NATURALNESS

A USER'S MANUAL

Warsaw, December 2015

MF and A. Urbano, Phys.Rev. D92 (2015) 015028 [arXiv:1504.05403]
arXiv:1510.03861

a dimension full parameter is natural
if its renormalised and bare values are of the same order

$$\delta m^2(\mu) = \frac{g^2}{(2\pi)^2}\left[M^2 + M^2 \ln \frac{M^2}{\mu^2}\right]$$

<ins>not</ins> a physical principle,
only a prejudice we use in model building

Naturalness is a problem of decoupling
in a theory with two widely separated energy scales

We want the low-energy parameters
to not depend on those at high-energy
(i.e., no fine-tuning in the effective theory)

scalar particles do not decouple

$$\delta m^2(\mu) = \frac{g^2}{(2\pi)^2}\left[M^2 + M^2 \ln\frac{M^2}{\mu^2}\right]$$

it is a threshold correction, finite and proportional to a mass scale

it is proportional to the coupling to the new physics above the threshold

1. it must be computable

the ``cutoff'' pitfall: $$\boxed{\delta m^2 = a\,\Lambda^2}$$

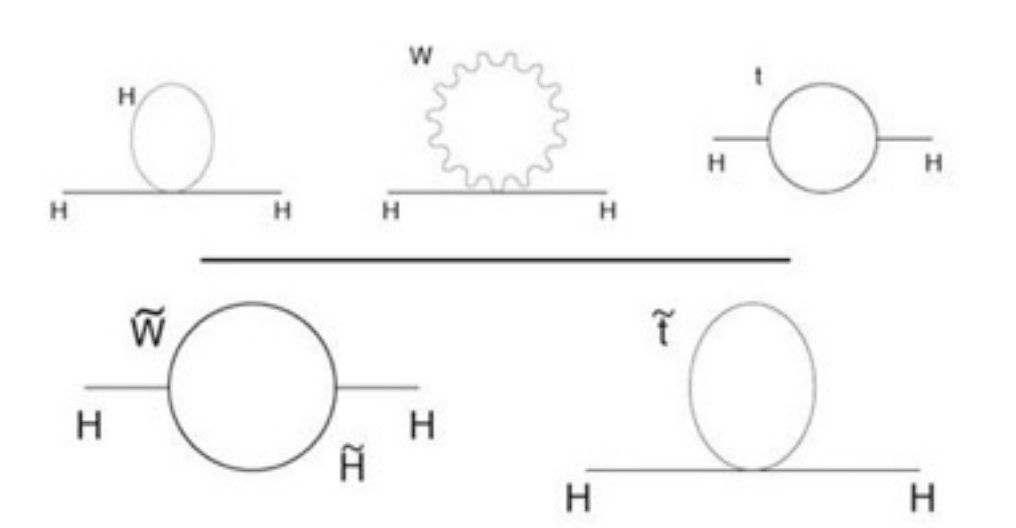
the Planck scale/Landau pole argument

Can we do a loop integral? $$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k-p)^2-m^2}\frac{1}{k^2-m^2}$$

integration to infinity probes quantum gravity
but what we do not know how to compute
is swept under the rug of renormalisation

2. there must be (at least) two scales

the ``top quark'' pitfall: $$\delta m^2 = -3\frac{\lambda_t^2}{2\pi}\Lambda^2$$



SUSY: the naive argument
little Higgs models
twin Higgs models

the SM by itself is natural

example: the neutrino seesaw mechanism

$$\frac{\Lambda^2}{16\pi^2}\lambda_t^2 \qquad \Lambda \simeq M < 10^3 \quad \text{GeV}$$

$$\frac{M^2}{16\pi^2}y_\nu^2 \qquad M \leq 10^7 \quad \text{GeV}$$

the cutoff in effective field theories refers to the external momenta (not the loop's!)

used in a momentum-dependent regularisation leads to potentially misleading results

e.g., Veltman formula

$$\boxed{m^2 = \frac{\Lambda^2}{8\pi^2}\left[3\lambda^2 + 3g'^2 + 6g^2 - 12\lambda_t^2\right]}$$

first, you find some (well-motivated) new physics
then, you check whether it is natural

e.g.: Neutrino seesaw, dark matter, GUT

Take dark matter…

<u>Minimal dark matter</u>: n-multiplet of SU(2)

