Impact of Gravity on the SM Vacuum Stability

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SCALARS 2015, WARSAW, December 3 - 7, 2015

Stability Condition of the Standard Model Vacuum

Stability Condition of our Universe

... a time honored subject ...

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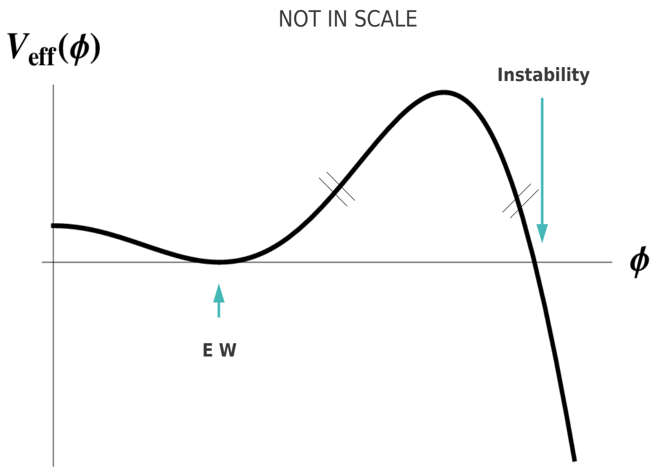
Top loop-corrections to the Higgs Effective Potential destabilize the electroweak vacuum...



EW Scale $= v \sim 246$ GeV

For $M_H \sim 125$ GeV , $M_t \sim 173$ GeV :

Instability Scale $\sim 10^{11}$ GeV

MOREOVER

Higgs boson $M_H \sim 125$ GeV

Experimental data consistent with SM predictions

No sign of New Physics

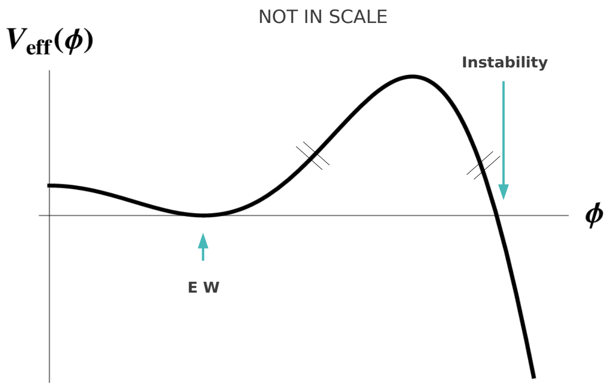
Boosted new interest and work on an old idea

... the possibility that ...

New Phyiscs shows up only at very high energies

... maybe Planck scale

Back to the Higgs One-Loop Effective Potential $V^{1l}(\phi)$

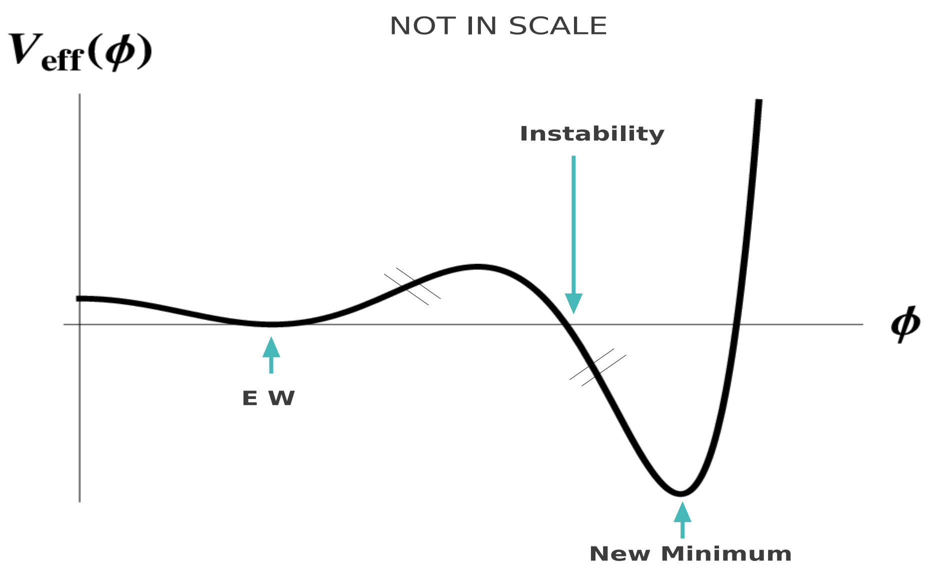


$$V^{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{1}{64\pi^2}\left[\left(m^2+\frac{\lambda}{2}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right)-\frac{3}{2}\right)\right.$$

$$+3\left(m^2+\frac{\lambda}{6}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{6}\phi^2}{\mu^2}\right)-\frac{3}{2}\right)+6\,\frac{{g_1}^4}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}{g_1}^2\phi^2}{\mu^2}\right)-\frac{5}{6}\right)$$

$$\left.+3\,\frac{\left({g_1}^2+{g_2}^2\right)^2}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}\left({g_1}^2+{g_2}^2\right)\phi^2}{\mu^2}\right)-\frac{5}{6}\right)-12\,h_t^4\phi^4\left(\ln\frac{g^2\phi^2}{\mu^2}-\frac{3}{2}\right)\right]$$

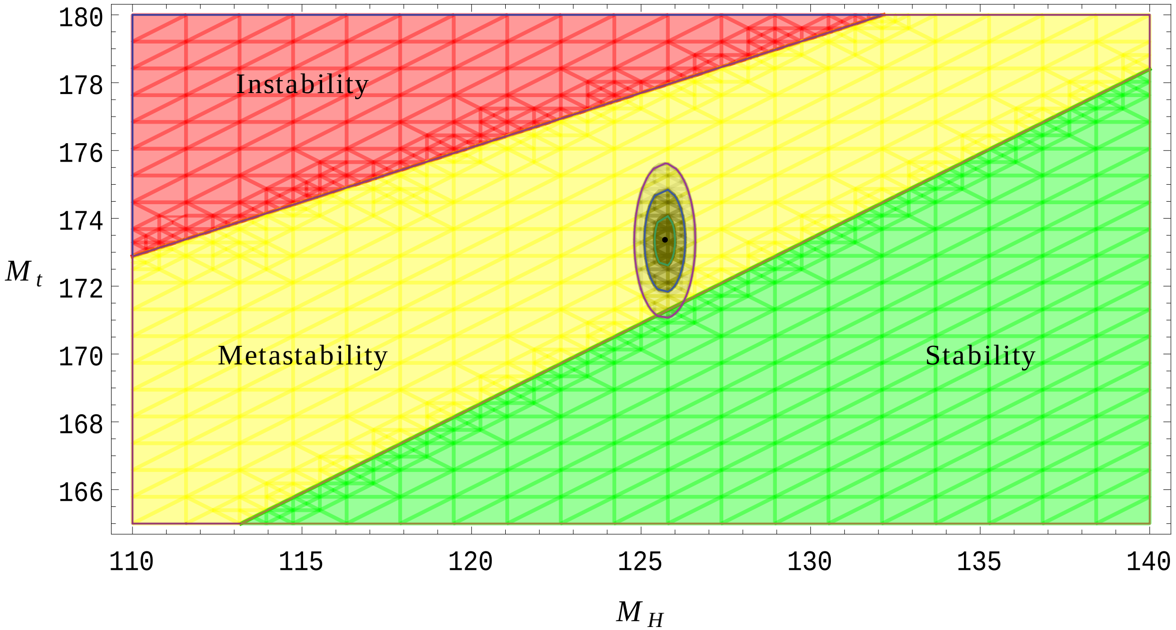
RG Improved Effective Potential $V_{_{RGI}}(\phi)$



Depending on M_H and M_t , the second minimum can be : (1) lower than the EW minimum (as in the figure) : This is the case for $M_H \sim 125$ GeV , $M_t \sim 173$ GeV ; (2) at the same level ... ; (3) higher ...

When the potential at the New Minimum is lower than the potential at the EW Minimum, compute the Tunnelling Time ...

...and we draw the Stability Diagram in the $M_H - M_t$ plane



Stability region : $V_{eff}(v) < V_{eff}(\phi^{(2)}_{min})$.

Meta-stability region : $V_{eff}(\phi^{(2)}_{min}) < V_{eff}(v)$ and $\tau > T_U$.

