Implications of Dark Matter bound states

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SCUOLA
NORMALE
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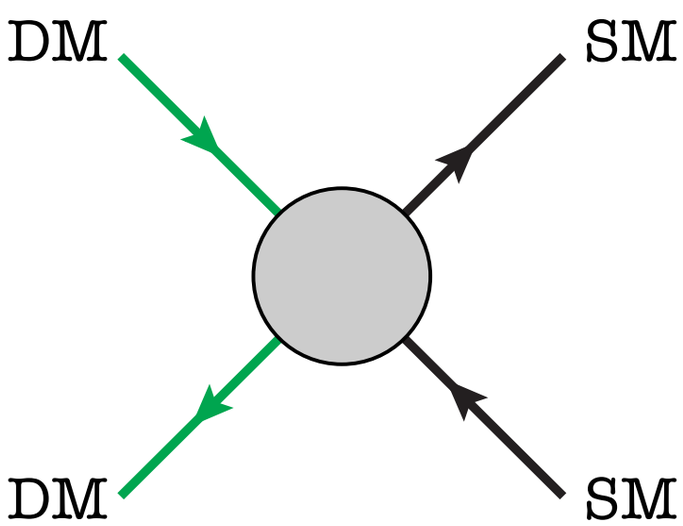
$$\Omega_{\mathrm{DM}} h^2 = 0.119$$

The standard (thermal) picture

$$\frac{dY_{DM}}{dz} = -\frac{s\,\langle\sigma v_{rel}\rangle}{Hz}\left(Y_{DM}^2 - Y_{DM}^{eq2}\right)$$



: $\langle\sigma v_{rel}\rangle = \sigma_0\,[M_\chi, \ldots]$

Imposing $\Omega_{\rm DM} h^2 = 0.119$ we get the DM mass or at least bounds on its value.

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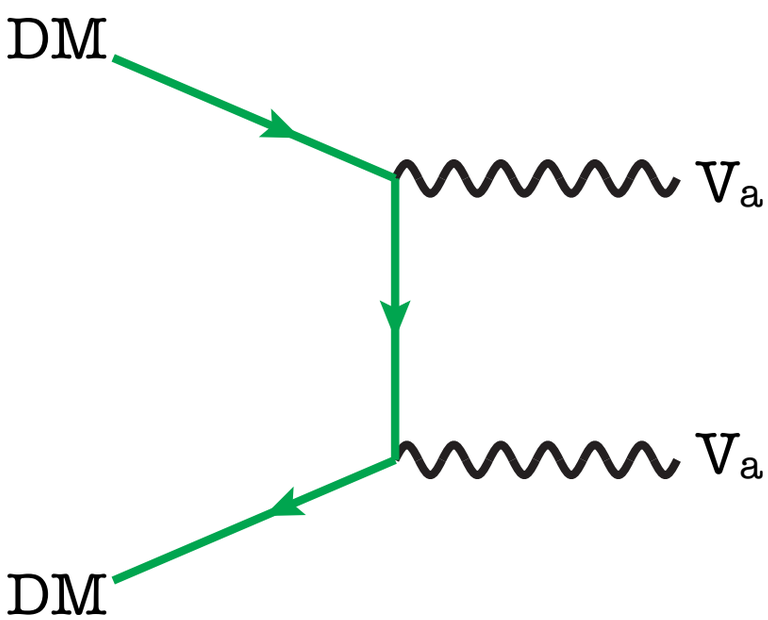

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$$\langle \sigma v_{rel} \rangle \simeq \frac{\pi \alpha^2}{M_\chi^2}$$

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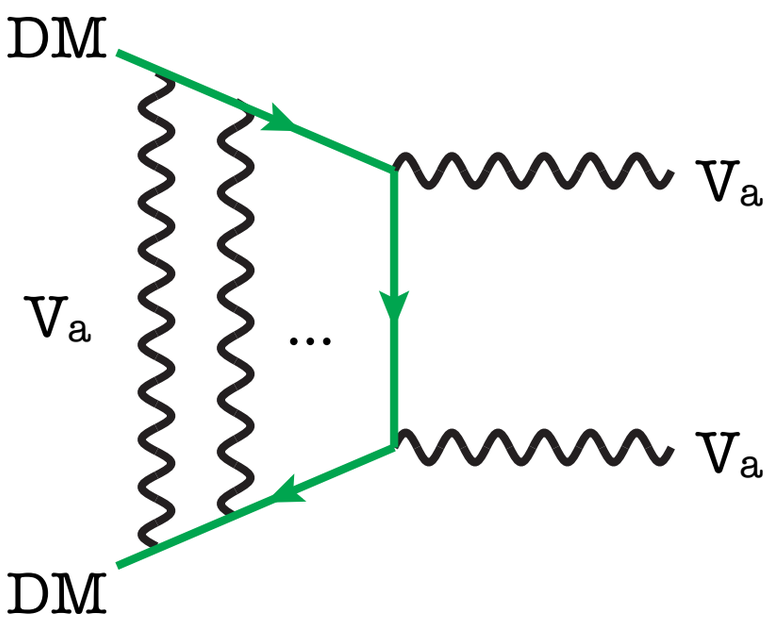
If the **interaction** is **long ranged** ($\alpha M_\chi > M_V$), non perturbative effects can spoil the perturbative results

[Sommerfeld (1931); Hisano et al. (2002); Cirelli et al. (2005); Fenget al. (2009); Slatyer et al. (2013);...]



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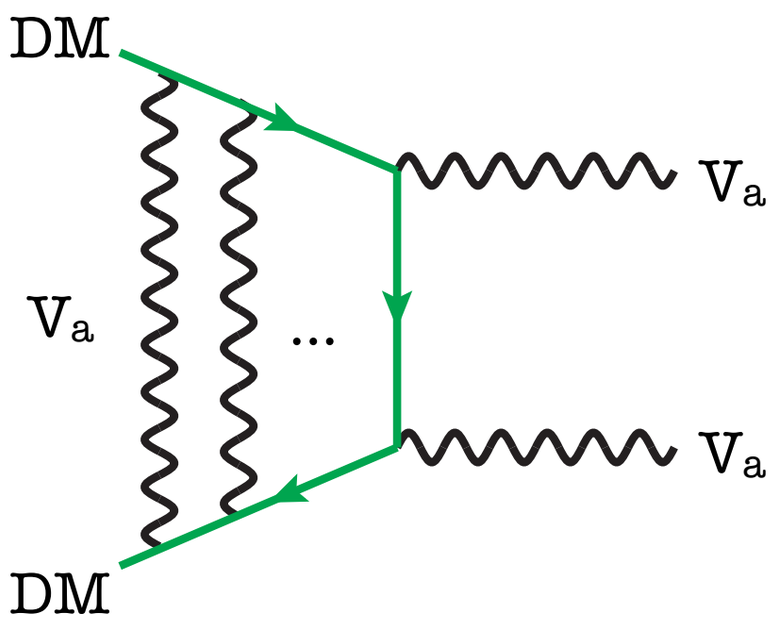

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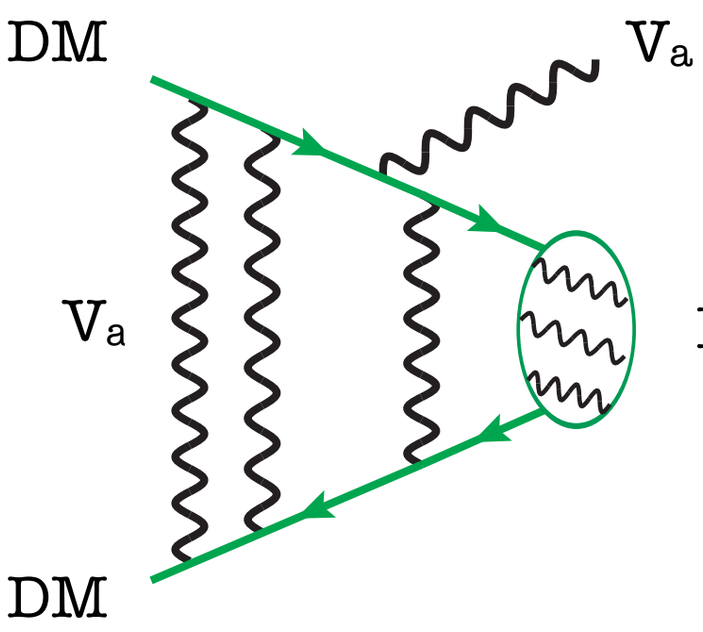
[Wise et al. (2014); Petraki et al. (2014); Ellis al. (2015); Slatyer et al. (2016);...]



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What is the cross section for the production of a bound state?

What is the fate of a bound state once it is formed?

