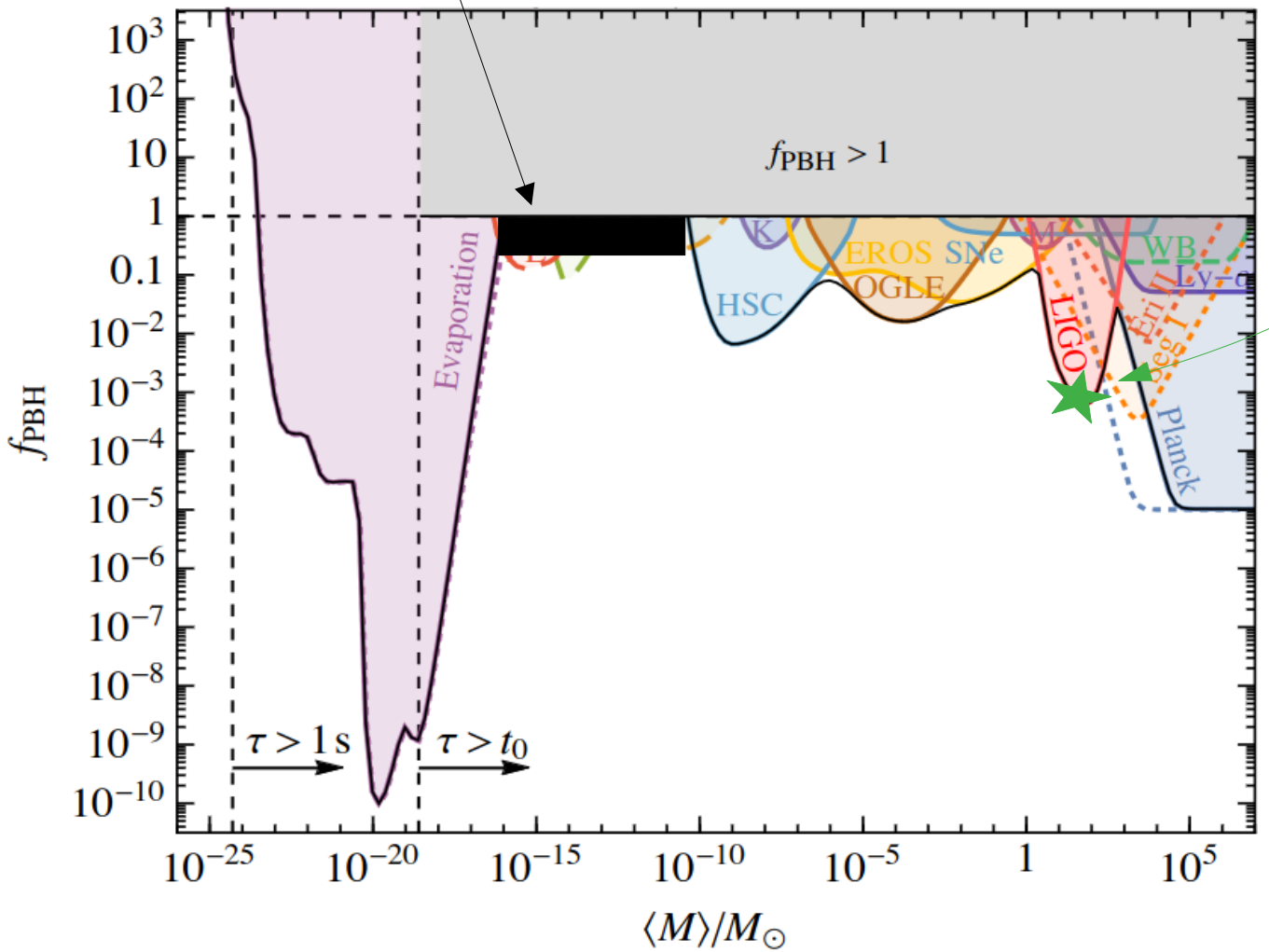
Observed PBH binaries
$M_{\rm PBH} \approx 1 - 100 M_\odot$
$f_{\rm PBH} \approx 10^{-3}$



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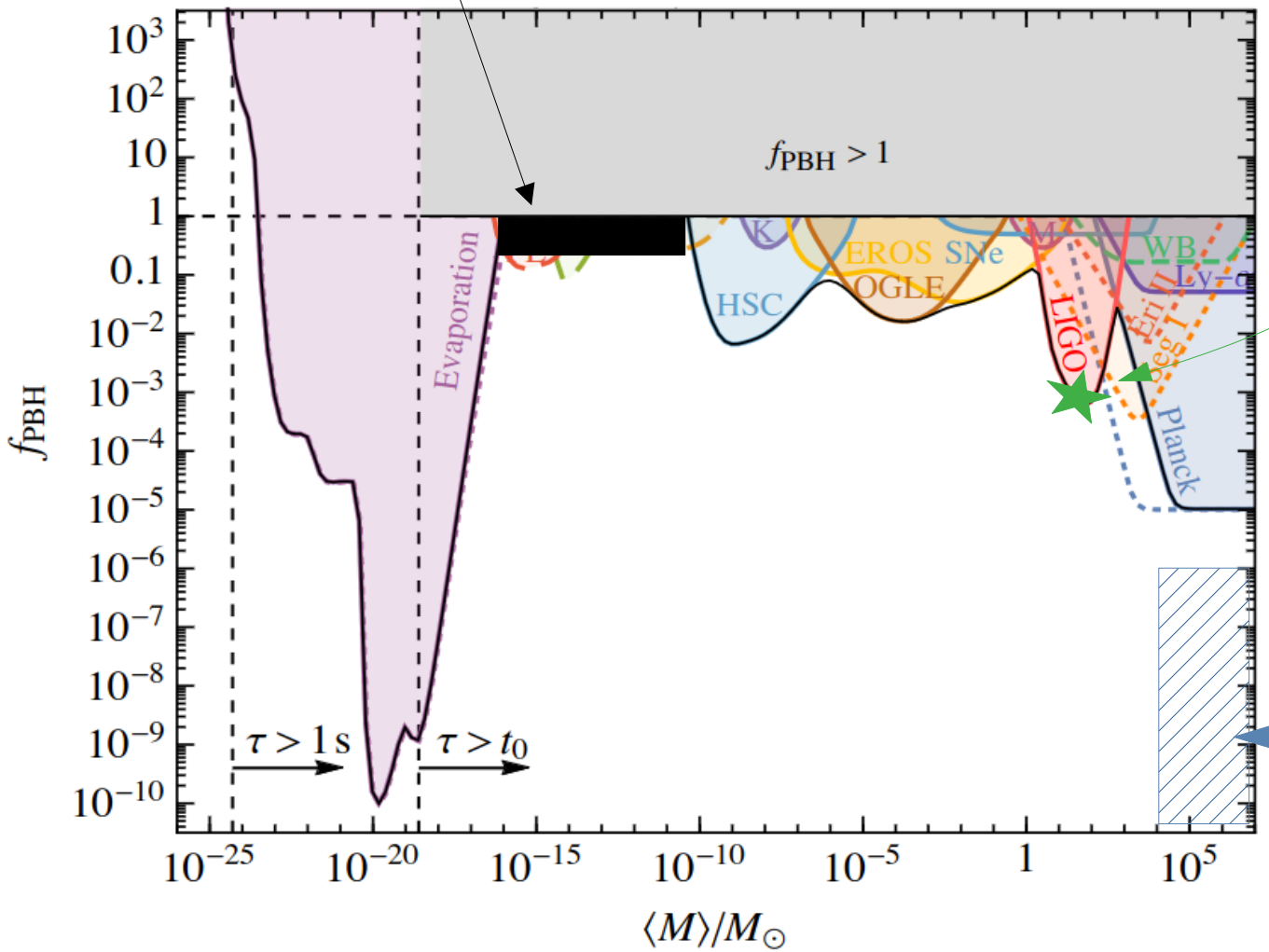
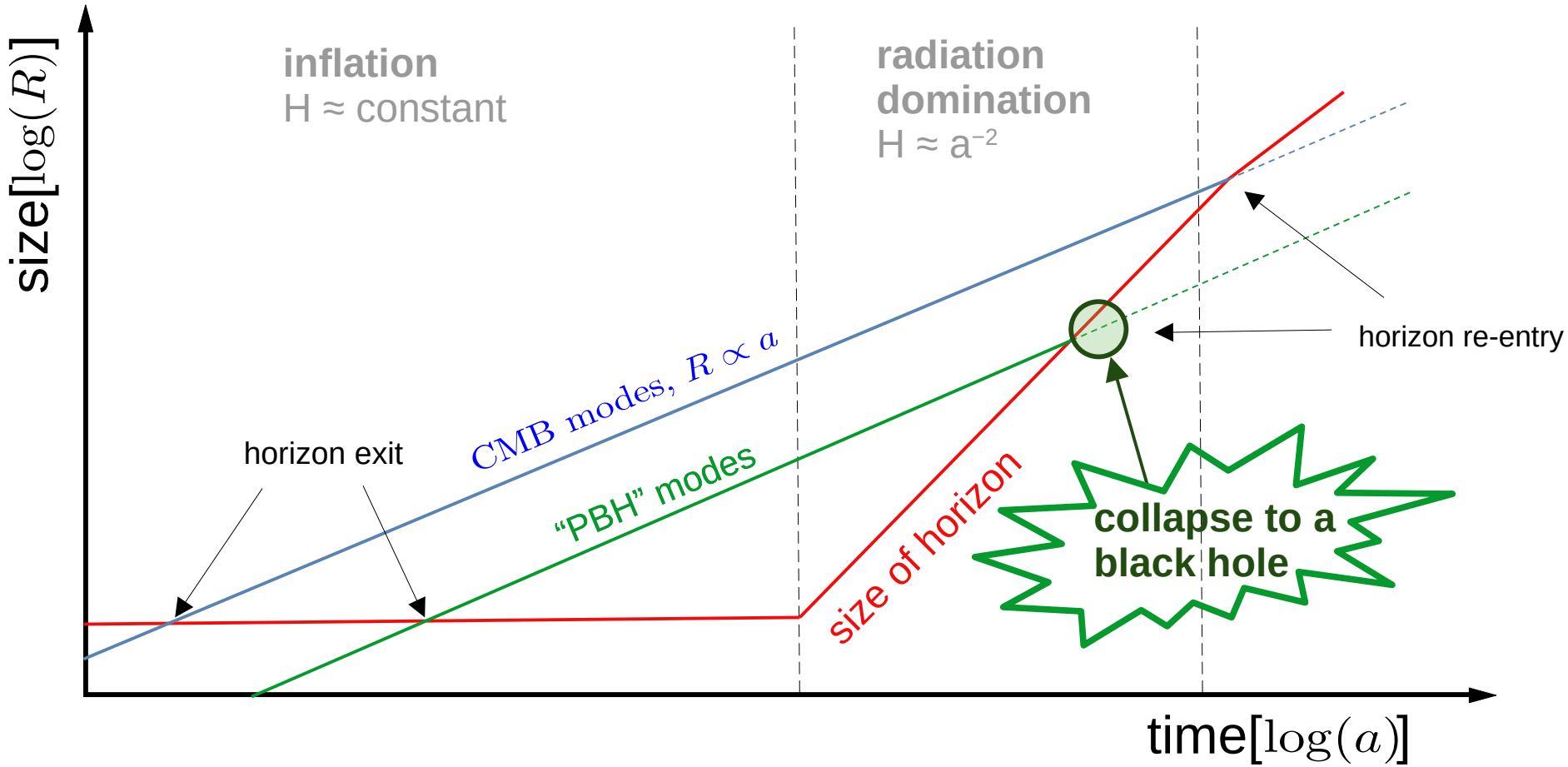


Observed PBH binaries
$M_{\rm PBH} \approx 1 - 100 M_\odot$
$f_{\rm PBH} \approx 10^{-3}$

PBHs as SMBH seeds
$M_{\rm PBH} \gtrsim 10^4 M_\odot$
$f_{PBH} \lesssim 10^{-6}$

PBH formation

general idea: collapse of large inhomogeneities in the early universe



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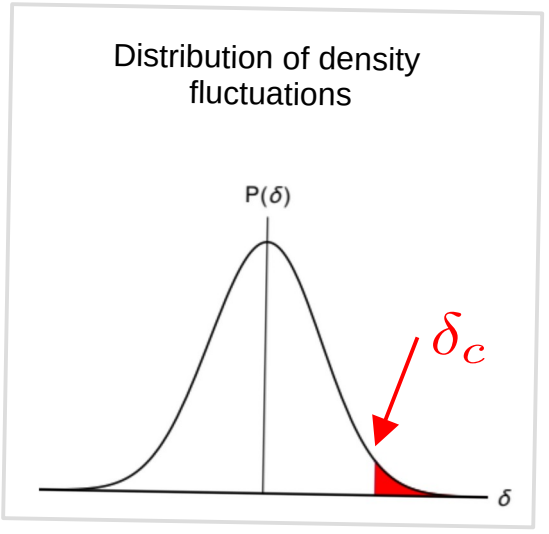


