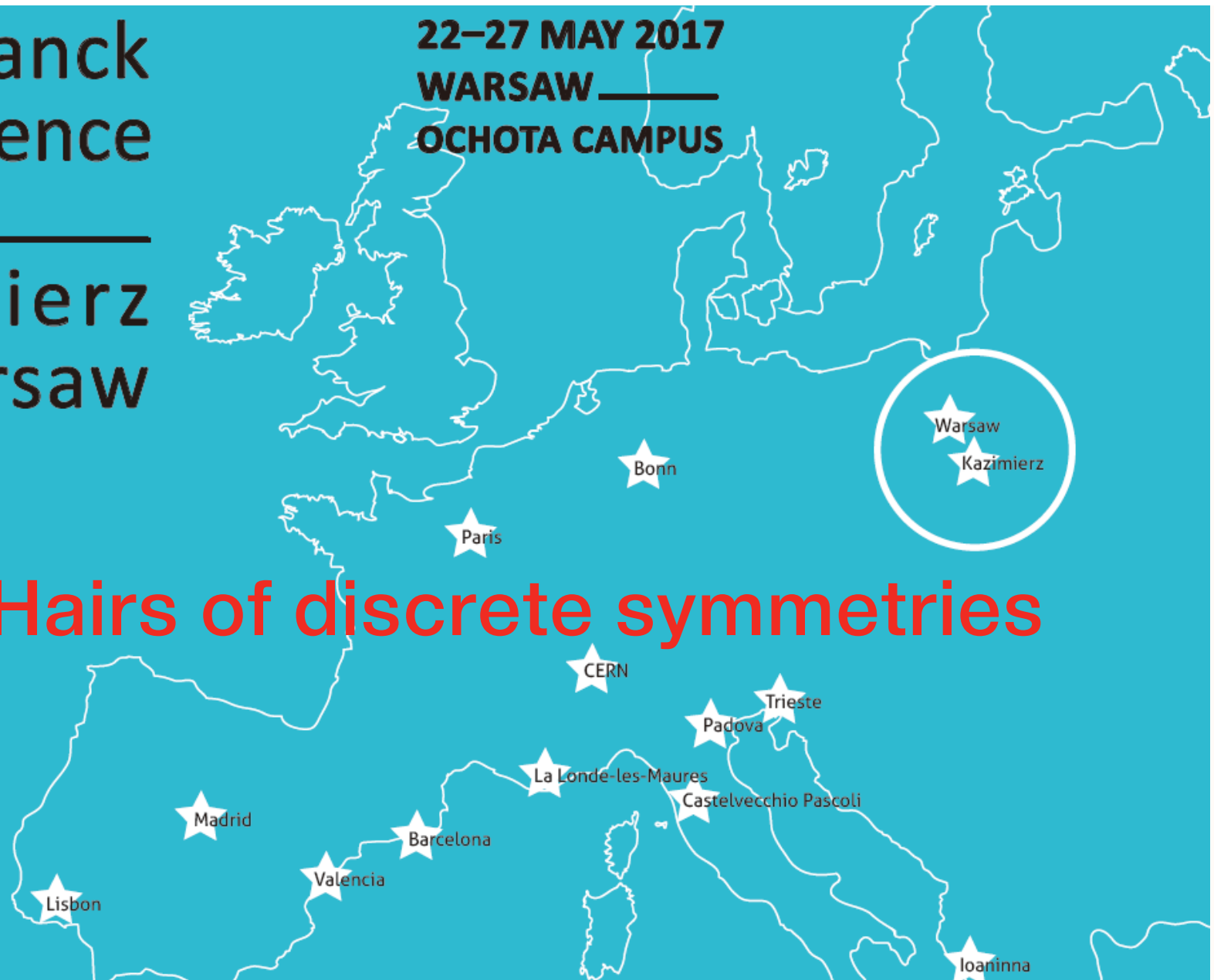


20<sup>th</sup> Planck  
Conference  
from \_\_\_\_\_  
Kazimierz  
to Warsaw

22–27 MAY 2017  
WARSAW \_\_\_\_\_  
OCHOTA CAMPUS

Hairs of discrete symmetries



# Hairs of discrete symmetries

(Top-down and bottom-up models)

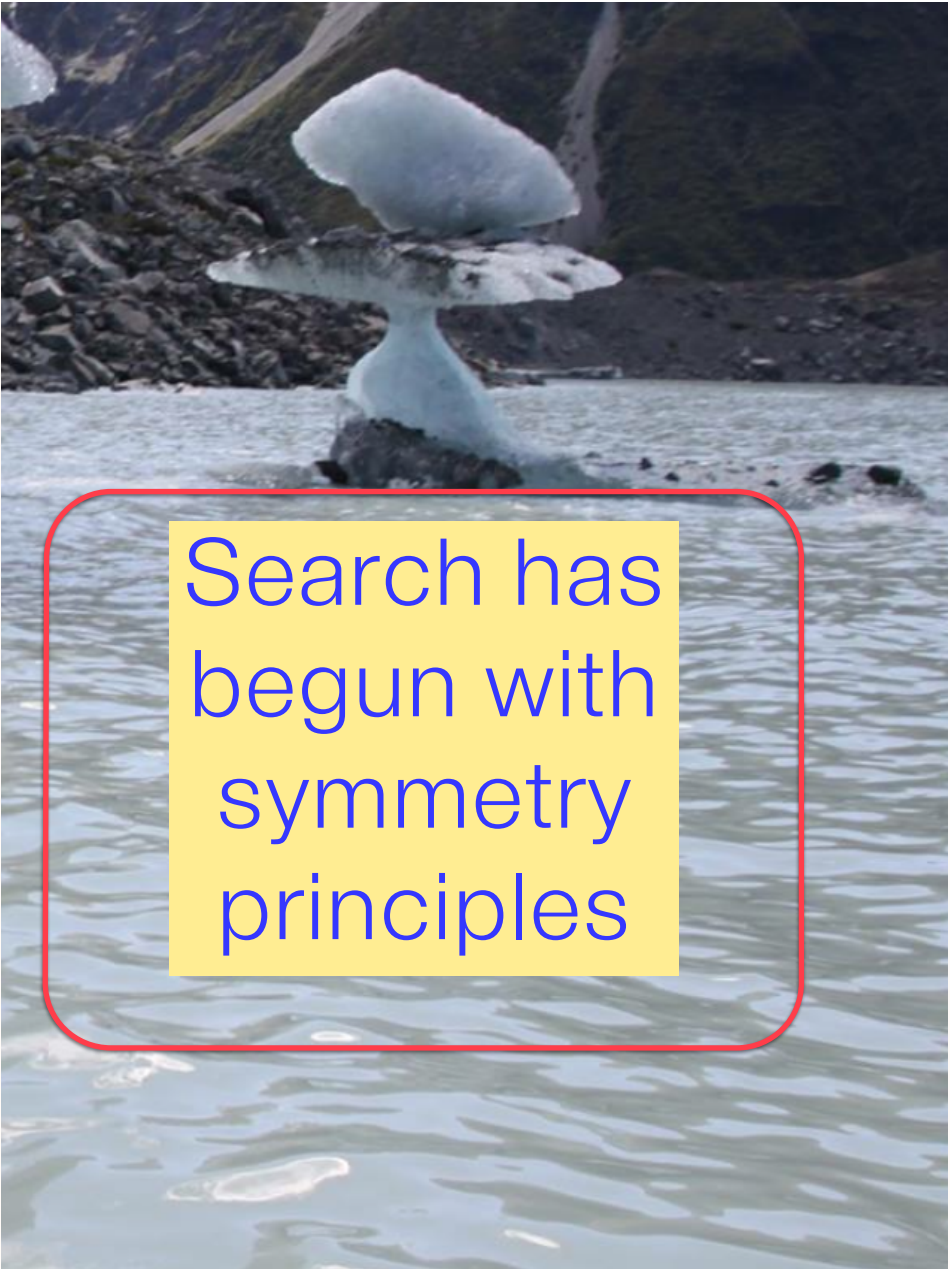
Jihn E. Kim

Kyung Hee University,  
CAPP, IBS,  
Seoul National Univ.

