General Extensions of the Standard Model with **New Scalars**

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Framework:
Effective Theories for New Physics

✓ Impose established symmetries: Lorentz, gauge, ...

✓ Assume "fundamental" scale Λ

✓ Choose fields to describe relevant d.o.f.

SM Effective Theory

Effective Theories w/ *Extra Particles*

✓ Write local symmetric operators of dim Δ with arbitrary coefficients of order $\Lambda^{4-\Delta}$

Effective Theory w/ *Extra Scalars*

J. de Blas, M. Chala, MPV, J. Santiago,
JHEP 1504 (2015) 078   [1412.8480]

[See also ph/9808484, 0908.1567, 1108.4027]

- Field content: SM (w/ Higgs) + extra spin 0
- Impose SU(3)xSU(2)xU(1) gauge invariance
- Power counting: start with renormalizable couplings
- Phenomenology: leading effects ➡ **linear** interactions
- General couplings and masses. Mass scales << Λ.

Outline

1. General new scalars and their interactions

2. Indirect effects & Effective Lagrangian

3. Limits on new scalars

Scalar Irreps

Color Singlets

$$S \in (1,1)_0$$

$$S_1 \in (1,1)_1$$

$$S_2 \in (1,1)_2$$

$$\varphi \in (1,2)_{\frac{1}{2}}$$

$$\Xi_0 \in (1,3)_0$$

$$\Xi_1 \in (1,3)_1$$

$$\Theta_1 \in (1,4)_{\frac{1}{2}}$$

$$\Theta_3 \in (1,4)_{\frac{3}{2}}$$

Color Triplets

$$\omega_1 \in (3,1)_{-\frac{1}{3}}$$

$$\omega_2 \in (3,1)_{\frac{2}{3}}$$

$$\omega_4 \in (3,1)_{-\frac{4}{3}}$$

$$\Pi_1 \in (3,2)_{\frac{1}{6}}$$

$$\Pi_7 \in (3,2)_{\frac{7}{6}}$$

$$\zeta \in (3,3)_{-\frac{1}{3}}$$

Color Sextets

$$\Omega_1 \in (6,1)_{\frac{1}{3}}$$

$$\Omega_2 \in (6,1)_{-\frac{2}{3}}$$

$$\Omega_4 \in (6,1)_{\frac{4}{3}}$$

$$\Upsilon \in (6,3)_{\frac{1}{3}}$$

Color Octet

$$\Phi \in (8,2)_{\frac{1}{2}}$$

Scalar Irreps

Color Singlets

$S \in (1,1)_0$

$S_1 \in (1,1)_1$

$S_2 \in (1,1)_2$

$\varphi \in (1,2)_{\frac{1}{2}}$

$\Xi_0 \in (1,3)_0$

$\Xi_1 \in (1,3)_1$

$\Theta_1 \in (1,4)_{\frac{1}{2}}$

$\Theta_3 \in (1,4)_{\frac{3}{2}}$

$$\left\{ \begin{array}{ll} \Xi_1^{++} & \Xi_1^{--} \\ \Xi_1^{+} & \Xi_1^{-} \\ \Xi_1^{0} & \Xi_1^{0\,\prime} \end{array} \right.$$

Color Triplets

$\omega_1 \in (3,1)_{-\frac{1}{3}}$

$\omega_2 \in (3,1)_{\frac{2}{3}}$

$(3,1)_{-\frac{4}{3}}$

$(3,2)_{\frac{1}{6}}$

$\Pi_7 \in (3,2)_{\frac{7}{6}}$

$\zeta \in (3,3)_{-\frac{1}{3}}$

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$\Omega_1 \in (6,1)_{\frac{1}{3}}$

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Color Octet

$\Phi \in (8,2)_{\frac{1}{2}}$

Lagrangian (Δ≤4)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_\sigma \eta_\sigma \left[(D_\mu\sigma)^\dagger D^\mu \sigma - M_\sigma^2 \sigma^\dagger \sigma \right] - V(\{\sigma\},\phi) - \sum_\sigma \left(\sigma^\dagger J_\sigma + \text{h.c.} \right)$$

$$\sim \overline{\psi_L} \otimes \xi_R \qquad \overline{\xi_R} \otimes \psi_L$$

E.g.

$(1,3)_1$

$$\begin{aligned} \mathcal{L} \supset\ & (y^l_{\Xi_1})_{ij}\, \Xi_1^a \overline{l_L^i} \sigma_a \varepsilon l_L^{j\,c} + \text{h.c.} \\ & + \kappa_{\Xi_1} \Xi_1^{a\,\dagger} (\tilde{\phi}^\dagger \sigma_a \phi) + \text{h.c.} \\ & + \lambda_{\Xi_1} (\Xi_1^{a\,\dagger} \Xi_1^a)(\phi^\dagger\phi) + \tilde{\lambda}_{\Xi_1} \frac{i}{\sqrt{2}} \epsilon_{abc} (\Xi_1^{a\,\dagger} \Xi_1^b)(\phi^\dagger \sigma_c \phi) \\ & + \ \ldots \end{aligned}$$

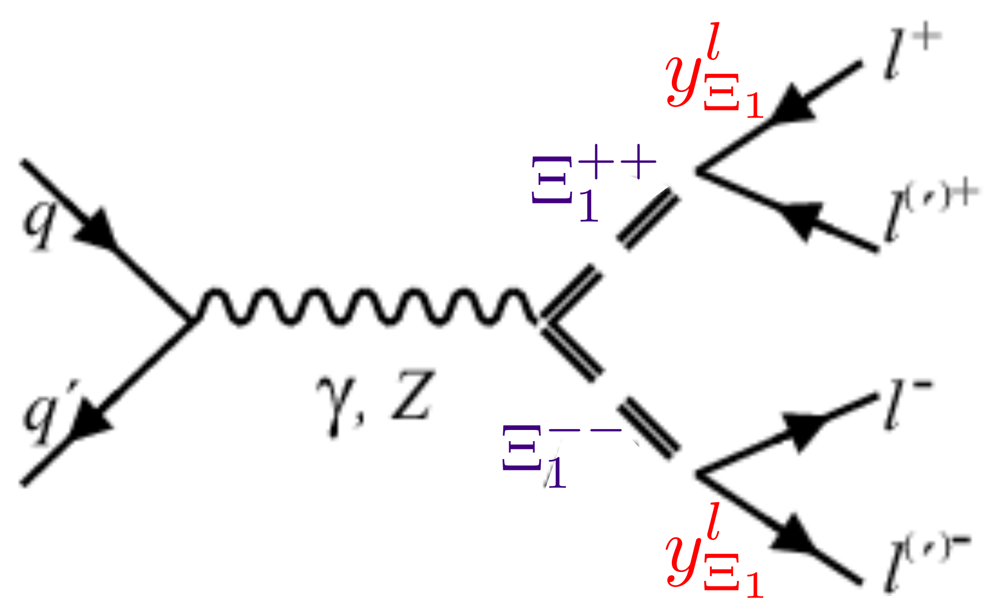
Lagrangian (Δ≤4)

