Particle production in time-dependent backgrounds

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in collaboration with
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based on: JHEP 1503 (2015) 113

SCALARS 2015

Basics

- **Bogoliubov transformation**

$$a_k^{\text{out}} = \alpha_k a_k^{\text{in}} + \beta_k a_k^{\text{in}\,\dagger}$$
$$a_k^{\text{out}\,\dagger} = \alpha_k^* a_k^{\text{in}\,\dagger} + \beta_k^* a_k^{\text{in}}$$

with

$$|\alpha_k|^2 \mp |\beta_k|^2 = 1.$$

and

$$n_k \equiv \langle 0^{\text{in}}|N_k|0^{\text{in}}\rangle = \langle 0^{\text{in}}|a_{\vec{k}}^{\text{out}\,\dagger} a_{\vec{k}}^{\text{out}}|0^{\text{in}}\rangle = V|\beta_k|^2.$$

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  It seems that if $\beta_k = 0$ particles are not produced.

- **Adiabaticity**

  Particles can be produced only when the background varies rapidly in time (WKB approximation is broken) - in the non-adiabatic region where $\dot{\omega}_k / \omega_k^2 > 1$.

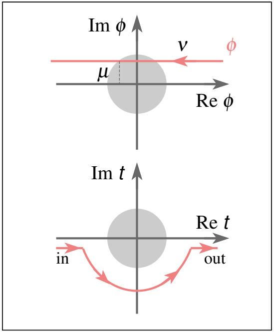
Method (L.Kofman et al., arXiv:hep-th/0403001)

$$V = \tfrac{1}{2} g^2 |\phi|^2 \chi^2$$

- asymptotically: $\langle \phi \rangle = vt + i\mu$, $\langle \chi \rangle = 0$
- non-adiabatic region: $|\phi| \lesssim \sqrt{v/g}$
- background field in non-adiabatic region: χ particles are produced
- produced particles induce a new linear potential

$$\rho_\chi = \int \frac{d^3k}{(2\pi)^3} n_k \sqrt{k^2 + g^2|\phi(t)|^2} \approx g|\phi(t)|n_\chi$$

and an attractive force ("oscillations")



- each time the occupation number of produced χ particles is:

$$n_k^\chi = V \cdot \left| e^{-i\int^t dt' \omega_k(t')} \right|^2 = V \cdot \exp\left(-\pi \frac{k^2 + g^2\mu^2}{gv} \right)$$

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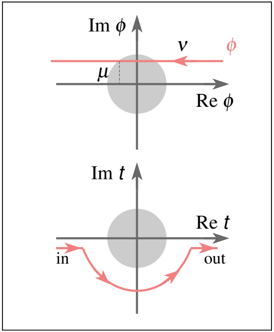

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We investigate the superpotential of the form: $W = \frac{g}{2} \Phi X^2$.

Role of interactions

So far:

produced particles just propagate and cause backreaction (induced potential) but they do not interact.

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Adding interactions gives us the generalized Bogoliubov transformation:

$$a_{\vec{k}}^{\text{out}} = \alpha_k a_{\vec{k}}^{\text{in}} + \beta_k a_{-\vec{k}}^{\text{in}\,\dagger} - i\sqrt{Z}\int d^4x e^{-i\vec{k}\cdot\vec{x}}\Big(-\beta_k \Psi_k^{\text{in}}(x^0) + \alpha_k \Psi_k^{\text{in}\,*}(x^0)\Big) J(x).$$

