Viable Models of Spontaneous CP Violation

G. C. Branco
CFTP/IST, Univ. Lisboa

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What is the Origin of CP Violation?

Open Question

There are 2 ways of breaking CP

(i) At the Lagrangian level

(ii) Spontaneous CP Violation

Lagrangian is CP invariant but the vacuum breaks CP

CP Violation at the Lagrangian level

What is the simplest (and safest) procedure to check whether a given $\mathcal{L}$ violates CP? Write the Lagrangian as:

1) Write the Lagrangian as:

$$\mathcal{L} = \mathcal{L}^{CP} + \mathcal{L}^{\text{remaining}}$$

where $\mathcal{L}^{CP}$ conserves CP (Typically gauge interaction)

4)

2) Write the most general CP transformation which leaves $\mathcal{L}^{CP}$ invariant.

Study whether CP invariance defined in this way imply any restrictions on $\mathcal{L}^{\text{remaining}}$. If such non-trivial CP restrictions exist, one can derive necessary conditions for CP invariance.

Violations of any of these necessary conditions imply CP violation.

Let us consider the case of the SM:

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CP invariance of Yukawa couplings imply:

$$U_L^\dagger Y_u U_R^u = Y_u^*$$
$$U_L^\dagger Y_d U_R^d = Y_d^*$$

J. Bernabeu
G.C.B, M. Gronau
1986

$$\Downarrow$$

$$U_L^\dagger (Y_u Y_u^\dagger) U_L = (Y_u Y_u^\dagger)^*$$
$$U_L^\dagger (Y_d Y_d^\dagger) U_L = (Y_d Y_d^\dagger)^*$$

$$\Rightarrow \quad \mathrm{tr}\left[H_{Y_u}, H_{Y_d}\right]^3 = 0$$

for any $n^{\underline{o}}$ of generations!!

$$H_{Y_{u,d}} \equiv (Y_{\substack{u\\d}} Y_{\substack{u\\d}}^\dagger)$$

$$I_Y^{CP} \equiv \mathrm{tr}\left[H_{Y_u}, H_{Y_d}\right]^3 = 0$$

Necessary Conditions for CP invariance for any number of fermion generations

For $n=3$ $\quad I_Y^{CP} = 0$ necessary and sufficient condition for CP invariance at Lagrangian level. After gauge sym. breaking

$$I^{CP} \equiv \Delta_{cu}\,\Delta_{ct}\,\Delta_{tu}\,\Delta_{sd}\,\Delta_{bs}\,\Delta_{bd}\,\mathrm{Im}\,Q$$

$$\Delta_{cu} = m_c^2 - m_u^2 \quad ; \quad Q = V_{us} V_{cb} V_{cs}^* V_{ub}^*$$

This result was obtained by us (J. Bernabeu, M. Gronau, GCB) in 1986. We just followed the idea of Lee and Yang. ... CP is defined by the part of the Lagrangian which respects CP.

Lee could have derived the above result in 1973 and would have won a second Nobel prize, with Kobayashi and Maskawa ...

Spontaneous CP Violation

One can impose CP invariance at the Lagrangian level but have a vacuum which is not CP invariant

TD Lee (1973)

Lee has shown that it is possible to have spontaneous CP violation in the context of a minimal extension of the SM with Two Higgs Doublets (THDM)

T. D. Lee has shown that if one introduces 2 Higgs doublets and imposes CP invariance of the Lagrangian then there is a region of parameters of the Higgs potential where the minimum is at

$$\langle\phi_1\rangle = \begin{bmatrix} 0 \\ v_1 e^{i\theta_1} \end{bmatrix} ; \langle\phi_2\rangle = \begin{bmatrix} 0 \\ v_2 e^{i\theta_2} \end{bmatrix}$$

For generic $\theta \equiv \theta_2 - \theta_1$ the vacuum is not CP invariant $\Rightarrow$ spontaneous CP violation

T.D.Lee (1973)

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In general two Higgs doublet models have FCNC

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour changing neutral currents (FCNC) are forbidden at tree level

- **in the gauge sector, no ZFCNC**
- **in the Higgs sector, no HFCNC**

Two Dogmas likely to be violated

Models with two or more Higgs doublets have potentially large HFCNC

Strict limits on FCNC processes!

Conjecture

Nature violates all the three dogmas put forward by the inventors of the SM:

- Neutrinos are not massless
- There are tree-level Z-FCNC in gauge sector
- There are tree-level HFCNC in the scalar sector

A viable model of spontaneous CP violation has to surmount two obstacles:

(i) The model should have a vacuum which violate CP.