Warsaw, 26 May 2017

[arXiv:1703.05389](https://arxiv.org/abs/1703.05389) [hep-th]

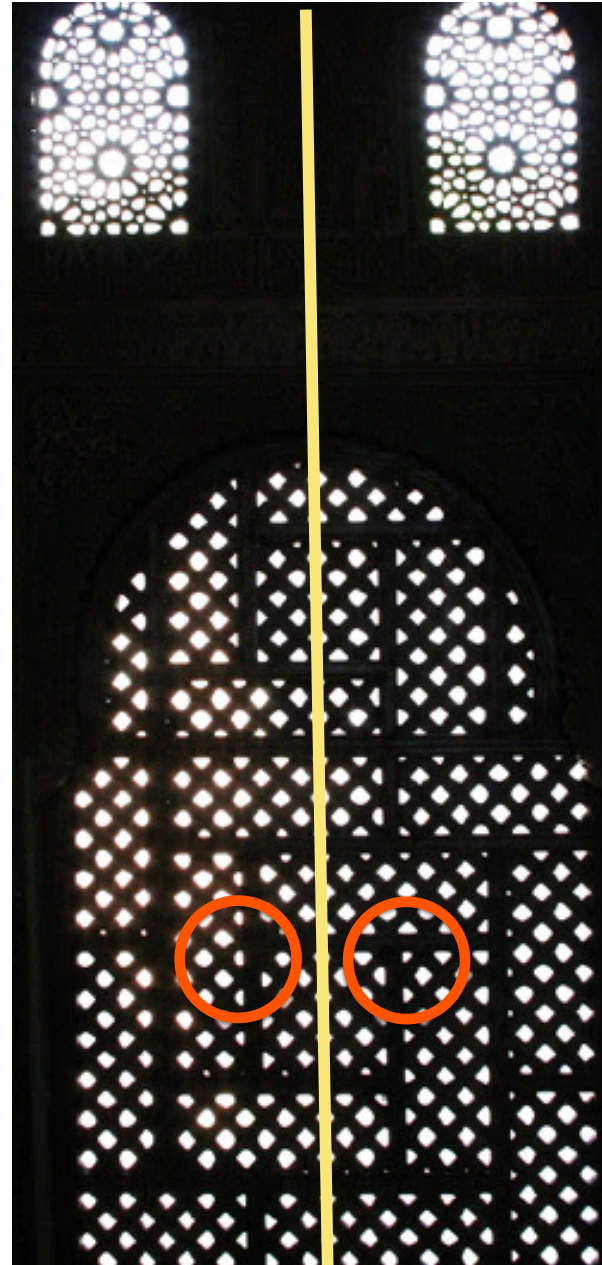
# 1. Discrete gauge symmetries



Search has  
begun with  
symmetry  
principles

Symmetry is  
beautiful: a  
framework,  
beginning with  
Gross' grand  
design.

Parity: Slightly  
broken!



*From top-down approach:*

*Discrete symmetries are better to be subgroups of gauge symmetries such that spontaneous breaking of the gauge symmetries to those discrete groups do not lead to any unsatisfactory behavior. [Krauss-Wilzek (1989)]*

This defines ‘discrete gauge symmetries’

Discrete gauge symmetries have been used practically in most model buildings afterwards.

*From top-down view of electroweak scale:*

*Chiral representations at the GUT scale are the key in obtaining the SM at low energy. The choice of 200 GeV as a low energy scale depended on the details of parameters.*

At this conference many other reasons were given for the scale of the SM.

The SM is a chiral model.

$$Y = \frac{\begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad e_L^c}{-1 \quad +1}{2}$$

$$\text{Tr } Y=0,$$

$$\text{Tr } Y^3 = 2 \left( -\frac{1}{2} \right)^3 + (+1)^3 = +\frac{3}{4}$$

Lepton part alone cannot give a chiral model.  
Fractionally charged quarks are needed.

$$Y = \frac{\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_L^c, \quad d_L^c}{+1 \quad -2 \quad +1}{6 \quad 3 \quad 3}$$

$$\text{Tr } Y=0,$$

$$\text{Tr } Y^3 = 6 \left( +\frac{1}{6} \right)^3 + 3 \left( -\frac{2}{3} \right)^3 + 3 \left( +\frac{1}{3} \right)^3 = -\frac{3}{4}$$

The Han-Nambu model also works. The color was first introduced. The problem was known in the paper of Okubo's Omega minus particle prediction.



$SU(3)$  : Vectorlike representations, hence not allowed,

$SU(2) \times SU(2)$  : No chiral theory with even number of doublets,

$U(1) \times U(1)'$  : Six conditions for the absence of anomalies,  $\{\text{Tr}Y, \text{Tr}Y', \text{Tr}Y^3, \text{Tr}Y'^3, \text{Tr}YY'^2, \text{Tr}Y^2Y'\} = 0$

$SU(2) \times U(1)$  : Two conditions with doublets and singlets,  $\{\text{Tr}Y, \text{Tr}Y^3\} = 0$ .

$$\sum_{i=1}^{4N} Q_i^3 = \left( \sum_{i=1}^{4N} Q_i \right) \left( \sum_{i=1}^{4N} Q_i^2 - \sum_{i \neq j}^{4N} Q_i Q_j \right) + 3 \sum_{i \neq j \neq k}^{4N} Q_i Q_j Q_k = 0.$$

$$\sum_{i=1}^{4N} Q_i = 0,$$

$$\sum_{i=1}^{4N} Q_i^3 = \left( \sum_{i=1}^{4N} Q_i \right) \left( \sum_{i=1}^{4N} Q_i^2 - \sum_{i \neq j}^{4N} Q_i Q_j \right) + 3 \sum_{i \neq j \neq k}^{4N} Q_i Q_j Q_k = 0.$$

There is a chiral model.

$$Q = \frac{1}{2} : \quad \ell_i = \begin{pmatrix} E_i \\ N_i \end{pmatrix}_{\frac{+1}{2}}, \quad \begin{matrix} E_{i,-1}^c \\ N_{i,0}^c \end{matrix}, \quad (i = 1, 2, 3)$$

$$Q = -\frac{3}{2} : \quad \mathcal{L} = \begin{pmatrix} \mathcal{E} \\ \mathcal{F} \end{pmatrix}_{\frac{-3}{2}}, \quad \begin{matrix} \mathcal{E}_{,+1}^c \\ \mathcal{F}_{,+2}^c \end{matrix}$$

$$\begin{aligned} \text{Tr } Q^3 = 3 & \left[ 2 \left( \frac{+1}{2} \right)^3 + (-1)^3 \right] & \frac{-9}{4} \\ & + 2 \left( \frac{-3}{2} \right)^3 + (+1)^3 + (+2)^3 & \frac{-27 + 4 + 32}{4} \end{aligned}$$

Three families and the new chiral model appear together in string compactification (orbifold  $Z(12-I)$ ).