$$
\begin{aligned}
\mathcal{L} = \quad & \mathcal{L}_{\rm SM} \\
& + \sum_\sigma \eta_\sigma \left[(D_\mu \sigma)^\dagger D^\mu \sigma - M_\sigma^2 \sigma^\dagger \sigma \right] \\
& - V(\{\sigma\}, \phi) - \sum_\sigma \left(\sigma^\dagger J_\sigma + \text{h.c.} \right)
\end{aligned}
$$

$$\sim \overline{\psi_L} \otimes \xi_R \qquad \overline{\xi_R} \otimes \psi_L$$

E.g.

$(1,3)_1$



Lagrangian (Δ≤4)

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E.g.

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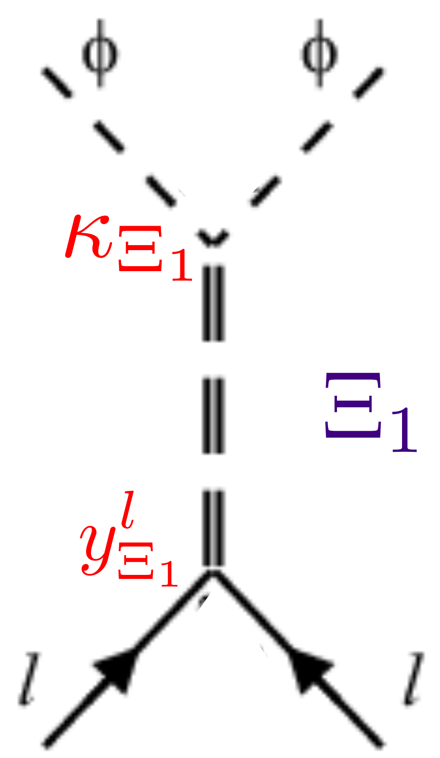


Lagrangian (Δ≤4)

$$\begin{aligned}\mathcal{L} = \quad & \mathcal{L}_{\rm SM} \\ & + \sum_\sigma \eta_\sigma \left[(D_\mu \sigma)^\dagger D^\mu \sigma - M_\sigma^2 \sigma^\dagger \sigma \right] \\ & - V(\{\sigma\}, \phi) - \sum_\sigma \left(\sigma^\dagger J_\sigma + \text{h.c.} \right)\end{aligned}$$

Also mixed interactions:

E.g.