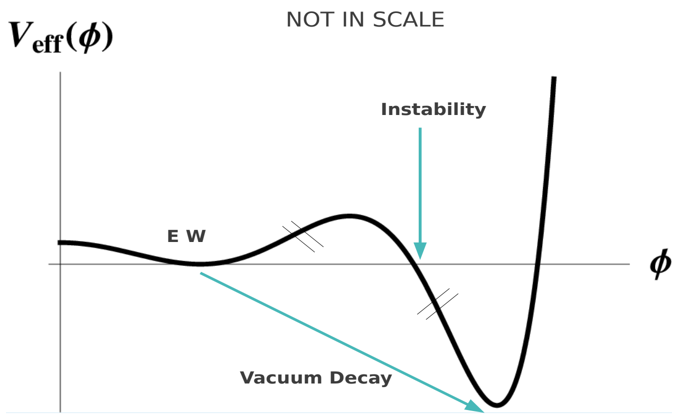
Instability region : $V_{eff}(\phi^{(2)}_{min}) < V_{eff}(v)$ and $\tau < T_U$.

Stability line : $V_{eff}(v) = V_{eff}(\phi^{(2)}_{min})$.

Instability line : M_H and M_t such that $\tau = T_U$.

Metastability Scenario

The second minimum is lower than EW $\Rightarrow$



Tunnelling between the Metastable EW Vacuum and the True Vacuum

As long as EW vacuum lifetime larger than the age of the Universe ...

.... we may well live in the Meta-Stable (EW) Vacuum ....

This is the case for the experimental values : $M_H \sim 125$ GeV, $M_t \sim 173$ GeV

How do we compute the tunneling time ?

EW vacuum lifetime (= Tunneling Time τ)

$$\Gamma = \frac{1}{\tau} = T_U^3 \, \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b) \right]}{\det \left[-\partial^2 + V''(v) \right]} \right|^{-1/2} e^{-S[\phi_b]}$$

$\phi_b(r)$ **:** **Bounce Solution**

Solution to the Euclidean Equation of Motion with appropriate boundary conditions

T. Banks, C. Bender , T. T. Wu, Phys. Rev. D 8 (1973) 3346

S. Coleman, Phys. Rev. D 15 (1977) 2929

C.G.Callan, S.Coleman, Phys. Rev. D 16 (1977) 1762

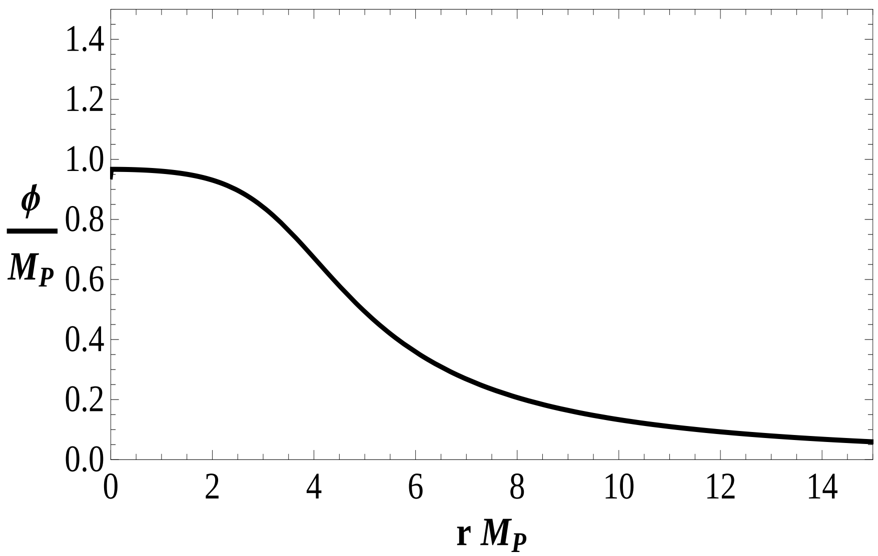
Bounce : solution to the Euclidean Equation of Motion

$$-\partial_\mu \partial_\mu \phi + \frac{d\,V(\phi)}{d\,\phi} = 0$$

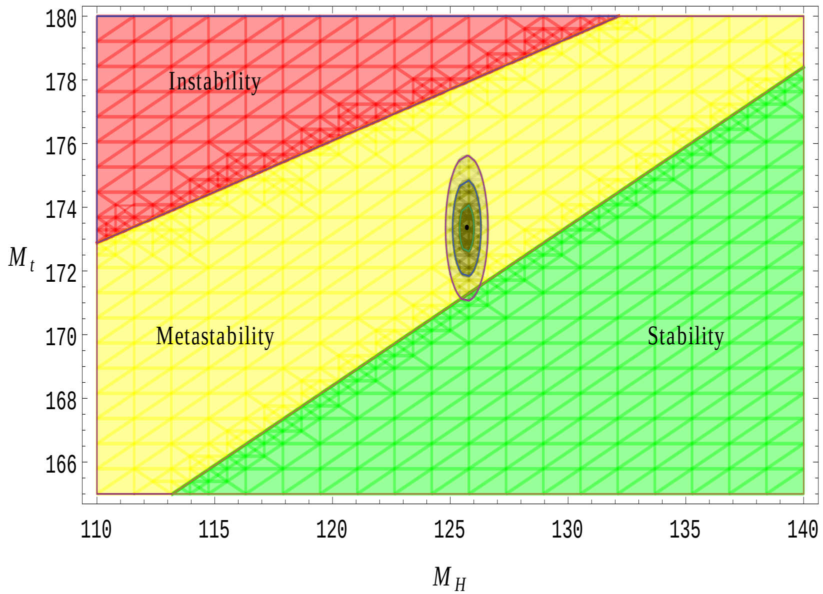
$O(4)$ **Symmetry** **(**$r =$ **euclidean radial coordinate)**

$$\Rightarrow \qquad -\frac{d^2\phi}{dr^2} - \frac{3}{r}\frac{d\phi}{dr} + \frac{d\,V(\phi)}{d\,\phi} = 0$$

Boundary conditions : $\phi'(0) = 0\ , \qquad \phi(\infty) = v \to 0$



With this Heavy Artilery we get the Stability Diagram

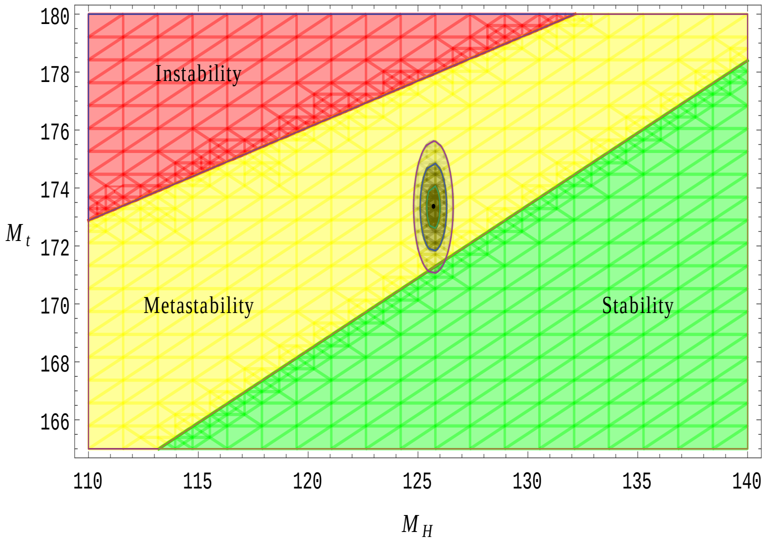


Important : People argued that even if at the **Planck scale** (or at some other **very high energy scale**) **New Physics** is expected, the latter has **no influence** on the **Stability Diagram**.