A hope to detect this new chiral representation, at LHC:

Kim, [arXiv:1703.10925](#) [hep-ph]

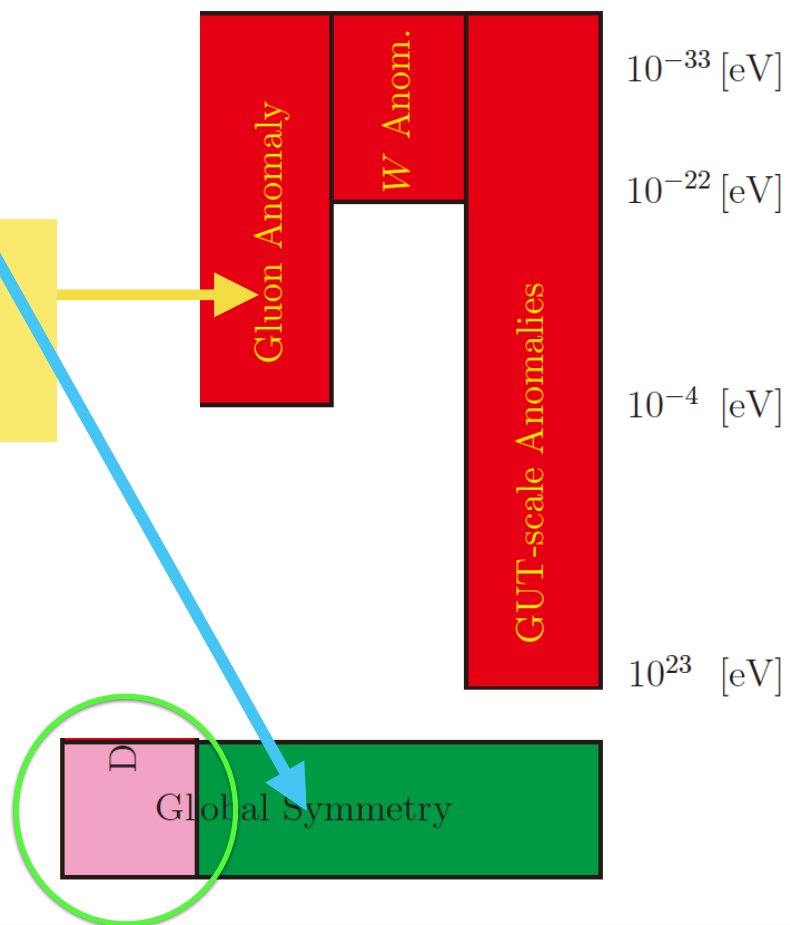
## 2. Approximate global symmetries

After the Brout-Englert-Higgs-Guralnik-Hagen-Kibble mechanism has been accepted, we do not talk about approximate gauge symmetries. But, even now we use the word, “approximate global symmetries”.

If the discrete symmetries are subgroups of gauge groups, then we do not talk about approximate discrete symmetries. But, if a discrete symmetry is a subgroup of global symmetries, then we can talk about approximate discrete symmetries.

From the exact  
global symmetry.

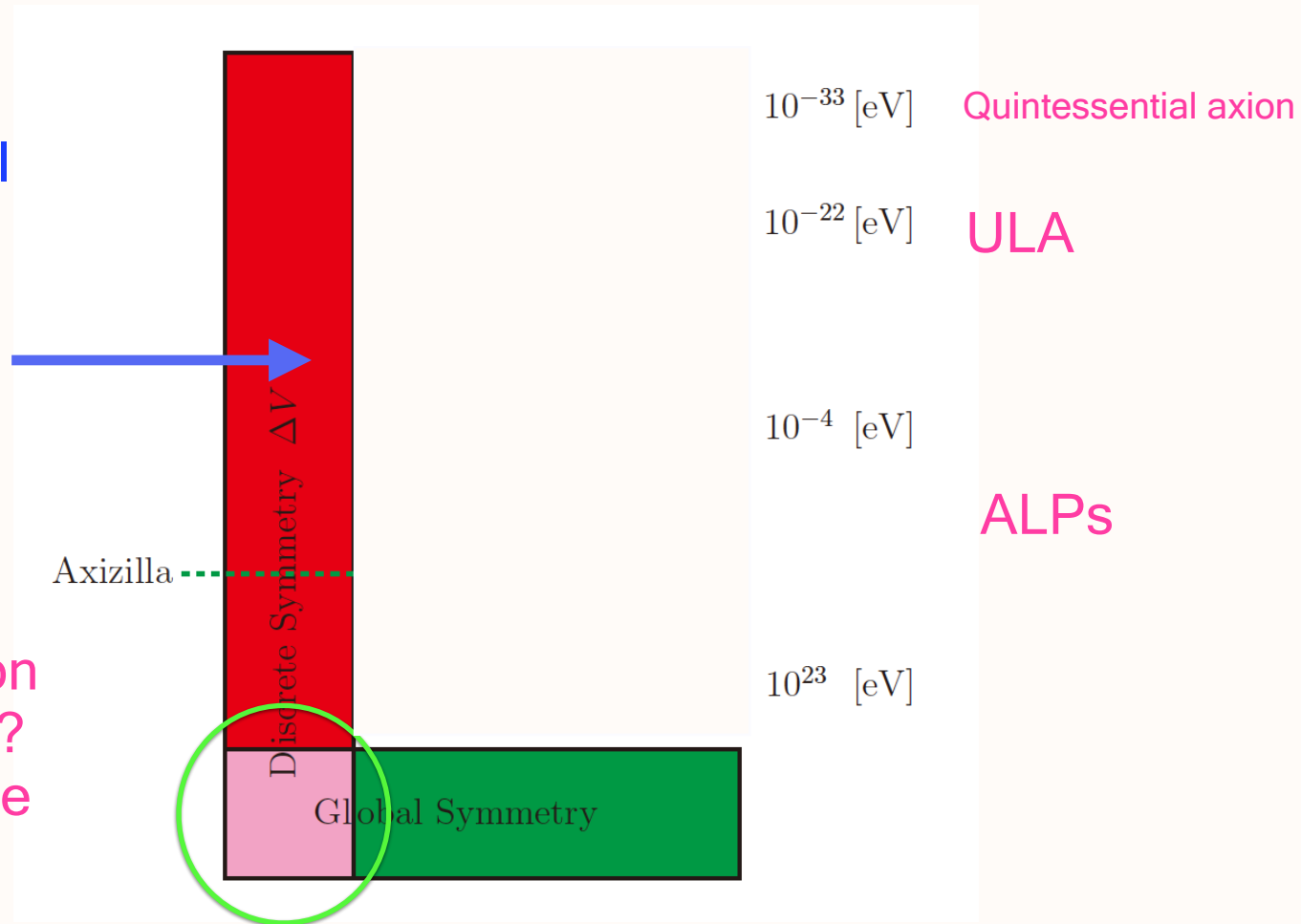
This anomaly  
breaks the PQ  
symmetry.