$(1,3)_1$

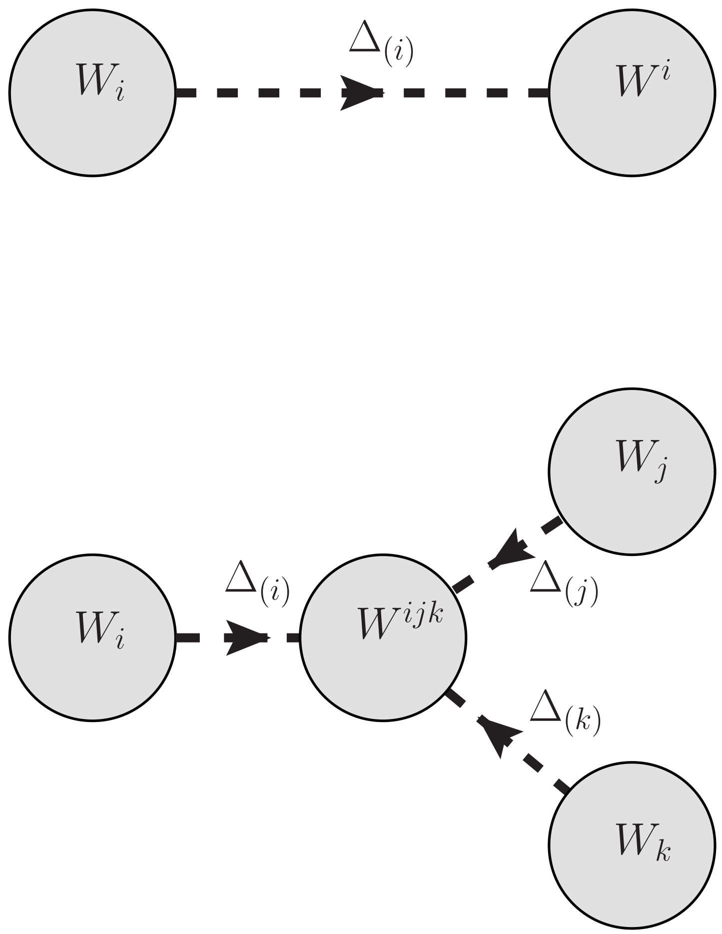
+

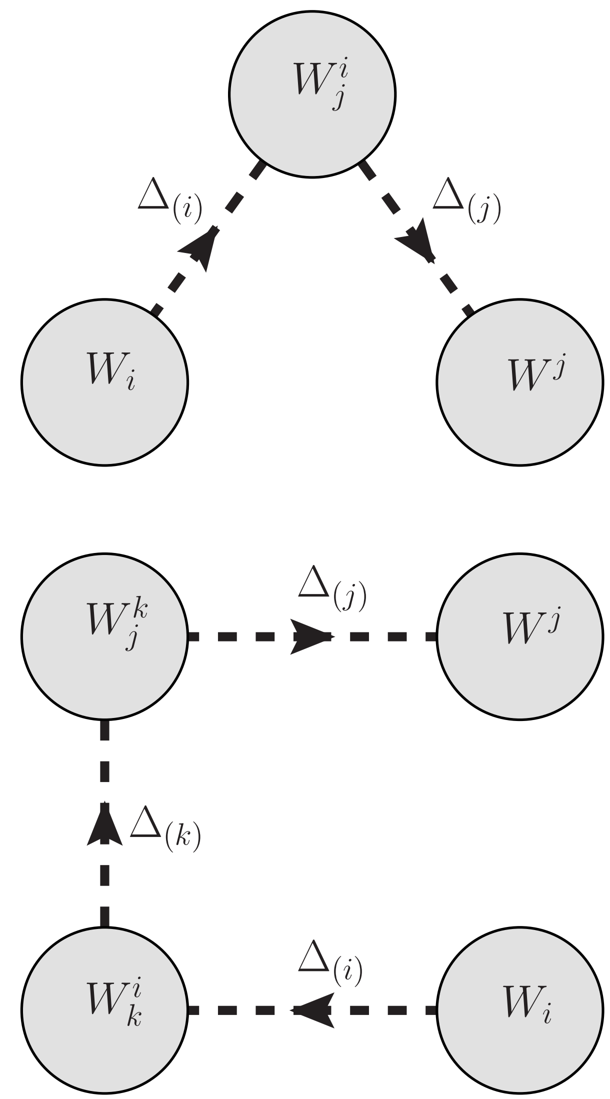
$(1,3)_0$

$$\mathcal{L} \supset \kappa^{ijk}_{\Xi_0 \Xi_1} \frac{i}{\sqrt{2}} \epsilon_{abc} \Xi^a_{0i} \Xi^{b\,\dagger}_{1j} \Xi^c_{1k}$$

Indirect effects

$$\mathcal{L}_{\rm int} = -\sum_{m,n} \sigma^\dagger_{j_1} \cdots \sigma^\dagger_{j_n} W^{j_1 \dots j_n}_{i_1 \dots i_m} \sigma^{i_1} \cdots \sigma^{i_m}$$



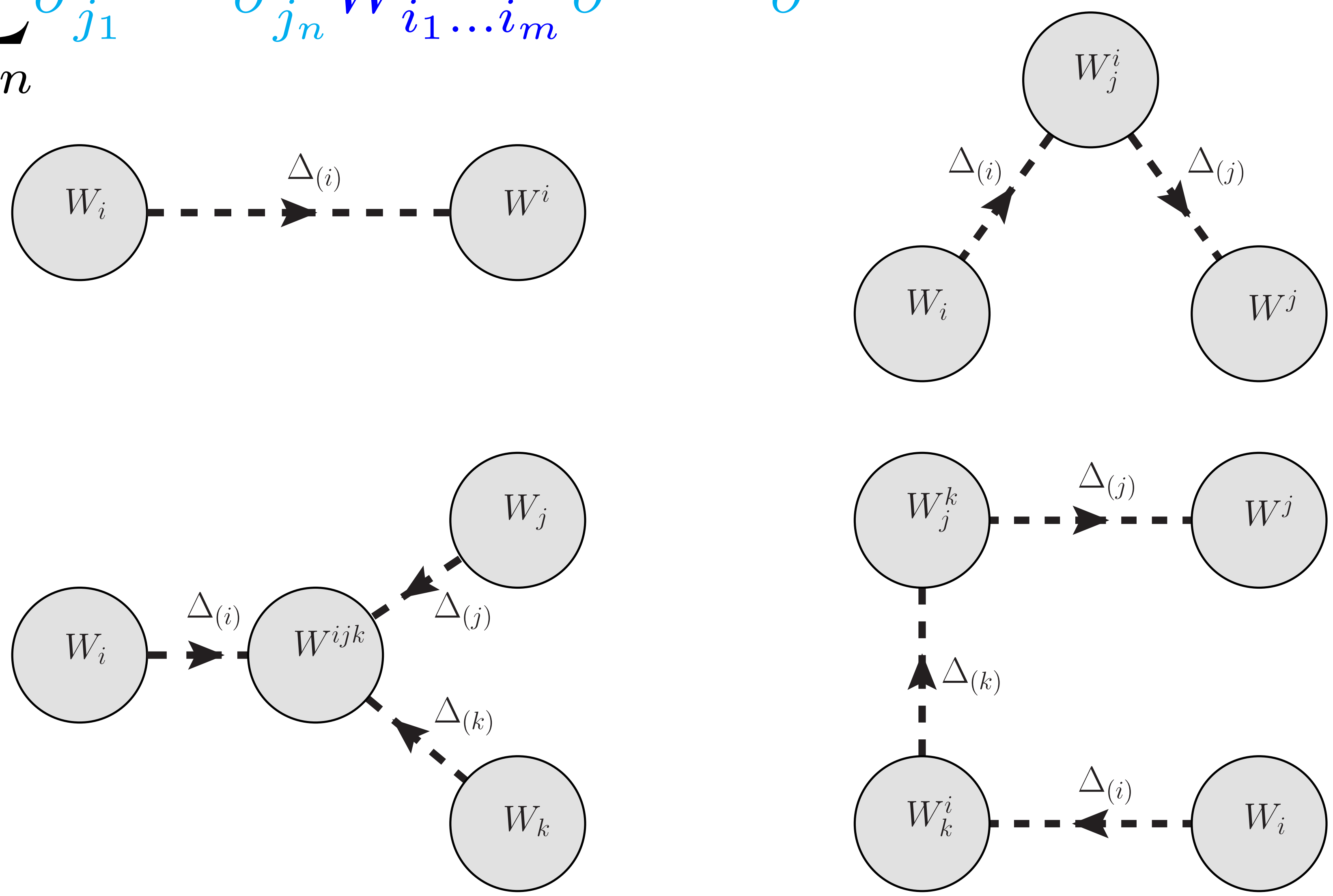


$$\Delta_{(i)} = -(D_i^2 + M_i^2)^{-1} = -\frac{1}{M_i^2}\left(1 - \frac{D_i^2}{M_i^2}\right) + \dots \xrightarrow[E \ll M_i]{}$$

particular SM Effective Theory

Indirect effects

$$\mathcal{L}_{\text{int}} = -\sum_{m,n} \sigma^\dagger_{j_1} \cdots \sigma^\dagger_{j_n} W^{j_1\ldots j_n}_{i_1\ldots i_m} \sigma^{i_1} \cdots \sigma^{i_m}$$