Accordingly, the **Tunnelling Time** for the **experimental values**, $M_H \sim 125$ GeV, $M_t \sim 173$ GeV :

$$\tau \sim 10^{600}\, T_U$$

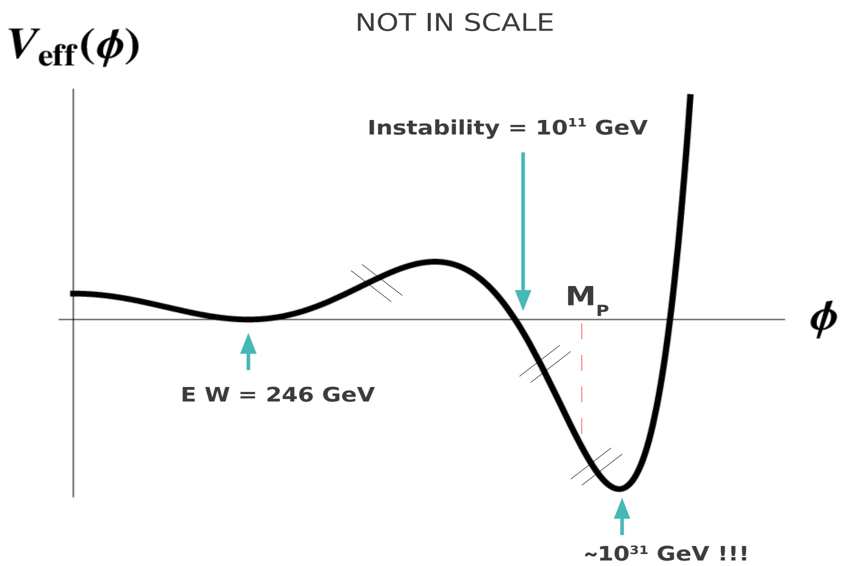
... However ... it has been shown that ...



contrary to this expectation, this Stability Diagram is not universal: even if New Physics shows up only at very high energies, the Stability Diagram depends on it ...

VB, E. Messina, *Phys.Rev.Lett.111, 241801 (2013)*; VB, E. Messina, A. Platania, *JHEP 1409 (2014) 182*; VB, E. Messina, M. Sher, *Phys.Rev.D91 (2015) 1, 013003*; VB, E. Messina, *arXiv:1507.08812*

Worth to know that for $M_H \sim 125$ GeV and $M_t \sim 173$ GeV, the Higgs RGI Effective Potential obtained with Standard Model interactions only

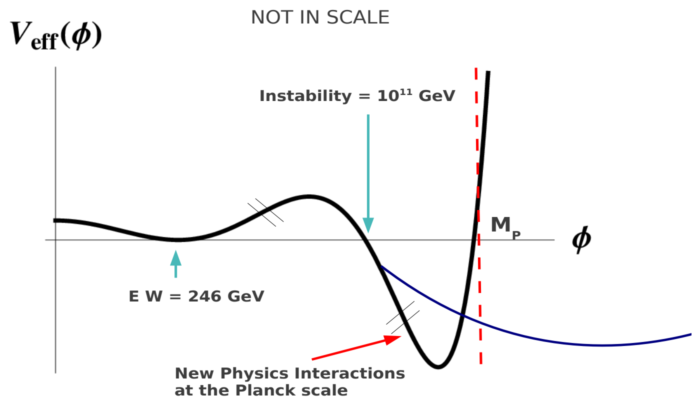


has the New Minimum at $\phi^{(2)}_{min} \sim 10^{30}$ GeV !

SM Higgs Effective Potential extrapolated well above M_P !

To make sense out of this potential, the following arguments were used

1. New Physics Interactions that is expected to appear at the Planck scale M_P eventually stabilize the potential around M_P ...



... that is, if in the Higgs Potential we take into account the presence of these new physics interactions at M_P

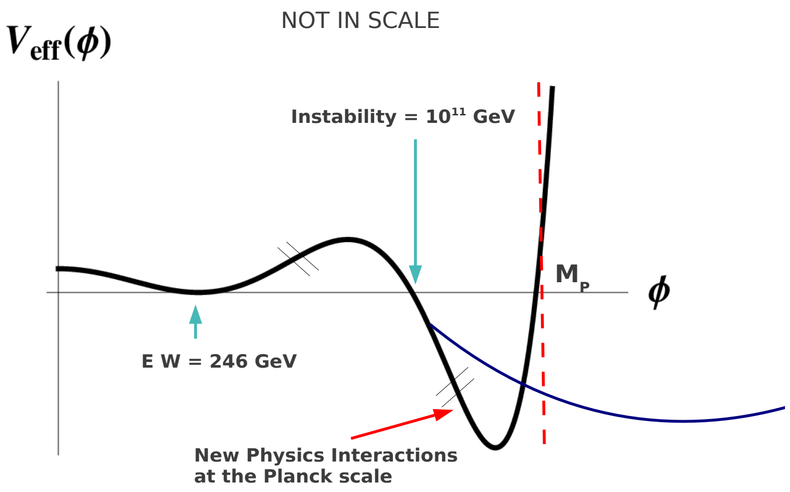
...these terms stabilize the Higgs potential around M_P...

2. These New Physics Interactions around the Planck scale do not affect the EW vacuum lifetime τ (can be neglected when computing τ) because

(a) - Instability scale, $\Lambda_{inst} \sim 10^{11}$ GeV, much lower than Planck scale $\Rightarrow$

$$\Rightarrow \quad \text{suppression} \quad \left(\frac{\Lambda_{inst}}{M_P}\right)^n \quad \text{expected}$$

(b) - For tunnelling, only the turning points do matter



... These arguments turn out to be incorrect ...

We can model the presence of New Physics at High energy scales in different ways

... Consider a (Very) Toy UV completion of the SM ...

Add to the SM potential a "New Boson" and a "New Fermion" :

$$\Delta V(\phi, S, \psi) = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \frac{g_S}{4}\phi^2 S^2 + M_f \bar{\psi}\psi + \frac{g_f}{\sqrt{2}}\phi\bar{\psi}\psi$$

with $M_f \sim 10^{17}$ **GeV and** $M_S \sim 10^{18}$ **GeV.**

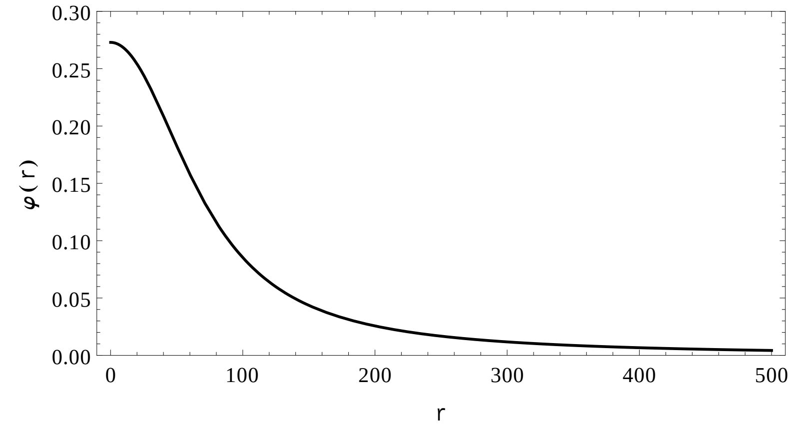
Integrating out this new scalar and fermion fields we get the

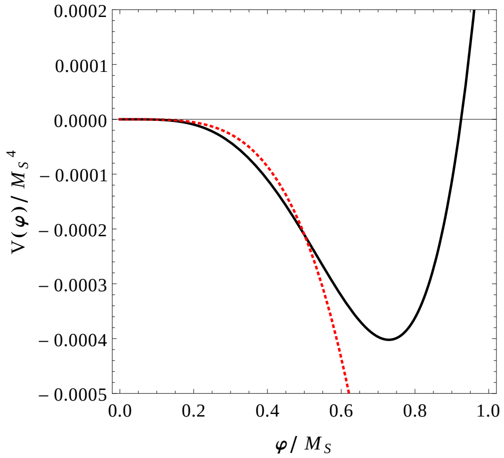
Modified Higgs Potential

$$\begin{aligned} V(\phi) &= \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{1}{64\pi^2}\left(M_S^2 + \frac{g_S}{2}\phi^2\right)^2 \left[\ln\left(\frac{M_S^2 + \frac{g_S}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right] \\ &- \frac{1}{16\pi^2}\left(M_f^2 + \frac{g_f^2}{2}\phi^2\right)^2 \left[\ln\left(\frac{M_f^2 + \frac{g_f^2}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right] \end{aligned}$$

... and compute the tunneling time ...