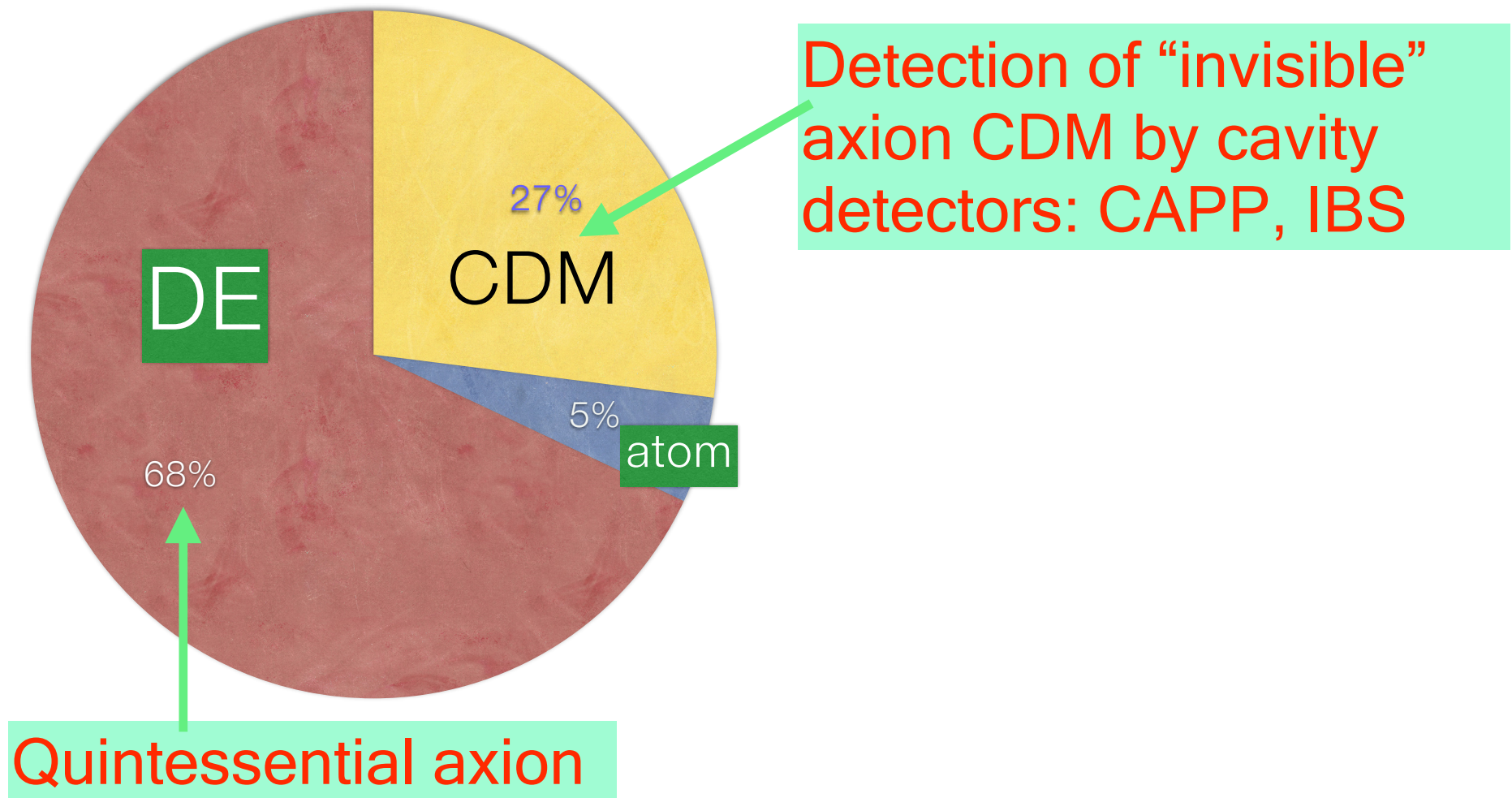


VEV of scalar  $\phi$   
gives  
the  $f_a$  scale.

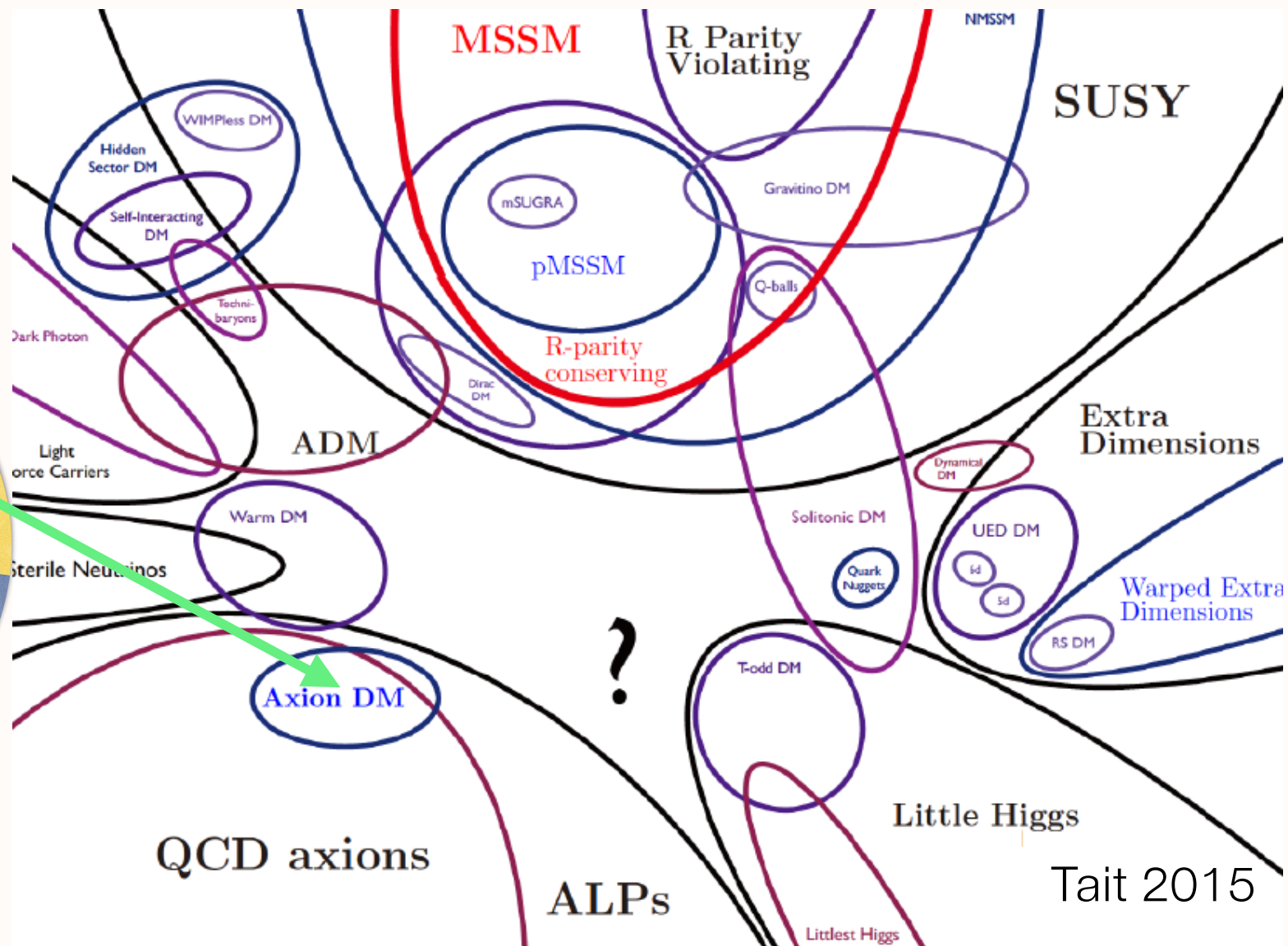
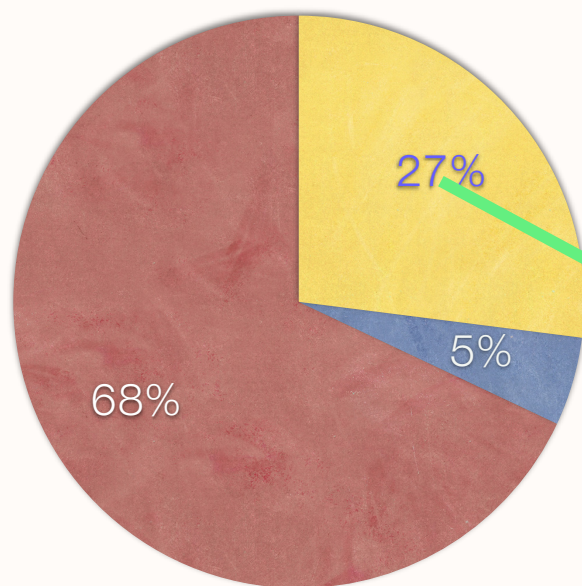
Except the anomalous  $U(1)$ , any global symmetry does not have anomalies from string theory. So, this  $V$  is present.