Assumption: in symmetric phase mass matrix has only one negative eigenvalue

$\xrightarrow{E \ll M_i}$

particular SM Effective Theory

SM Effective theory

w/ coefficients given by the parameters of "complete" theory

Example: SM + one isotriplet $(1,3)_1$

$$\mathcal{L}_{\rm eff} \supset -2\frac{\kappa_{\Xi_1}(y^{l\,\dagger}_{\Xi_1})_{ij}}{M^2_{\Xi_1}}\,\overline{l^{i\,c}_L}\tilde{\phi}^*\tilde{\phi}^\dagger l^j_L$$

$$+\frac{(y^l_{\Xi_1})_{ki}(y^l_{\Xi_1})_{jl}}{M^2_{\Xi_1}}\,(\overline{l^i_L}\gamma_\mu l^j_L)(l^k_L\gamma^\mu l^l_L)$$

$$+4\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}\,(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) + 4\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}\,(\phi^\dagger\phi)((D_\mu\phi)^\dagger D^\mu\phi)$$

$$+\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}(-2\lambda_{\Xi_1}+\sqrt{2}\tilde{\lambda}_{\Xi_1})\,(\phi^\dagger\phi)^3$$

SM Effective theory

w/ coefficients given by the parameters of "complete" theory

Example: SM + one isotriplet $(1,3)_1$

$$\mathcal{L}_{\text{eff}} \supset -2\frac{\kappa_{\Xi_1}(y^{l\,\dagger}_{\Xi_1})_{ij}}{M^2_{\Xi_1}}\,\overline{l^{i\,c}_L}\tilde{\phi}^*\tilde{\phi}^\dagger l^j_L$$

GIMR basis

$$+\frac{(y^l_{\Xi_1})_{ki}(y^l_{\Xi_1})_{jl}}{M^2_{\Xi_1}}\,(\overline{l^i_L}\gamma_\mu l^j_L)(l^k_L\gamma^\mu l^l_L) \quad +2\frac{|\kappa_{\Xi_1}|^2 y^e_{ii}}{M^4_{\Xi_1}}\,(\phi^\dagger\phi)(\overline{l^i_L}\phi e^i_R)$$

$$+2\frac{|\kappa_{\Xi_1}|^2 y^d_{ii}}{M^4_{\Xi_1}}\,(\phi^\dagger\phi)(\overline{q^i_L}\phi d^i_R) \; +2\frac{|\kappa_{\Xi_1}|^2 V^\dagger_{ij}y^u_{jj}}{M^4_{\Xi_1}}\,(\phi^\dagger\phi)(\overline{q^i_L}\tilde{\phi} u^i_R)$$

$$+4\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}\,(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) \; +2\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}\,(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$$

$$-\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}(2\lambda_{\Xi_1} - \sqrt{2}\tilde{\lambda}_{\Xi_1} - 4\lambda_\phi)\,(\phi^\dagger\phi)^3$$

SM Effective theory

w/ coefficients given by the parameters of "complete" theory

Example: SM + one isotriplet $(1,3)_1$

$$\mathcal{L}_{\rm eff} \supset -2\frac{\kappa_{\Xi_1}(y^{l\,\dagger}_{\Xi_1})_{ij}}{M^2} \overline{l^{i\,c}_L}\tilde{\phi}^*\tilde{\phi}^\dagger l^j_L$$

ν mass

LEP2, Møller

$$+\frac{(y^l_{\Xi_1})_{ki}(y^l_{\Xi_1})_{jl}}{M^2_{\Xi_1}}(\overline{l^i_L}\gamma_\mu l^j_L)(\overline{l^k_L}\gamma^\mu l^l_L) \quad +2\frac{|\kappa_{\Xi_1}|^2 y^e_{ii}}{M^4}(\phi^\dagger\phi)(\overline{l^i_L}\phi e^i_R)$$

$$+2\frac{|\kappa_{\Xi_1}|^2 y^d_{ii}}{M^4_{\Xi_1}}(\phi^\dagger\phi)(\overline{q^i_L}\phi d^i_R) \quad \frac{\ldots y_{jj}}{M^4_{\Xi_1}}(\phi^\dagger\phi)(\overline{q^i_L}\tilde{\phi}u^i_R)$$

Higgs-fermions

$$+4\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) +2\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$$

Higgs w.f.

T parameter

$$-\frac{|\kappa_{\Xi_1}|^2}{M^4_{\Xi_1}}(2\lambda_{\Xi_1}-\sqrt{2}\tilde{\lambda}_{\Xi_1}-4\lambda_\phi)(\phi^\dagger\phi)^3$$

Higgs selfcoupling.

SM Effective theory

w/ coefficients given by the parameters of "complete" theory

Matching with SM effective theory

- Can use EFT results to constrain parameters.
- Or, better, fit in terms of "complete" parameters to take correlations into account.
- Can combine with other contributions to EFT

Limits from indirect effects: Observables

✓ $\not{B}$ & $\not{L}$

✓ Flavor physics (hadronic & leptonic)

✓ EWPD: Z-pole, low-energy, W mass & width, t and H mass, $\Delta\alpha^{(5)}_{\rm had}$, α_s , LEP2, CKM unitarity

✓ LHC dilepton & dijet searches

✓ Higgs observables

Limits from indirect effects: Observables

- ✓ $\not{B}$ & $\not{L}$ ← Avoid simultaneous dangerous couplings
- ✓ Flavor physics (hadronic & leptonic) ← Align
- ✓ EWPD: Z-pole, low-energy, W mass & width, t and H mass, $\Delta\alpha^{(5)}_{\rm had}$, α_s , LEP2, CKM unitarity
- ✓ LHC dilepton & dijet searches
- ✓ Higgs observables

Limits from indirect effects:

Colorless Scalars

Scalar	Parameter	95% C.L. Bound [TeV^{-1}]
$\mathcal{S}$	$\frac{\lvert\kappa_{\mathcal{S}}\rvert}{M_{\mathcal{S}}^2}$	1.55
$\mathcal{S}_1$	$\frac{\lvert y^l_{\mathcal{S}_1}\rvert}{M_{\mathcal{S}_1}}$	$\begin{pmatrix} - & 0.08 & - \\ 0.08 & - & - \\ - & - & - \end{pmatrix}$
$\mathcal{S}_2$	$\frac{\lvert y^e_{\mathcal{S}_2}\rvert}{M_{\mathcal{S}_2}}$	$\begin{pmatrix} 0.36 & 0.19 & 0.28 \\ 0.19 & - & - \\ 0.28 & - & - \end{pmatrix}$
φ	$\frac{\lvert y^e_{\varphi}\rvert}{M_{\varphi}}$	$\begin{pmatrix} 0.26 & 0.56 & 0.79 \\ 0.56 & - & - \\ 0.79 & - & - \end{pmatrix}$
	$\frac{\lvert (y^d_{\varphi})_{11}\rvert}{M_{\varphi}}$	0.61
	$\frac{\lvert (y^u_{\varphi})_{11}\rvert}{M_{\varphi}}$	0.44
Ξ_0	$\frac{\lvert\kappa_{\Xi_0}\rvert}{M_{\Xi_0}^2}$	0.11
Ξ_1	$\frac{\lvert\kappa_{\Xi_1}\rvert}{M_{\Xi_1}^2}$	0.04
	$\frac{\lvert y^l_{\Xi_1}\rvert}{M_{\Xi_1}}$	$\begin{pmatrix} 0.33 & 0.09 & 0.18 \\ 0.09 & - & - \\ 0.18 & - & - \end{pmatrix}$

Limits from indirect effects:

Colored Scalars

Scalar	Parameter	95% C.L. Bound [TeV^{-1}]
ω_1	$\frac{\lvert y^{ql}_{\omega_1}\rvert}{M_{\omega_1}}$	$\begin{pmatrix} 0.19 & 0.53 & - \\ 0.40 & - & - \\ - & - & - \end{pmatrix}$
	$\frac{\lvert (y^{qq}_{\omega_1})_{11}\rvert}{M_{\omega_1}}$	0.24
	$\frac{\lvert y^{eu}_{\omega_1}\rvert}{M_{\omega_1}}$	$\begin{pmatrix} 0.27 & 0.49 & - \\ 0.48 & - & - \\ - & - & - \end{pmatrix}$
	$\frac{\lvert (y^{du}_{\omega_1})_{11}\rvert}{M_{\omega_1}}$	0.47
ω_4	$\frac{\lvert y^{ed}_{\omega_4}\rvert}{M_{\omega_4}}$	$\begin{pmatrix} 0.28 & 0.98 & 0.98 \\ 0.42 & - & - \\ - & - & - \end{pmatrix}$
Π_1	$\frac{\lvert y^{ld}_{\Pi_1}\rvert}{M_{\Pi_1}}$	$\begin{pmatrix} 0.27 & 1.80 & 1.80 \\ 0.48 & - & - \\ - & - & - \end{pmatrix}$

Scalar	Parameter	95% C.L. Bounds [TeV^{-1}]
Π_7	$\frac{\lvert y^{lu}_{\Pi_7}\rvert}{M_{\Pi_7}}$	$\begin{pmatrix} 0.27 & 1.04 & - \\ 0.33 & - & - \\ - & - & - \end{pmatrix}$
	$\frac{\lvert y^{eq}_{\Pi_7}\rvert}{M_{\Pi_7}}$	$\begin{pmatrix} 0.29 & 0.93 & 1.06 \\ 0.32 & - & - \\ - & - & - \end{pmatrix}$
Ω_1	$\frac{\lvert (y^{ud}_{\Omega_1})_{11}\rvert}{M_{\Omega_1}}$	0.78
Ω_2	$\frac{\lvert (y^{d}_{\Omega_2})_{11}\rvert}{M_{\Omega_2}}$	0.68
Ω_4	$\frac{\lvert (y^{u}_{\Omega_4})_{11}\rvert}{M_{\Omega_4}}$	0.47
ζ	$\frac{\lvert y^{ql}_{\zeta}\rvert}{M_{\zeta}}$	$\begin{pmatrix} 0.21 & 0.30 & - \\ 0.66 & - & - \\ 0.47 & - & - \end{pmatrix}$
Φ	$\frac{\lvert (y^{qu}_{\Phi})_{11}\rvert}{M_{\Phi}}$	0.88
	$\frac{\lvert (y^{dq}_{\Phi})_{11}\rvert}{M_{\Phi}}$	1.12
Υ	$\frac{\lvert y^{q}_{\Upsilon}\rvert}{M_{\Upsilon}}$	0.32

Limits from indirect effects: cancellations

- Among scalars
- Among scalars and fermions
- Among scalars and vectors

	$\mathcal{S}_1$	$\mathcal{S}_2$	Ξ_1	ω_1	ω_2	ω_4	ζ	Ω_1	Ω_2	Ω_4	Υ
$\mathcal{B}_\mu$	$+\alpha_{ll}^{(1)}$	$\pm\alpha_{ee}$	$-\alpha_{ll}^{(1)}$	$-\alpha_{qq}^{(1)}$	$+\alpha_{dd}^{(1)}$	$+\alpha_{uu}^{(1)}$	$+\alpha_{qq}^{(1)}$	$+\alpha_{qq}^{(1)}$	$-\alpha_{dd}^{(1)}$	$-\alpha_{uu}^{(1)}$	$-\alpha_{qq}^{(1)}$
	$(-)$	$(-\alpha_{ee})$	$(-\alpha_{ll}^{(1)})$	$(-\alpha_{qq}^{(1)})$	$(-)$	$(-)$	$(-)$	$(-)$	$(-\alpha_{dd}^{(1)})$	$(-\alpha_{uu}^{(1)})$	$(-\alpha_{qq}^{(1)})$
$\mathcal{W}_\mu$	$\pm\alpha_{ll}^{(1)}$		$\pm\alpha_{ll}^{(1)}$	$\pm\alpha_{qq}^{(1)}$			$\pm\alpha_{qq}^{(1)}$	$\pm\alpha_{qq}^{(1)}$			$\pm\alpha_{qq}^{(1)}$
				$\pm\alpha_{qq}^{(8)}$			$\pm\alpha_{qq}^{(8)}$	$\pm\alpha_{qq}^{(8)}$			$\pm\alpha_{qq}^{(8)}$
	$(-)$		$(-\alpha_{ll}^{(1)})$	$(+\alpha_{qq}^{(1)})$			$(-)$	$(-)$			$(+\alpha_{qq}^{(1)})$
				$(+\alpha_{qq}^{(8)})$			$(-)$	$(-)$			$(-\alpha_{qq}^{(8)})$
$\mathcal{G}_\mu$				$+\alpha_{qq}^{(8)}$	$\pm\alpha_{dd}^{(1)}$	$\pm\alpha_{uu}^{(1)}$	$-\alpha_{qq}^{(8)}$	$+\alpha_{qq}^{(8)}$	$\pm\alpha_{dd}^{(1)}$	$\pm\alpha_{uu}^{(1)}$	$-\alpha_{qq}^{(8)}$
				$(+\alpha_{qq}^{(8)})$	$(-)$	$(-)$	$(-)$	$(-)$	$(-\alpha_{dd}^{(1)})$	$(-\alpha_{uu}^{(1)})$	$(-\alpha_{qq}^{(8)})$
$\mathcal{H}_\mu$				$-\alpha_{qq}^{(1)}$			$+\alpha_{qq}^{(1)}$	$+\alpha_{qq}^{(1)}$			$-\alpha_{qq}^{(1)}$
				$\pm\alpha_{qq}^{(8)})$			$\pm\alpha_{qq}^{(8)}$	$\pm\alpha_{qq}^{(8)}$			$\pm\alpha_{qq}^{(8)}$
				$(-\alpha_{qq}^{(1)})$			$(-)$	$(-)$			$(-\alpha_{qq}^{(1)})$
				$(-\alpha_{qq}^{(8)})$			$(-)$	$(-)$			$(+\alpha_{qq}^{(8)})$