Critical collapse

$$M_{\rm PBH} = kM_k(\delta - \delta_c)^\gamma$$

* collapse parameters depend on the shape of perturbations
* density contrast non-linearly related to curvature fluctuations
* non-gaussianities (e.g. exponential tails)

Average PBH mass

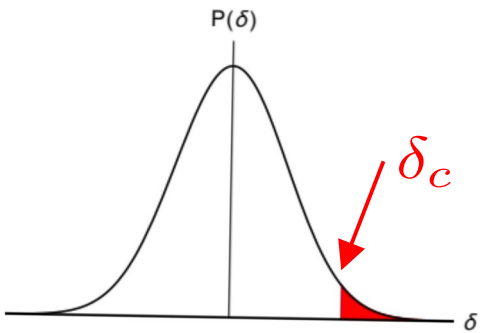
$$\langle M_{\rm PBH} \rangle \approx \mathcal{O}(3M_k)$$

SIZE of perturbations: $\mathcal{P}_{\mathcal{R},\rm peak} \approx 10^{-3} - 10^{-2}$

SCALE of perturbations: $k_{\rm peak} \lesssim 5 \times 10^{14} {\rm Mpc}^{-1}$

PBH formation

Distribution of density fluctuations



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Average PBH mass

$$\langle M_{\rm PBH} \rangle \approx \mathcal{O}(3M_k)$$

SIZE of perturbations:

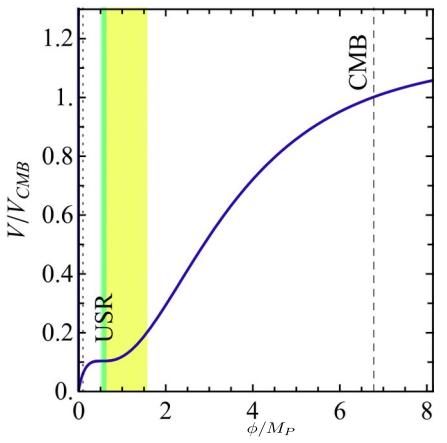
$$N \lesssim 37$$

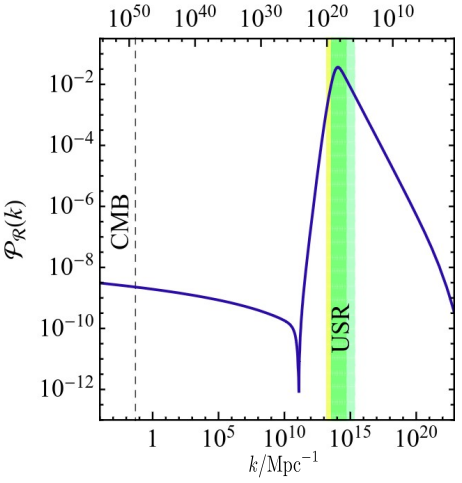
e-folds between the exit CMB and PBH modes

SCALE of perturbations: $k_{\rm peak} \sim 3 \times 10^{-2}\,{\rm Mpc}^{-1}$

Inflationary timeline

[2205.13540 Karam et al]



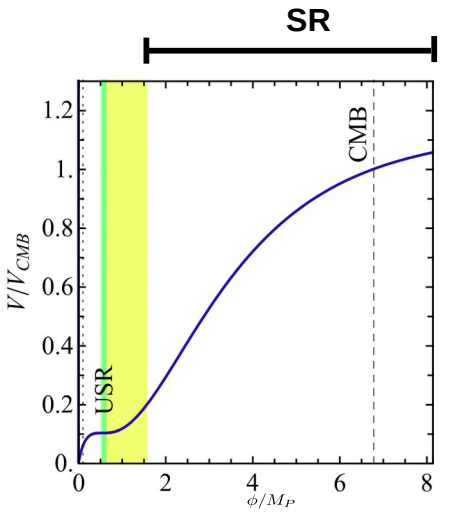


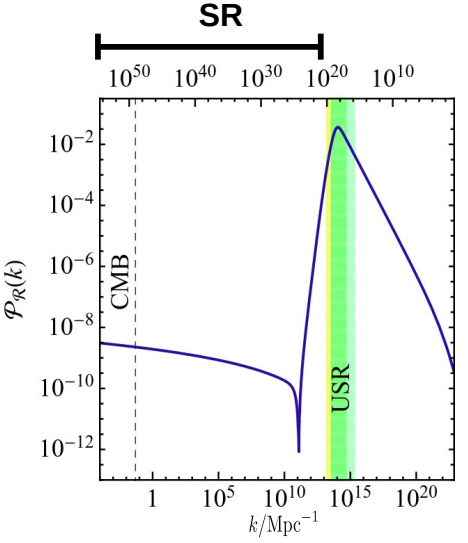
Inflationary timeline

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

lasts $\mathcal{O}(30)$ e-folds





Inflationary timeline

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1. SLOW-ROLL (SR)

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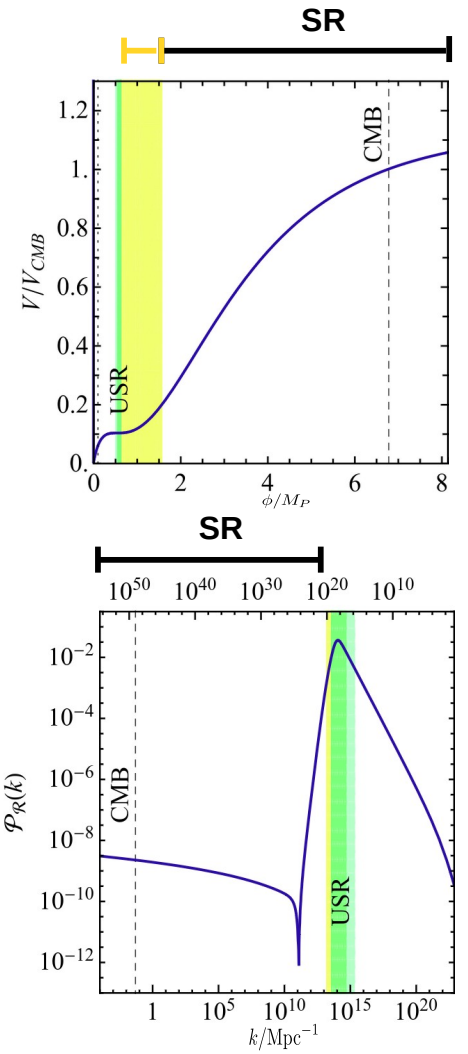
2. Transition from SR to USR

lasts $\lesssim 1$ e-folds



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1. SLOW-ROLL (SR)

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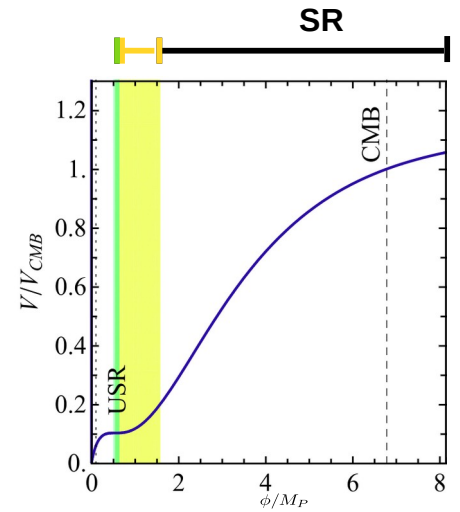
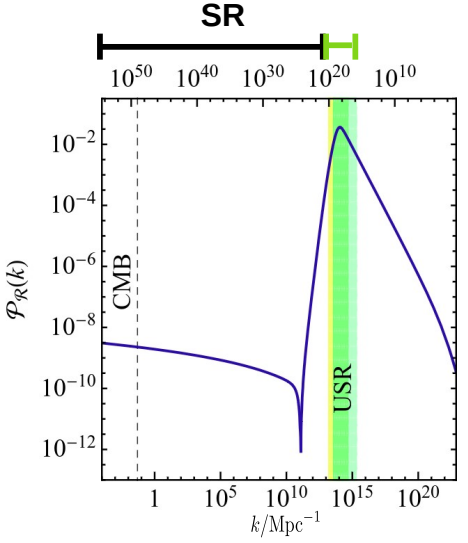
2. Transition from SR to USR

lasts $\lesssim 1$ e-folds

3. ULTRA SLOW-ROLL (USR)

lasts $\mathcal{O}(3)$ e-folds





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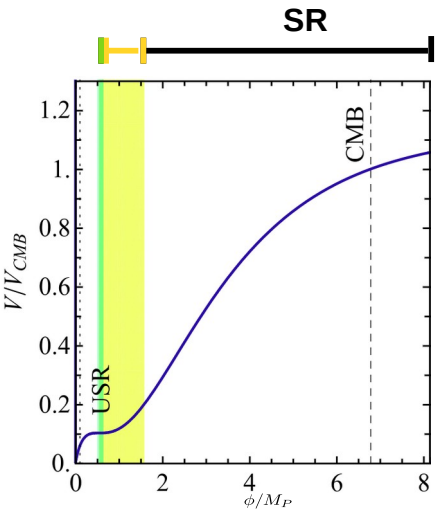
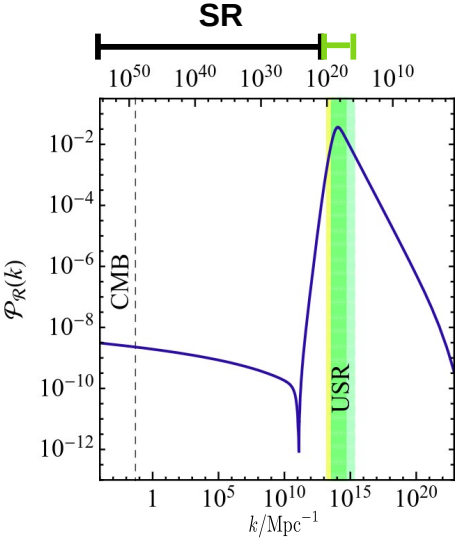
4. Transition from USR to CR





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lasts $\lesssim 1$ e-folds

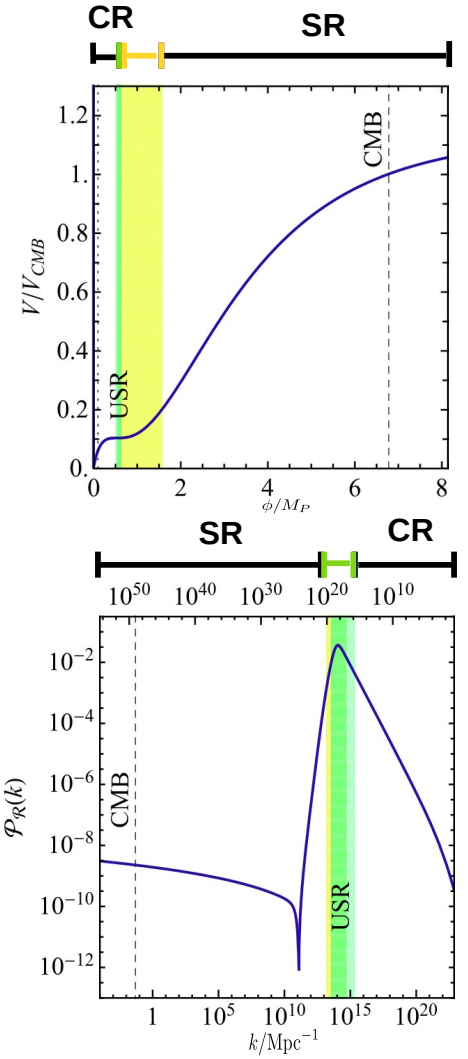
3. ULTRA SLOW-ROLL (USR)