Still the question is at what level? If one allows the discrete symmetry from string.



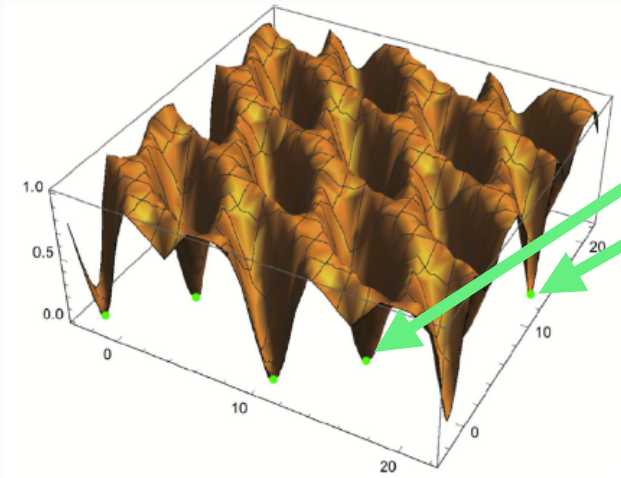






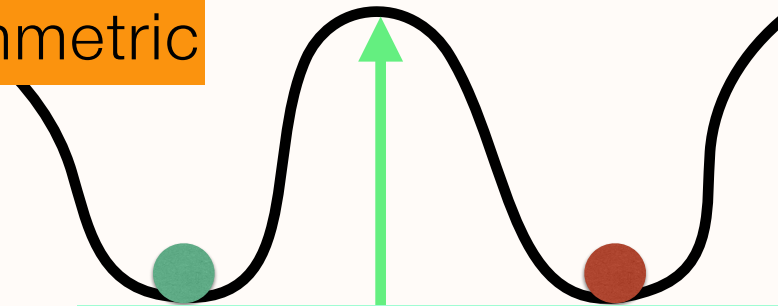
Tait 2015

## 2. Discrete symmetries and domain walls



Vacua settle in the evolving Universe lead to domain walls. Basically, the discrete symmetry is not assumed to be broken. Spontaneous breaking of the discrete symmetry is in the evolving Universe.

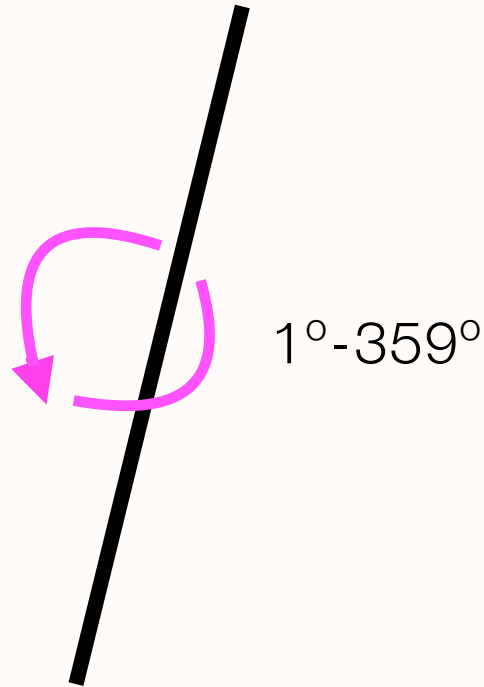
Parity symmetric

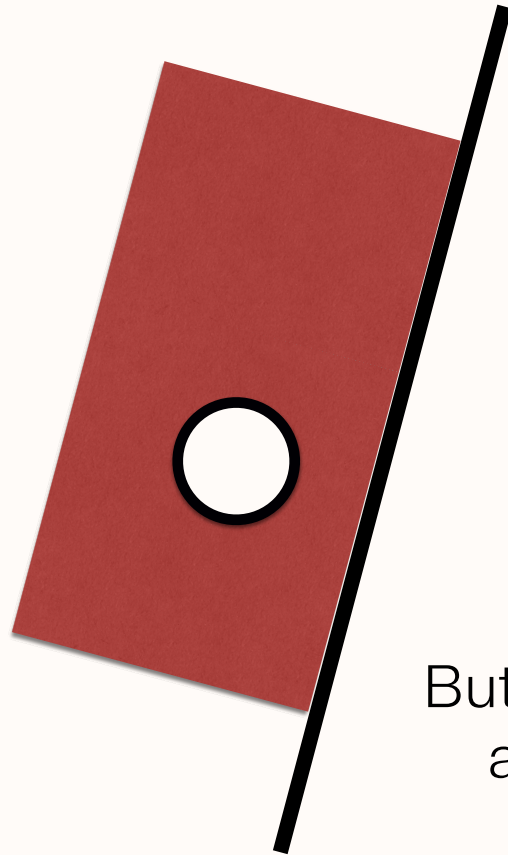


Domain wall energy

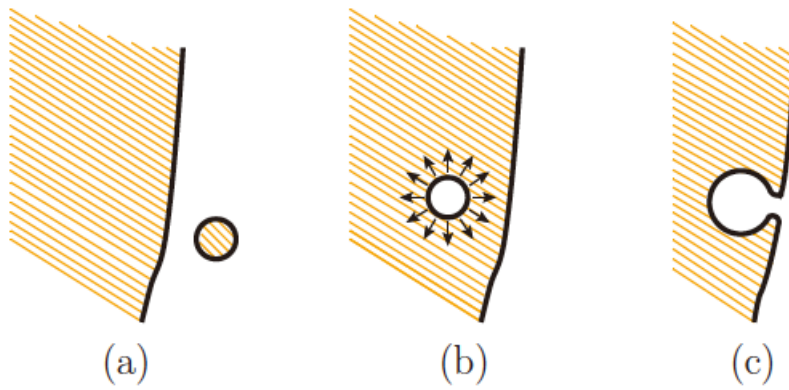
[Zeldovich-Kobzarev-Okun (1974)]