1. *only gauge interactions with SM*
2. *mass is only free parameter*

n=5 Majorana fermion

Cirelli, Fornengo, Strumia, Nucl. Phys. B753 (2006) 178 [hep-ph/0512090]
Farina, Pappadopulo, Strumia, JHEP 1308 (2013) 022 [arXiv:1303.7244 [hep-ph]]
Cirelli, Strumia, Tamburini, Nucl. Phys. B787 (2007) [arXiv: 0706.4071 [hep-ph]]

corrections to Higgs boson mass:

$$\delta m_h^2\big|_\psi = 30\,\frac{g^4 M^2}{(4\pi)^4}\left[6\ln\frac{M^2}{\mu^2}-1\right],$$

$$\delta m_h^2\big|_\phi = -30\,\frac{g^4 M^2}{(4\pi)^4}\left[\frac{3}{2}\ln^2\frac{M^2}{\mu^2}+2\ln\frac{M^2}{\mu^2}-\frac{7}{2}\right]$$

naturalness

$$M < 1.5\ \mathrm{GeV}\ (\mathrm{Fermion})$$

$$M < 4.2\ \mathrm{GeV}\ (\mathrm{Scalar})$$

relic density

after Sommerfeld enhancement

$$M = 10\ \mathrm{TeV}$$

protecting the Higgs boson mass
by a partial implementation of SUSY

$$\boxed{\Phi^A_{\mathrm{DM}} = \phi^A + \theta \cdot \psi^A + F^A \theta^2 \,,}$$

$$\Phi^a_{H_{u,d}} = H^a_{u,d} + \theta \cdot \tilde{h}^a_{u,d} + G^a_{u,d}\theta^2 \,,$$

$$V^\alpha = \theta \cdot (\sigma^\mu \bar{\theta})\, W^\alpha_\mu + \bar{\theta}^2\, \theta \cdot \lambda^\alpha + \theta^2\, \bar{\theta} \cdot \bar{\lambda}^\alpha + \frac{1}{2}\theta^2\, \bar{\theta}^2 D^\alpha$$

$$\mathcal{L}_{WZ} = \int d^2\theta d^2\bar{\theta}\left[\Phi^\dagger_{\mathrm{DM}}\, e^{2gV}\,\Phi_{\mathrm{DM}} + \Phi^\dagger_{H_k}\, e^{2gV}\,\Phi_{H_k}\right]$$
$$+\left[\frac{1}{2}\int d^2\theta\,\mathrm{Tr}(\mathrm{W}\cdot\mathrm{W}) + \int d^2\theta\,\mathcal{W}(\Phi_{\mathrm{DM}}) + h.c.\right]$$

$$\mathcal{W}(\Phi_{\mathrm{DM}}) = \frac{M}{2}(\epsilon_{\mathbf{5}})_{AB}\Phi^A_{\mathrm{DM}}\Phi^B_{\mathrm{DM}}$$