lasts $\mathcal{O}(3)$ e-folds

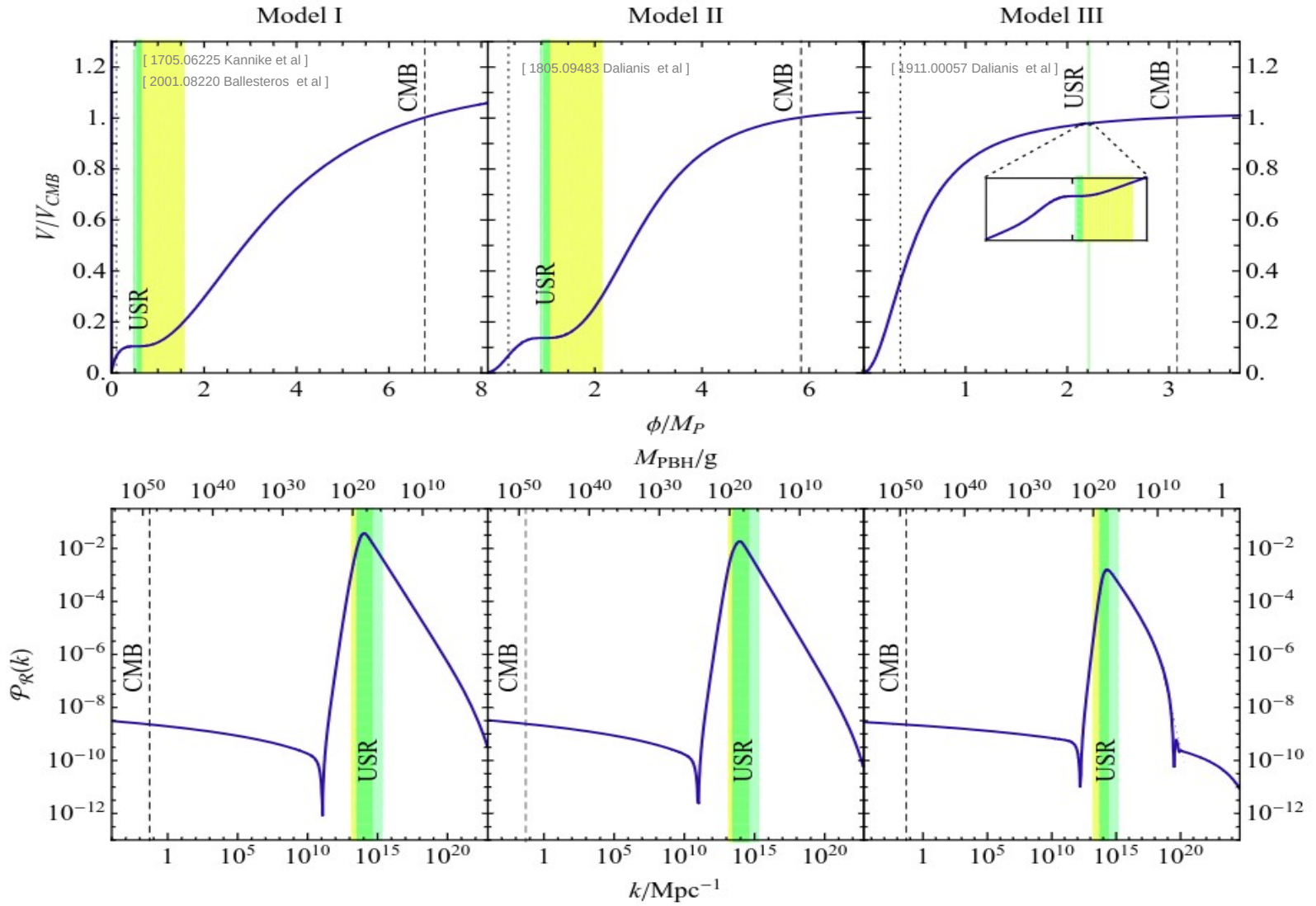
4. Transition from USR to CR

5. CONSTANT-ROLL (CR)

Can also be SR

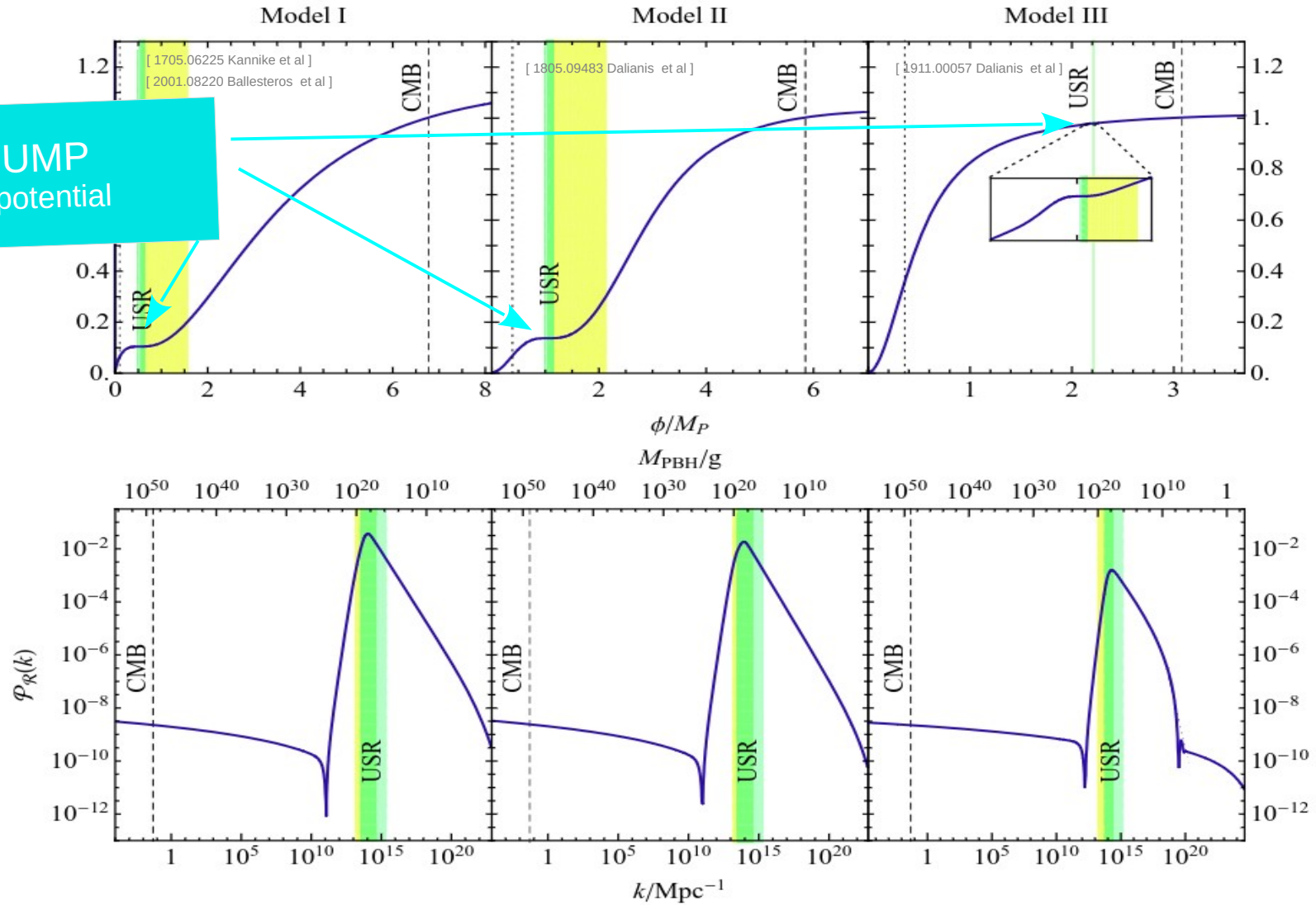


TYPICAL MODELS



TYPICAL MODELS

tiny BUMP
in the potential

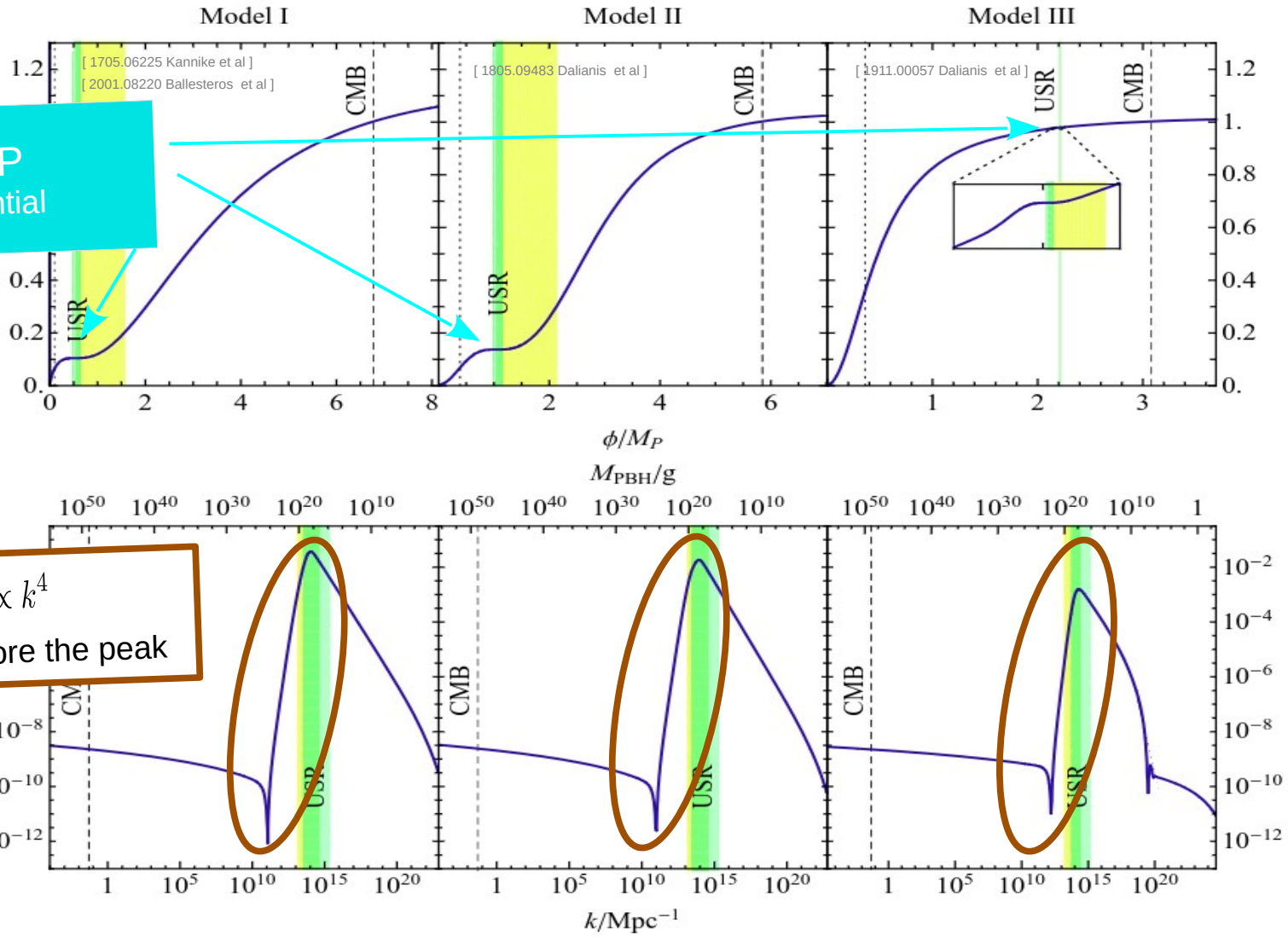


TYPICAL MODELS

tiny BUMP
in the potential

$\mathcal{P}_\mathcal{R} \propto k^4$

growth before the peak



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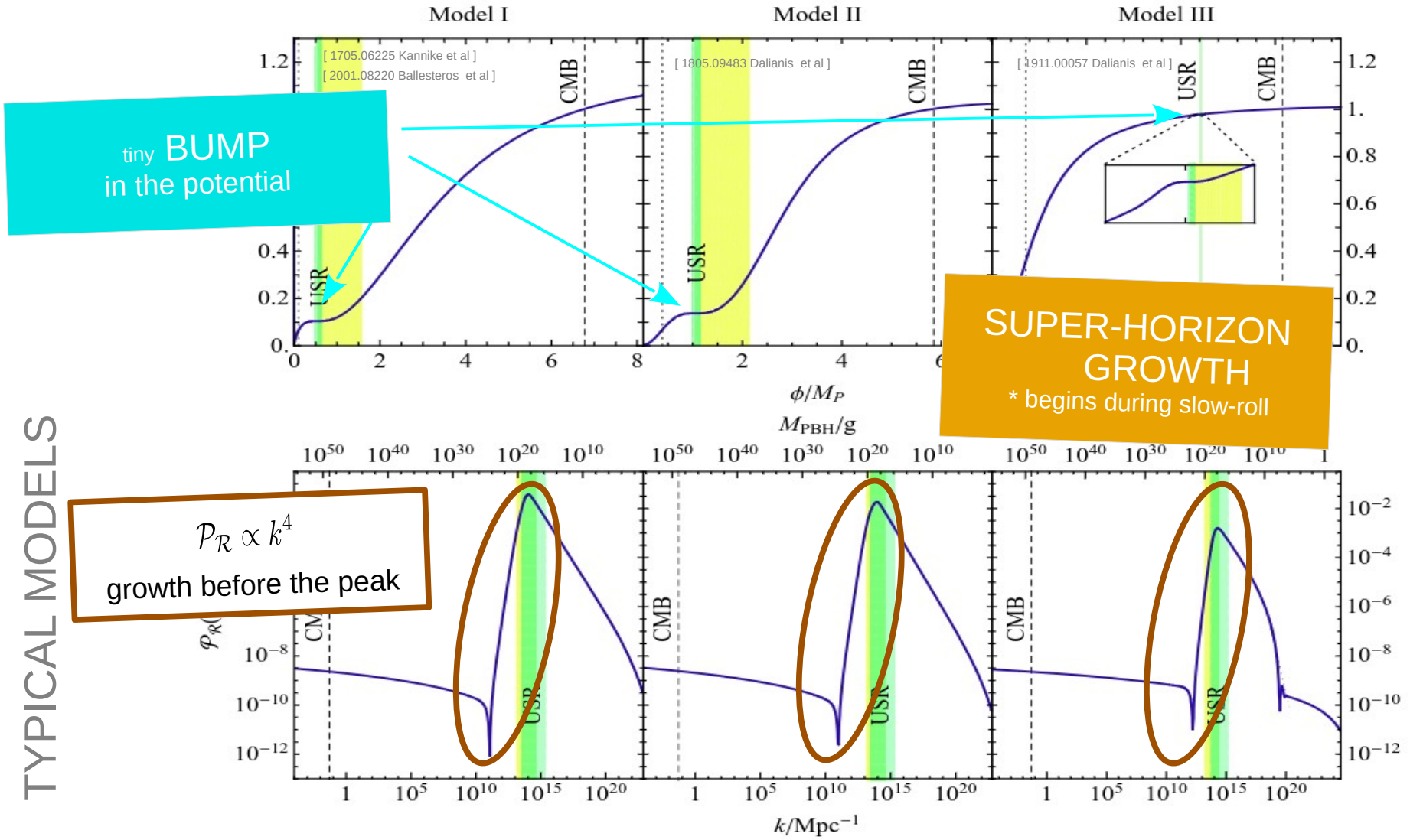