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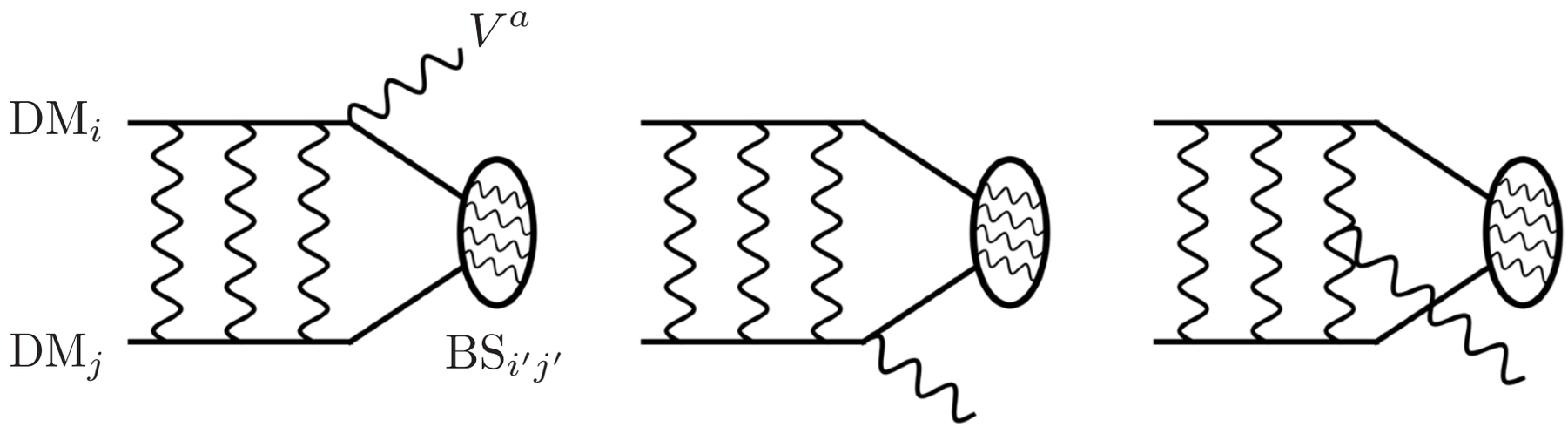
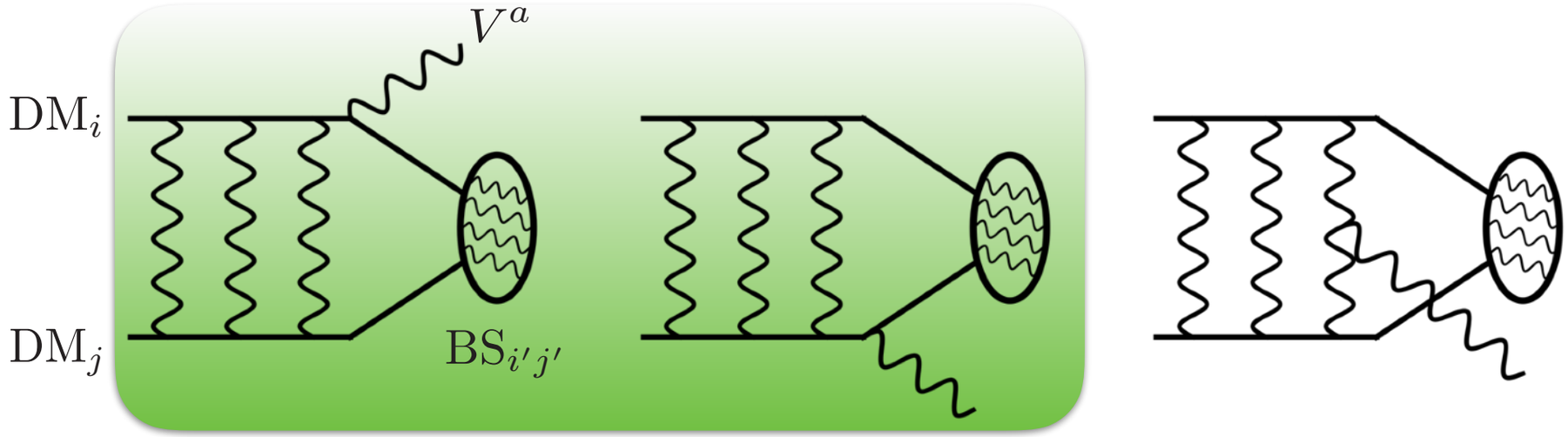
A couple of remarks:

- We only considered single boson emission $\Longrightarrow \Delta\ell = \pm 1 \quad \& \quad \Delta S = 0$
- The process is kinematically possible only if $P^2/M_\chi + E_B \geq M_V$



$$H_I = -\frac{g}{M_\chi}\left(\vec{A}^a \cdot \vec{p}_1 T^a_{i'i}\delta_{jj'} + \vec{A}^a \cdot \vec{p}_2 \overline{T}^a_{j'j}\delta_{ii'}\right) - \left(g\alpha\vec{A}^a(0)\cdot\hat{r}\, e^{-M_a r}\right) T^b_{i'i}\overline{T}^c_{j'j} f^{abc}$$



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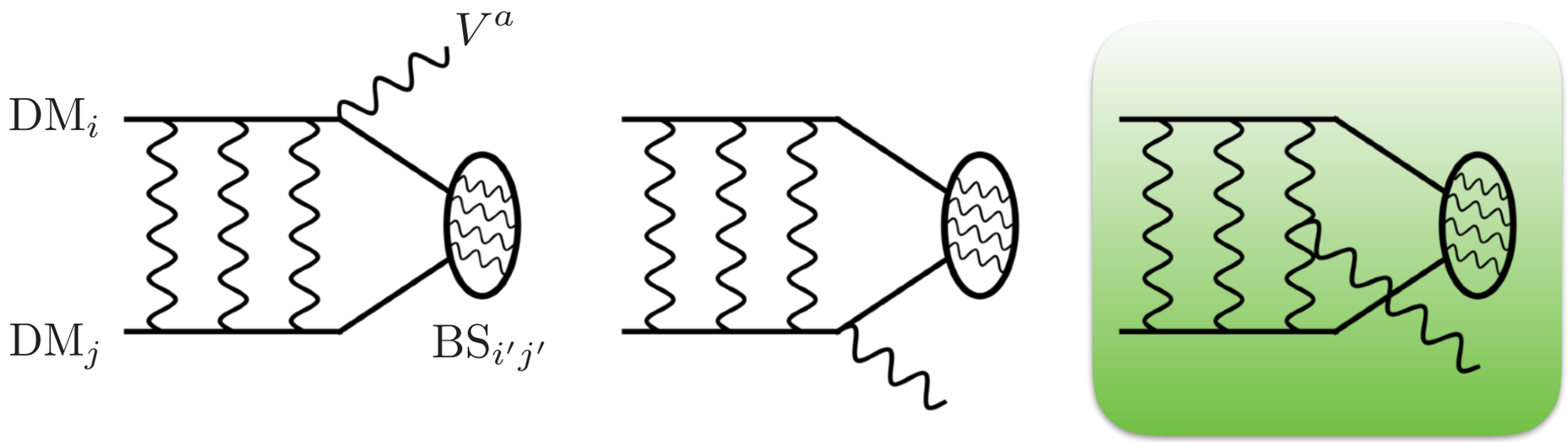

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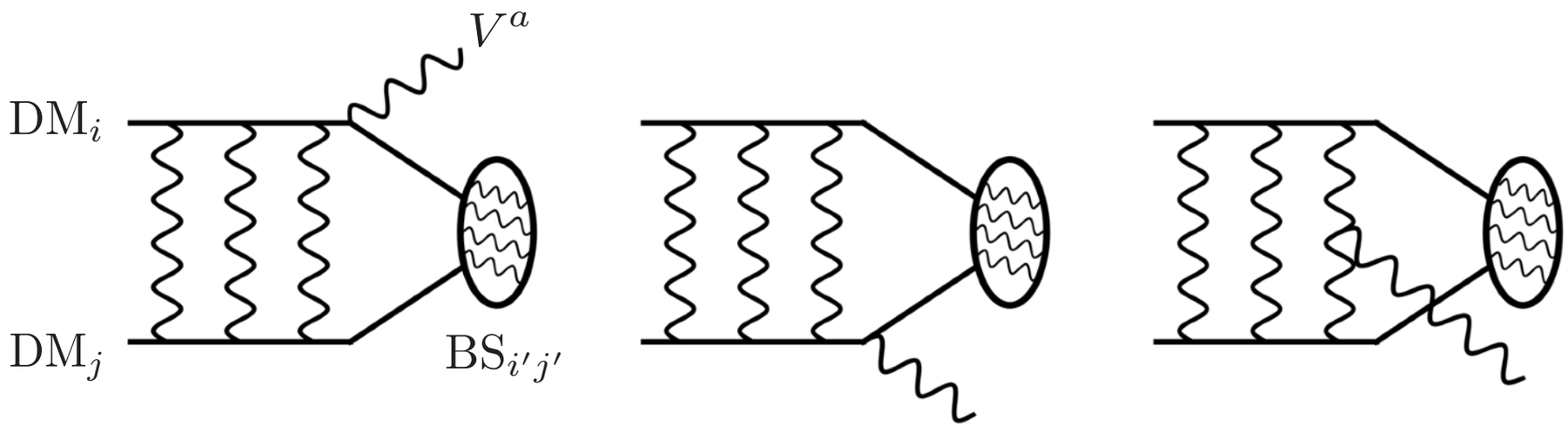