$Z_N$  models,  $N=2,3, \dots$ , lead to domain walls. If  $Z_N$  arises from a subgroup of  $U(1)$  creating strings and broken by anomaly, we can think of  $N$  domain walls, going around the string 360 degrees,

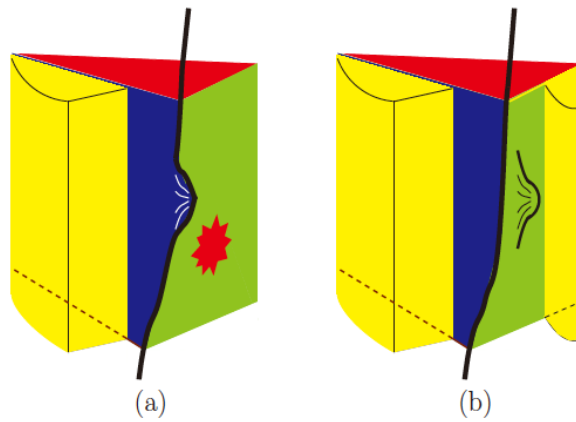




But,  $\text{NDW}=1$  does not have  
a serious cosmological  
DW problem.



$N=1$



$N=2$

But,  $NDW > 1$  have serious cosmological  
DW problems. [Sikivie (1982)]

Sakikawa (Tokyo group) talk at Patras 2017, with  $NDW=1$ :

Estimates of  $\xi$  and  $\epsilon$  may differ by an order of magnitude.  
Accordingly, the prediction for the PQ scale is ambiguous.

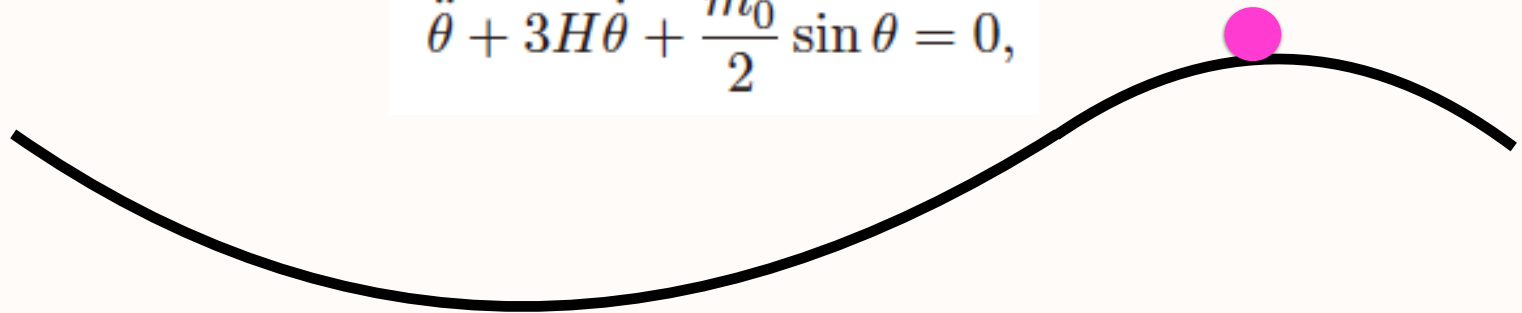
$$\Omega_a = \Omega_{\text{CDM}} \quad \rightarrow \quad 7 \times 10^9 \text{ GeV} \lesssim F_a \lesssim 3 \times 10^{12} \text{ GeV} (?)$$

Median:  $5 \times 10^{11} \text{ GeV}$

Roughly,  $10^{11} \text{ GeV}$  gives about 20% of axion dark matter with  
axion mass around  $0.6 \times 10^{-4} \text{ eV}$ .

Lattice calculation of temperature dependence between 100 MeV to 1 GeV got interest at the Patras conference. Here, I point out the **axion bottle neck problem**.

$$\ddot{\theta} + 3H\dot{\theta} + \frac{m_0^2}{2} \sin \theta = 0,$$



Anharmonic effect: Stays there long time until T is sufficiently lowered.



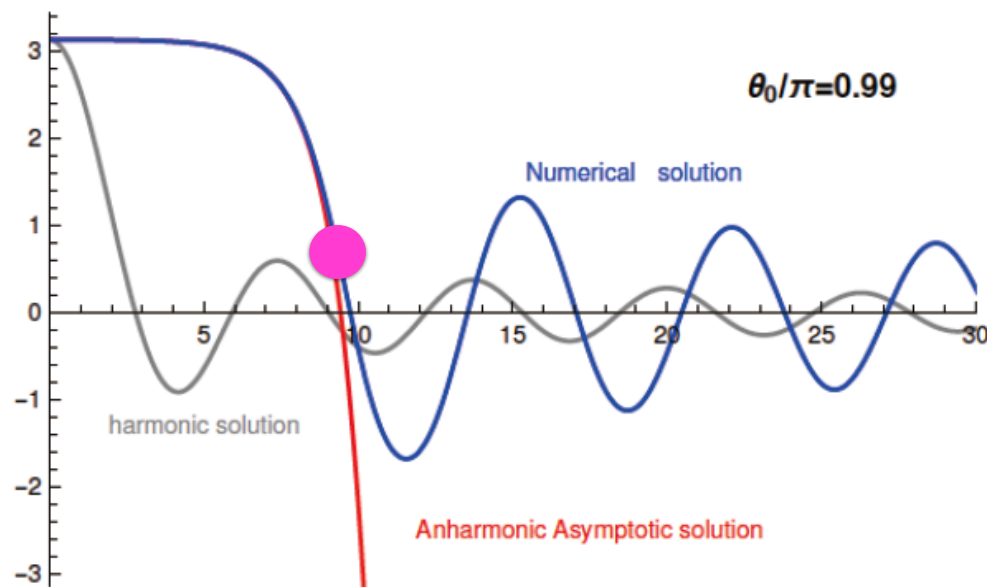
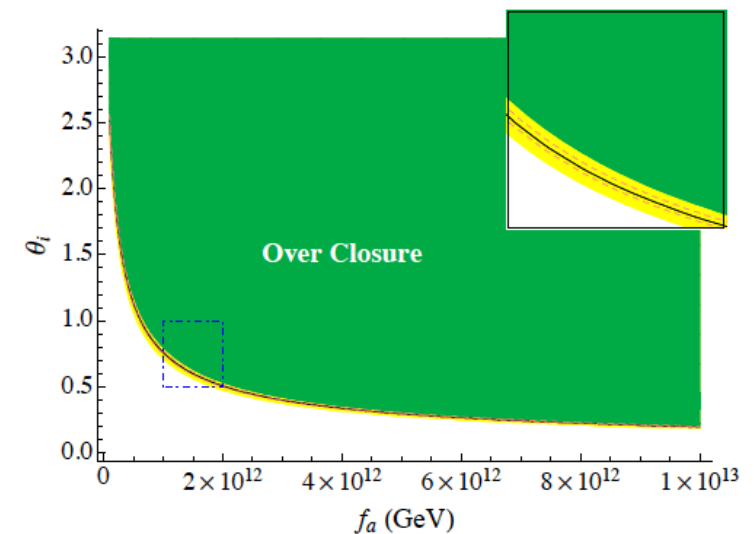


Figure 6.2: The numerical solution (blue) in the anharmonic regime  $0.99\pi$  [6].