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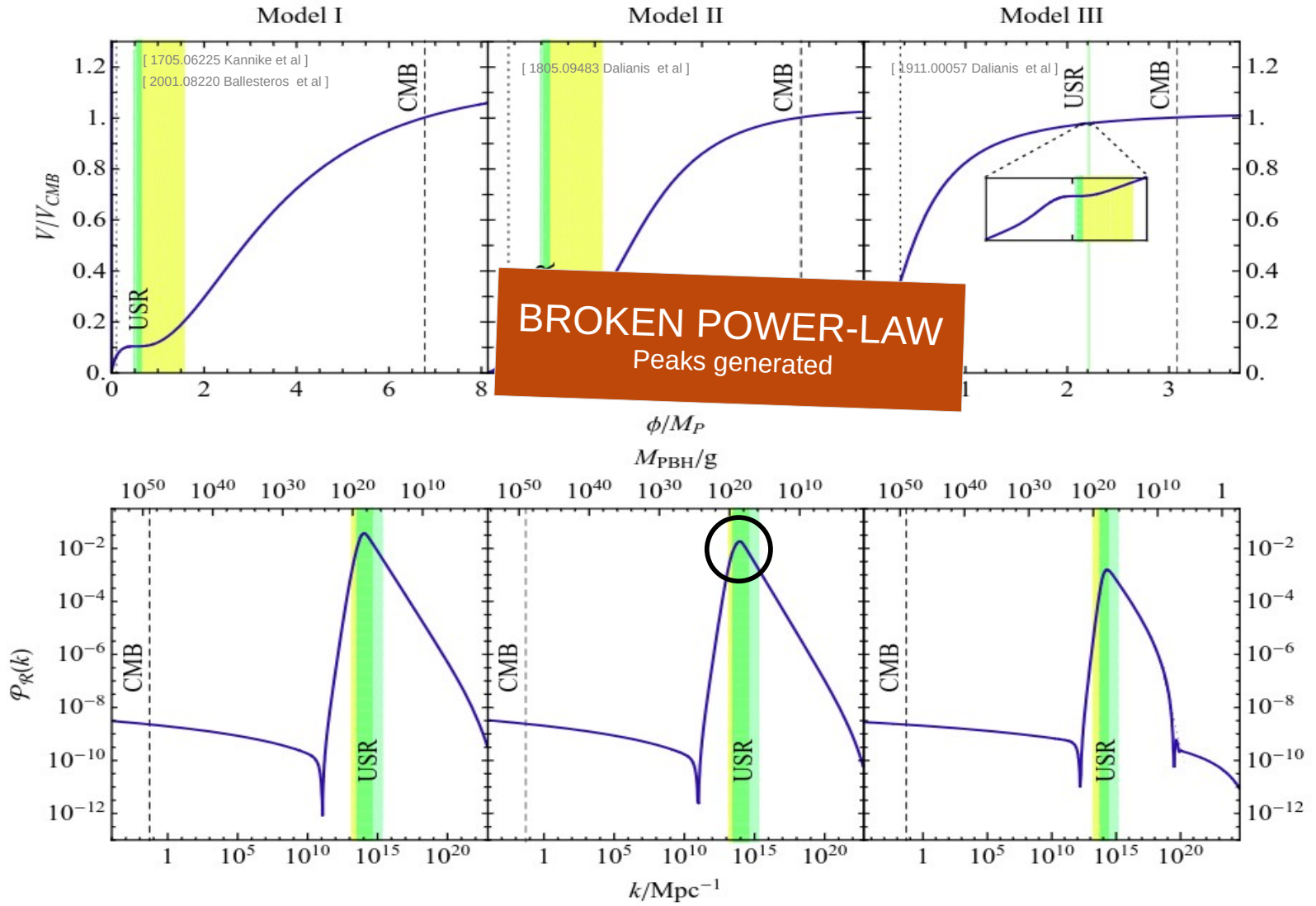

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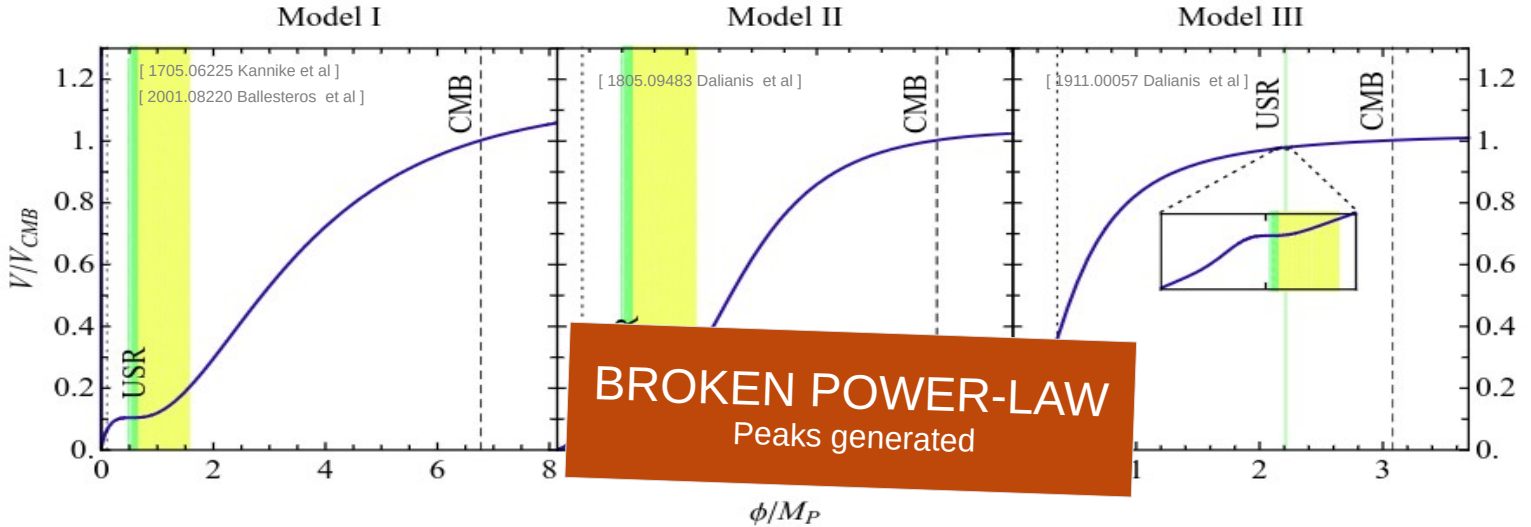
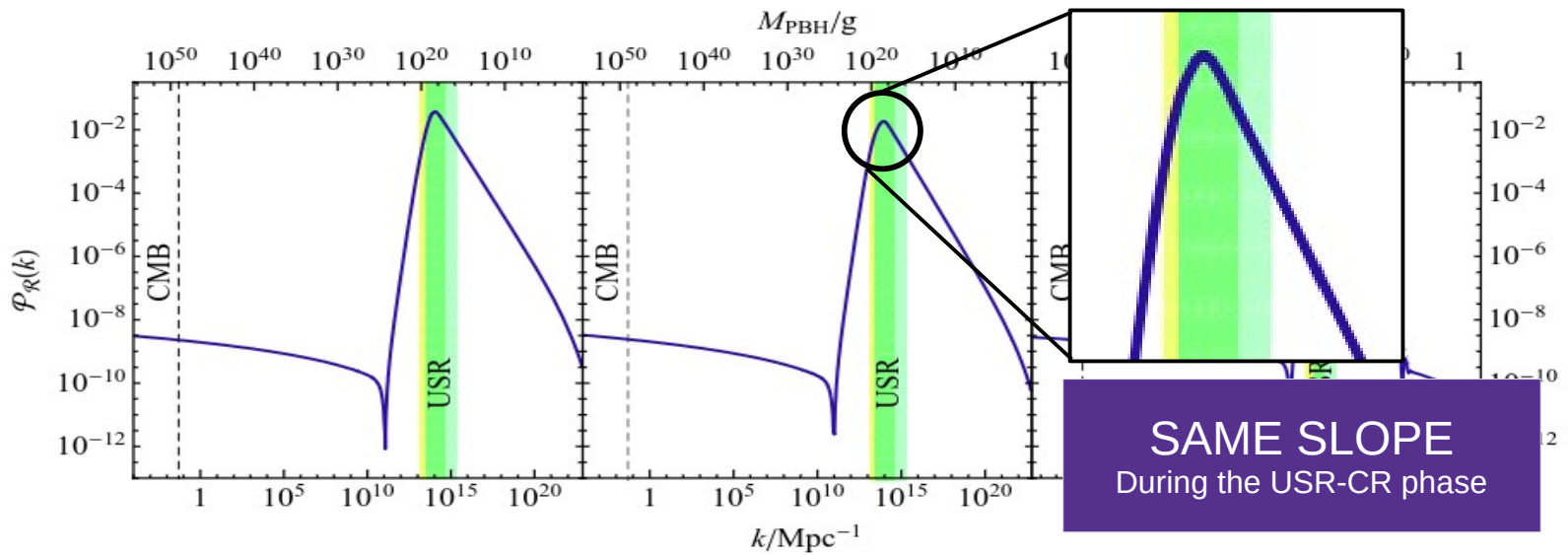