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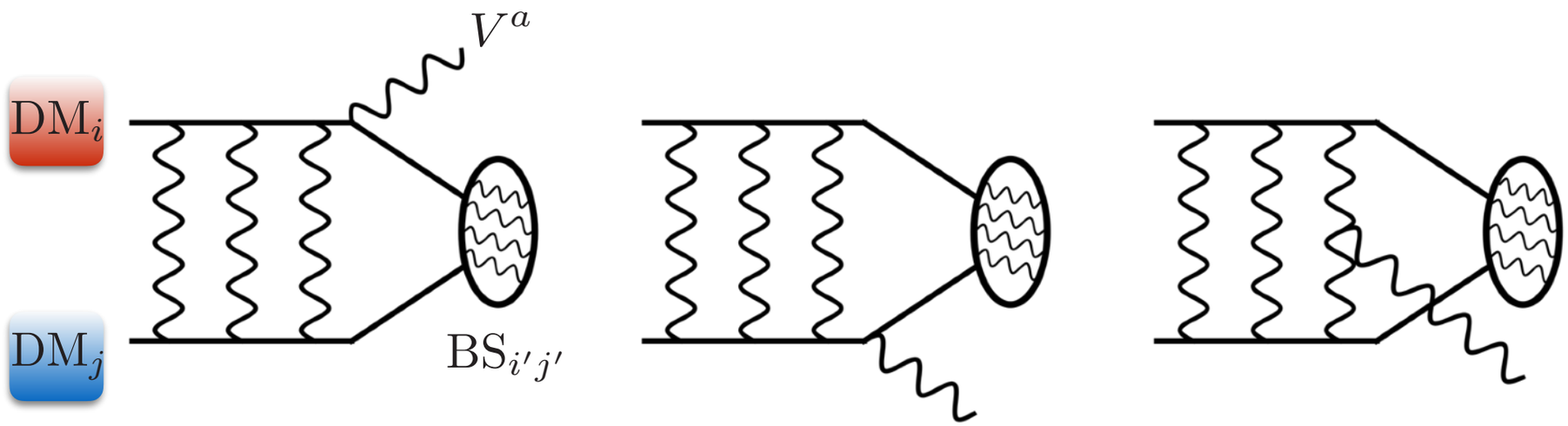

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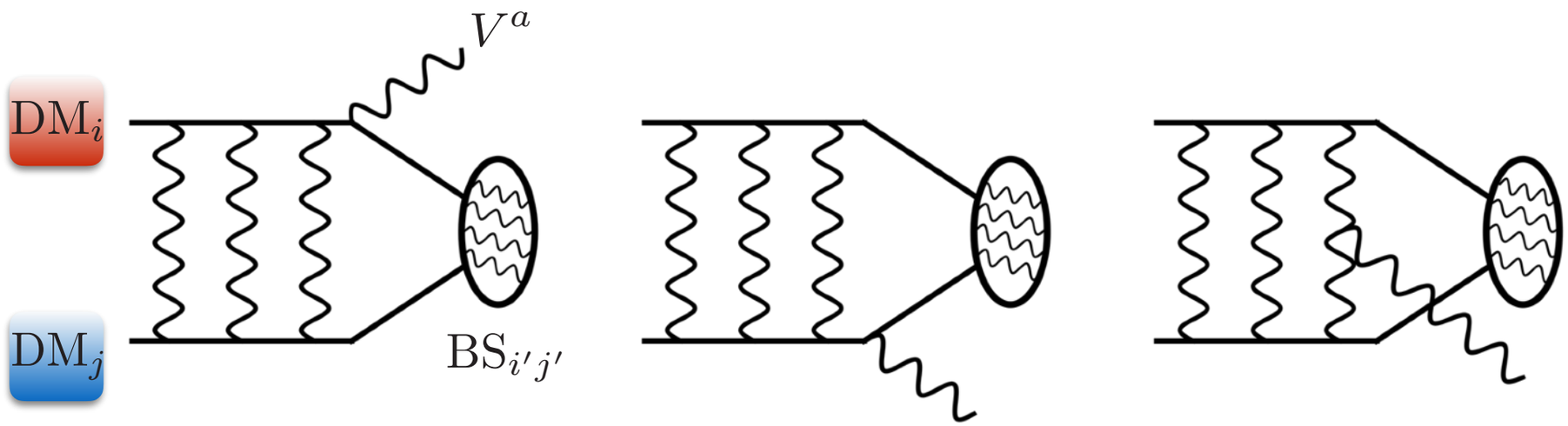
$$R \otimes R' = \sum_J J$$



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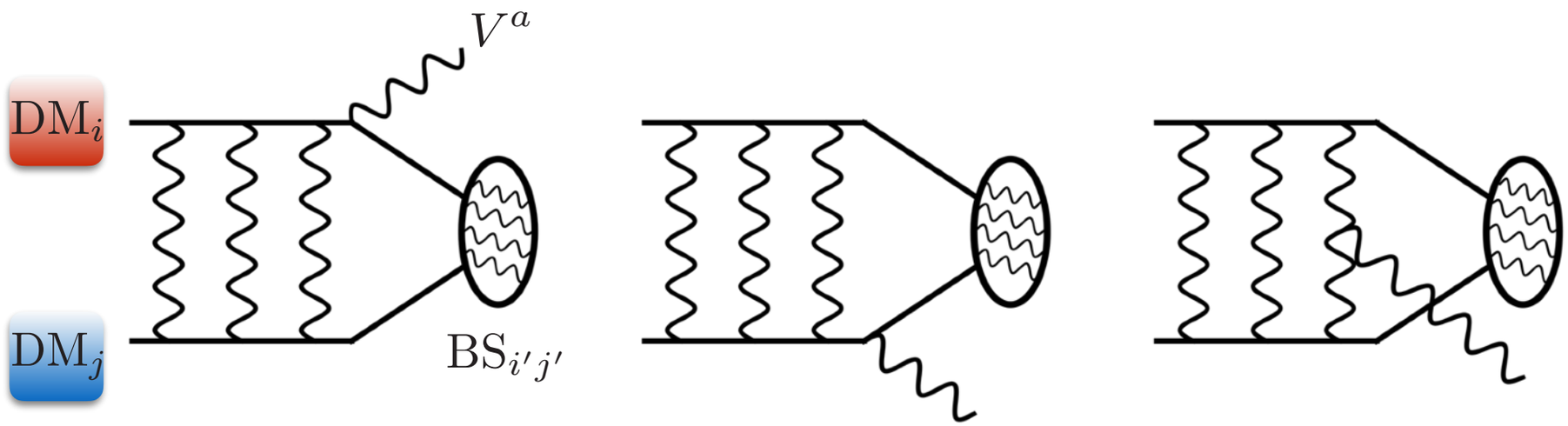

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- The wave functions are solutions of $\quad -\frac{\nabla^2 \psi}{M_\chi} + V\psi = E\psi$

In general BS cross sections have to be computed numerically, however for $\mathrm{M_V} = 0$ we can get an analytic expression, e.g.

$$(\sigma v_{\rm rel})^{n=1,\ell=0}_{\rm bsf} = \sigma_0 \lambda_i (\lambda_f \zeta)^5 \frac{2S+1}{g_\chi^2} \frac{2^{11}\pi(1+\zeta^2\lambda_i^2)e^{-4\zeta\lambda_i \mathrm{arccot}(\zeta\lambda_f)}}{3(1+\zeta^2\lambda_f^2)^3\,(1-e^{-2\pi\zeta\lambda_i})} \times \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \frac{1}{\lambda_f} C_{\mathcal{T}}^{aMM'} \right|^2$$

Physics becomes more clear in the limit $v_{\rm rel} \ll \alpha \Leftrightarrow \zeta \gg 1$

$$(\sigma v_{\rm rel})^{n=1,\ell=0}_{\rm bsf} \propto \frac{\pi\alpha^2}{M_\chi^2} \times \frac{\lambda_i^3 \alpha}{\lambda_f v_{\rm rel}} \times \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \frac{1}{\lambda_f} C_{\mathcal{T}}^{aMM'} \right|^2$$

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What is the fate of a bound state once it is formed?