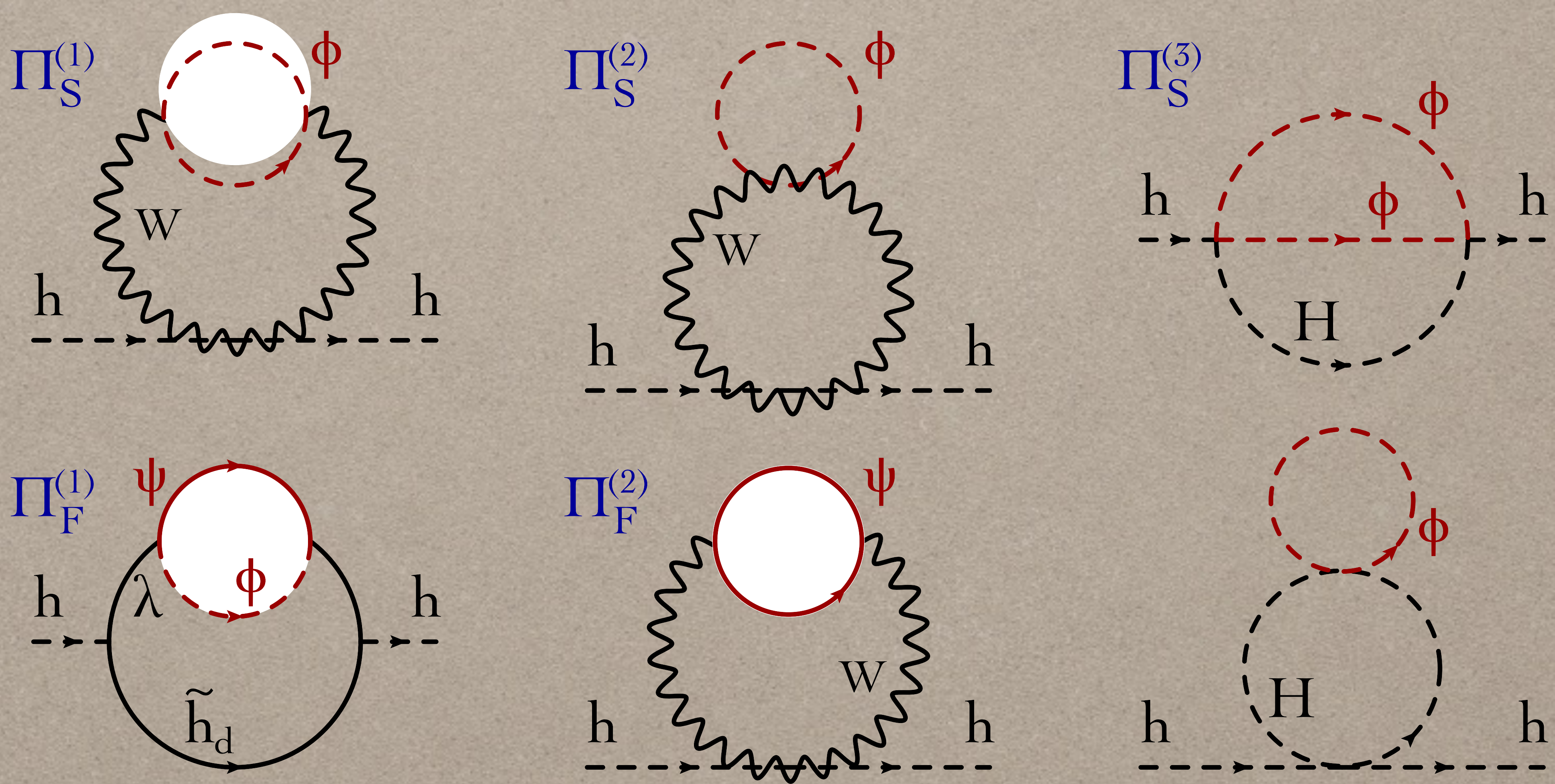
$$\mathcal{L}_{\mathrm{nMDM}} = \mathcal{L}_{SM} + \mathcal{L}_{WZ} - \frac{1}{2}\tilde{m}_\lambda(\lambda^\alpha\cdot\lambda^\alpha + h.c.)$$
$$+\mu(\epsilon_{\alpha\beta}\tilde{h}^\alpha_u\cdot\tilde{h}^\beta_d + h.c.)$$