BROKEN POWER-LAW
Peaks generated



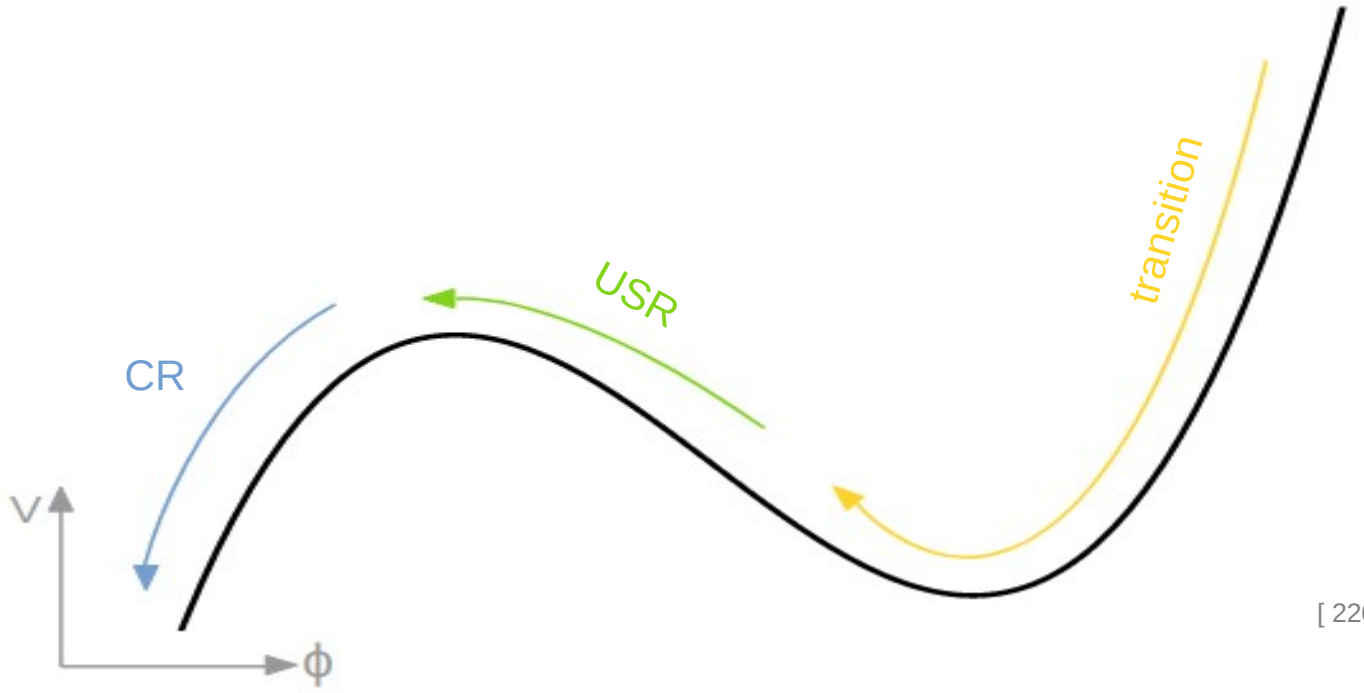
SAME SLOPE
During the USR-CR phase

Wands duality

New attractor phase begins

inflaton slows down

inflaton speeds up

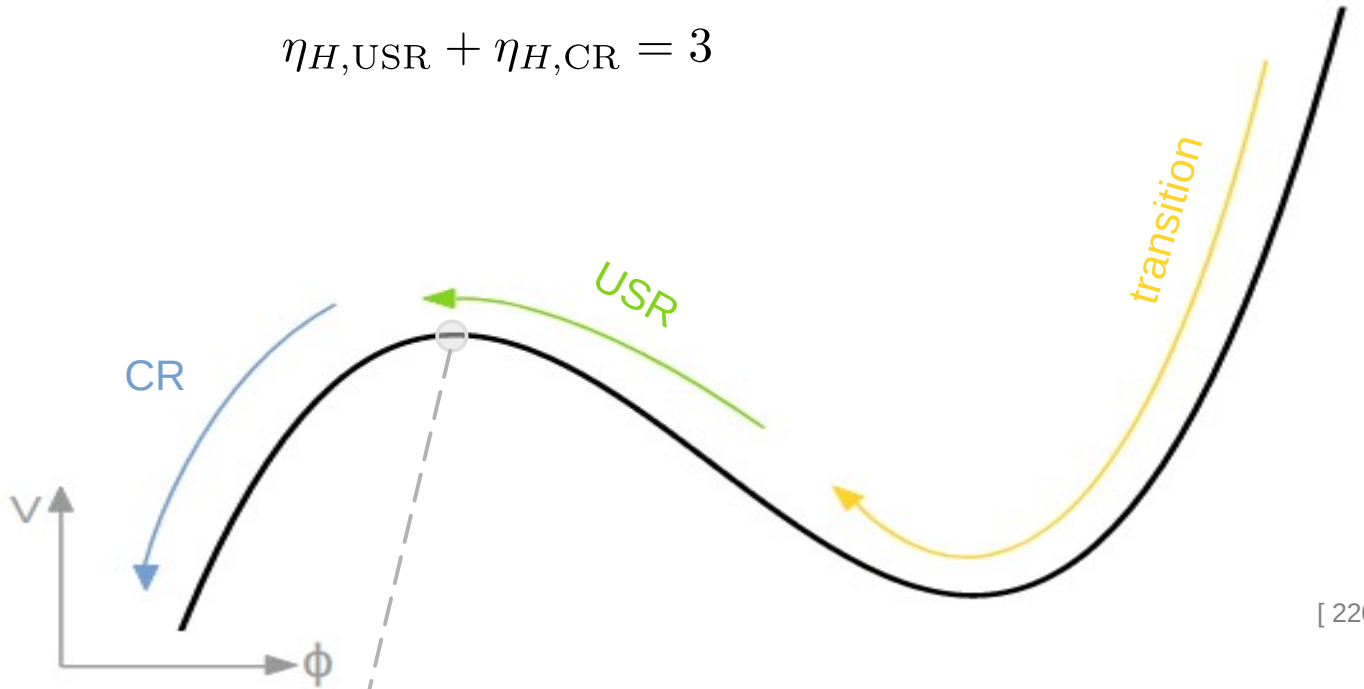


[2205.13540 Karam et al]

Wands duality

New attractor phase begins | inflaton slows down | inflaton speeds up

$$\eta_{H,\mathrm{USR}} + \eta_{H,\mathrm{CR}} = 3$$



[2205.13540 Karam et al]

expansion around maximum:

$$\partial_N^2\phi + 3\partial_N\phi \approx -\eta_{V,c}(\phi - \phi_c)$$

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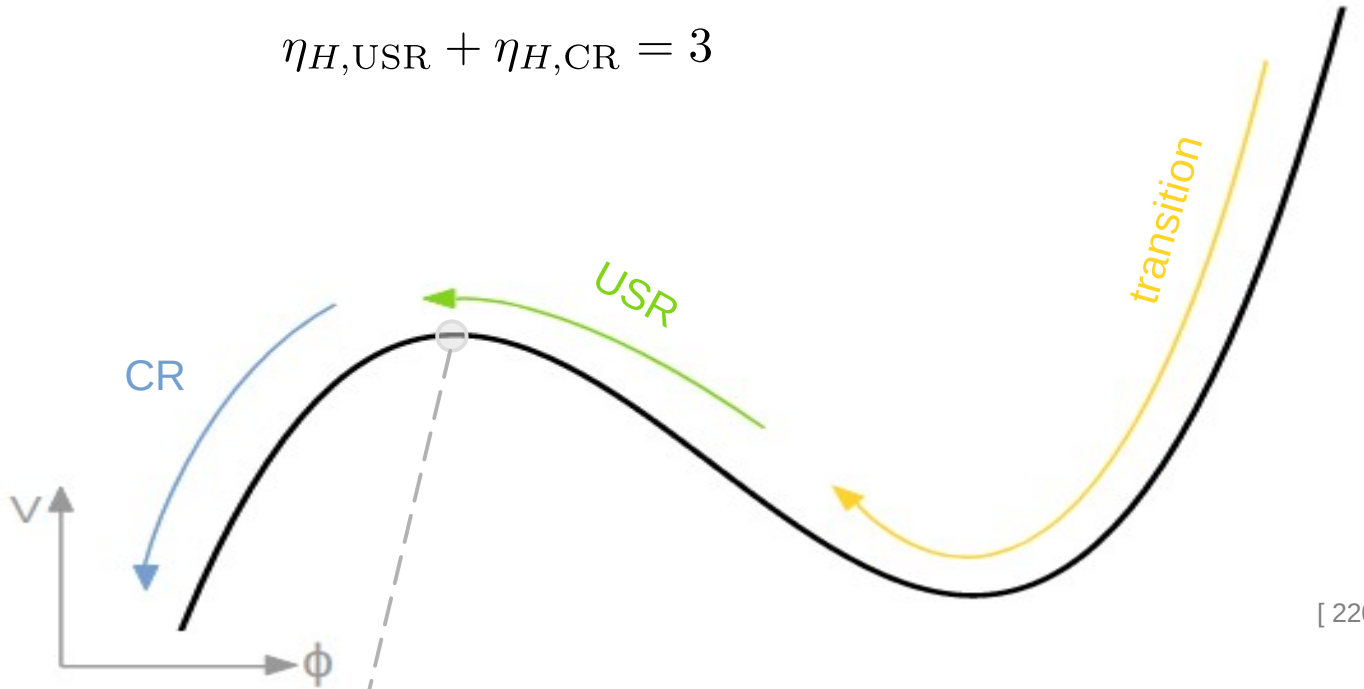

[2205.13540 Karam et al]

expansion around maximum:

$$\partial_N^2\phi + 3\partial_N\phi \approx -\eta_{V,c}(\phi - \phi_c) \quad \Rightarrow \quad u_k'' + \left(\frac{z''}{z} - k^2\right) u_k = 0 \qquad z \equiv a\partial_N\phi$$

same evolution of modes exiting during USR and CR

Wands duality

New attractor phase begins

inflaton speeds up

Smoothness => DUALITY

[2205.13540 Karam et al]

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A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITION

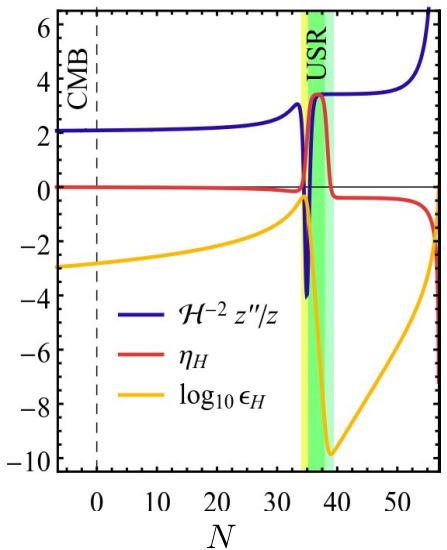
THE ANSATZ

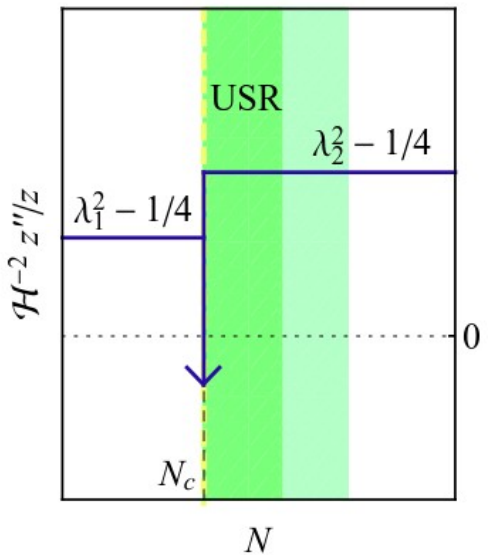
1. instantaneous SR to USR transition

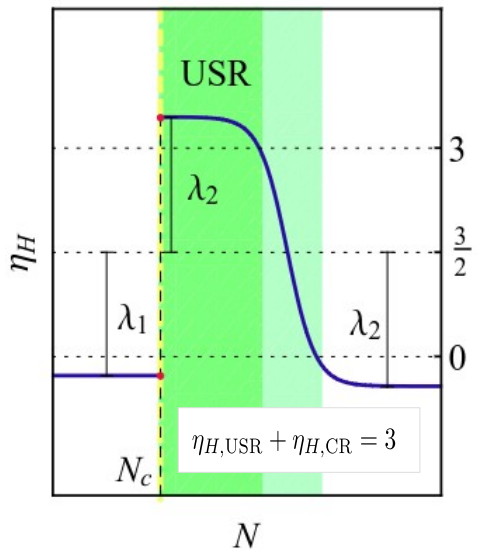
2. respects the dual relation

$$\eta_{H,\mathrm{USR}} + \eta_{H,\mathrm{CR}} = 3$$

[2205.13540 Karam et al]







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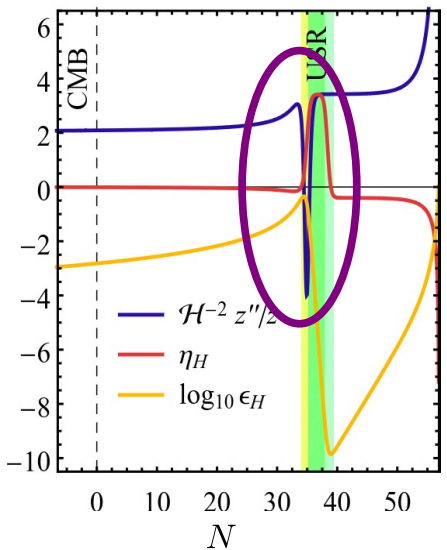