EXAMPLE 1 : τ modified and $\tau > T_U$





$$M_H \sim 125 \;\; \textbf{GeV and} \; M_t \sim 173 \;\;\; \textbf{GeV}$$

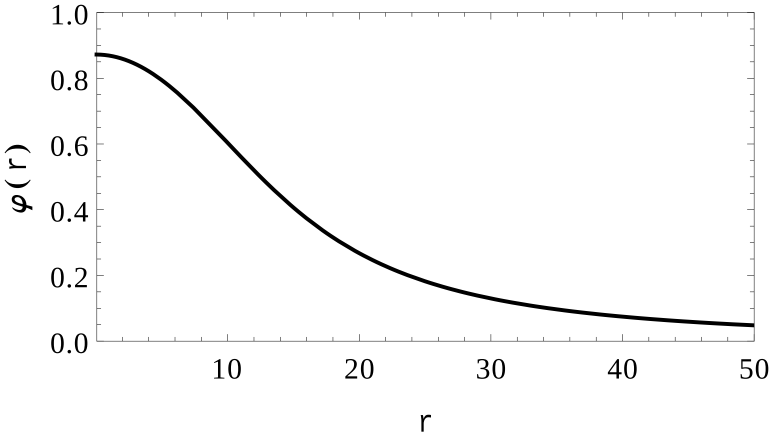
Right panel, black solid line : potential modified by the presence of M_f and M_S for certain values of of M_S, M_f, g_f and g_S

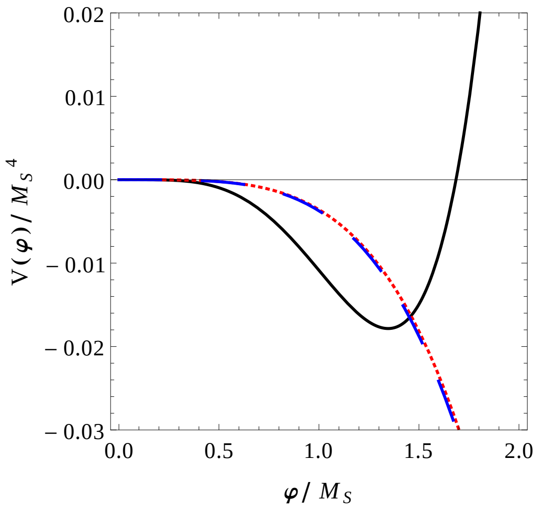
Remember : without New Physics $\tau \sim 10^{600}\, T_U$

Here : with New Physics $\tau \sim 10^{180} T_U$

$$(M_S = 1.2 \cdot 10^{18} \; \textbf{GeV}, \; M_f = 0.6 \cdot 10^{17} \; \textbf{GeV}, \; \lambda_{SM}(M_S) = -0.0151, \; \textbf{...} \;)$$

EXAMPLE 2 : τ modified and $\tau < T_U$





$$M_H \sim 125 \ \textbf{GeV and}\ M_t \sim 173 \ \ \textbf{GeV}$$

Left panel : New Bounce Solution. Right panel : SM-alone Effective Potential (dashed - dotted line); potential modified by the presence of M_f and M_S (black solid line), for other values of M_S, M_f, g_f and g_S

Remember : without New Physics $\tau \sim 10^{600}\, T_U$

Here : with New Physics $\tau \sim 10^{-65} T_U$

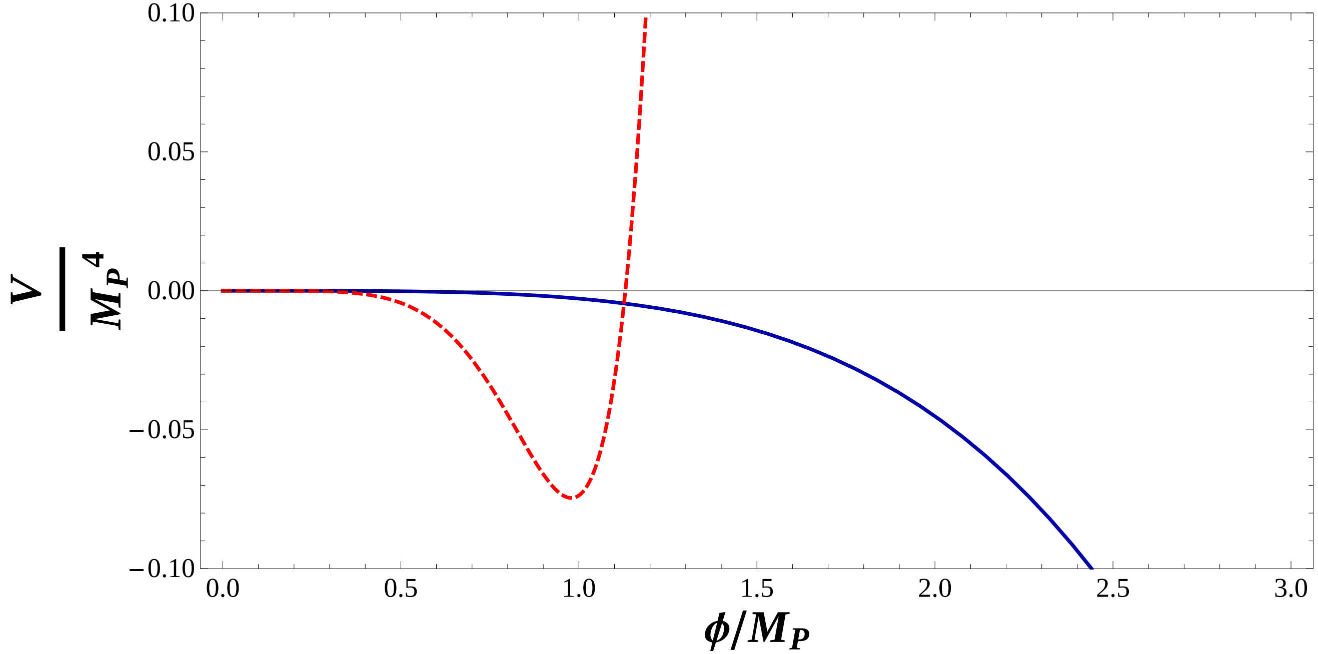
$$(M_S = 1.2 \cdot 10^{18}\ \textbf{GeV},\ M_f = 2.4 \cdot 10^{15}\ \textbf{GeV},\ \lambda_{SM}(M_S) = -0.0151,\ \ldots\ )$$

Another way of parametrizing New Physics at very high energy scales, around M_P :

Add, for instance, ϕ^6 and ϕ^8 interactions to the SM Higgs potential at M_P :

$$V(\phi) \ = \ \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$

Zoom around M_P



Blue line : $V_{eff}(\phi)$ **no new physics terms (SM alone)**

Red line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = -2 \quad \lambda_8(M_P) = 2.1$

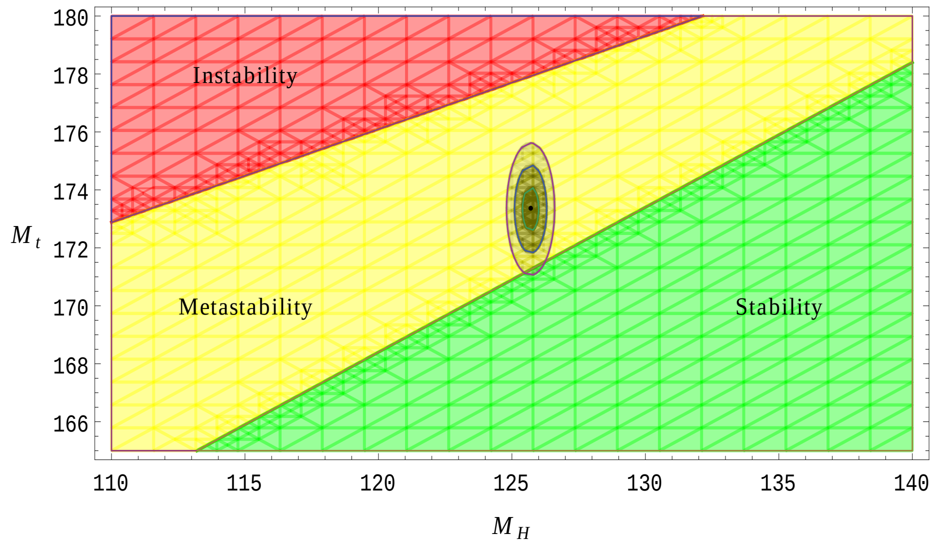
For this example $\tau \sim 10^{-200}\, T_U$ **:** τ **modified**, **and** $\tau << T_U$

