Yukawa alignment in multi-Higgs doublet models and family symmetries

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Multi-Higgs doublet models (MHDM)
Multi-Yukawas $\rightarrow$ FCNCs

See Higgs basis: only H_1 has VEV ; $M_f = v_1 Y_1^f$
Now go also to fermion basis where M_f is diagonal,
other Yukawas $Y^f_{A\neq 1}$ in general have off-diagonal entries

$H^0_{A\neq 1}$ mediate FCNCs

Solution (proposed by Pich Tuzón) :
Yukawa alignment
Symmetry?

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Renormalization-group constraints on Yukawa alignment in multi-Higgs-doublet models

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Abstract

We write down the renormalization-group equations for the Yukawa-coupling matrices in a general multi-Higgs-doublet model. We then assume that the matrices of the Yukawa couplings of the various Higgs doublets to right-handed fermions of fixed quantum numbers are all proportional to each other. We demonstrate that, in the case of the two-Higgs-doublet model, this proportionality is preserved by the renormalization-group running only in the cases of the standard type-I, II, X, and Y models. We furthermore show that a similar result holds even when there are more than two Higgs doublets: the Yukawa-coupling matrices to fermions of a given electric charge remain proportional under the renormalization-group running if and only if there is a basis for the Higgs doublets in which all the fermions of a given electric charge couple to only one Higgs doublet.

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Yukawa sector of Multi Higgs Doublet Models in the presence of Abelian symmetries

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A general method for classifying the possible quark models of a multi-Higgs-doublet model, in the presence of Abelian symmetries, is presented. All the possible sets of textures that can be present in a given sector are shown, thus turning the determination of the flavor models into a combinatorial problem. Several symmetry implementations are studied for two and three Higgs doublet models. Some models implementations are explored in great detail, with a particular emphasis on models known as Branco-Grimus-Lavoura and nearest-neighbour-nnteraction. Several considerations on the flavor changing neutral currents of multi-Higgs models are also given.

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I. INTRODUCTION

The Standard Model (SM) of strong and electroweak interactions is very successful phenomenologically and the discovery of a Higgs-like particle [1] was the missing piece in order to establish it as the best model available. However, there is a general consensus that this should not be the final theory because it does not explain basic issues such as dark matter, neutrino masses, number of families, and many others.

One possible extension of the SM is the addition of extra copies of the Higgs field, just like in the fermionic sector. The most common scenario is the two Higgs doublet model (2HDM), which has been extensively studied in the literature; for a review see [2]. Models with three or more Higgs bosons have also been considered, but the lack of information on these extension is much larger. With the addition of extra scalar doublets the number of parameters, in the scalar and Yukawa sector, increases largely. In these N Higgs doublet models (NHDM) it is very common to add symmetries to help tackle the problem. For the 2HDM, Ivanov [3] has shown that, no matter what combination of flavor symmetries and/or generalized CP symmetries one imposes on the scalar potential, one always ends up with one of six distinct classes of potentials. Later, this issue was studied further by Ferreira, Haber, and Silva [4]. The recent studies of Ivanov and Vdovin [5] have extended these analyses to the three Higgs doublet models (3HDM). The study of Abelian symmetries in the NHDM scalar sector was done by Ivanov, Keus, and Vdovin [6]. Despite the extensive general studies of symmetries in the scalar potential of NHDM, the Yukawa sector has been left partially apart. There are several particular flavor models in literature with two, three, or more Higgs fields, but there is a lack of a general approach as the one existing for the scalar sector.

The study of the Yukawa sector in NHDM tends to be a little involved since, besides the scalar fields, we have three types of fermions (Q_L, n_R, and p_R) repeated 3 times. This enlarges significantly the number of choices for the representations of a given group. Recently, a general study of 3HDM in the presence of A_4 and S_4 was done [7]. These are two interesting non-Abelian groups since they lead to a scalar potential highly symmetric, allowing the complete determination of the global minimums [8]. While, the study of non-Abelian symmetries in the Yukawa sector depends strongly of the irreducible representations (irreps) and the way we attribute them, for the Abelian case we only have one-dimensional irreps. Using this feature, Ferreira and Silva [9] have presented a general study of Abelian symmetries in the Yukawa sector of the 2HDM. The aim of this work is to extend this study to the NHDM case.