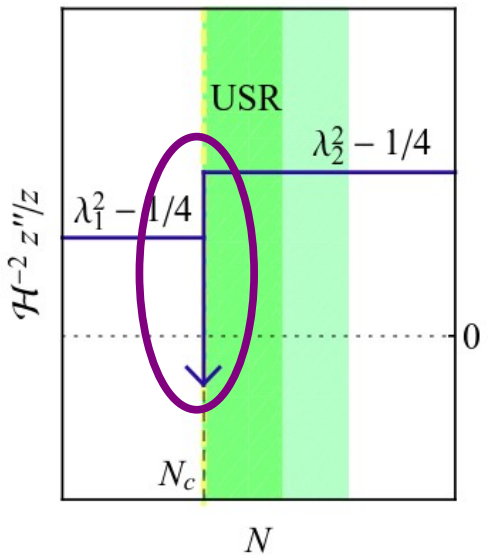

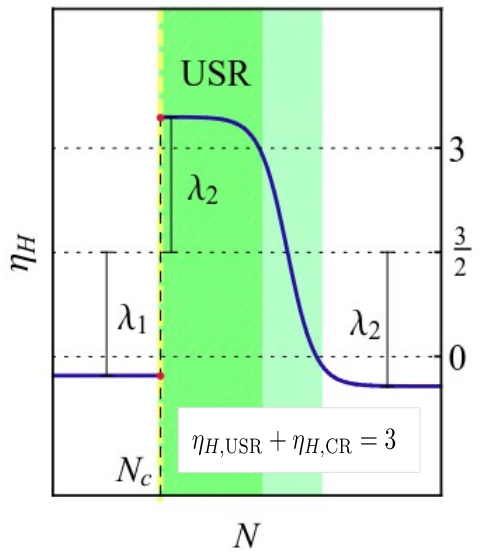

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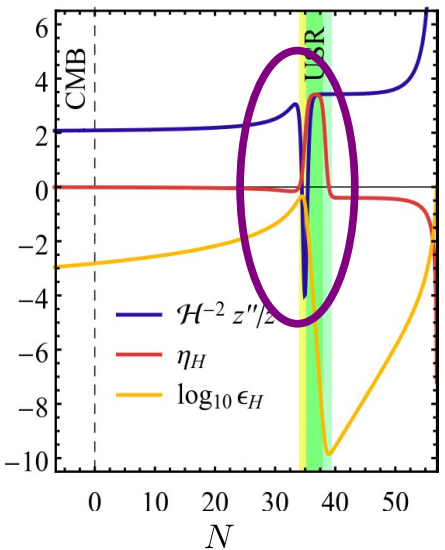
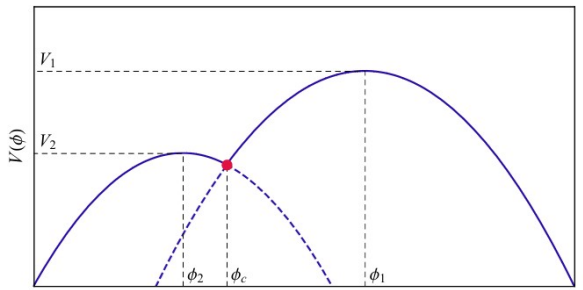
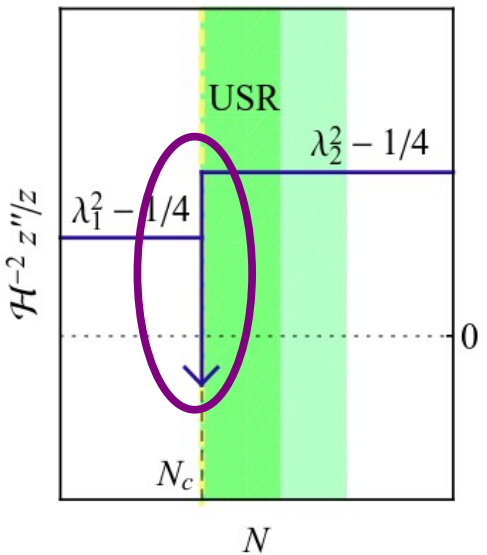
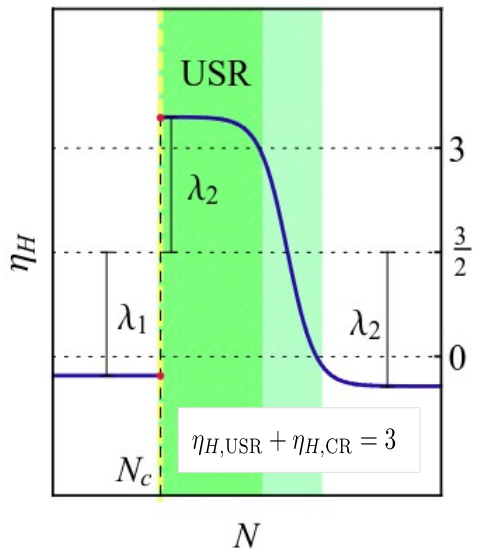


DERIVABLE FROM A POTENTIAL



NOT JUST AN ANSATZ ON THE BACKGROUND!





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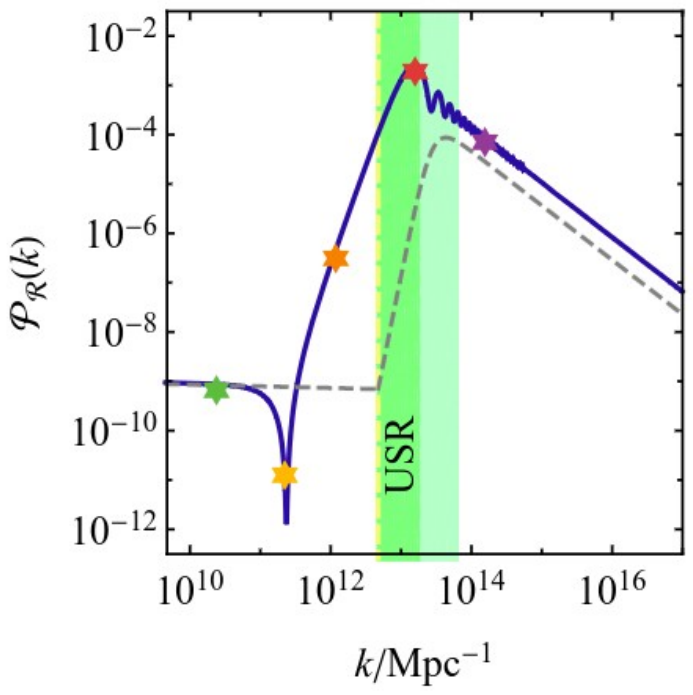
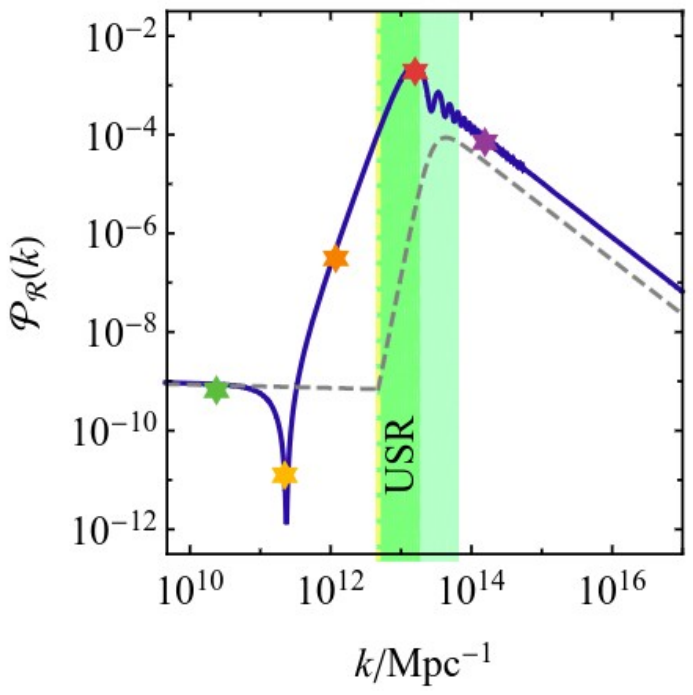


CURVATURE POWER SPECTRUM

A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITION

[2205.13540 Karam et al]

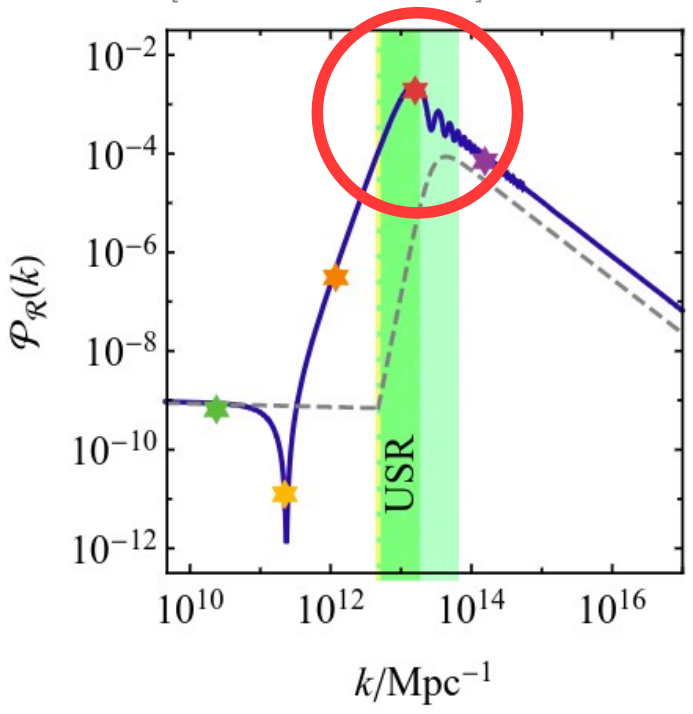


CURVATURE POWER SPECTRUM

1. **can be found analytically**
2. **contains power spectra viable for both PBHs and CMB**
3. **approximates quasi-inflection point models quite well**

A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITION

[2205.13540 Karam et al]



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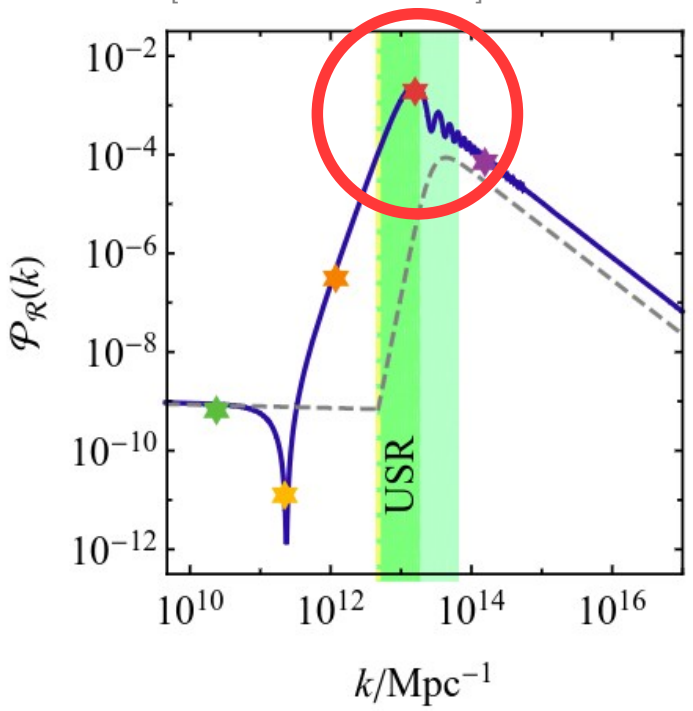

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=> PEAK SHAPE DEPENDS ON THE SR to USR TRANSITION

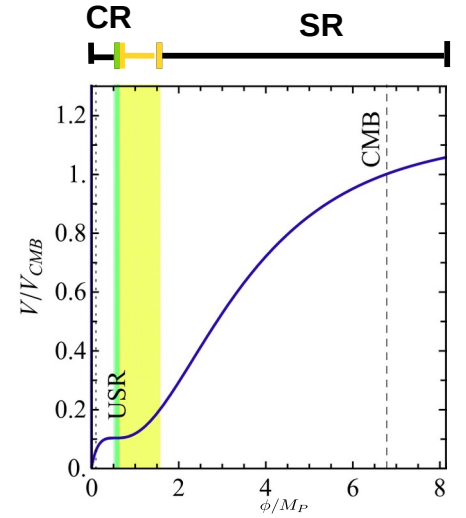
Inflationary timeline

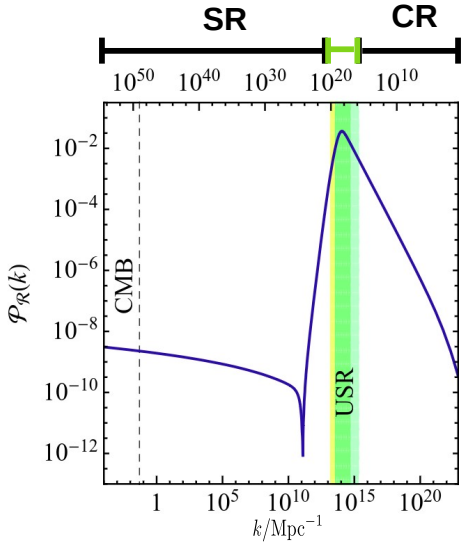
[2205.13540 Karam et al]