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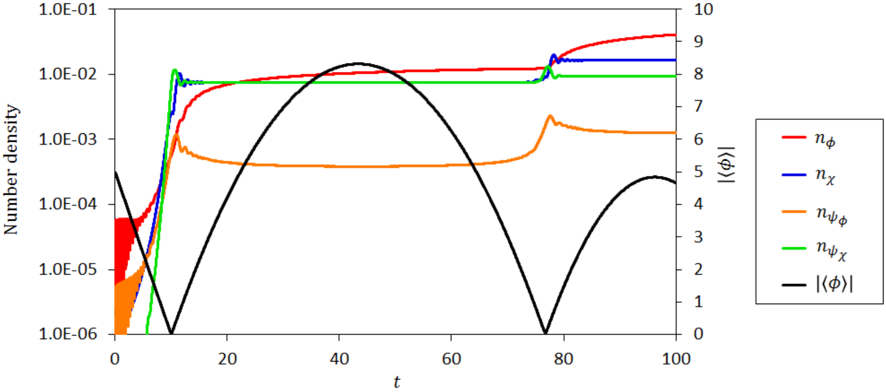
Occupation number is now

$$n_k = \begin{cases} V|\beta_k|^2 \; + \; \dots & (\beta_k \neq 0) \\ 0 \; + \; Z|\int d^4x e^{-i\vec{k}\cdot\vec{x}} \Psi_k^{\text{in}\,*} J|0^{\text{in}}\rangle|^2 & (\beta_k = 0) \end{cases}$$

Particles are produced even if $\beta_k = 0$.

How big is that effect?

Role of interactions



$$\phi(t=0) = 5 + 0.05i,\ \dot{\phi}(t=0) = -0.5,\ g = 1$$

at $t = 30$: $n_\phi = 7.82 \cdot 10^{-4}$, $n_\chi = 2.77 \cdot 10^{-3}$, $n_{\psi_\phi} = 4.26 \cdot 10^{-5}$, $n_{\psi_\chi} = 2.78 \cdot 10^{-3}$

Non-negligible production of massless particles.

Summary

Conclusions:

- general method of investigating nonperturbative particle production is presented (in the suitable limit we recover parametric resonance - see the talk by S. Enomoto)
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Our current interests

- models with more than one coupling
- the case of $\langle \chi \rangle \neq 0$
- higher-order interactions
- applications
- ...

Back up slides

Adiabaticity

What is the condition under which particle production occurs?

We choose adiabatic vacuum as it gives us the minimal production: particular set of modes v_k is chosen.

Mode functions have to fulfill the equation of motion

$$\ddot{v}_{\vec{k}} + \omega^2_{\vec{k}}(t) v_{\vec{k}} = 0.$$

It can be solved in two regimes:

- adiabatic region: $\dot{\omega}_k/\omega_k^2 < 1$ slowly varying background

  $n_k(t) \approx \text{const}$

- non-adiabatic region: $\dot{\omega}_k/\omega_k^2 > 1$ rapidly varying background

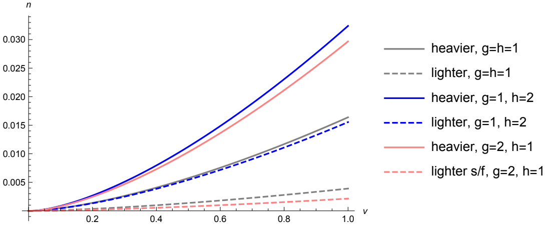
  $n_k(t) \neq \text{const}$ **particle production occurs**

Simple supersymmetric models

Superpotentials:

- $W = \frac{g}{2}\Phi X^2$ (then: $V_{\text{scalar}} = \frac{g^2}{4}|\chi|^4 + g^2|\phi|^2|\chi|^2$)
- $W = \frac{g}{2}\Phi X^2 + h\Phi\Psi X$

Once again we can choose: $\langle\phi\rangle = vt + i\mu$.



Conclusion: heavier states are produced more efficiently.

Why supersymmetry?