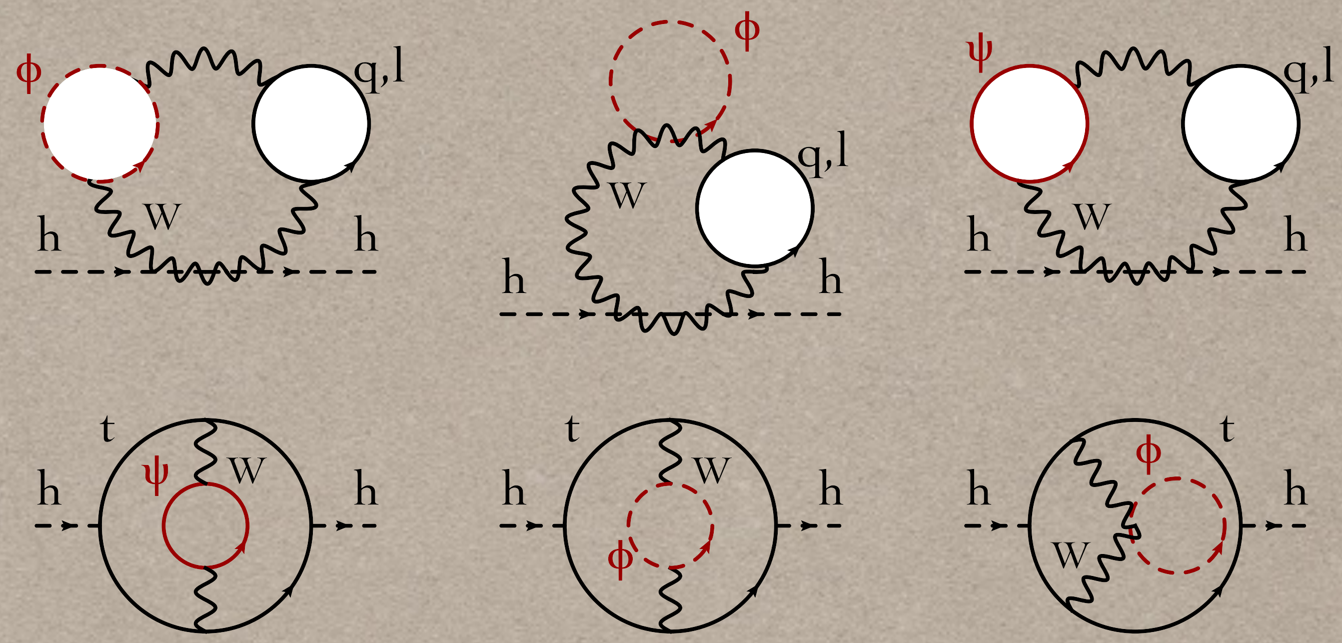
stability of MDM

$$\phi HHH^*H^*/M$$

$$\psi LHH^*/M^2$$

2-loop Higgs boson mass



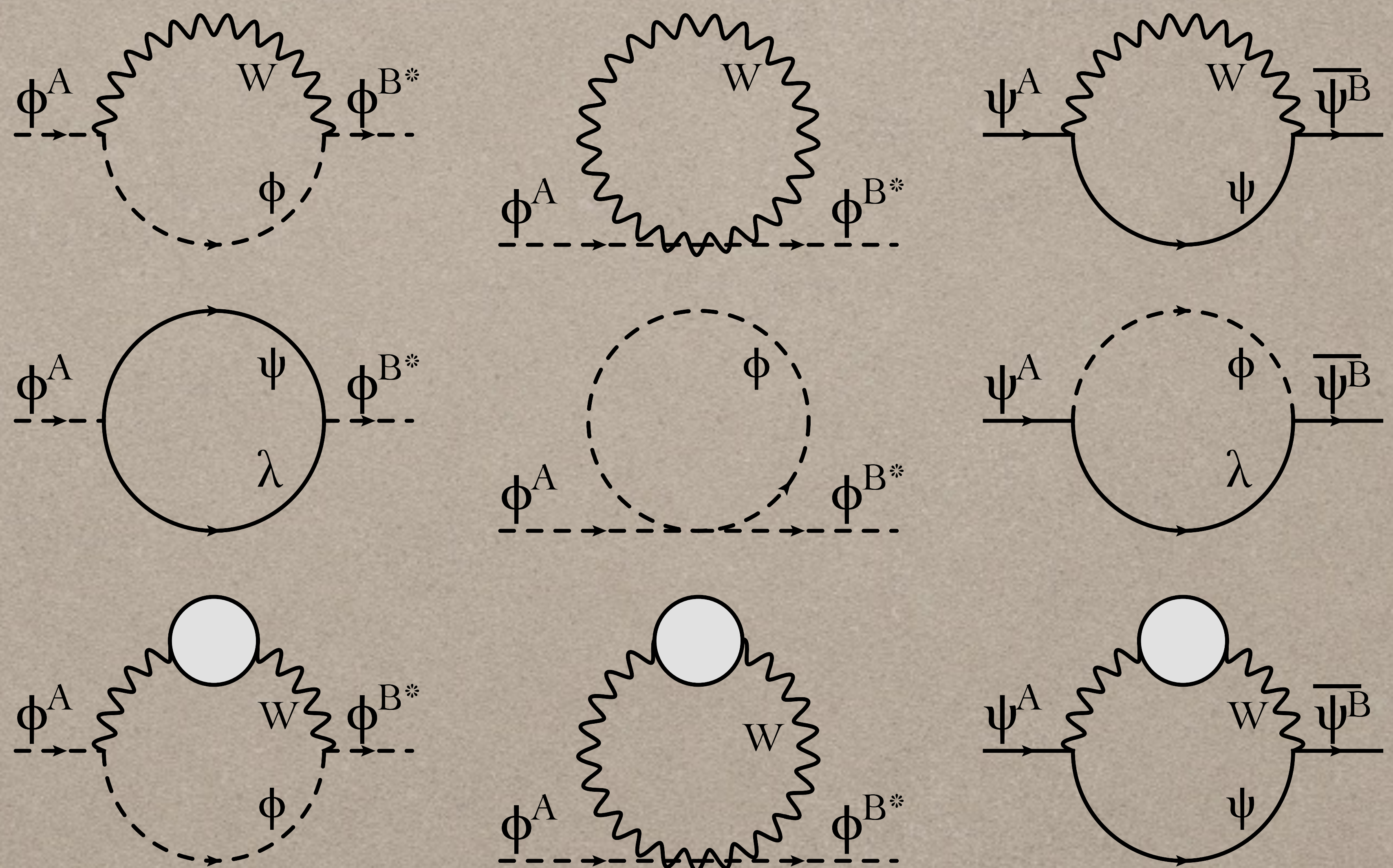


$$\delta m_h^2 \sim g^6 M^2/(4\pi)^6$$

naturalness

$$M < 15\ \mathrm{TeV}\ \ (\mathrm{Fermion})$$
$$M < 42\ \mathrm{TeV}\ \ (\mathrm{Scalar})$$