But we can also have : τ **modified**, **and** $\tau > T_U$

Or even : $\tau \sim \tau_{SM}$

Back to the Stability Diagram with $\lambda_6 = 0$ and $\lambda_8 = 0$

Literature case



If we now add Add New Physics at the Planck scale

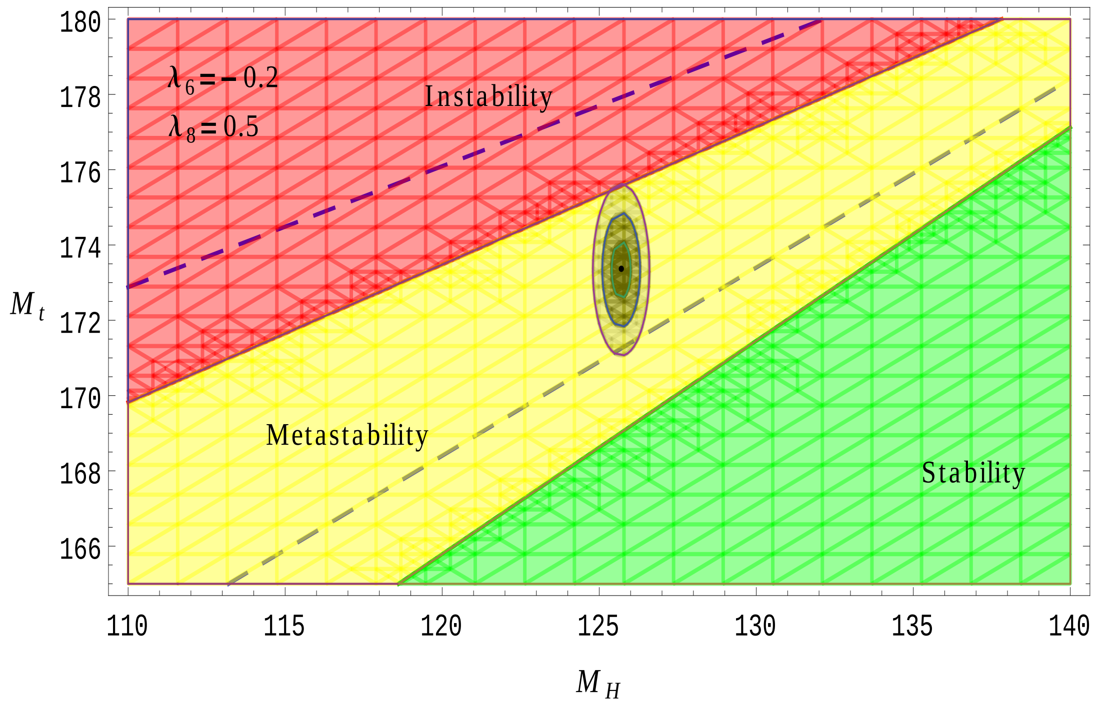
from what we have just learned we expect ...

Modification of the Instability Line (red)

Modification of the Stability Line (green)

Phase diagram with $\lambda_6 = -0.2$ and $\lambda_8 = 0.5$

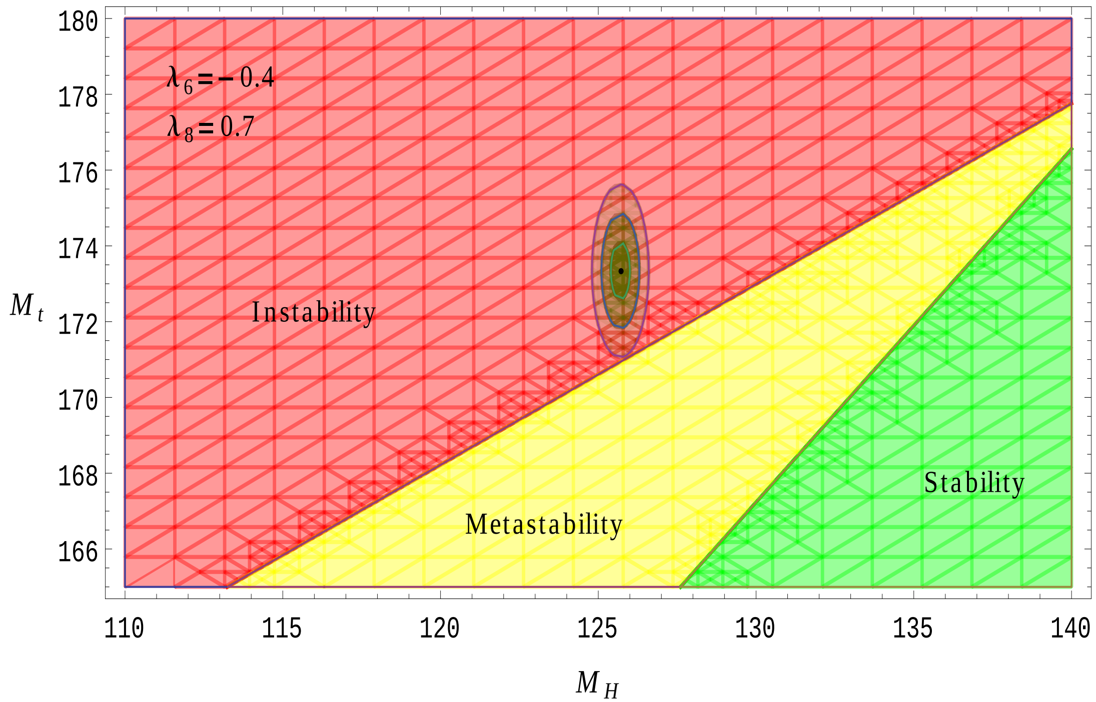
(This is like "Example 1" : $\tau << \tau_{SM}$, but still $\tau > T_U$)



The strips move downwards ... The Experimental Point no longer at $3\,\sigma$ from the stability line !!! ... Stability Diagram depends on new physics !

Phase diagram with $\lambda_6 = -0.4$ and $\lambda_8 = 0.7$

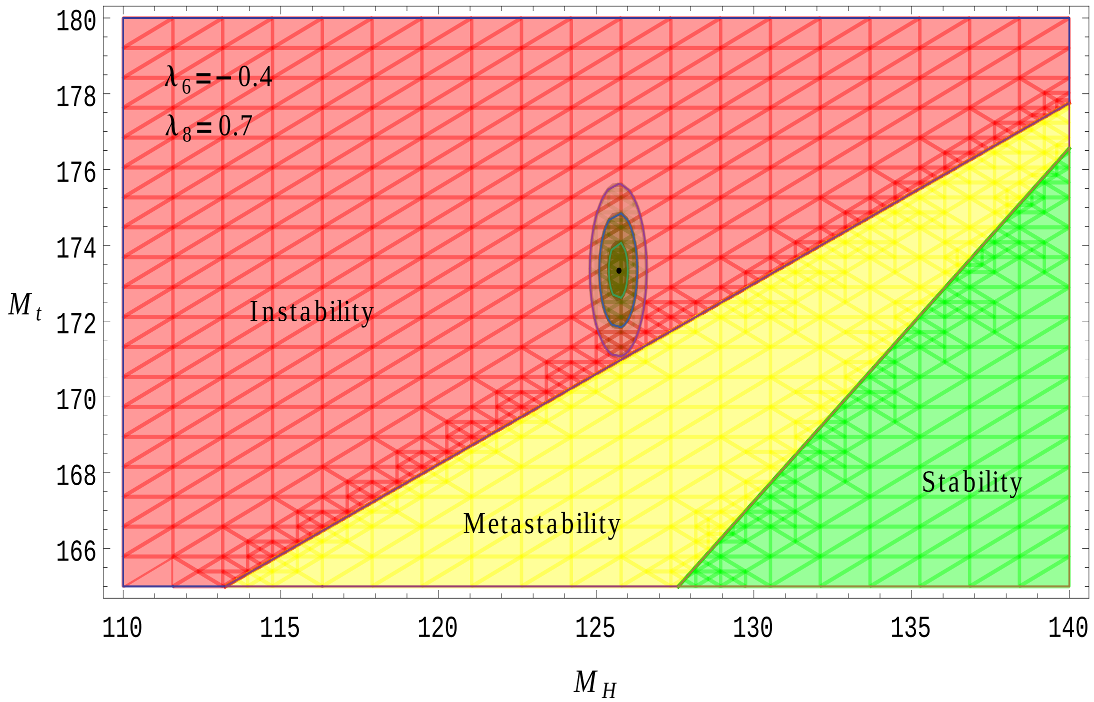
(This is like "Example 2" : $\tau << \tau_{SM}$, and $\tau < T_U$)



Stability Diagram depends on new physics !