This article is organized as follows. In Sec. II we introduce our notation and show how the action of Abelian symmetries constrains the Yukawa textures. In Sec. III we show the possible combinations of textures, i.e. chains, that can be built in Abelian models, as well as the possible Higgs fields transformations and associated textures. In Sec. IV we explain how to make the connection between the up-quark and down-quark sectors, allowing us to build explicit models for the quark sector. In Sec. V we extend our previous analyses to cases where the Abelian group is a direct product of cyclic groups. In Sec. VI explicit model implementations are studied in detail, in particular, the well-known Branco-Grimus-Lavoura (BGL) and nearest-neighbour-interaction (NNI) models. We draw our conclusions in Sec. VII.

II. ABELIAN SYMMETRIES VERSUS YUKAWA TEXTURES

The most general and renormalizable scalar potential constructed with N copies of the $SU(2)_L \otimes U(1)_Y$ doublet Φ_a $(a = 1, \cdots, N)$ is

$$V = Y_{ab} \left(\Phi_a^\dagger \Phi_b\right) + Z_{abcd} \left(\Phi_a^\dagger \Phi_b\right) \left(\Phi_c^\dagger \Phi_d\right) \tag{1}$$

with

$$Y_{ab} = Y_{ba}^* , \quad Z_{abcd} = Z_{cdab} = Z_{badc}^* , \tag{2}$$

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These models correspond to some of the ones presented in Eq. (56a). We can have direct models, where Γ_1 is connected with Δ_1 and Γ_0 with Δ_0, or cross models, where Γ_1 is connected with Δ_0 and vice versa. For any number N of Higgs fields the minimal symmetry group that can be used to implement these models is Z_2. Therefore we can always implement NFC without the introduction of accidental symmetries.

There are other ways of implementing NFC in NHDM; however, these cannot be implemented through a symmetry. One common example is the Yukawa alignment in 2HDM [14]. In this case NFC is achieved by requiring that all the Yukawa couplings, for each sector, are proportional,

$$\mathbf{\Gamma}_i = c_i \mathbf{\Gamma}_j \quad \text{and} \quad \mathbf{\Delta}_i = c_i \mathbf{\Delta}_j \,, \quad \forall_{i,j} \,. \tag{92}$$

As shown in [15], no symmetry implementation can be used to implement this requirement. In [16], alignment was seen as a low-energy effect of NFC models, while in [17] its origin was related with flavor symmetries.

Family symmetries and alignment in multi-Higgs doublet models

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The exact alignment of the Yukawa structures on multi-Higgs doublet models provides cancellation of tree-level flavour changing couplings of neutral scalar fields. We show that family symmetries can provide a suitable justification for the Yukawa alignment.

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I. INTRODUCTION

A Multi-Higgs Doublet Model (MHDM) consists in a straightforward generalisation of the Standard Model (SM) where extra $SU(2)$ doublet scalars are added to the field content. With just one extra doublet added, the two-Higgs doublet model is a particular case of the MHDM [1, 2].

In the MHDM there are Yukawa couplings associated to each Higgs doublet for each family of fermions - up quarks, down quarks, charged leptons and neutrinos if Right-Handed (RH) neutrinos are also added. It is well known that this presents the potential for large unobserved flavour changing processes such as Flavour Changing Neutral Currents (FCNCs). One can see this in the Higgs basis in which the mass matrices of the fermions come from only those Yukawa matrices associated with a particular doublet. For a given family, that mass matrix can be diagonalised - but without further assumption the other Yukawa matrices of the family are arbitrary complex matrices which would enable large tree-level FCNCs. It has been noted [3] that these unobserved processes completely cancel in the exact alignment limit i.e. all Yukawa matrices of a given family are perfectly aligned. It was shown [4] that such alignment can not be preserved by renormalisation unless additional symmetries are imposed, although the contributions to the unobserved processes due to this misalignment can be compatible with the current experimental constraints for regions of the parameter space [5].