Decay

Decays happen mostly in the $\ell = 0$ states

Spin 0

$$\Gamma_d\left(B_{n0} \to VV\right) \propto \alpha_{eff}^5 M_\chi$$

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Break

The breaking rate is related to the formation cross section by the Milne relation

$$2 n_B \Gamma_b = (n_\chi^{eq})^2 \langle \sigma_{\rm bsf} v_{rel} \rangle$$

In the non rel. limit this reduces to

$$\Gamma_b \propto (M_\chi T)^{3/2} e^{-E_B/T} \langle \sigma_{\rm bsf} v_{rel} \rangle$$

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Bound state formation is important when $\Gamma_d \gg \Gamma_b$

How bound states enter in the Boltzmann equations?

Each bound states is identified by an index I which collectively denotes all its quantum numbers.

$$\frac{dY_{DM}}{dz} = -\frac{s}{Hz}\left[\langle\sigma v_{rel}\rangle\left(Y_{DM}^2 - Y_{DM}^{eq2}\right) + \sum_I \langle\sigma_I v_{rel}\rangle\left(Y_{DM}^2 - Y_{DM}^{eq2}\frac{Y_I}{Y_I^{eq}}\right)\right]$$

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Typically $\Gamma_{I,d}, \Gamma_{I\to J} \gg H$ so we can neglect the expansion of the Universe and solve for $Y_I/Y_I^{\rm eq}$

How bound states enter in the Boltzmann equations?

$$\frac{dY_{\rm DM}}{dz} = -\frac{\langle \sigma_{\rm eff} v_{\rm rel}\rangle s}{Hz}\left(Y_{\rm DM}^2 - Y_{\rm DM}^{\rm eq2}\right)$$

In the simple case of a single bound state

$$\langle \sigma_{\rm eff} v_{\rm rel}\rangle = \langle \sigma v_{\rm rel}\rangle + {\rm BR}\,\langle \sigma_{\rm bsf} v_{\rm rel}\rangle \qquad {\rm BR} = \frac{\langle \Gamma_{\rm d}\rangle}{\langle \Gamma_{\rm d} + \Gamma_{\rm b}\rangle}$$

(Finally) Results!

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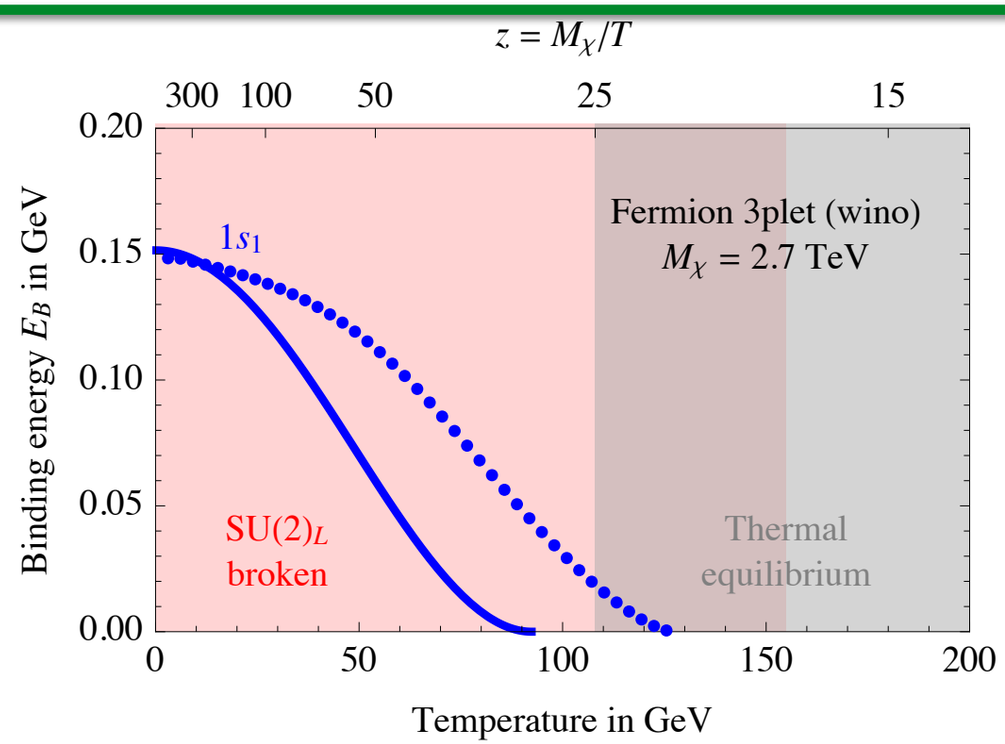