1. SLOW-ROLL (SR)
   CMB
2. Transition from SR to USR
   PEAK SHAPE

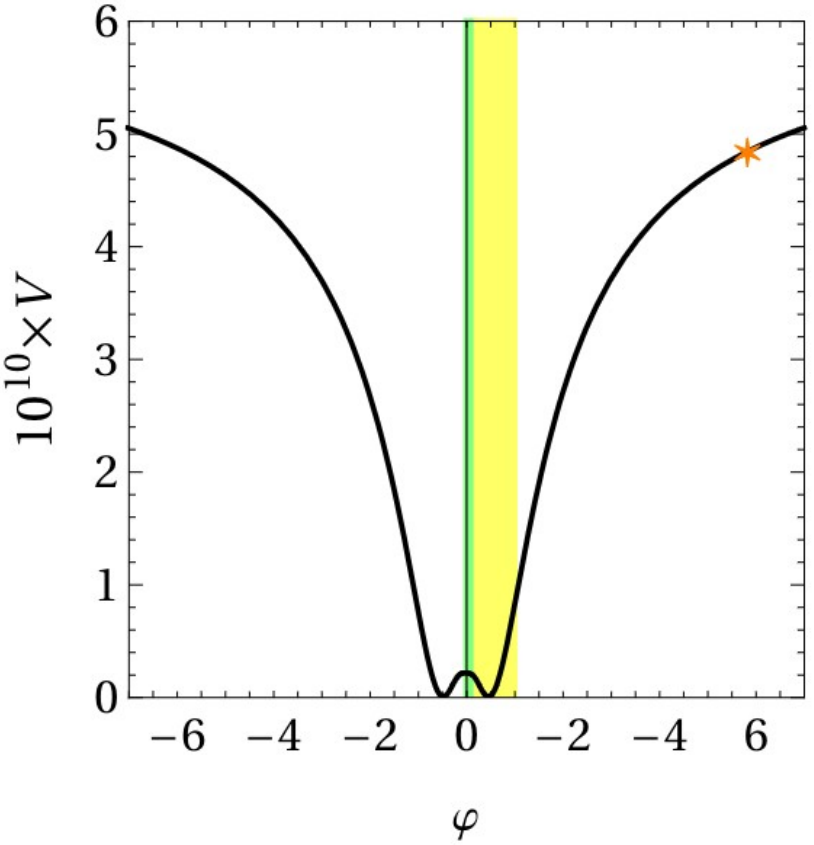
DUALITY

3. ULTRA SLOW-ROLL (USR)
4. Transition from USR to CR
5. CONSTANT-ROLL (CR)





DOUBLE-WELL POTENTIALS

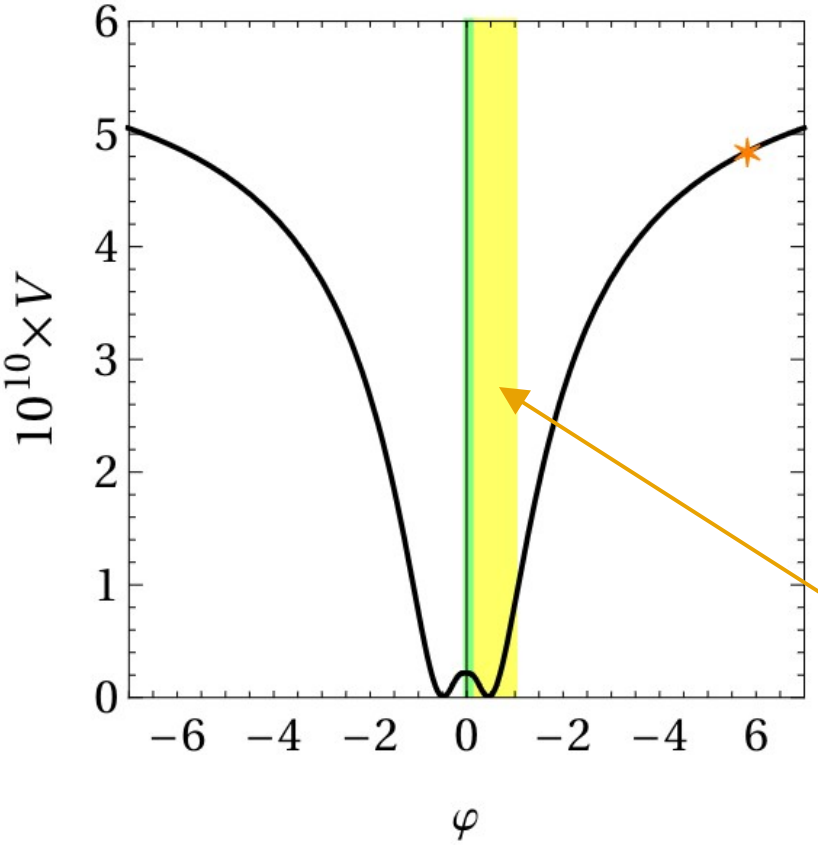


[2305.09630 Karam et al]

same inflationary timeline: SR to USR to CR

DOUBLE-WELL POTENTIALS

[2305.09630 Karam et al]

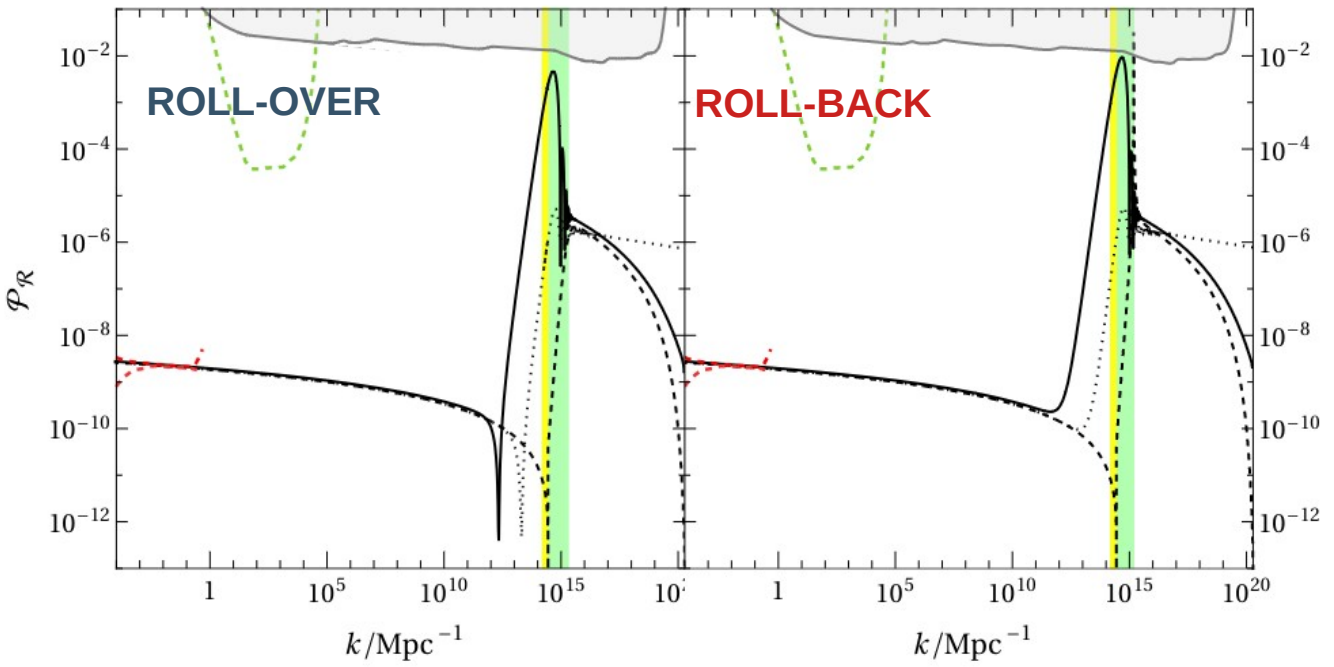


inflation temporarily halted!

same inflationary timeline: SR to USR to CR

DOUBLE-WELL POTENTIALS

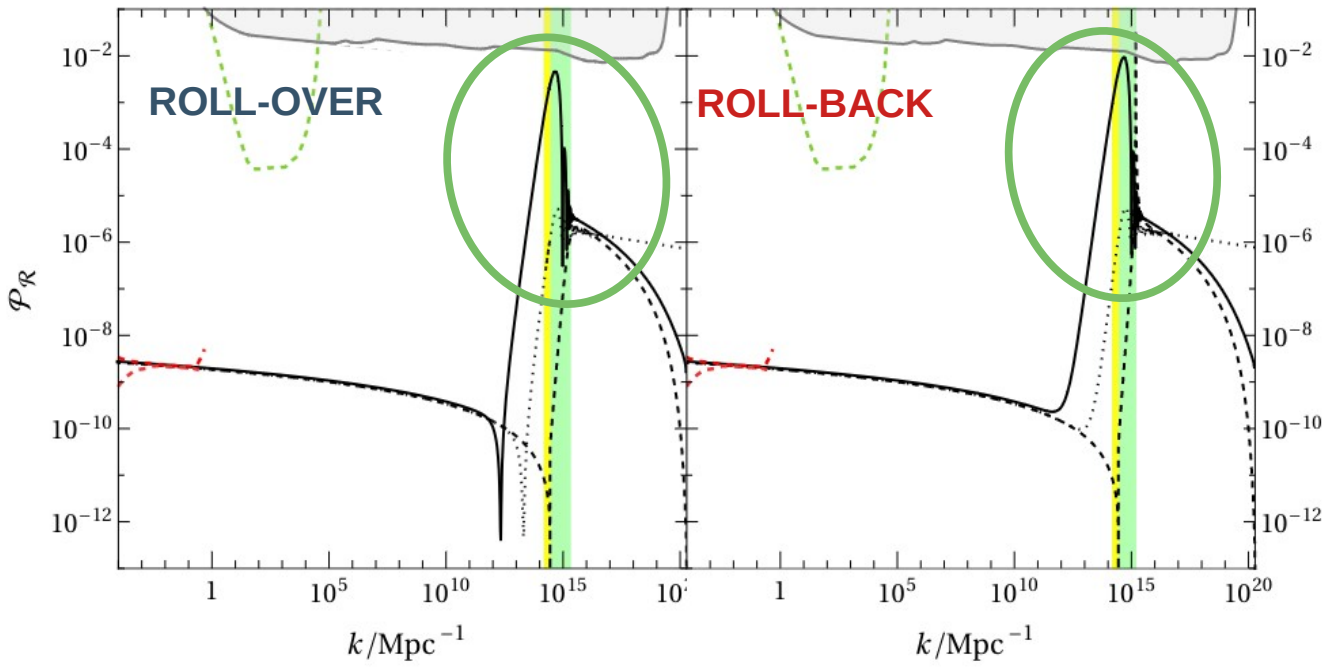
[2305.09630 Karam et al]



DOUBLE-WELL POTENTIALS

temporarily halted inflation => enhanced oscillatory features

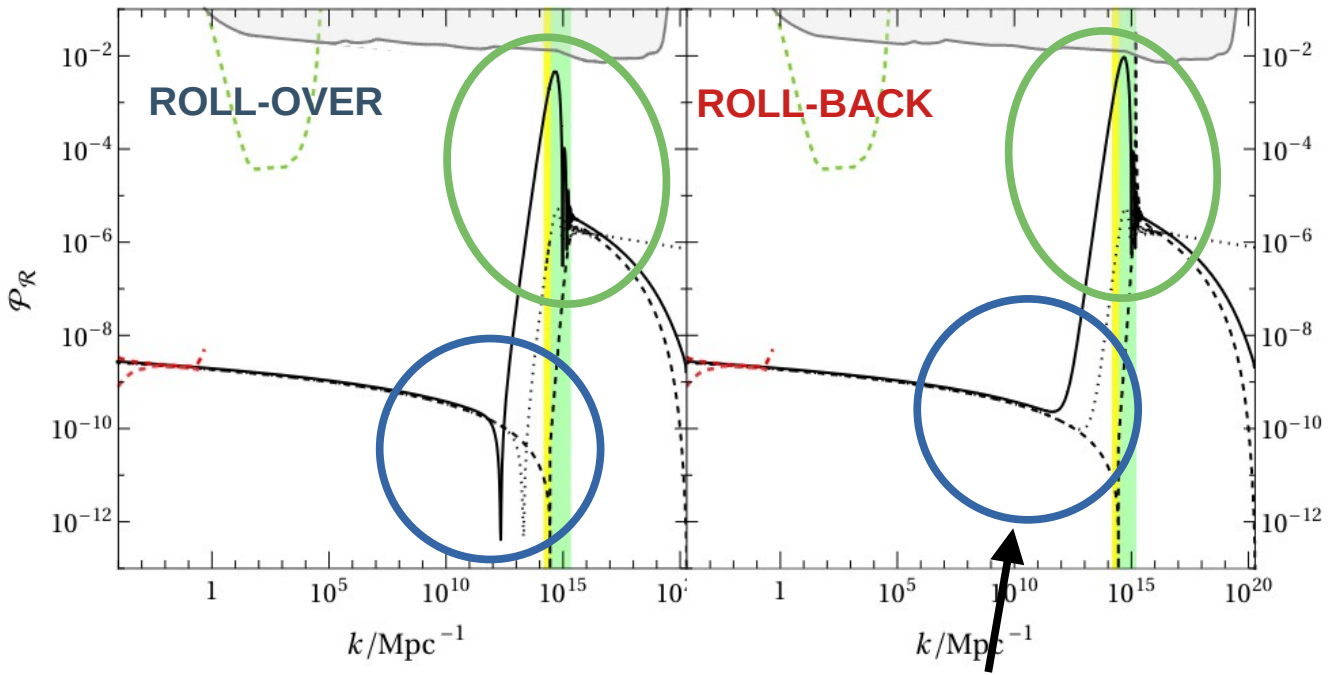
[2305.09630 Karam et al]



DOUBLE-WELL POTENTIALS

non-adiabaticity => enhanced oscillatory features

[2305.09630 Karam et al]

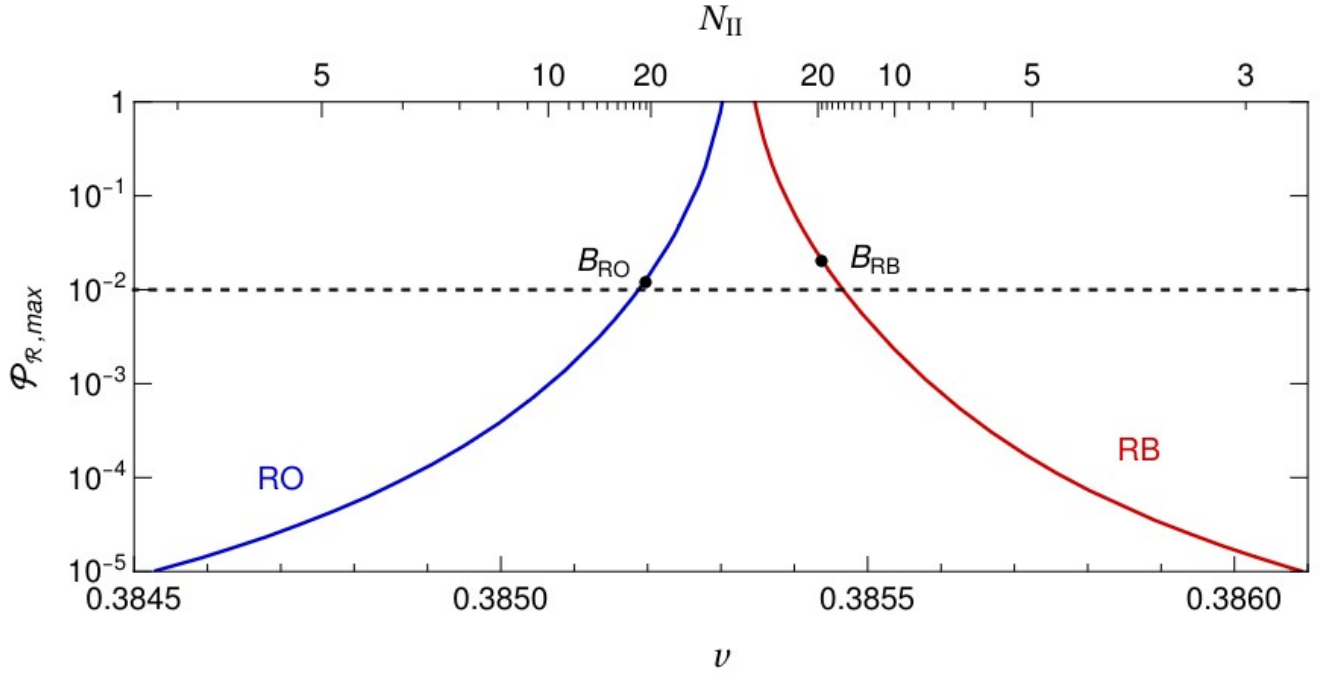


dip disappears!