SUSY multiplet splitting



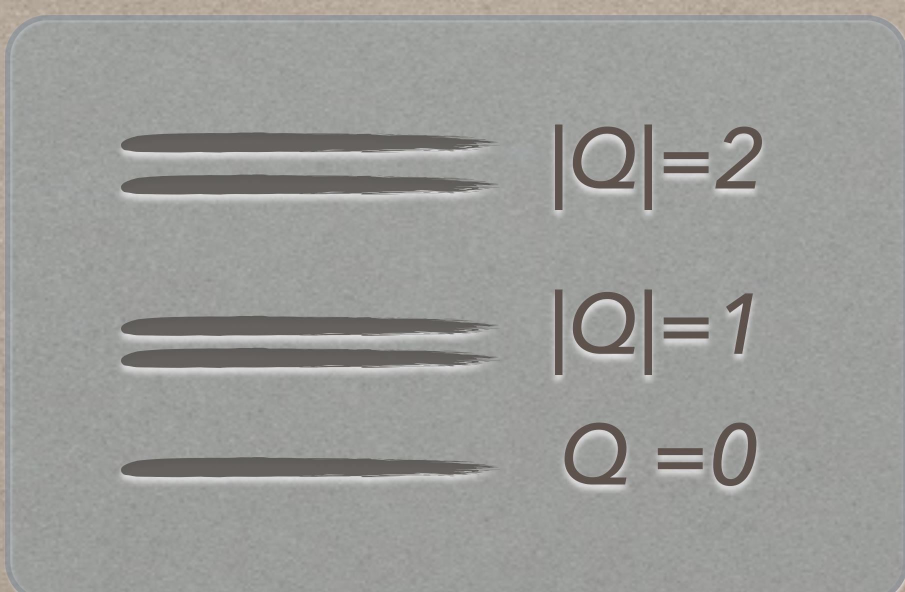
$$\Delta M^{(\phi,\psi)} \simeq g^2 M/(16\pi^2) \simeq 2.5 \times 10^{-3}\ M$$

multiplet splitting

$$M_\phi^{(Q)} = M_\phi + Q^2 \Delta M_g + \Delta M_{\rm SUSY}^\phi - Q(s_\beta^2 - c_\beta^2)\frac{g^2 v^2}{8 M_\phi}$$