It is reasonable to expect that problematic processes will be suppressed when there is approximate alignment. In analogy to the solution of the SUSY flavour problem where Family Symmetries (FSs) provide approximate alignment of fermions and sfermions (see e.g. [6–8]), a

In the most extreme cases, the FS can provide perfect alignment and protect it from renormalisation effects. Here we present models where the FS is used solely to address the potential flavour problems of the MHDM by achieving perfect alignment for the given families - without attempting to ameliorate the flavour issues of the SM.

The exact Yukawa alignment in the MHDM is achieved through a specific strategy that combines two requirements: only one FS Invariant Combination (FSIC) is allowed for each family; all the Higgs $SU(2)$ doublets are singlets of the FS, such that the single allowed FSIC can be made invariant under the SM through coupling to any of the Higgs doublets. We then argue that as a generalisation of this strategy, dropping the constraining single FSIC requirement while maintaining the Higgs as singlets of the FS is a promising approach to achieve approximate Yukawa alignment.

II. SIMPLE ALIGNMENT EXAMPLE

In order to illustrate the proposed strategy we use $SU(3)_{[]} \otimes SU(3)_{()} \otimes C_n$ as the FS. The fermions are assigned as triplets. To allow a FSIC, familons (SM singlet scalars) are added and assigned as anti-triplets under the FS. The requirement that each family has a single allowed coupling is simple conceptually, but it can be quite difficult to implement. In order to do so, an auxiliary Abelian factor (C_n) is not enough. This is why we resort to two distinct $SU(3)$ (c.f. [9, 10]) as with two non-Abelian factors the Left-Handed (LH) sectors (Q and L) are separated from the RH sectors (u^c, d^c, e^c and ν^c). C_n is then sufficient to keep both Q separate from L and each RH sector separate from one another.

We start with the goal of Yukawa alignment for a single

$$SU(3)_F \times SU(3)_f'$$

$$\mathcal{L}_u = \sum_{A=1}^{N} c^u_A H^\dagger_A \underbrace{[\phi^i_Q Q_i]}_{\text{LH } [\;]} \underbrace{(\phi^j_u u^c_j)}_{\text{RH } (\;)} + h.c.$$

Just up Quarks, single FS. Inv. contraction (FSIC)

$Q_i = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}, 3$; $\phi_Q^i = (\phi_Q^1 \; {}^2 \; {}^3)$

Up & down quarks: single FSIC each sector

$$\mathcal{L}_Q = \sum_{A=1}^{N} [\phi^i_Q Q_i] \left(c^d_A H_A (\phi^j_d d^c_j) + c^u_A H^\dagger_A (\phi^j_u u^c_j) \right) + h.c.$$

Generalise for Leptons . . .

$$\mathcal{L} = \sum_{A=1}^{N} \Big(c_A^d H_A [\phi_Q^i Q_i](\phi_d^j d_j^c) + c_A^u H_A^\dagger [\phi_Q^i Q_i](\phi_u^j u_j^c) + c_A^e H_A [\phi_L^i L_i](\phi_e^j e_j^c) + c_A^\nu H_A^\dagger [\phi_L^i L_i](\phi_\nu^j \nu_j^c) \Big) + h.c.$$

$\hookrightarrow$ $$Y_{fA}^{ij} = c_A^f \langle \phi_F^i \rangle \langle \phi_f^j \rangle$$

Alignment enforced by Symmetry!