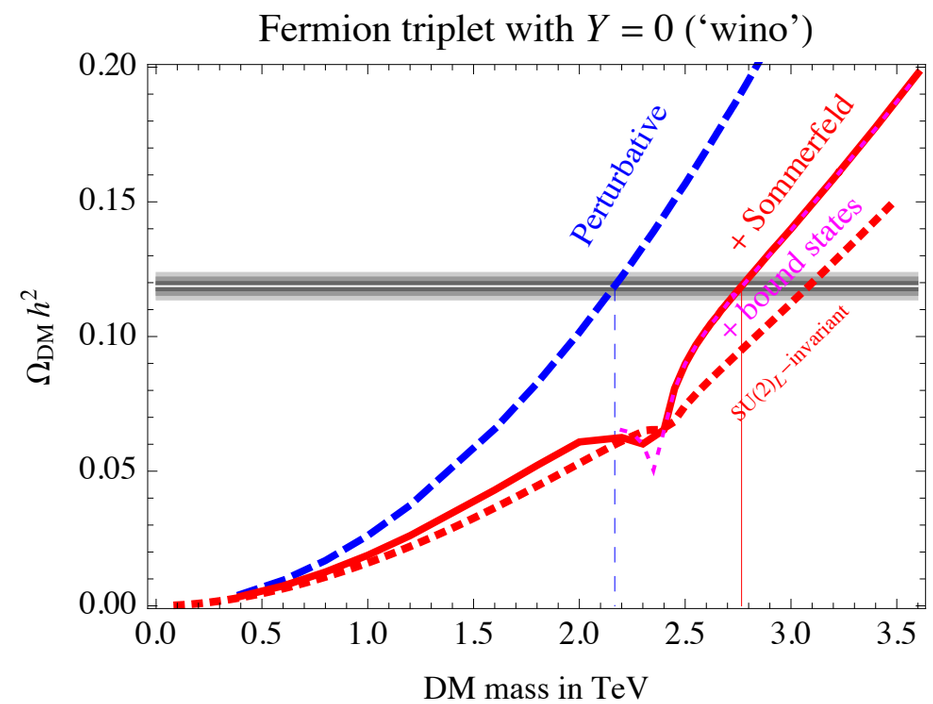

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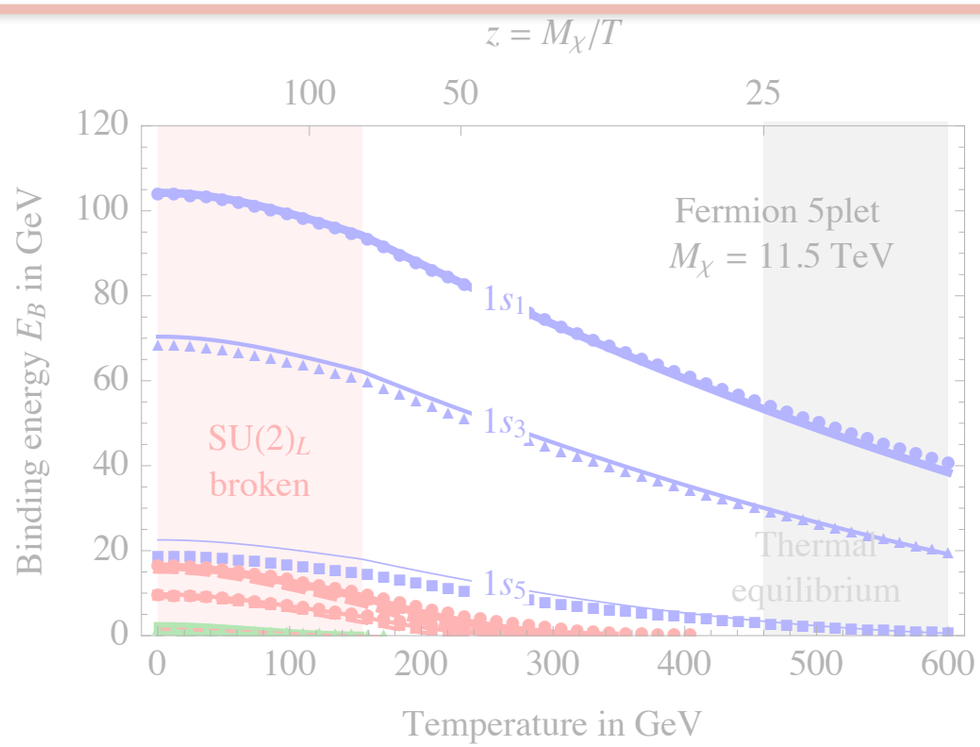
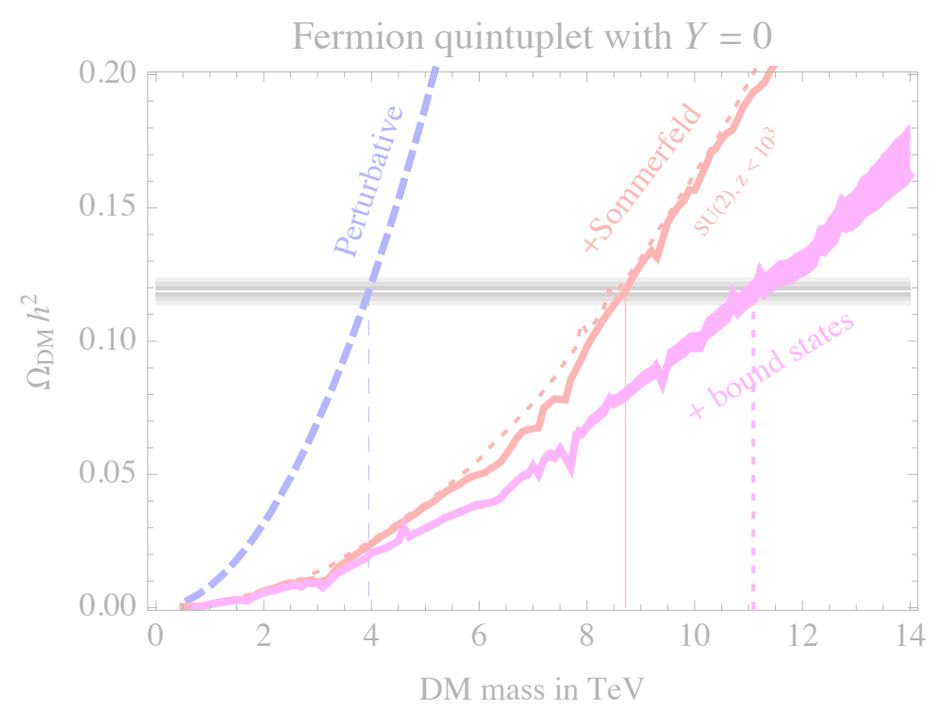
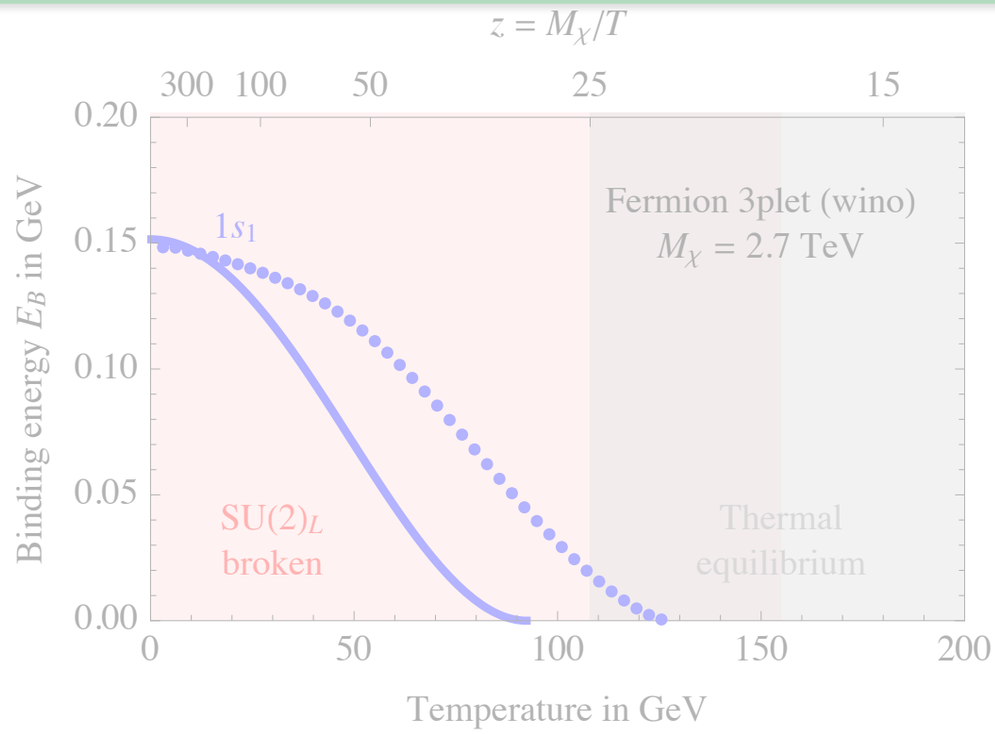
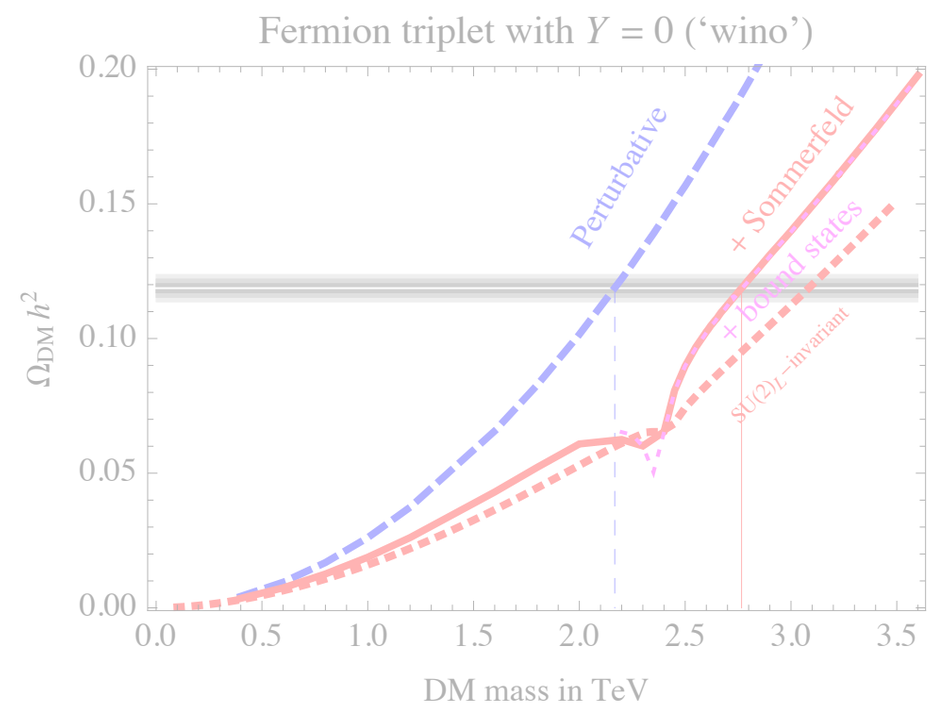




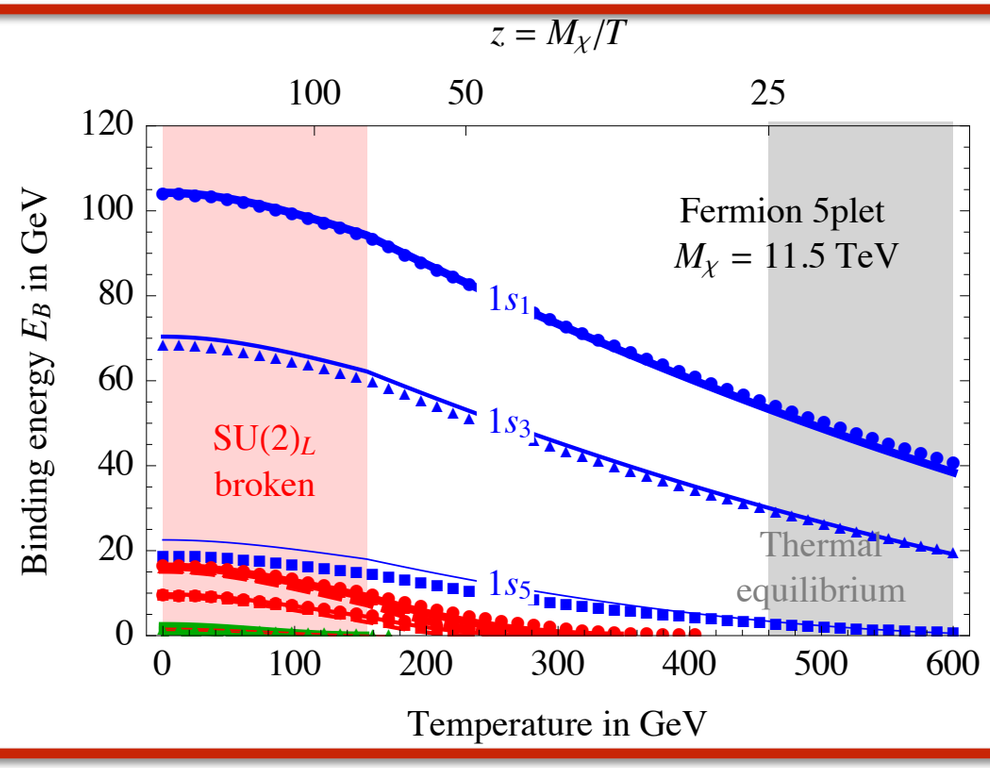
(Finally) Results!