In particular

The tunnelling time depends on New Physics, even if the latter shows up only at very High Energies !



These results came as a surprise to the community ...

It was thought, in fact, that New Physics that lives at very high energies (Planck Mass, or GUT scale, or ...) should not have an impact in the computation of the tunnelling time and more generally in establishihg the Stability Diagram

Why is that new physics at M_P has such an impact on τ ?

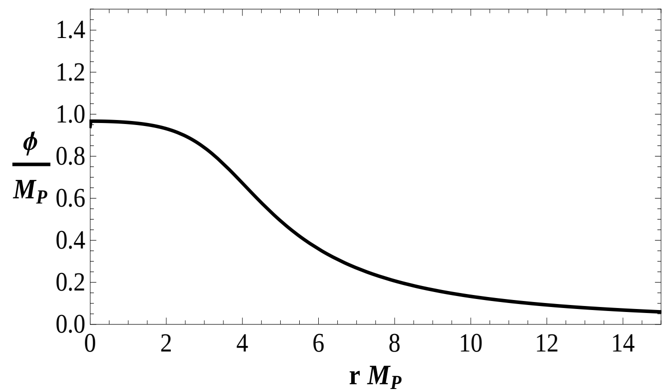
Why the decoupling arguments do not apply ?

1. New physics appears in terms of higher dimension operators $\frac{\phi^n}{M_P^n}$, and observing that $\Lambda_{inst} \sim 10^{11} M_P$ GeV people expected their contribution to be suppressed as $(\frac{\Lambda_{inst}}{M_P})^n$ But: Tunnelling is a non-perturbative phenomenon. We first select the saddle point, i.e. compute the bounce (tree level), and then compute the quantum fluctuations (loop corrections) on the top of it.

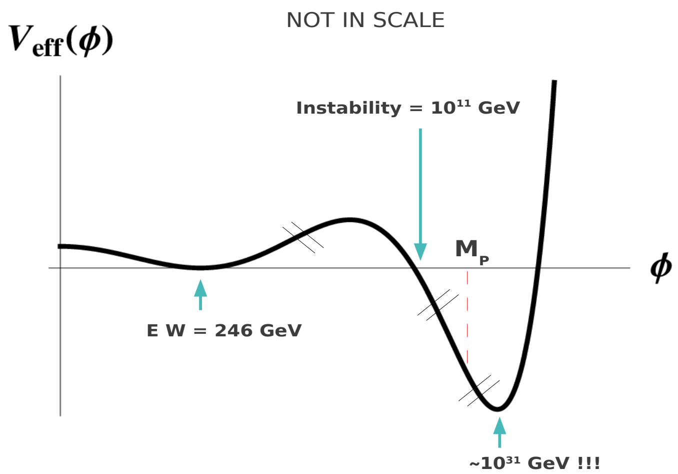
Suppression in terms of inverse powers of M_P (power counting theorem) concerns the loop corrections, not the selection of the saddle point (tree level).

Remember : $$\tau \sim e^{S[\phi_b]}$$

New bounce $\phi_b^{(new)}(r)$, New action $S[\phi_b^{(new)}]$, New τ



2. Turning points...



This is QFT with "very many" dof, not 1 dof QM $\Rightarrow$ the potential is not $V(\phi)$ in figure with 1 dof, but...

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - V(\phi) = \frac{1}{2}\dot{\phi}(\vec{x},t)^2 - U(\phi(\vec{x},t))$$

where $U(\phi(\vec{x},t))$ **is :** $\quad U(\phi(\vec{x},t)) = V(\phi(\vec{x},t)) + \frac{1}{2}(\vec{\nabla}\phi(\vec{x},t))^2$

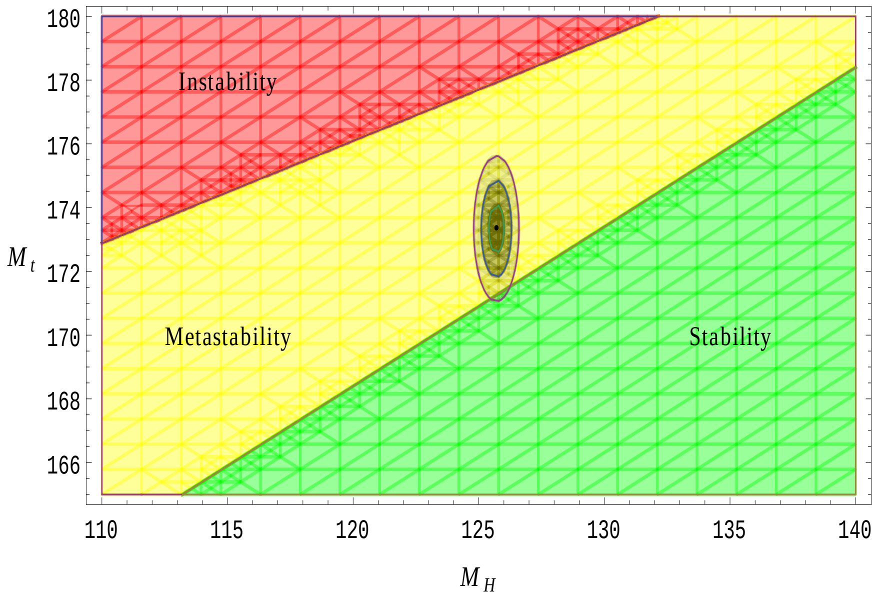
Very many dof, not 1 dof... The Potential is : $\sum_{\vec{x}} U(\phi(\vec{x},t))$

The bounce is not a constant configuration ... Gradients do matter a lot!

Lessons

From this analysis the false vacuum stability in flat space-time we learn that ...

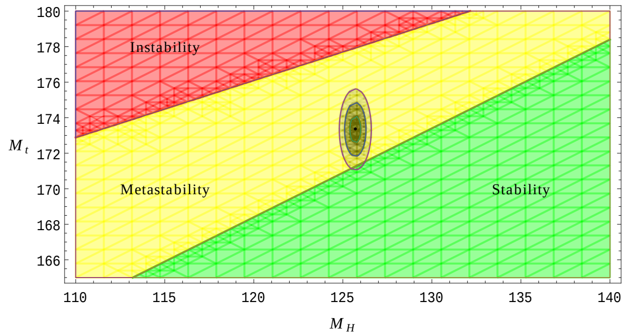
The Phase Diagram

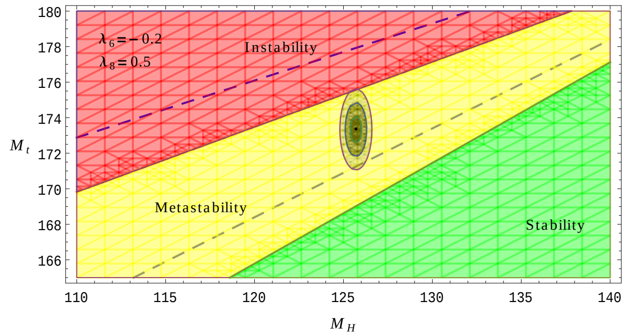


in not Universal !

... one out of different possibilities ...