Problem: Single FSIC → M_f is rank 1

can "SU(3)-rotate" each flavon to (0,0,1) direction

$$\mathcal{L} = \sum_{A=1}^{N} \Big(c_A^d H_A [\phi_Q^i Q_i](\phi_d^j d_j^c) + c_A^u H_A^\dagger [\phi_Q^i Q_i](\phi_u^j u_j^c) + c_A^e H_A [\phi_L^i L_i](\phi_e^j e_j^c) + c_A^\nu H_A^\dagger [\phi_L^i L_i](\phi_\nu^j \nu_j^c) \Big) + h.c.$$

Remains as leading order strategy up to corrections (model-dependent)

Comment from Constanza

$$M_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ; \quad M_d = ?$$

$$t \longrightarrow b$$

$\searrow$ Orthogonal b

Question about F.N.

$$\begin{array}{cccccc} 0 & 0 & & +1 & & +3 \\ Q_3 & \mu_3 & ; & \mu_2 & & \mu_1 \\ \hline Q_3 & d_3 & \phi & & & \\ 0 & +1 & -1 & & & \end{array}$$

$$Y_A^{\mu}$$

Similar idea, different approach

Residual Symmetry method

[Re]constructing Finite Flavour Groups: Horizontal Symmetry Scans from the Bottom-Up

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Abstract: We present a novel procedure for identifying discrete, leptonic flavour symmetries, given a class of unitary mixing matrices. By creating explicit 3D representations for generators of residual symmetries in both the charged lepton and neutrino sector, we reconstruct large(r) non-abelian flavour groups using the GAP language for computational finite algebra. We use experimental data to construct only those generators that yield acceptable (or preferable) mixing patterns. Such an approach is advantageous because it 1) can reproduce known groups from other 'top-down' scans while elucidating their origins from residuals, 2) find new previously unconsidered groups, and 3) serve as a powerful model building tool for theorists wishing to explore exotic flavour scenarios. We test our procedure on a generalization of the canonical tri-bimaximal (TBM) form.

Bottom-Up Discrete Symmetries for Cabibbo Mixing

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We perform a bottom-up search for discrete non-Abelian symmetries capable of quantizing the Cabibbo angle that parameterizes CKM mixing. Given a particular Abelian symmetry structure in the up and down sectors, we construct representations of the associated residual generators which explicitly depend on the degrees of freedom present in our effective mixing matrix. We then discretize those degrees of freedom and utilize the *Groups, Algorithms, Programming* (GAP) package to close the associated finite groups. This short study is performed in the context of recent results indicating that, without resorting to special model-dependent corrections, no small-order finite group can simultaneously predict all four parameters of the three-generation CKM matrix and that only groups of $\mathcal{O}(10^2)$ can predict the analogous parameters of the leptonic PMNS matrix, regardless of whether neutrinos are Dirac or Majorana particles. Therefore a natural model of flavour might instead incorporate small(er) finite groups whose predictions for fermionic mixing are corrected via other mechanisms.

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Residual symmetry method: Multi-Higgs Models

$$\mathcal{L}_e = \sum_{A=1}^{N} Y_A^e H_A \bar{L} e_R + h.c.$$

Higgs basis

$$\mathcal{L}_e = Y_1^e v_1 \bar{L} e_R + \sum_{A=1}^{N} Y_A^e H_A^0 \bar{L} e_R + h.c.$$

Impose residual symmetries $H_A \xrightarrow{T_l} e^{i\phi_l} H_A$; $H_A \xrightarrow{T_e} e^{i\phi_e} H_A$

$$L \to T_l\, l_L, \qquad e_R \to T_e\, e_R$$

$$H_1 \bar{L} Y_1^e e_R \to H_1 \bar{L} T_l^\dagger Y_1^e T_e e_R e^{i(\phi_l + \phi_e)}$$

Invariance requires $Y_A^e \to T_l Y_A^e T_e^\dagger e^{-i(\phi_l + \phi_e)}$

Make $G = Y^\dagger Y$; $I = Y Y^\dagger$

Invariance requires :

$$T_l I_1^e T_l^\dagger = I_1^e$$

$$T_e^\dagger G_1^e T_e = G_1^e$$

(no ϕ_l, ϕ_e dependence but needed for Y invariance)