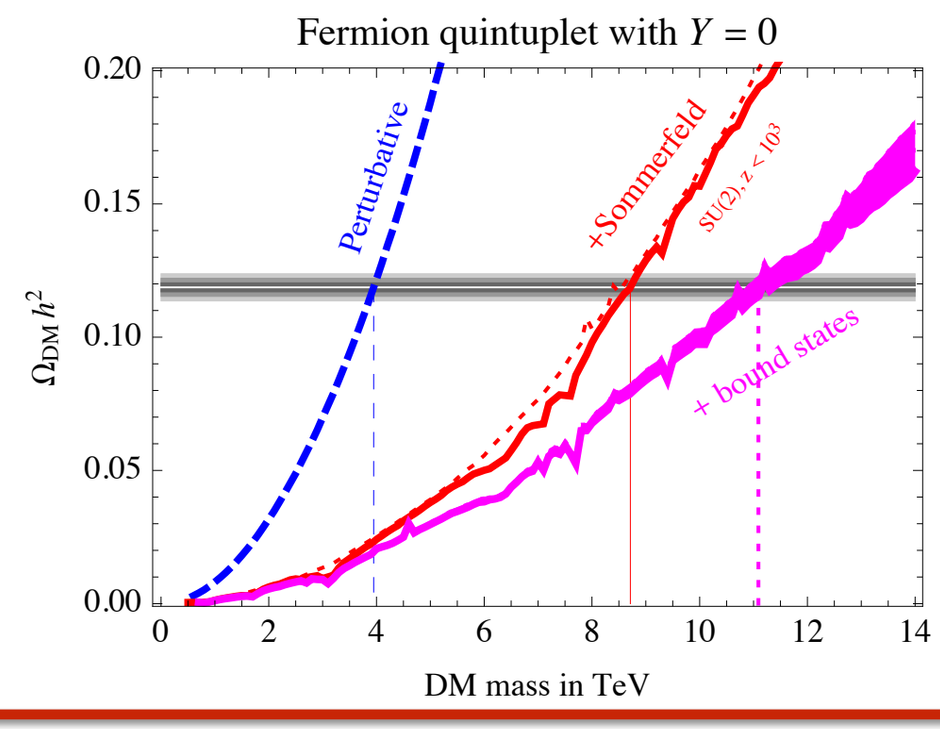
Wino





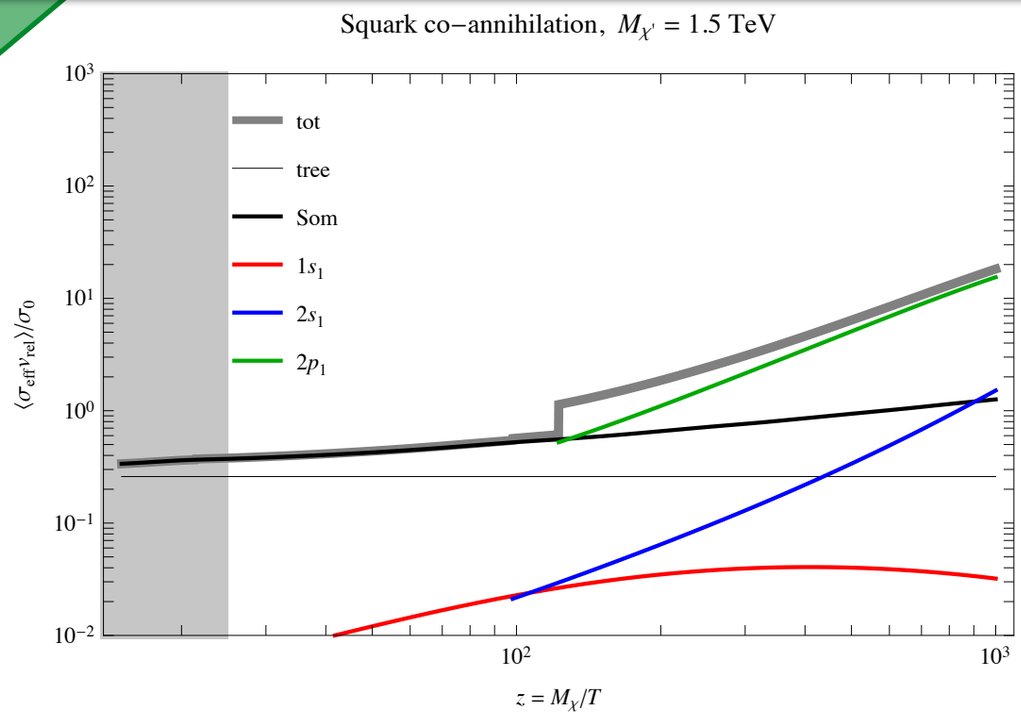
5-plet

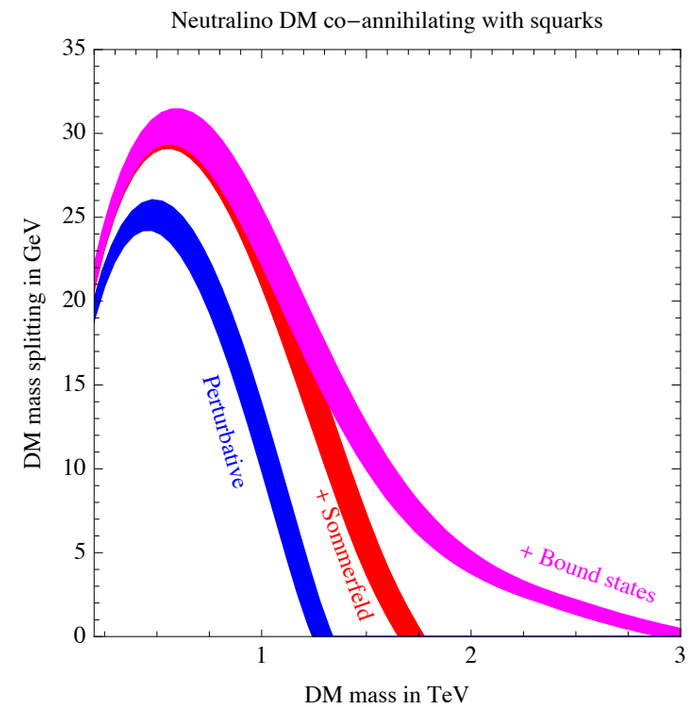




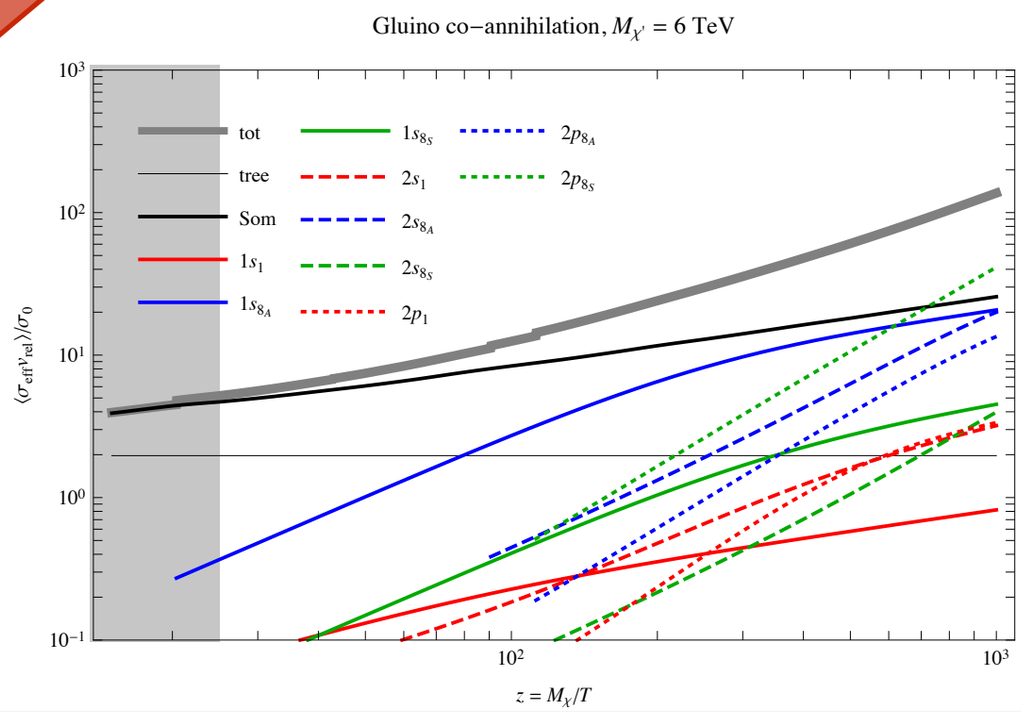
More results: neutralino co-annihilating with...