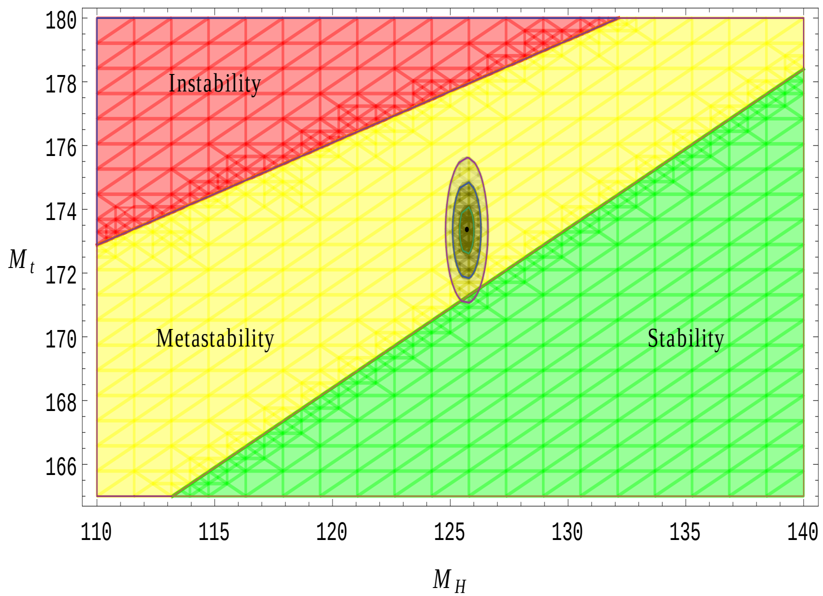
"Precision Measurements of M_H and M_t"





Precision measurements of M_t (and M_H) cannot discriminate between stability, metastability or criticality ... The knowledge of M_t and M_H alone is not sufficient to decide of the EW vacuum stability condition. We need informations on NEW PHYSICS in order to asses this question ...

“Near-Criticality”



The “near-criticality” picture is easily screwed up by any small seed of New Physics ... Strong sensitivity to new physics, No Universality ... Models based on “Near Criticality” ... ? ...

... But probably the biggest surprise is still to come ...

Up to now we have computed the lifetime of the false vacuum state

neglecting the impact of Gravity

Euclidean Equation of Motion

$$-\partial_\mu \partial_\mu \phi + \frac{d\,V(\phi)}{d\,\phi} = 0$$

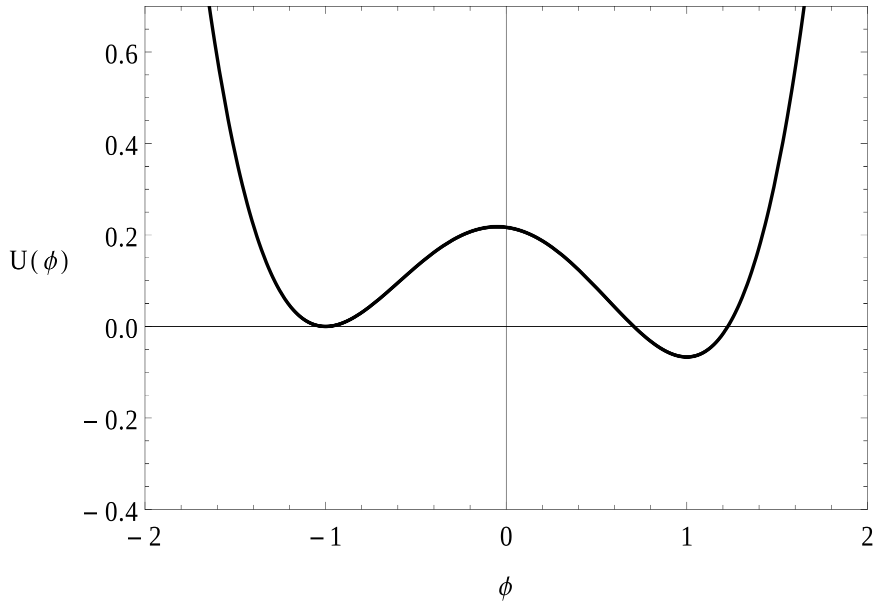
$O(4)$ **Symmetry** **(**$r =$ **euclidean radial coordinate)**

$$\Rightarrow \qquad \frac{d^2\phi}{dr^2} + \frac{3}{r}\frac{d\phi}{dr} = \frac{d\,V(\phi)}{d\,\phi}$$

Boundary conditions : $\phi'(0) = 0\ , \qquad \phi(\infty) = 0$

$$\Gamma \ \sim \ e^{-S[\phi_b]}$$

False Vacuum Decay



Coleman studied in Minkowski flat space-time (1977)

Later (1980) Coleman - De Luccia considered the impact of gravity

In both cases... Thin Wall ...

In the presence of gravitational background

we have to couple to the Bounce equation the Einstein equation and solve for the system of coupled equations

Then we can compare the Action B in the gravitational background with the Action B_0 in flat space-time

In a gravitational background - Thin Wall Approximation

Comparing the action B in the gravitational background with the action B_0 in flat space-time

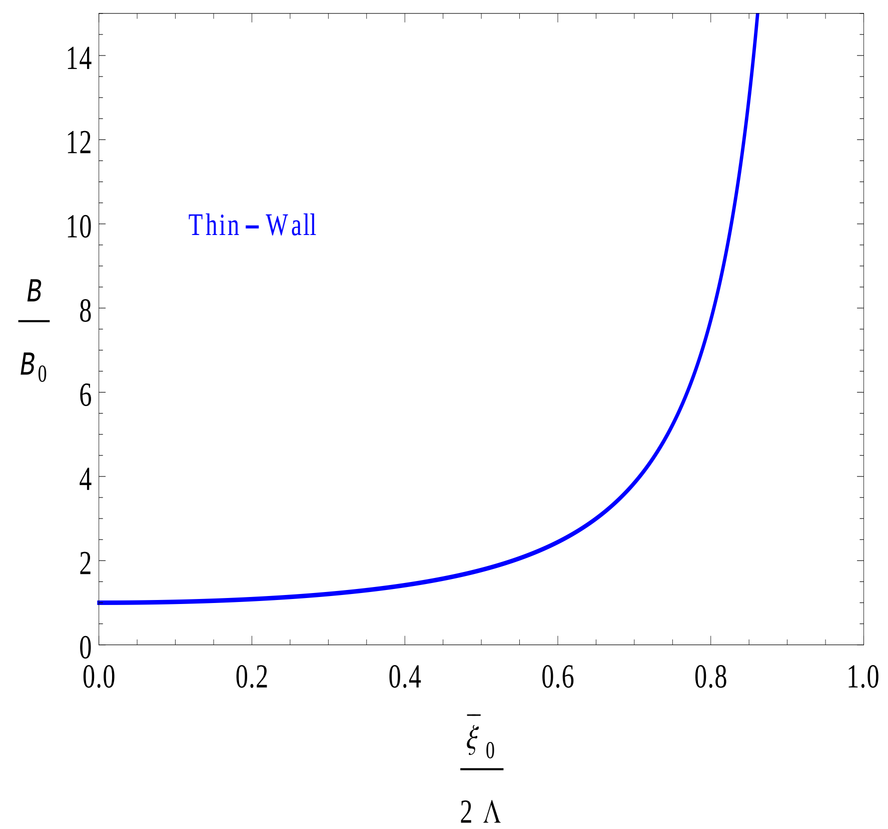
$$B = \frac{B_0}{\left[1 - \left(\bar{\xi}_0/(2\Lambda)\right)^2\right]^2}$$