Diagonal mass basis $\longrightarrow$ $T_{l,e}$ diagonal

In Higgs & diagonal mass basis

$$T_{l,e} = diag(\alpha_{l,e}, \beta_{l,e}, \gamma_{l,e}) \rightarrow \begin{array}{c} T_l I_1^e T_l^\dagger = I_1^e \\ T_e^\dagger G_1^e T_e = G_1^e \end{array}$$

✓

$$T_l I_1^e T_l^\dagger = I_1^e$$
$$T_e^\dagger G_1^e T_e = G_1^e$$

$$\underbrace{T_l\, I_1^e}\, T_l^\dagger = I_1^e$$

$$I_1^e = \begin{pmatrix} m_e^2 & & \\ & m_\mu^2 & \\ & & m_\tau^2 \end{pmatrix} / v_1^2$$

diagonal
Higgs basis
& charged lepton
basis

For other Y_A^e and respective I_A^e
Invariance requires

$$T_l\, I_A^e\, T_l^\dagger = I_A^e$$

$A \neq 1$

$$\begin{pmatrix} e^{i(\alpha_l-\alpha_l)}\, I_{11} & e^{i(\alpha_l-\beta_l)}\, I_{12} & e^{i(\alpha_l-\gamma_l)}\, I_{13} \\ e^{i(\beta_l-\alpha_l)}\, I_{21} & e^{i(\beta_l-\beta_l)}\, I_{22} & e^{i(\beta_l-\gamma_l)}\, I_{23} \\ e^{i(\gamma_l-\alpha_l)}\, I_{31} & e^{i(\gamma_l-\beta_l)}\, I_{32} & e^{i(\gamma_l-\gamma_l)}\, I_{33} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}$$

To satisfy this equation with $\alpha_l \neq \beta_l \neq \gamma_l$

$I_{ij} = 0 \,, \quad i \neq j$

$T_l \, I_A \, T_l^{\dagger} = I_A$ ✓

Repeat for G_A, ...

$$Y_A = \begin{pmatrix} y_A^{e_1} & & \\ & y_A^{e_2} & \\ & & y_A^{e_3} \end{pmatrix}$$

in Higgs & diagonal mass basis

FCNCs controlled by residual symmetries

Note: ratios are not fixed!

$$\frac{m_e}{m_\mu} \neq \frac{y_A^{e_1}}{y_A^{e_2}} \quad \text{etc.}$$

Back to Y_A invariance

$$T_\ell^\dagger Y_A T_e e^{-i(\phi_\ell + \phi_e)} = Y_A$$

So really, $T_\ell = T_e$ required,

and $\phi_\ell = \phi_e = 0$

One residual for each sector,
all H_A should be trivial under residual

In essence:

$$\bar{\ell}_{L_1} \longrightarrow e^{-i\alpha}\,\bar{\ell}_{L_1} \qquad \ell_{R_1} \longrightarrow e^{i\alpha}\,\ell_{R_1}$$

$$\bar{\ell}_{L_2} \longrightarrow e^{-i\beta}\,\bar{\ell}_{L_2} \qquad \ell_{R_2} \longrightarrow e^{i\beta}\,\ell_{R_2}$$

$$\bar{\ell}_{L_3} \longrightarrow e^{-i\gamma}\,\bar{\ell}_{L_3} \qquad \ell_{R_3} \longrightarrow e^{i\gamma}\,\ell_{R_3}$$

So regardless of H_A $\quad (\alpha \neq \beta \neq \gamma)$

$$\left(y_A^{e_1}\,\bar{\ell}_{L_1}\ell_{R_1} + y_A^{e_2}\,\bar{\ell}_{L_2}\ell_{R_2} + y_A^{e_3}\,\bar{\ell}_{L_3}\ell_{R_3}\right) H_A$$

are the only invariants

Discussion

$u_{L_1} \to e^{i\alpha_u} u_{L_1}$ $\qquad$ $d_{L_1} \to e^{i\alpha_d}$

$G_F \times SU(2)_L$ (SM)

$G_1 \to \underline{G_a}, \underline{G_B}, G \ldots$

e_L, e_R, ν_L, ν_R

$U_{PMNS} = f$(mismatch between G_c, G_e)