- natural way of introducing fermions
- cancellation of UV divergences
- it's simple but still nontrivial (2 scalars + 2 fermions, 2 massive + 2 massless)

Yang-Feldman equation

A scalar field Ψ has equation of motion of the form

$$\left(\partial^2 + M^2(x)\right)\Psi(x) + J(x) = 0.$$

Its solution is called Yang-Feldman equation

$$\Psi(x) = \sqrt{Z}\Psi^{as}(x) - iZ \int_{t_{as}}^{x^0} dy^0 \int d^3y [\Psi^{as}(x), \Psi^{as}(y)] J(y),$$

where the integral part plays the role of retarded potential and

$$\Psi(t^{as}, \vec{x}) = \sqrt{Z}\Psi^{as}(t^{as}, \vec{x}).$$

Calculation

We consider the simple supersymmetric model with superpotential

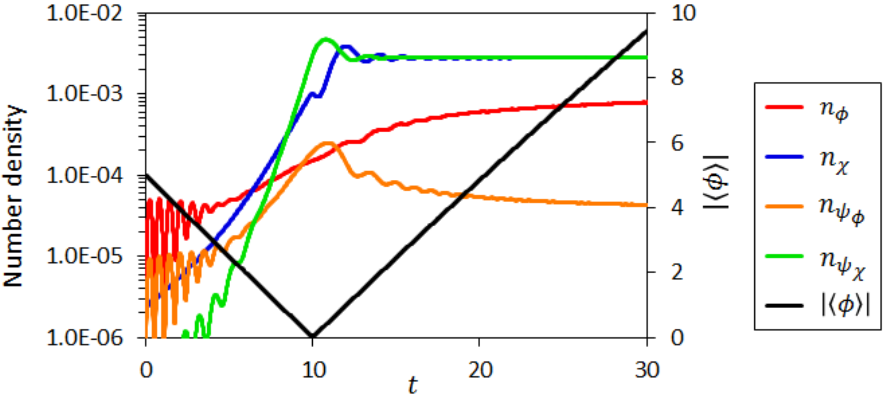
$$W = \frac{g}{2}\Phi X^2.$$

Plan of calculation

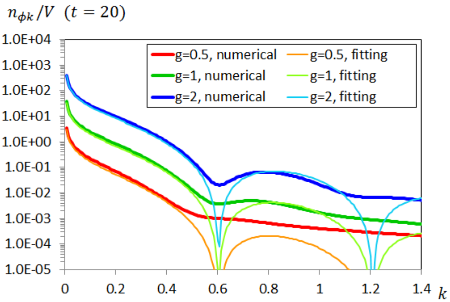
- Yang-Feldman equations for all the species ϕ, χ, ψ_ϕ, ψ_χ
- relation between "in" and "out" fields (Bogoliubov transformation)
- identifying β_k coefficients what leads to number density estimation

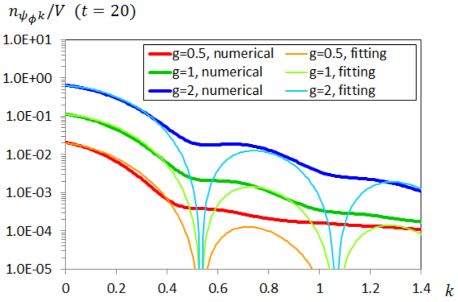
- for massive particles (χ, ψ_χ): fully analytical
- for massless particles (ϕ, ψ_ϕ): WKB approximation and numerical analysis

Results: one transition



Distribution functions for massless particles





$$n_{\phi_k}/V \sim \text{const}_1 \cdot \frac{g^2}{4\pi} \cdot \frac{1}{e^{\sqrt{\pi k^2/g|v|}} - 1} \cdot f_1(t) \cdot \text{"oscillating part"}$$

$$n_{\psi_{\phi_k}}/V \sim \text{const}_2 \cdot \frac{g^2}{4\pi} \cdot \frac{1}{e^{\sqrt{\pi k^2/g|v|}} + 1} \cdot f_2(t) \cdot \text{"oscillating part"}$$

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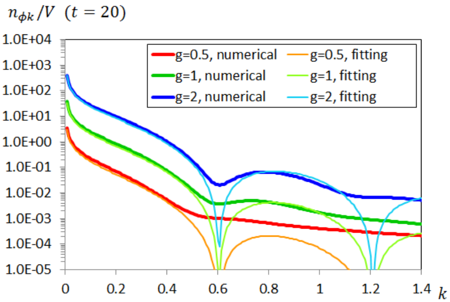