$$M_\psi^{(Q)} = M_\psi + Q^2 \Delta M_g + \Delta M_{\rm SUSY}^\psi$$

$$\Delta M_g \simeq 166 \text{ MeV}$$



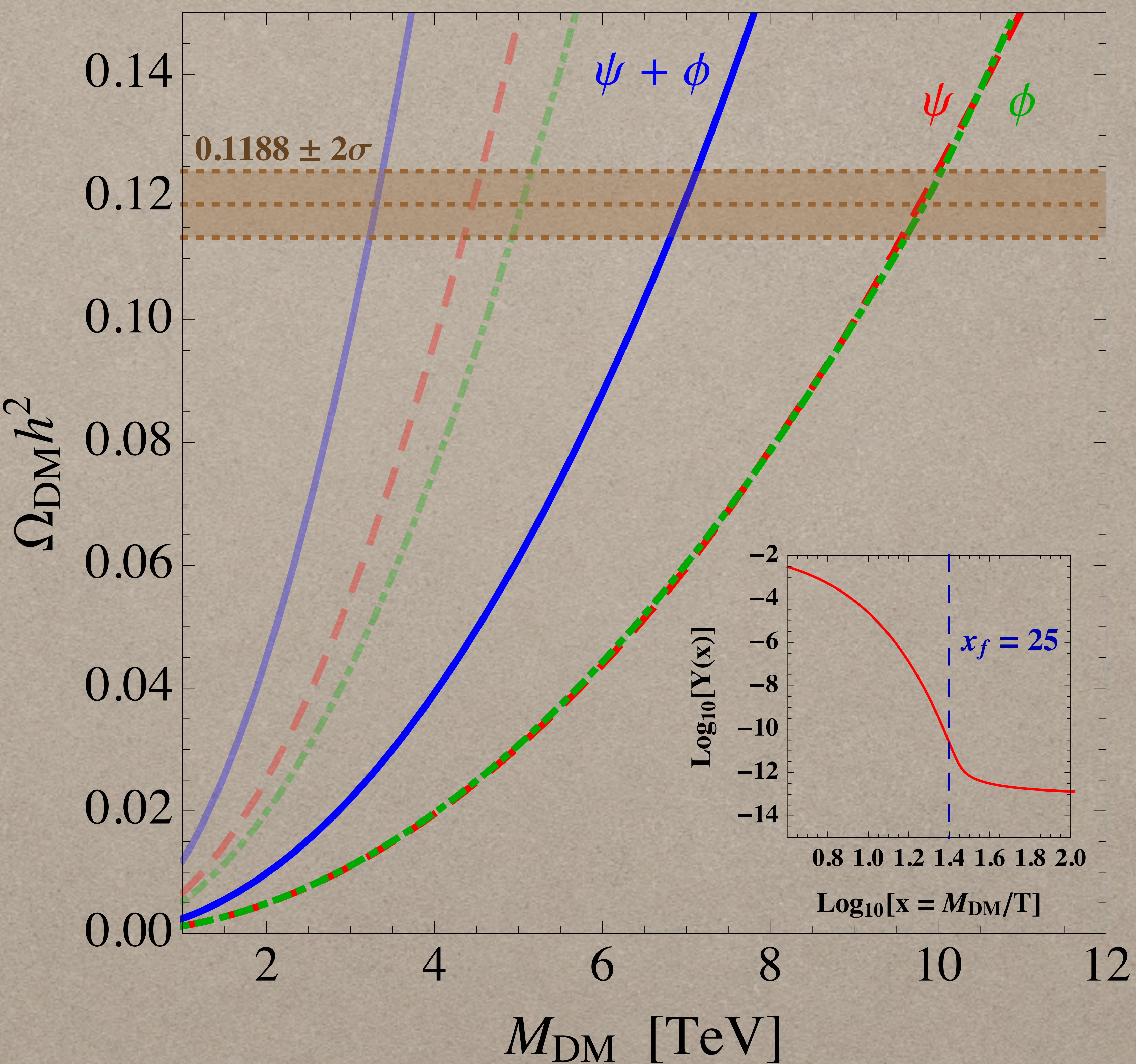
Boltzmann equation

$$\langle \sigma v \rangle_\phi = \frac{g^4}{64\pi M^2 2n}\left[3 - 4n^2 + n^4 + \frac{n^2-1}{2}\right]$$

$$\langle \sigma v \rangle_\psi = \frac{g^4}{64\pi M^2 2n}\left[2n^4 + 17n - 19\right]$$

$$\Omega_{\mathrm{DM}} h^2 = \frac{Y_\infty M s_0}{\rho_c^0 h^{-2}} = 0.1188 \pm 0.0022$$

with Sommerfeld enhancement



0.1188 ± 2σ
ψ + φ
ψ
φ
$\Omega_{\rm DM}h^2$
$M_{\rm DM}$ [TeV]
x_f = 25
$\mathrm{Log_{10}[Y(x)]}$
$\mathrm{Log_{10}[x} = M_{\rm DM}\mathrm{/T]}$