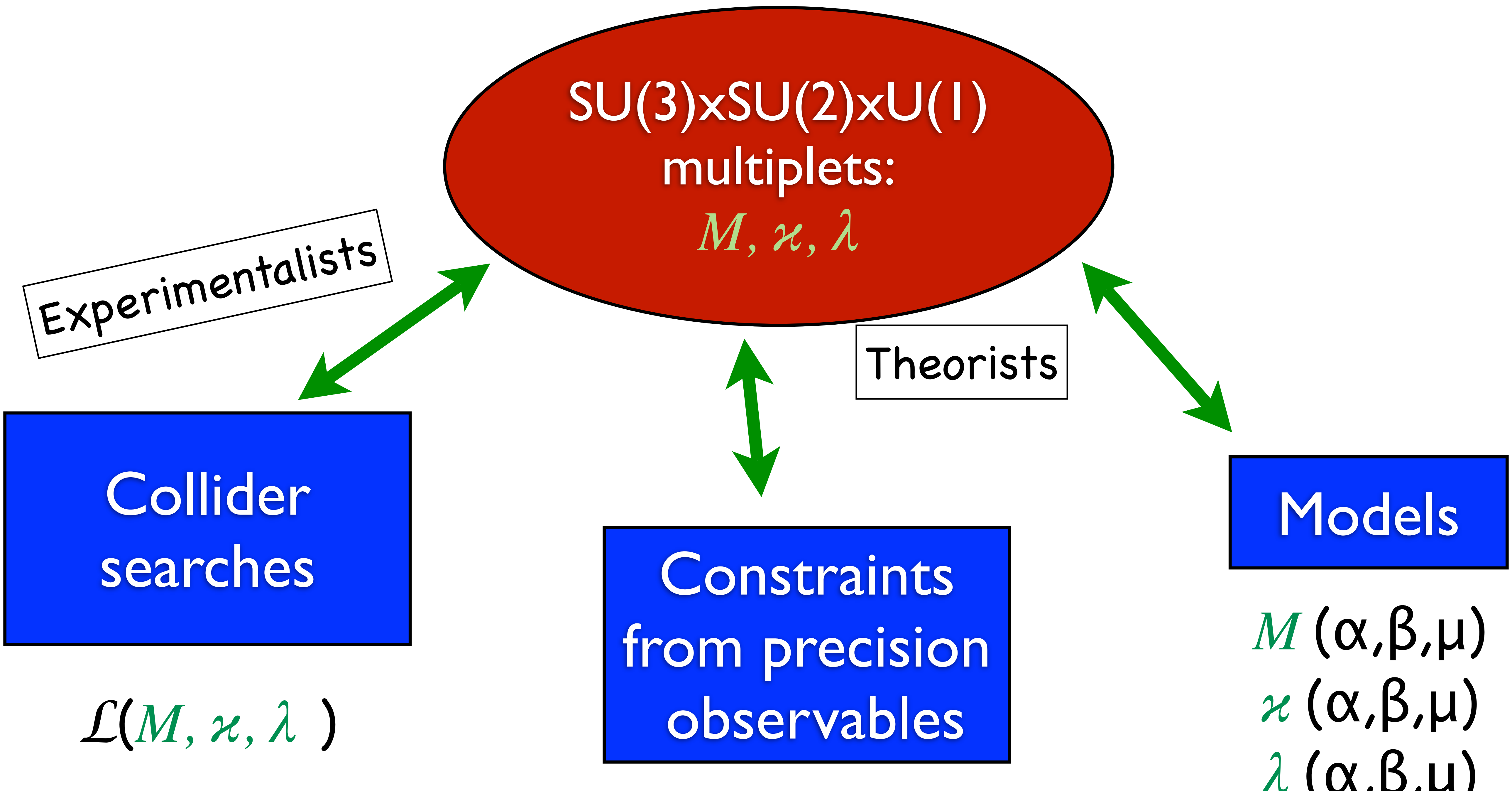
Four fermions, i=l, j=k

+: same sign

-: opposite sign

Global picture

A standard gauge-invariant approach allows for a model-independent interpretation/guide of particle searches



Global picture

Linear renormalizable interactions;

- ✓ (Vectorlike) Quarks
- ✓ (Vectorlike) Leptons
- ✓ Vector Bosons
- ✓ Scalars
- Combined Fermions + Scalars
- Loops
- Non-renormalizable interactions