$m_0 = \text{constant}$ ,  $T=0$  value. More than one period. For  $T$ -dependent mass, more time is needed.

$m_0 = T$  dependent.

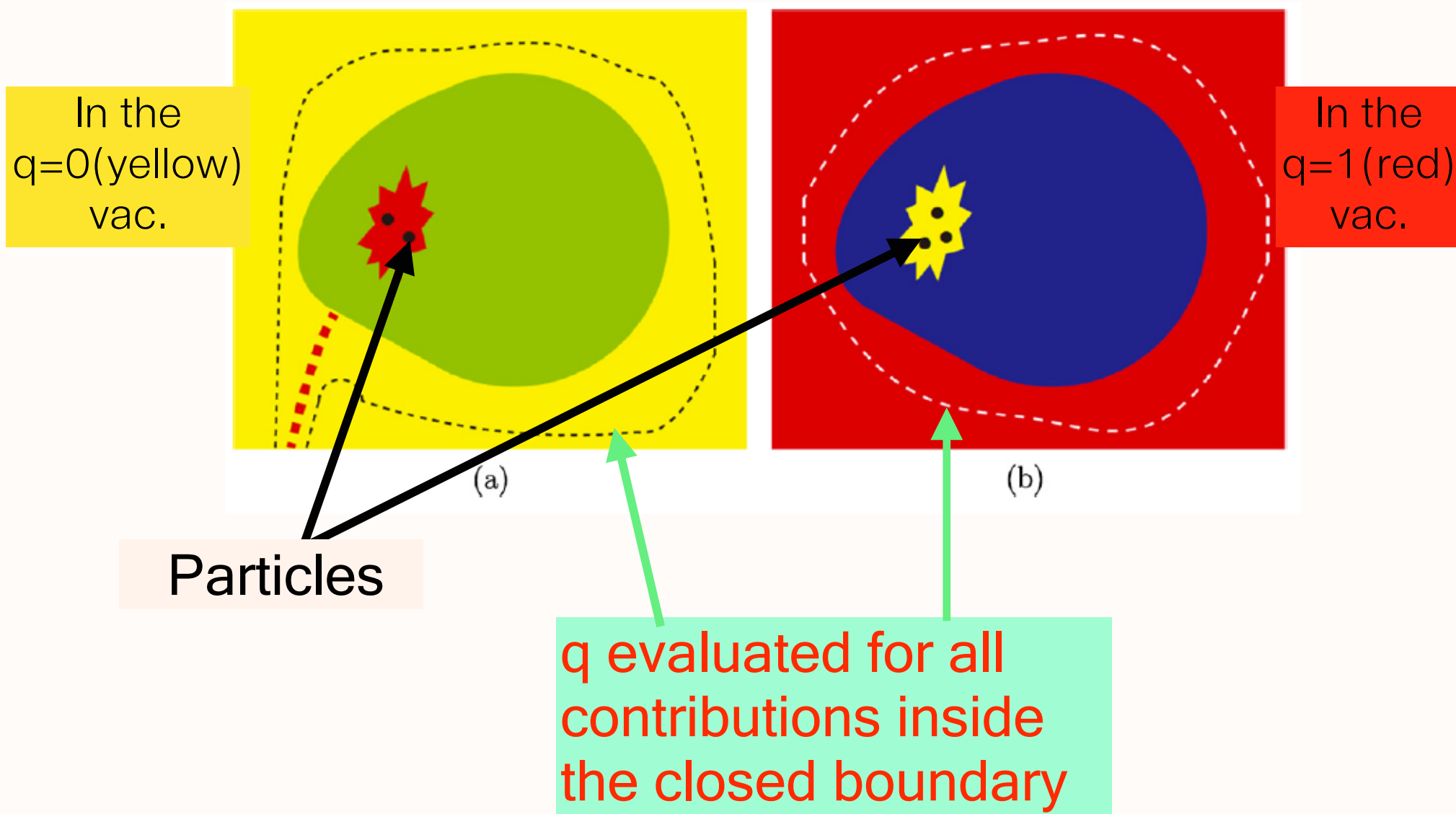
Bae-Huh-K, 0806.0497 [hep-ph]  
K, RMP 82, 557 (2010)

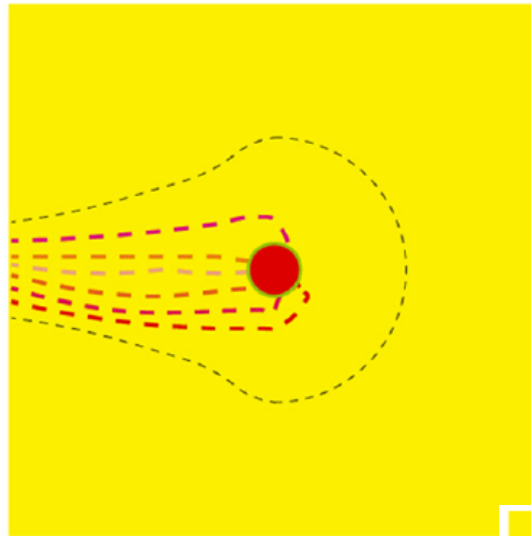
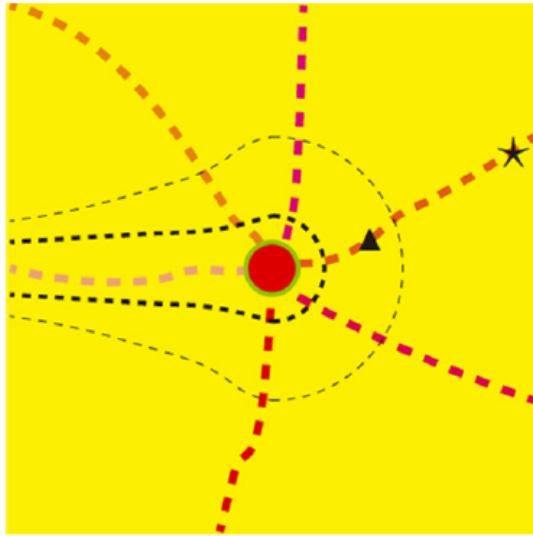