new physics is SUSY, SM breaks the symmetry

<u>telescoping SUSY models</u>

*<u>little hierarchy</u>: just Higgs sector (**neutrino seesaw**)*

*plus weak gauge bosons (**MDM**)*

SUSY signatures

mostly in electroweak sector with no color

winos

$$\mathcal{L}_{\chi^0} = -\frac{1}{2}(\tilde{G}^0)^T \begin{pmatrix} \tilde{m}_\lambda & -\frac{gv}{2}s_\beta & \frac{gv}{2}c_\beta \\ -\frac{gv}{2}s_\beta & 0 & -\mu \\ \frac{gv}{2}c_\beta & -\mu & 0 \end{pmatrix} \tilde{G}^0 + h.c.$$

$$\equiv -\frac{1}{2}(\tilde{G}^0)^T \mathcal{M}_{\chi^0} \tilde{G}^0 + h.c.\,,$$

$$\mathcal{L}_{\chi^\pm} = -\frac{1}{2}\left[(\tilde{g}^+)^T \mathcal{M}^T \tilde{g}^- + (\tilde{g}^-)^T \mathcal{M} \tilde{g}^+\right] + h.c.\ ,$$

$$\mathcal{M} = \begin{pmatrix} \tilde{m}_\lambda & gvs_\beta/\sqrt{2} \\ gvc_\beta/\sqrt{2} & \mu \end{pmatrix}$$

2HDM

$$H_u = \begin{pmatrix} \frac{1}{\sqrt{2}}(H^0 c_\alpha - h_0 s_\alpha + iA^0 s_\beta - iG^0 c_\beta) \\ H^- s_\beta - G^- c_\beta \end{pmatrix},$$

$$H_d = \begin{pmatrix} H^+ c_\beta + G^+ s_\beta \\ \frac{1}{\sqrt{2}}(H^0 s_\alpha + h_0 c_\alpha + iA^0 c_\beta + iG^0 s_\beta) \end{pmatrix}$$

you decide the scale,
the scale decides
the degree of SUSY required

telescoping models:

- only extra sleptons and higgsinos
- also weak gauginos
- also squarks
- also gluinos

*<u>intermediate hierarchy</u>: Higgs, plus weak gauge bosons, plus quarks (**neutrino/leptogenesis**)*

*<u>large hierarchy</u>: whole SM (**GUT/gauge unification**)*

SUSY signature in electroweak sector and color (only quarks or also gluinos)

backup slides

it is similar but not identical
to the concept of fine tuning

e.g.: Higgs boson mass in SUSY models

$$m_Z^2 2\cos^2 2\beta + \frac{3}{(4\pi)^2}\frac{m_t^2}{v^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_t^2}\left(1 - \frac{X_t^2}{12m_t^2}\right)\right]$$