Back slides

Interactions of new scalars

$$\mathcal{S} \sim (1,1)_0$$

$$V_{\mathcal{S}} = \kappa_{\mathcal{S}}\ \mathcal{S}\phi^\dagger\phi + \lambda_{\mathcal{S}}\ \mathcal{S}^2\phi^\dagger\phi + \kappa_{\mathcal{S}^3}\ \mathcal{S}^3$$

$$\mathcal{S}_1 \sim (1,1)_1$$

$$J_{\mathcal{S}_1} = (y^l_{\mathcal{S}_1})_{ij}\ \overline{l^i_L} i\sigma_2 l^{c\,j}_L \quad ((y^l_{\mathcal{S}_1})_{ij} = -(y^l_{\mathcal{S}_1})_{ji})$$

$$\mathcal{S}_2 \sim (1,1)_2$$

$$J_{\mathcal{S}_2} = (y^e_{\mathcal{S}_2})_{ij}\ \overline{e^i_R} e^{c\,j}_R \quad ((y^e_{\mathcal{S}_2})_{ij} = (y^e_{\mathcal{S}_2})_{ji})$$

$$\varphi \sim (1,2)_{\frac{1}{2}}$$

$$J_\varphi = (y^e_\varphi)_{ij}\ \overline{e^i_R} l^j_L + (y^d_\varphi)_{ij}\ \overline{d^i_R} q^j_L + (y^u_\varphi)_{ij}\ i\sigma_2 \overline{q^i_L}^T u^j_R$$
$$V_\varphi = \lambda_\varphi\ (\varphi^\dagger\phi)(\phi^\dagger\phi) + \text{h.c.}$$

$$\Xi_0 \sim (1,3)_0$$

$$V_{\Xi_0} = \kappa_{\Xi_0}\ \phi^\dagger \Xi^a_0 \sigma_a \phi + \lambda_{\Xi_0}\ (\Xi^a_0 \Xi^a_0)\,(\phi^\dagger\phi)$$

$$\Xi_1 \sim (1,3)_1$$

$$J_{\Xi_1} = (y^l_{\Xi_1})_{ij}\ \overline{l^i_L} \sigma_a i\sigma_2 l^{c\,j}_L \quad ((y^l_{\Xi_1})_{ij} = (y^l_{\Xi_1})_{ji})$$
$$V_{\Xi_1} = \left(\kappa_{\Xi_1}\ \Xi^{a\,\dagger}_1 \left(\tilde{\phi}^\dagger \sigma_a \phi\right) + \text{h.c.}\right) + \lambda_{\Xi_1}\ \left(\Xi^{a\,\dagger}_1 \Xi^a_1\right)(\phi^\dagger\phi) + \tilde{\lambda}_{\Xi_1}\ f_{abc}\left(\Xi^{a\,\dagger}_1 \Xi^b_1\right)(\phi^\dagger\sigma_c\phi)$$

$$\Theta_1 \sim (1,4)_{\frac{1}{2}}$$

$$V_{\Theta_1} = \lambda_{\Theta_1}\ (\phi^\dagger\sigma_a\phi)\, C^I_{a\beta}\tilde{\phi}_\beta \epsilon_{IJ}\Theta^J_1 + \text{h.c.}$$

$$\Theta_3 \sim (1,4)_{\frac{3}{2}}$$

$$V_{\Theta_3} = \lambda_{\Theta_3}\ \left(\phi^\dagger\sigma_a\tilde{\phi}\right) C^I_{a\beta}\tilde{\phi}_\beta \epsilon_{IJ}\Theta^J_3 + \text{h.c.}$$

$$\omega_1 \sim (3,1)_{-\frac{1}{3}}$$

$$J_{\omega_1} = (y^{ql}_{\omega_1})_{ij}\,\overline{q^{c\,i}_L} i\sigma_2 l^j_L + (y^{qq}_{\omega_1})_{ij}\,\varepsilon_{ABC}\,\overline{q^{i\,B}_L} i\sigma_2 q^{c\,j\,C}_L + (y^{eu}_{\omega_1})_{ij}\,\overline{e^{c\,i}_R} u^j_R + (y^{du}_{\omega_1})_{ij}\,\varepsilon_{ABC}\,\overline{d^{i\,B}_R} u^{c\,j\,C}_R$$

$$((y^{qq}_{\omega_1})_{ij} = (y^{qq}_{\omega_1})_{ji})$$

$$\omega_2 \sim (3,1)_{\frac{2}{3}}$$

$$J_{\omega_2} = (y^{d}_{\omega_2})_{ij}\;\varepsilon_{ABC}\,\overline{d^{i\,B}_R} d^{c\,j\,C}_R \quad ((y^{d}_{\omega_2})_{ij} = -(y^{d}_{\omega_2})_{ji})$$

$$\omega_4 \sim (3,1)_{-\frac{4}{3}}$$

$$J_{\omega_4} = (y^{ed}_{\omega_4})_{ij}\;\overline{e^{c\,i}_R} d^j_R + (y^{uu}_{\omega_4})_{ij}\;\varepsilon_{ABC}\,\overline{u^{i\,B}_R} u^{c\,j\,C}_R \quad ((y^{uu}_{\omega_4})_{ij} = -(y^{uu}_{\omega_4})_{ji})$$

$$\Pi_1 \sim (3,2)_{\frac{1}{6}}$$

$$J_{\Pi_1} = (y^{ld}_{\Pi_1})_{ij}\;\; i\sigma_2 \overline{l^i_L}^T d^j_R$$

$$\Pi_7 \sim (3,2)_{\frac{7}{6}}$$

$$J_{\Pi_7} = (y^{lu}_{\Pi_7})_{ij}\; i\sigma_2 \overline{l^i_L}^T u^j_R + (y^{eq}_{\Pi_7})_{ij}\;\overline{e^i_R} q^j_L$$

$$\zeta \sim (3,3)_{-\frac{1}{3}}$$

$$J_\zeta = (y^{ql}_\zeta)_{ij}\;\overline{q^{c\,i}_L} i\sigma_2\sigma_a l^j_L + (y^{qq}_\zeta)_{ij}\;\varepsilon_{ABC}\,\overline{q^{i\,B}_L}\sigma_a i\sigma_2 q^{c\,j\,C}_L \quad \left((y^{qq}_\zeta)_{ij} = -(y^{qq}_\zeta)_{ji}\right)$$

$$\Omega_1 \sim (6,1)_{\frac{1}{3}}$$

$$J_{\Omega_1} = (y^{ud}_{\Omega_1})_{ij}\;\overline{u^{c\,i\,(A|}_R} d^{j\,|B)}_R + (y^{qq}_{\Omega_1})_{ij}\;\overline{q^{c\,i\,(A|}_L} i\sigma_2 q^{j\,|B)}_L \quad \left((y^{qq}_{\Omega_1})_{ij} = -(y^{qq}_{\Omega_1})_{ji}\right)$$