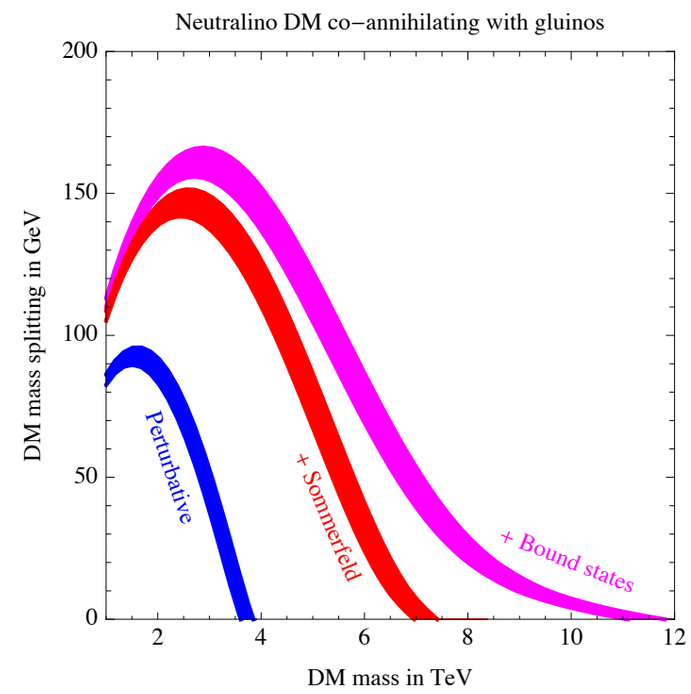
Squark





Gluino





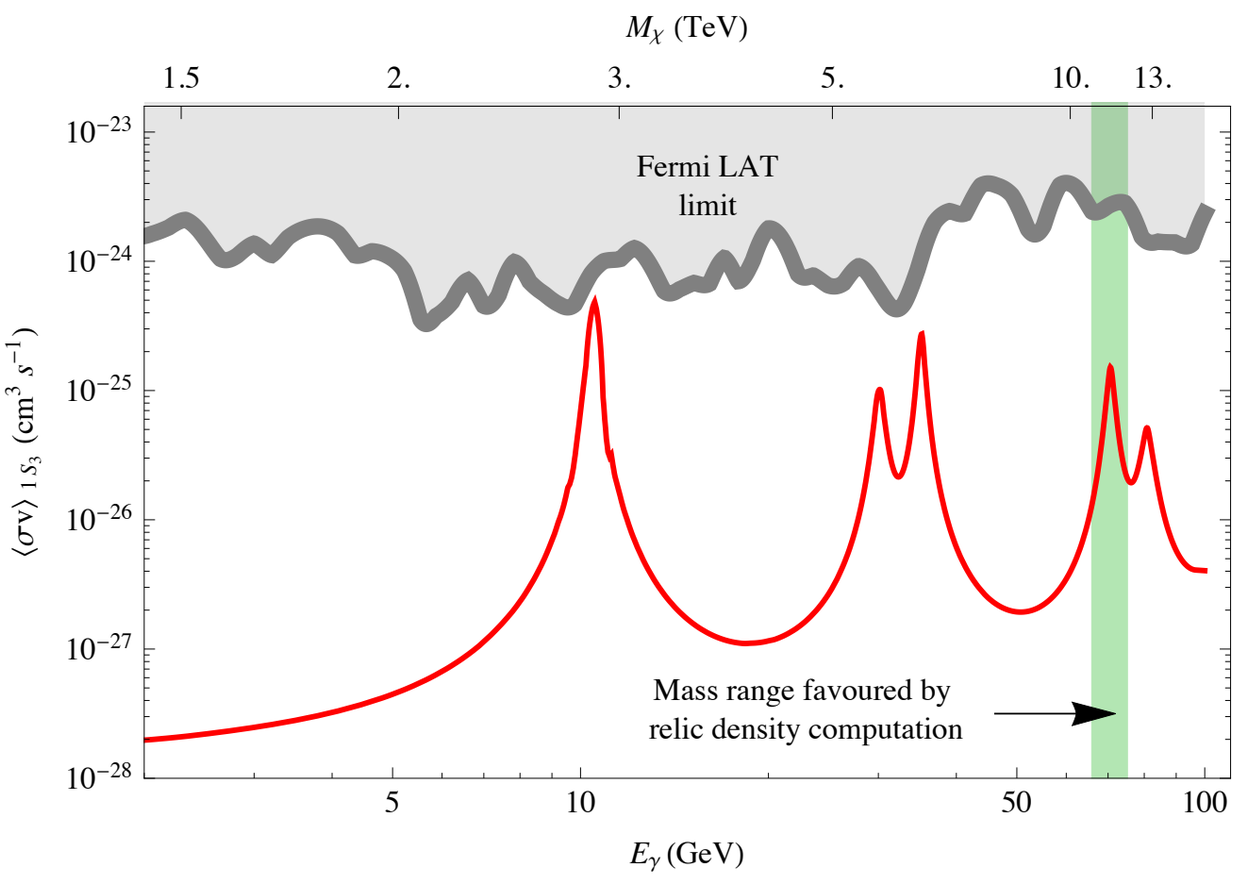
Outlook

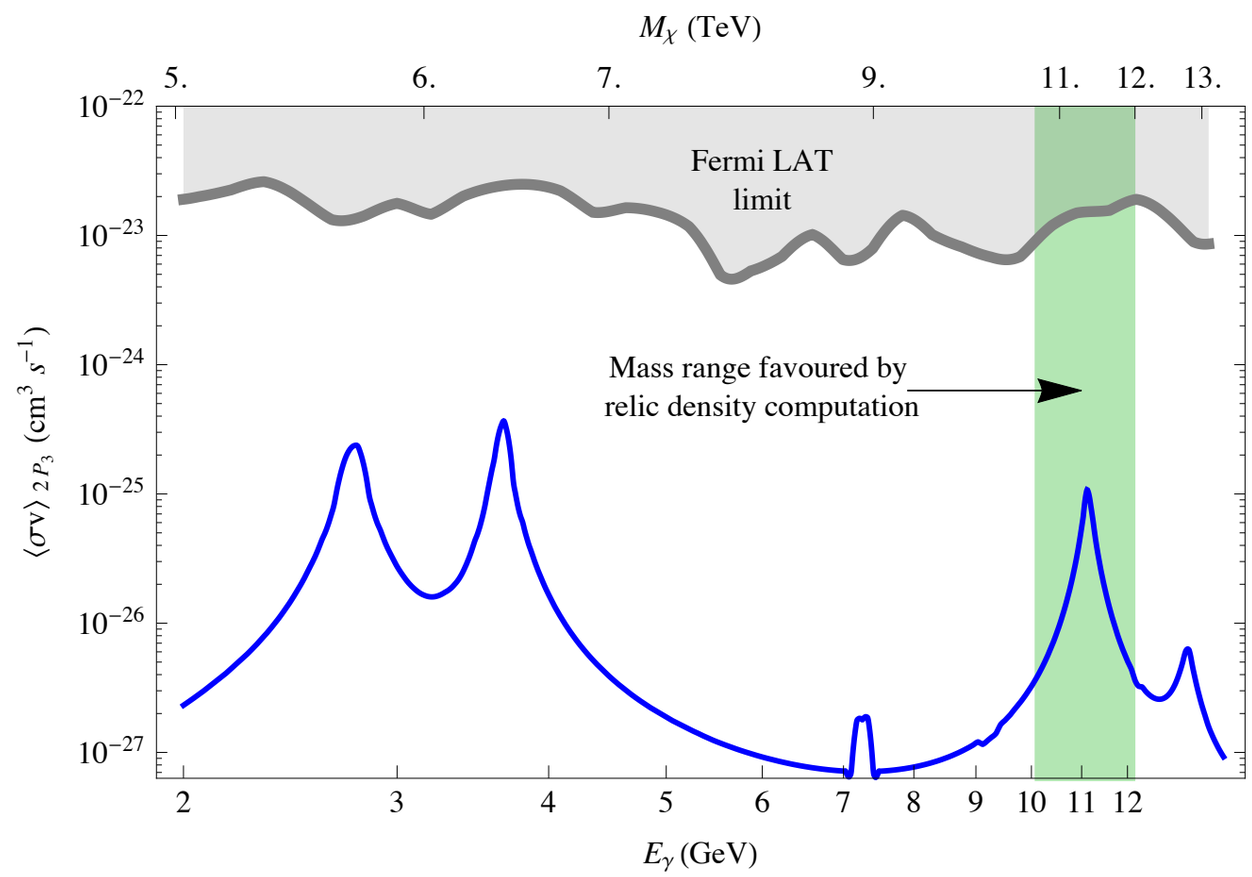
Bound State formation is an additional non- perturbative effect which affects the DM annihilation cross section

In models with massless or light ($\alpha M_\chi > M_V$) mediators it can be the dominant effect setting the relic density

? **Today bound states formation can lead to observable indirect detection signals and give precision information about dark matter?** ?