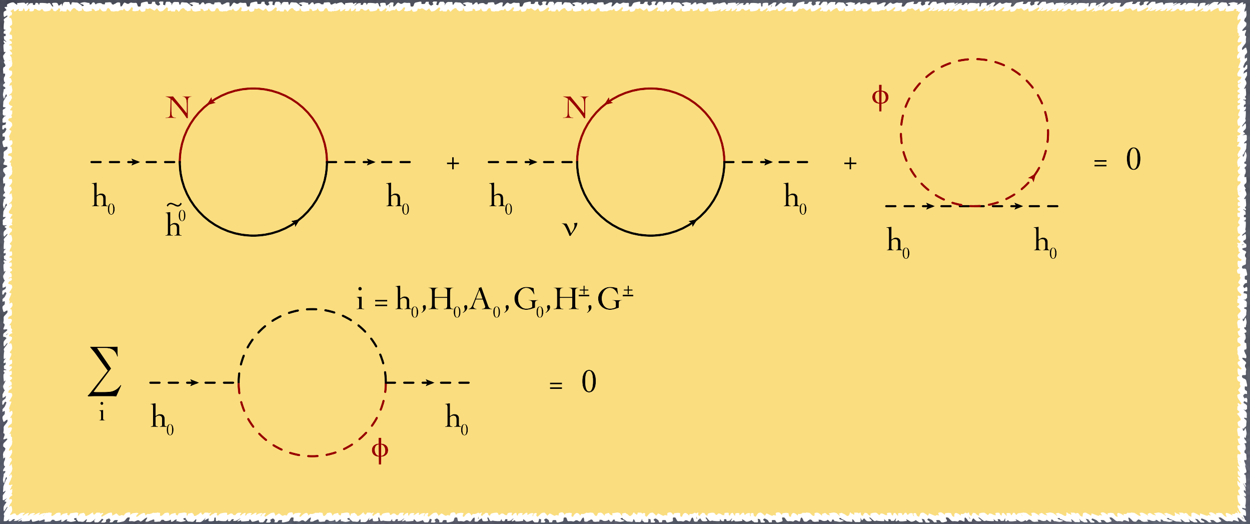
toy model: 1 generation

$$\boxed{\begin{aligned}
\Phi^1_\alpha &= H^u_\alpha + \theta \cdot L_\alpha + \theta^2 F^u_\alpha \\
\Phi^2_\alpha &= H^d_\alpha + \theta \cdot \tilde{h}^d_\alpha + \theta^2 F^d_\alpha \\
\Phi_{\mathrm{NP}} &= \phi + \theta \cdot N + \theta^2 F
\end{aligned}}$$

$$\mathcal{W}(\Phi_i) = \epsilon_{\alpha\beta}\Phi_1^\alpha\Phi_2^\beta\Phi_{\rm NP} + \frac{M}{2}\Phi_{\rm NP}^2$$

$$\begin{aligned}\mathcal{L}_{\rm int} = \quad & -\eta^2(\epsilon_{\alpha\beta}H^u_\alpha H^d_\beta)(\epsilon_{\alpha'\beta'}H^{u\,*}_{\alpha'}H^{d\,*}_{\beta'}) - \eta^2|\phi|^2 H^d_\alpha H^{d\,*}_\alpha - \eta^2|\phi|^2 H^u_\alpha H^{u\,*}_\alpha \\ & -\eta\epsilon_{\alpha\beta}\left(M\phi^* H^u_\alpha H^d_\beta + L_\alpha\cdot\tilde{h}^d_\beta\phi + \tilde{h}^d_\beta\cdot N H^u_\alpha + \underline{L_\alpha\cdot N H^d_\beta} + h.c.\right)\end{aligned}$$

+ SM

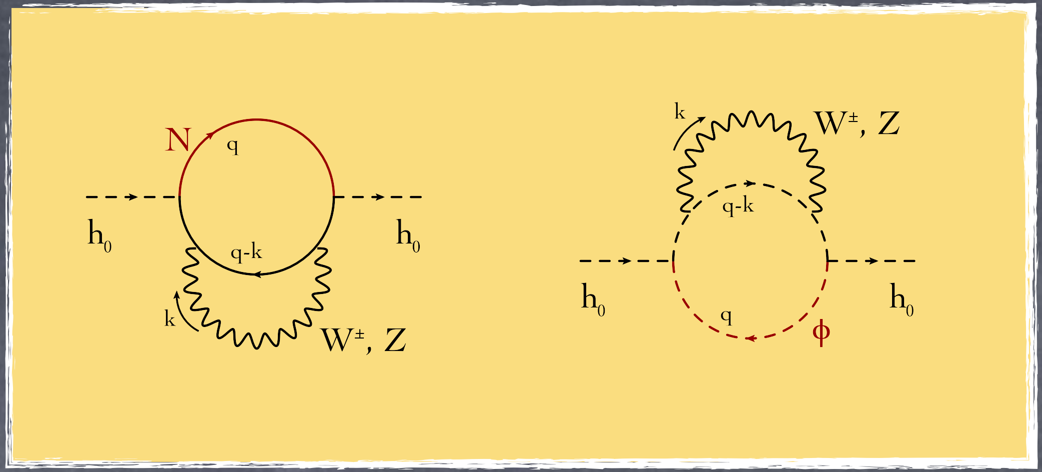


Higgs mass safe at the 1 loop

→ little hierarchy solved

$\boxed{\text{SM hard SUSY breaking}}$

it starts at 2-loop level



only NP SUSY

$$\delta m_H^2 \simeq \frac{\alpha}{4\pi}\frac{y_\nu^2}{16\pi^2}M^2$$

$$M \simeq 10^8 \text{ GeV}$$

3-loop level

NP + weak gauge bosons SUSY

$$\delta m_H^2 \simeq \left(\frac{\alpha}{4\pi}\right)^2 \frac{y_\nu^2}{16\pi^2}M^2$$

$$M \simeq 10^9 \text{ GeV}$$

the usual type-1 seesaw cannot
gives rise to baryogengesis via leptogenisis

$$\boxed{M \leq 10^7 \qquad\qquad M \geq 10^8}$$

the model provides
the missing order of magnitude