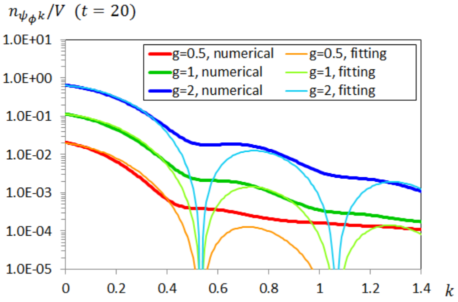

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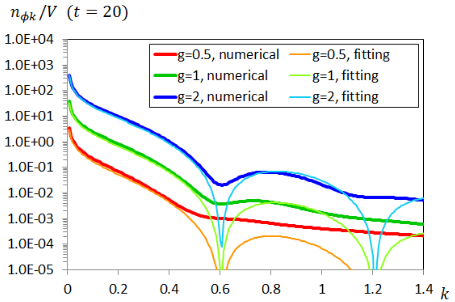

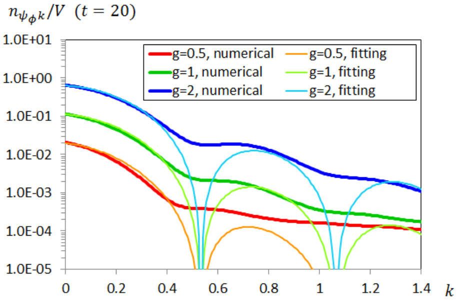

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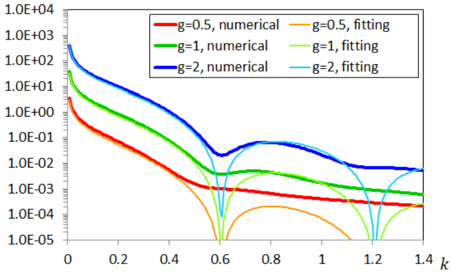

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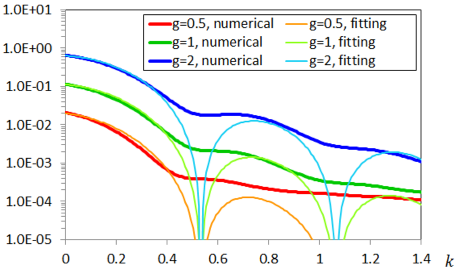

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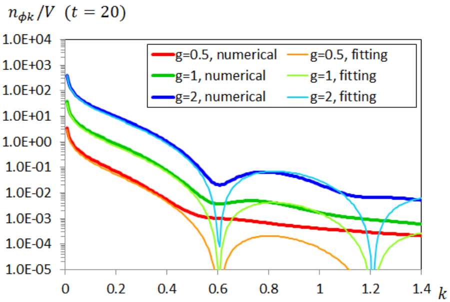
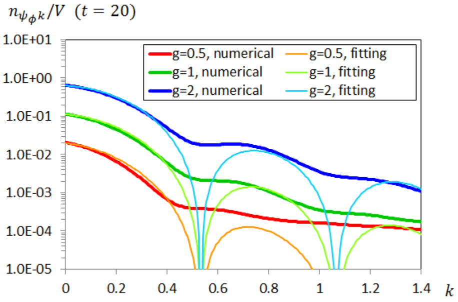
$$f_1(t) \propto t^2 \qquad f_2(t) \propto t$$

Distribution functions for massless particles





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After one transition:

- $n_\phi \approx 0$, $n_{\psi_\phi} \approx 0$
- $n_{\psi_\chi} = 2\frac{(gv)^{3/2}}{(2\pi)^3} e^{-\pi g\mu^2/v}$

- $n_\chi^{\text{broken before}} = 2\frac{(gv)^{3/2}}{(2\pi)^3} e^{-\frac{\pi(g^2\mu^2+m^2)}{gv}}$
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Cut-off momentum (for larger k we leave the non-adiabatic region):

$$k_{\text{max}}^2 = \frac{gv}{\pi}\ln(V) - g^2\mu^2$$