For the quintuplet there is hope





More precise studies are needed ... stay tuned!

Thanks for your attention!

$$C_{\mathcal{J}}^{aMM'} = \frac{1}{2}\mathrm{Tr}\Big[\mathrm{CG}^{M'}\{\mathrm{CG}^{M}, T^a\}\Big]$$

$$C_{\mathcal{T}}^{aMM'} = -i\mathrm{Tr}\Big[\mathrm{CG}^{M'}\, T^b\, \mathrm{CG}^{M}\, T^c\Big] f^{abc}$$

$$\sigma_{bsf} \propto \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \gamma\, C_{\mathcal{T}}^{aMM'} \right|^2$$

$$R \otimes R' = \sum_J J$$

$$\mathrm{CG}_{ij}^{M} \equiv \langle J, M | R, i; R', j \rangle$$

Wino

$I_J \rightleftarrows I_{J'}$	$\sum_{aMM'} \lvert C_{\mathcal{J}}^{aMM'} + \gamma C_{\mathcal{T}}^{aMM'} \rvert^2$
$1 \rightleftarrows 3$	$2\,\lvert 1 \pm \gamma \rvert^2$
$3 \rightleftarrows 5$	$\frac{5}{2}\,\lvert 1 \pm 2\gamma \rvert^2$

5-plet

$I_J \rightleftarrows I_{J'}$	$\sum_{aMM'} \lvert C_{\mathcal{J}}^{aMM'} + \gamma C_{\mathcal{T}}^{aMM'} \rvert^2$
$1 \rightleftarrows 3$	$6\,\lvert 1 \pm \gamma \rvert^2$
$3 \rightleftarrows 5$	$\frac{21}{2}\,\lvert 1 \pm 2\gamma \rvert^2$
$5 \rightleftarrows 7$	$12\,\lvert 1 \pm 3\gamma \rvert^2$
$7 \rightleftarrows 9$	$9\,\lvert 1 \pm 4\gamma \rvert^2$

$$C_{\mathcal{J}}^{aMM'} = \frac{1}{2}\mathrm{Tr}\Big[\mathrm{CG}^{M'}\{\mathrm{CG}^{M}, T^a\}\Big] \qquad C_{\mathcal{T}}^{aMM'} = -i\mathrm{Tr}\Big[\mathrm{CG}^{M'}\, T^b\, \mathrm{CG}^{M}\, T^c\Big] f^{abc}$$

$$\sigma_{bsf} \propto \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \gamma\, C_{\mathcal{T}}^{aMM'} \right|^2$$

$$3 \otimes 3 = 1_S \oplus 3_A \oplus 5_S \qquad \mathrm{CG}_{ij}^{M} \equiv \langle J, M | R, i; R', j \rangle$$

Wino

$I_J \rightleftarrows I_{J'}$	$\sum_{aMM'} \lvert C_{\mathcal{J}}^{aMM'} + \gamma C_{\mathcal{T}}^{aMM'} \rvert^2$
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5-plet

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$$C_{\mathcal{J}}^{aMM'} = \frac{1}{2}\mathrm{Tr}\Big[\mathrm{CG}^{M'}\{\mathrm{CG}^{M}, T^a\}\Big] \qquad C_{\mathcal{T}}^{aMM'} = -i\mathrm{Tr}\Big[\mathrm{CG}^{M'}\, T^b\, \mathrm{CG}^{M}\, T^c\Big] f^{abc}$$

$$\sigma_{bsf} \propto \sum_{aMM'} \left| C_{\mathcal{J}}^{aMM'} + \gamma\, C_{\mathcal{T}}^{aMM'} \right|^2$$

$$5 \otimes 5 = 1_S \oplus 3_A \oplus 5_S \oplus 7_A \oplus 9_S \qquad \mathrm{CG}_{ij}^{M} \equiv \langle J, M | R, i; R', j \rangle$$

Wino

$I_J \rightleftarrows I_{J'}$	$\sum_{aMM'} \lvert C_{\mathcal{J}}^{aMM'} + \gamma C_{\mathcal{T}}^{aMM'} \rvert^2$
$1 \rightleftarrows 3$	$2\,\lvert 1 \pm \gamma \rvert^2$
$3 \rightleftarrows 5$	$\frac{5}{2}\,\lvert 1 \pm 2\gamma \rvert^2$

5-plet

$I_J \rightleftarrows I_{J'}$	$\sum_{aMM'} \lvert C_{\mathcal{J}}^{aMM'} + \gamma C_{\mathcal{T}}^{aMM'} \rvert^2$
$1 \rightleftarrows 3$	$6\,\lvert 1 \pm \gamma \rvert^2$
$3 \rightleftarrows 5$	$\frac{21}{2}\,\lvert 1 \pm 2\gamma \rvert^2$
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$7 \rightleftarrows 9$	$9\,\lvert 1 \pm 4\gamma \rvert^2$

Bound states summary

5-plet

Name	I	S	n	ℓ	λ	$\Gamma_{\rm ann}/M_\chi$	$\Gamma_{\rm dec}/M_\chi$	Produced from
$1s_1$	1	0	1	0	6	$3240\ \alpha_2^5$	0	p_3
$1s_3$	3	1	1	0	5	$15625\ \alpha_2^5/48$	0	$p_1,\ p_5$
$1s_5$	5	0	1	0	3	$567\alpha_2^5/4$	0	$p_3,\ p_7$
$2s_1$	1	0	2	0	6	$405\ \alpha_2^5$	$\mathcal{O}(\alpha_2^4\alpha_{\rm em}^2)$	p_3
$2s_3$	3	1	2	0	5	$15625\alpha_2^5/384$	$\mathcal{O}(\alpha_2^4\alpha_{\rm em}^2)$	p_1, p_5
$2s_5$	5	0	2	0	3	$567\alpha_2^5/32$	$\mathcal{O}(\alpha_2^4\alpha_{\rm em}^2)$	p_3, p_7
$2p_1$	1	1	2	1	6	$\mathcal{O}(\alpha_2^7)$	$\approx 2\,\alpha_2^4\alpha_{\rm em}$	s_3
$2p_3$	3	0	2	1	5	$\mathcal{O}(\alpha_2^7)$	$\approx 1\,\alpha_2^4\alpha_{\rm em}$	s_1, s_5
$2p_5$	5	1	2	1	3	$\mathcal{O}(\alpha_2^7)$	$\approx 0.2\,\alpha_2^4\alpha_{\rm em}$	s_3, s_7

Gluino

Name	R	S	n	ℓ	λ	$\Gamma_{\rm ann}/M_\chi$	$\Gamma_{\rm dec}/M_\chi$	Produced from
$1s_1$	1_S	0	1	0	3	$243\alpha_3^5/4$	0	p_{8_A}
$1s_{8_A}$	8_A	1	1	0	3/2	$243\alpha_3^5/64$	0	$p_1,\ p_{8_S},\ p_{27_S}$
$1s_{8_S}$	8_S	0	1	0	3/2	$243\alpha_3^5/128$	0	$p_{8_A},\ p_{10_A}$
$2s_1$	1_S	0	2	0	3	$243\alpha_3^5/32$	$\mathcal{O}(\alpha_3^6)$	p_{8_A}
$2s_{8_A}$	8_A	1	2	0	3/2	$243\alpha_3^5/512$	$\mathcal{O}(\alpha_3^6)$	$p_1, p_{8_S},\ p_{27_S}$
$2s_{8_S}$	8_S	0	2	0	3/2	$243\alpha_3^5/1024$	$\mathcal{O}(\alpha_3^6)$	p_{8_A}, p_{10_A}
$2p_1$	1_S	1	2	1	3	$\mathcal{O}(\alpha_3^7)$	$\approx\ \alpha_3^6$	s_{8_A}
$2p_{8_A}$	8_A	0	2	1	3/2	$\mathcal{O}(\alpha_3^7)$	$\approx 0.1\alpha_3^5$	$s_1, s_{8_S},\ s_{27_S}$
$2p_{8_S}$	8_S	1	2	1	3/2	$\mathcal{O}(\alpha_3^7)$	$\approx\ 0.1\alpha_3^5$	s_{8_A}, s_{10_A}