# 3. DW boundaries : Hairs

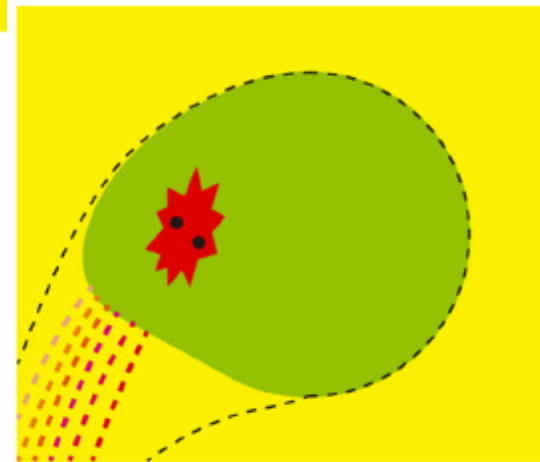
Charge inside a ball:  
vacuum charge

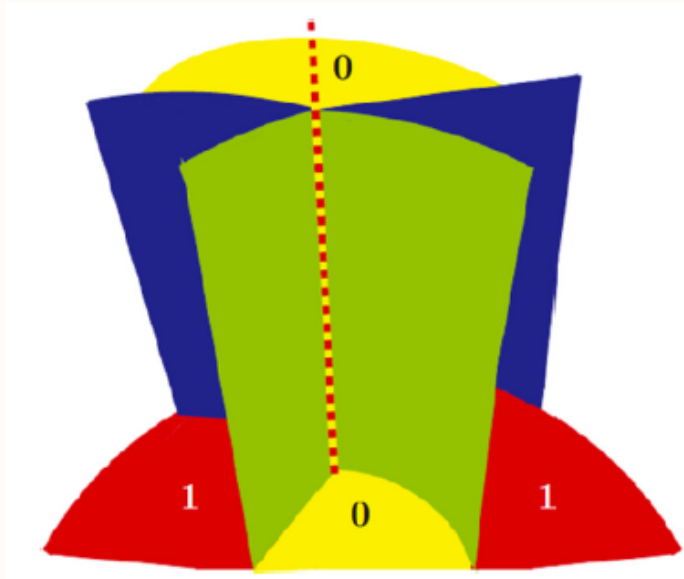
$$q = \frac{1}{2i} \int d^3x (\Phi^* \partial_t \Phi - \Phi \partial_t \Phi^*).$$





Hairs: thickness is the same at any distance from the surface.





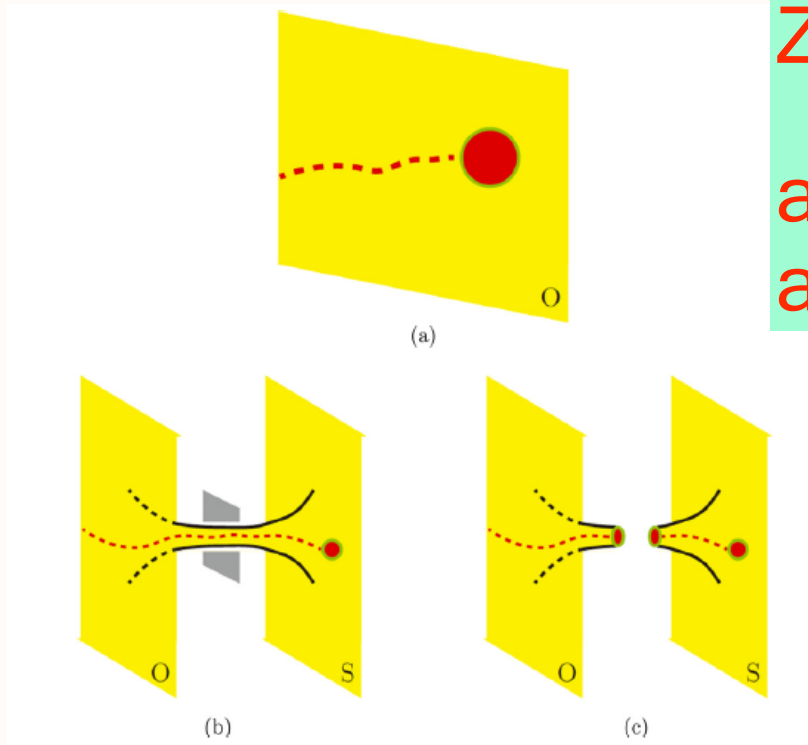
Boundary of domain walls looks like a hair.

$$j^{\theta\varphi}(\theta, \varphi) = \frac{1}{r^2} \delta(\cos \theta - \cos \theta_0) \delta(\varphi - \varphi_0).$$

The surface integral over the closed surface gives 1, the charge inside the surface.

**1703.05389 [hep-th]**

## Bottom-up approach:



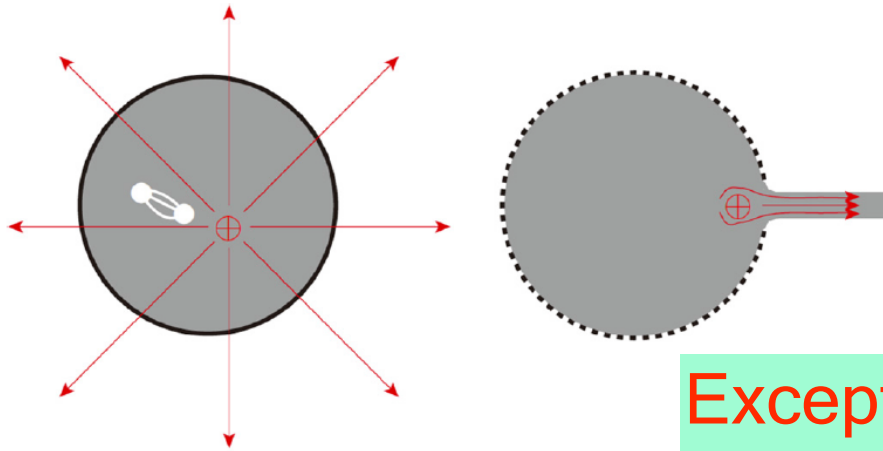
$Z_2$  tadpole ends:  
at head or  
at horizon

Wormholes do not  
break the discrete  
symmetry.

# Blackhole hairs



It must be applicable to  
BH also:



$$r_+ = \frac{1}{2} \left( r_s + \sqrt{r_s^2 - 4r_Q^2} \right)$$

Reissner-Nordstroem BH radius:  
E cannot end inside the horizon.  
But mass took into account this.  
So, the field energy outside  
horizon must be subtracted.

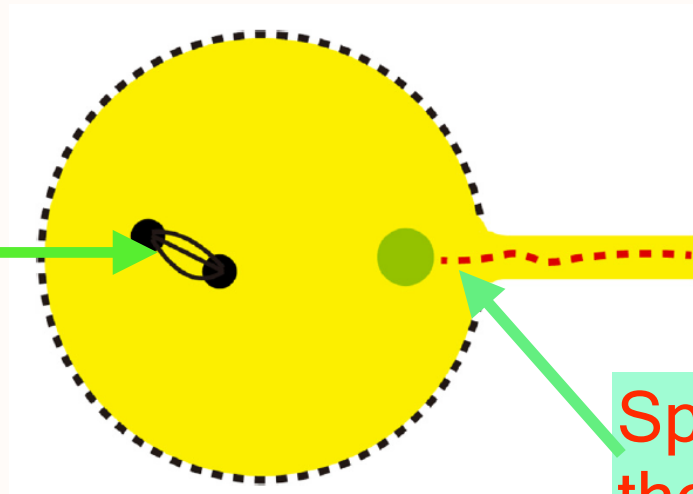
Except a tiny hole, the  
BH horizon makes sense.

The bundle of flux lines is  
like our hair.

## With discrete symmetry:

Spin 2 graviton respects the horizon

Spin 1 photon discussed above



Spin 0 boson depends on the vacuum structure, a tadpole.

It gives a hair also near the BH.

# 5. Conclusion

1. Symmetries.
2. Domain walls.
3. Intersection of DW boundaries: hairs