with

$$\Lambda = (8\pi\, G \cdot \Delta U/3)^{-1/2}$$

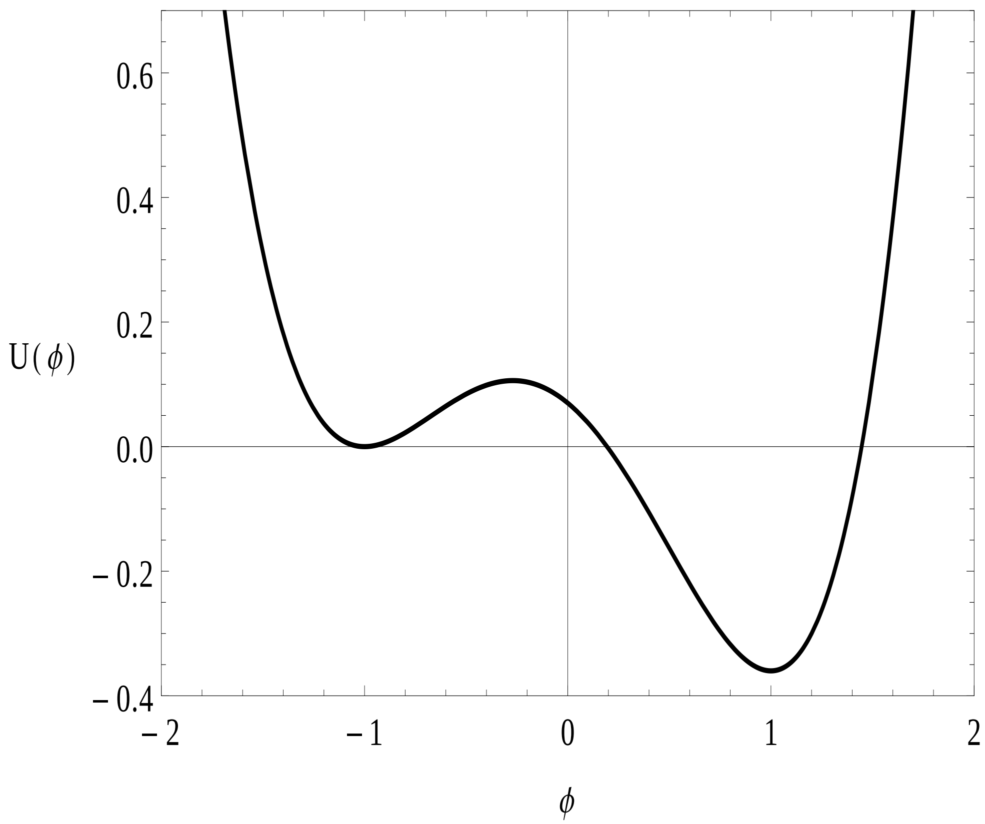
and

$$\Delta U = U(\phi_{fv}) - U(\phi_{tv})$$



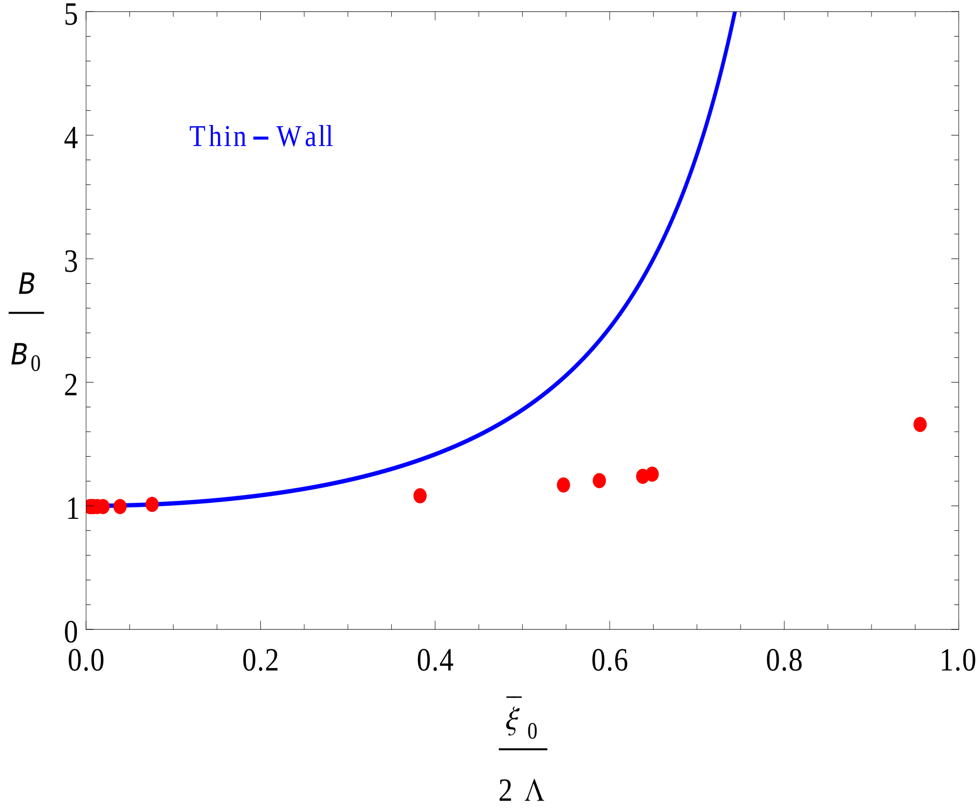
Thin − Wall
14
12
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0

Out of Thin Wall



Comparing the action B in the gravitational background with the action B_0 in flat space-time

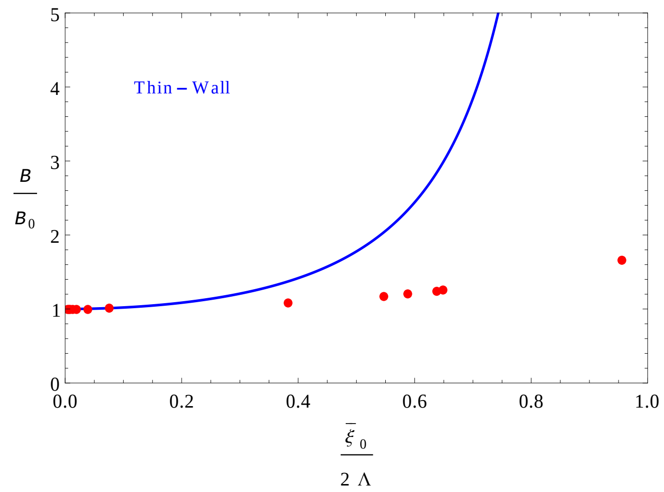
In the Thin Wall Approximation and Out of “Thin Wall”



LESSON

When $U(\phi_{fv}) - U(\phi_{tv})$ is not small, the intuition that we have developed from the Coleman-DeLuccia analysis on the Impact of Gravity does not apply !

It is no longer true that when the Bounce becomes larger and larger, the probability of materialization of the bounce becomes smaller and smaller ... eventually vanishing ...



Summary and Conclusions

Computation of the false vacuum lifetime in flat space-time

- **The Stability Phase Diagram of the EW vacuum strongly depends on New Physics even if it shows up at very high energies ($\sim M_P$)**

- **Precision Measurements of the Top and Higgs Masses will not allow to discriminate between stability, metastability or criticality of the EW vacuum. Phase Diagram too sensitive to New Physics ...**

- **Criticality ? ... Any small seed of new physics screws up the conditions**

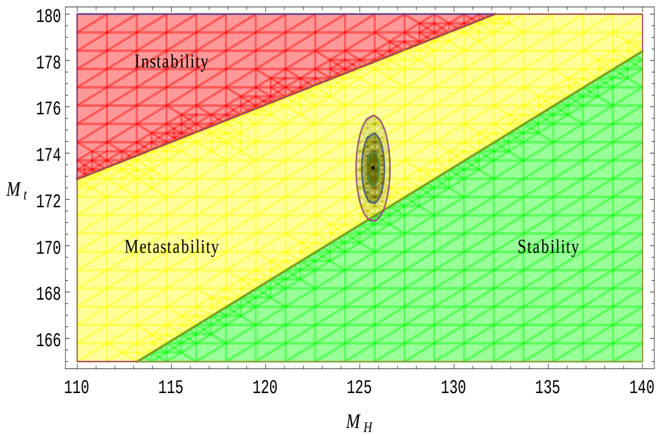
$$\lambda(M_P) \sim 0 \quad \text{and} \quad \beta(\lambda(M_P)) = \left(\mu \, \frac{d\,\lambda(\mu)}{d\,\mu}\right)_{\mu=M_P} \sim 0$$

- **"BSM stability test". A BSM is acceptable if it provides either a stable EW vacuum or a metastable one, with lifetime larger than the age of the universe (No $\tau << T_U$!!).**

- **Gravity does not wash out these conclusions! .... $B \sim B_0$**

Last point ... on the status of this Diagram ...

It seems to me that the "Stability Diagram" below



has lost its interest

This Diagram had an interest as long as it was thought (and this lasted for long time) that it has a Universal Meaning : Irrespectively of the form of New Physics at the Planck Scale, this was considered to be the Stability Diagram. ...

BACK UP SLIDES

... On the "Old View" ...

From: J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002

"For most of the relevant values of the top and Higgs masses, the instability scale Λ_{inst} is sufficiently smaller than the Planck mass, justifying the hypothesis of neglecting effects from unknown Planckian physics."

From: Isidori, Ridolfi, Strumia, Nucl.Phys. B609 (2001) 387

"The SM potential is eventually stabilized by unknown new physics around M_P : because of this uncertainty, we cannot really predict what will happen after tunnelling has taken place. Nevertheless, a computation of the tunnelling rate can still be performed, this result does not depend on the unknown new physics at the Planck scale."