FINE TUNING of the potential

NECESSITY for TUNING:

1. abundance **exponentially** sensitive to **peak height**
2. **peak height** sensitive to parameters of the model



FINE TUNING of the potential

NECESSITY for TUNING:

1. abundance **exponentially** sensitive to **peak height**
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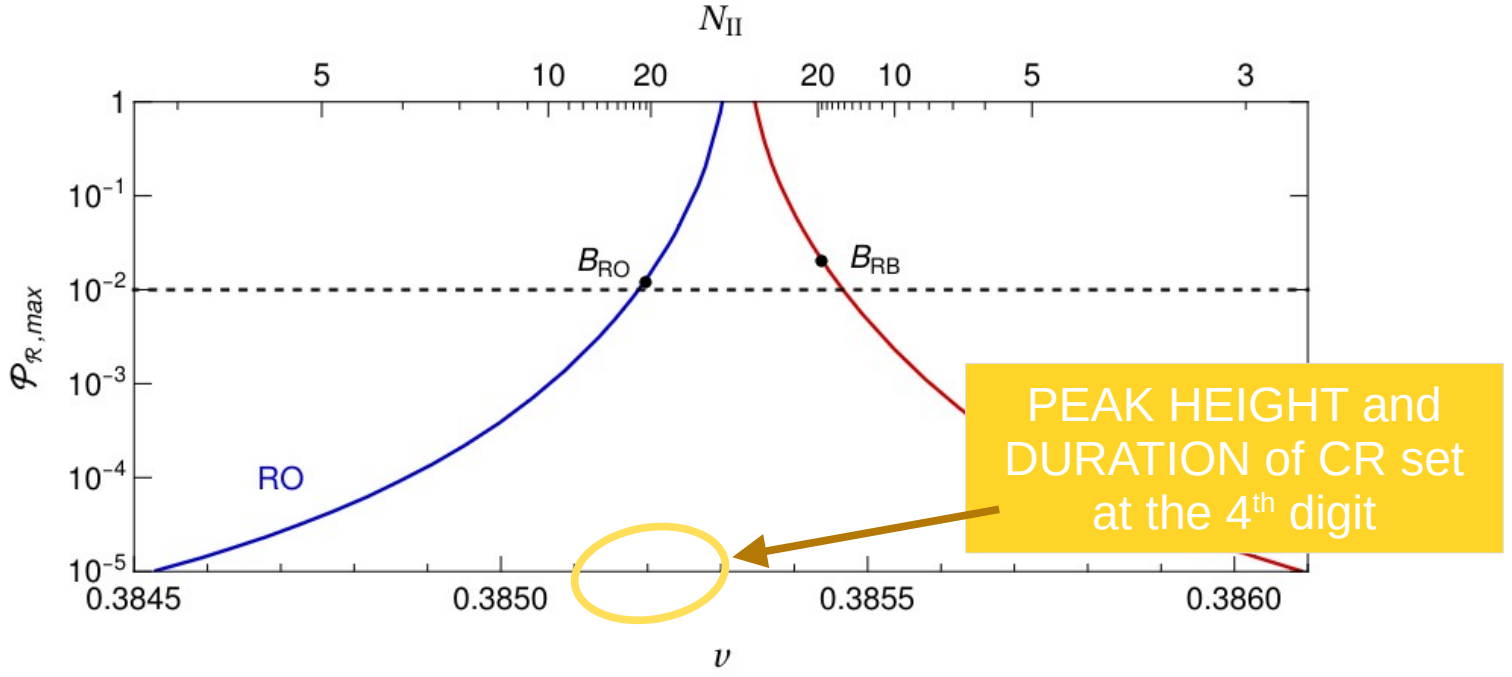

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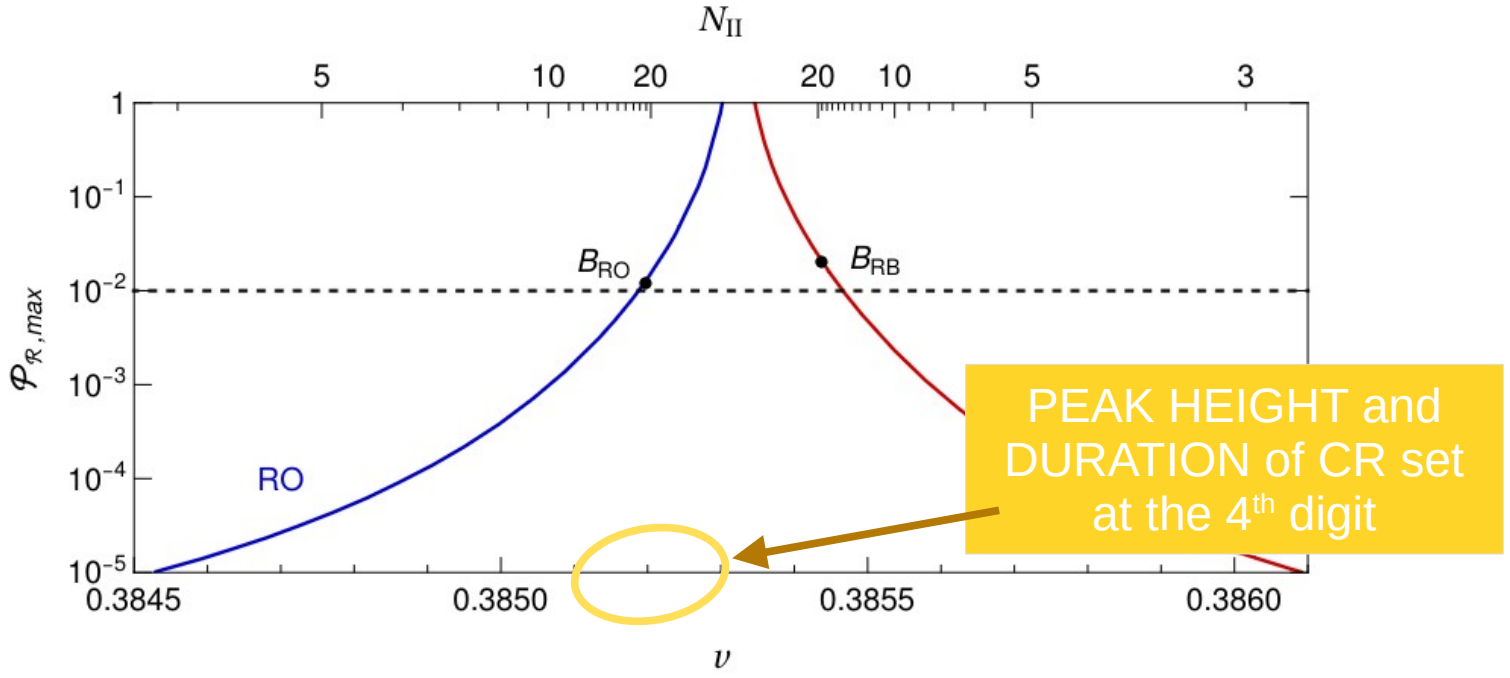
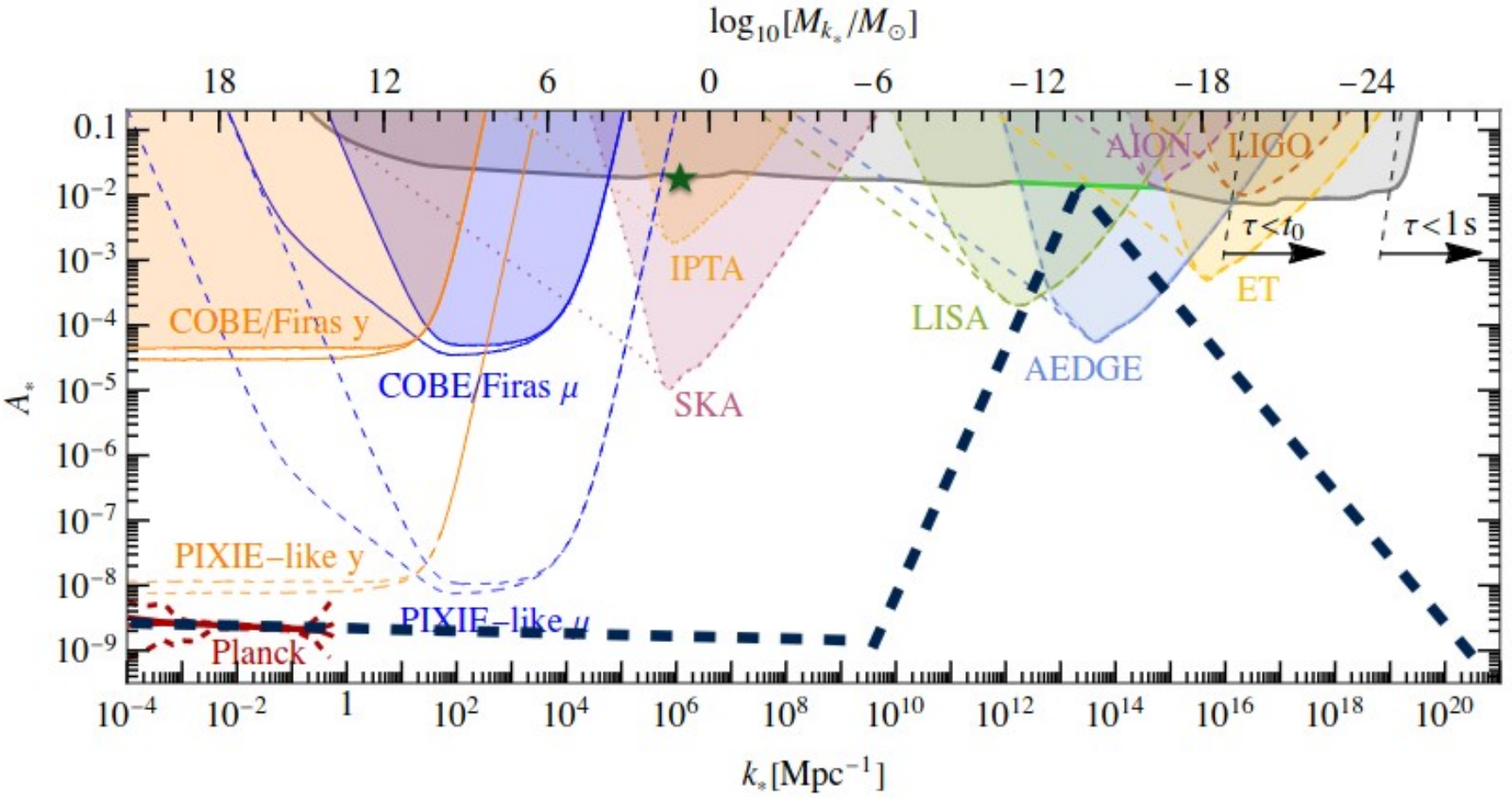


PEAK HEIGHT and DURATION of CR set at the 4th digit

less tuning with double-well potentials

Scalar induced GWs



SUMMARY

- **DM** as a byproduct of inflation

  (when $\mathcal{P}_{\mathcal{R},\mathrm{peak}} \approx 10^{-3} - 10^{-2}$ at $k \approx 10^{11} - 10^{15}\mathrm{Mpc}^{-1}$, testable in coming decades)

- quasi-inflection point models well understood

  (**USR-CR duality**, superhorizon enhancement, analytic approximations, k^4 growth)

- peak shape depends on the SR to USR transition

  (oscillatory features, non-adiabatic enhancement, double-well vs quasi-inflection point, amount of tuning)