$$\Omega_2 \sim (6,1)_{-\frac{2}{3}}$$

$$J_{\Omega_2} = (y^d_{\Omega_2})_{ij}\ \overline{d^{c\,i\,(A|}_R} d^{j\,|B)}_R \qquad ((y^d_{\Omega_2})_{ij} = (y^d_{\Omega_2})_{ji})$$

$$\Omega_4 \sim (6,1)_{\frac{4}{3}}$$

$$J_{\Omega_4} = (y^u_{\Omega_4})_{ij}\ \overline{u^{c\,i\,(A|}_R} u^{j\,|B)}_R \qquad ((y^u_{\Omega_4})_{ij} = (y^u_{\Omega_4})_{ji})$$

$$\Upsilon \sim (6,3)_{\frac{1}{3}}$$

$$J_{\Upsilon} = (y^q_{\Upsilon})_{ij}\ \overline{q^{c\,i\,(A|}_L} i\sigma_2\sigma_a q^{j\,|B)}_L \qquad ((y^q_{\Upsilon})_{ij} = (y^q_{\Upsilon})_{ji})$$

$$\Phi \sim (8,2)_{\frac{1}{2}}$$

$$J_{\Phi} = (y^{qu}_{\Phi})_{ij}\ i\sigma_2 \overline{q^i_L}^T T_A u^j_R + (y^{dq}_{\Phi})_{ij}\ \overline{d^i_R} T_A q^j_L$$

Mixed contributions from $\{\mathcal{S},\ \varphi,\ \Xi_0,\ \Xi_1,\ \Theta_1,\ \Theta_3\}$

$$\begin{aligned}
\Delta\mathcal{L}_{\rm int} = &-\left(\varphi^\dagger_i J_{\varphi_i} + \Xi^{a\,\dagger}_{1i} J^a_{\Xi_{1i}} + \text{h.c.}\right) - \kappa^i_{\mathcal{S}} \mathcal{S}_i \phi^\dagger\phi - \kappa^{ijk}_{\mathcal{S}^3} \mathcal{S}_i\mathcal{S}_j\mathcal{S}_k - \kappa^i_{\Xi_0} \Xi^a_{0i} \phi^\dagger \sigma_a \phi \\
&- \left(\kappa^i_{\Xi_1} \Xi^{a\,\dagger}_{1i} \tilde{\phi}^\dagger \sigma_a \phi + \kappa^{ij}_{\mathcal{S}\varphi} \mathcal{S}_i \varphi^\dagger_j \phi + \text{h.c.}\right) - \kappa^{ijk}_{\mathcal{S}\Xi_0} \mathcal{S}_i \Xi^a_{0j}\Xi^a_{0k} - \kappa^{ijk}_{\mathcal{S}\Xi_1} \mathcal{S}_i \Xi^{a\,\dagger}_{1j} \Xi^a_{1k} \\
&-\kappa^{ijk}_{\Xi_0^3} f_{abc} \Xi^a_{0i}\Xi^b_{0j}\Xi^c_{0k} - \kappa^{ijk}_{\Xi_0\Xi_1} f_{abc} \Xi^a_{0i} \Xi^{b\,\dagger}_{1j} \Xi^c_{1k} - \left(\kappa^{ij}_{\Xi_0\varphi} \Xi^a_{0i} \left(\varphi^\dagger_j \sigma_a \phi\right) + \kappa^{ij}_{\Xi_1\varphi} \Xi^{a\,\dagger}_{1i} \left(\tilde{\varphi}^\dagger_j \sigma_a \phi\right) + \text{h.c.}\right) \\
&- \left(\kappa^{ij}_{\Xi_0\Theta_1} \Xi^a_{0i} C^I_{a\beta} \tilde{\phi}_\beta \epsilon_{IJ} \Theta^J_{1j} + \kappa^{ij}_{\Xi_1\Theta_1} \Xi^{a\,\dagger}_{1i} C^I_{a\beta} \phi_\beta \epsilon_{IJ} \Theta^J_{1j} + \kappa^{ij}_{\Xi_1\Theta_3} \Xi^{a\,\dagger}_{1i} C^I_{a\beta} \tilde{\phi}_\beta \epsilon_{IJ} \Theta^J_{3j} + \text{h.c.}\right) \\
&-\lambda^{ij}_{\mathcal{S}} \mathcal{S}_i\mathcal{S}_j \left(\phi^\dagger\phi\right) - \left(\lambda^i_\varphi \left(\varphi^\dagger_i \phi\right)\left(\phi^\dagger\phi\right) + \text{h.c.}\right) - \lambda^{ij}_{\Xi_0} \Xi^a_{0i}\Xi^a_{0j} \left(\phi^\dagger\phi\right) - \tilde{\lambda}^{ij}_{\Xi_0} \Xi^a_{0i}\Xi^b_{0j} f_{abc} \left(\phi^\dagger \sigma_c \phi\right) \\
&-\lambda^{ij}_{\Xi_1} \Xi^{a\,\dagger}_{1i} \Xi^a_{1j} \left(\phi^\dagger\phi\right) - \tilde{\lambda}^{ij}_{\Xi_1} f_{abc} \Xi^{a\,\dagger}_{1i} \Xi^b_{1j} \left(\phi^\dagger \sigma_c \phi\right) - \lambda^{ij}_{\mathcal{S}\Xi_0} \mathcal{S}_i \Xi^a_{0j} \left(\phi^\dagger \sigma_a \phi\right) \\
&- \left(\lambda^{ij}_{\mathcal{S}\Xi_1} \mathcal{S}_i \Xi^{a\,\dagger}_{1j} \left(\tilde{\phi}^\dagger \sigma_a \phi\right) + \lambda^{ij}_{\Xi_1\Xi_0} f_{abc} \Xi^{a\,\dagger}_{1i} \Xi^b_{0j} \left(\tilde{\phi}^\dagger \sigma_c \phi\right) + \text{h.c.}\right) \\
&- \left(\lambda^i_{\Theta_1} \left(\phi^\dagger \sigma_a \phi\right) C^I_{a\beta} \tilde{\phi}_\beta \epsilon_{IJ} \Theta^J_{1i} + \lambda^i_{\Theta_3} \left(\phi^\dagger \sigma_a \tilde{\phi}\right) C^I_{a\beta} \tilde{\phi}_\beta \epsilon_{IJ} \Theta^J_{3i} + \text{h.c.}\right)
\end{aligned}$$