(ii) The model should have a mechanism to suppress FCNC so that they are in agreement with experiment.

In two Higgs Doublet Models (2HDM) Flavour Changing Neutral Currents (FCNC) have to be eliminated at tree level or naturally suppressed, in order to conform to experiment.

- Z_2 symmetry leading to Natural Flavour Conservation (NFC)

  Glashow and Weinberg (1977)

- Attempt at generalising NFC : R. Gatto

Can one have a framework where there are FCNC at tree level, but naturally suppressed?

Is it possible to have a framework where the FCNC exist, but are only functions of V^{CKM} and the ratio v_2/v_1?

The suppression of FCNC could be related to the smallness of some of the V^{CKM} elements.

- Naturally suppressed FCNC as a result of a symmetry of the Lagrangian. The suppression is due to small V^{CKM} elements

G.C.B, Grimus, Lavoura (1996) (BGL)

- Extension to the leptonic sector

F. Botella, GCB, MN Rebelo
F. Botella, GCB., MNRebelo, M.Nebot

- Thorough Phenomenological analysis

F. Botella, GCB, A. Carmona, M.Nebot, L. Pedro, M.N. Rebelo

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Notation

Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q_L^0}\,\Gamma_1\Phi_1 d_R^0 - \overline{Q_L^0}\,\Gamma_2\Phi_2 d_R^0 - \overline{Q_L^0}\,\Delta_1\tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\,\Delta_2\tilde{\Phi}_2 u_R^0 + \text{h.c.}$$

$$\tilde{\Phi}_i = -i\tau_2\Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\theta}\Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2 e^{-i\theta}\Delta_2),$$

Diagonalised by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag}\,(m_d, m_s, m_b)\,,$$
$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag}\,(m_u, m_c, m_t)\,.$$

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Leptonic Sector

Charged leptons

$$-\overline{L_L^0}\ \Pi_1 \Phi_1 \ell_R^0 - \overline{L_L^0}\ \Pi_2 \Phi_2 \ell_R^0\ + \text{h.c.}$$

$$\left(\ -\overline{L_L^0}\ \Sigma_1 \tilde{\Phi}_1 \nu_R^0 - \overline{L_L^0}\ \Sigma_2 \tilde{\Phi}_2 \nu_R^0 + \text{h.c.}\ \right)$$

Neutrino Dirac

$$\left(\ \frac{1}{2} {\nu_R^0}^T C^{-1} M_R \nu_R^0 + \text{h.c.}\ \right)$$

Neutrino Majorana

Expansion around the vev's

$$\Phi_j = \begin{pmatrix} \phi_i^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}, \qquad j = 1, 2$$

We perform the following transformation:

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = U \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = U \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix},$$

$$U = \frac{1}{v} \begin{pmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{pmatrix}; \qquad v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-\frac{1}{2}} \simeq 246\text{GeV}$$

U singles out

H^0 **with couplings to quarks proportional to mass matrices**

G^0 **neutral pseudo-Goldstone boson**

G^+ **charged pseudo-Goldstone boson**

Physical neutral fields are combinations of $H^0 \quad R \quad I$

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Neutral and charged Higgs Interactions for the quark sector

$$\mathcal{L}_Y(\text{quark, Higgs}) = -\overline{d_L^0}\frac{1}{v}\left[M_d H^0 + N_d^0 R + i N_d^0 I\right] d_R^0 \\ -\overline{u_L^0}\frac{1}{v}\left[M_u H^0 + N_u^0 R + i N_u^0 I\right] u_R^0 \\ -\frac{\sqrt{2}H^+}{v}\left(\overline{u_L^0} N_d^0\, d_R^0 - \overline{u_R^0} N_u^{0\dagger}\, d_L^0\right) + \text{h.c.}$$

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\theta}\Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1 e^{-i\theta}\Delta_2).$$

Flavour structure of quark sector of 2HDM characterised by:

four matrices $M_d,\ M_u,\ N_d^0,\ N_u^0.$

Likewise for Leptonic sector, Dirac neutrinos:

$$M_\ell,\ M_\nu,\ N_\ell^0,\ N_\nu^0.$$

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Yukawa Couplings in terms of quark mass eigenstates

for H^+, H^0, R, I

$$\mathcal{L}_Y(\text{quark, Higgs}) =$$

$$- \frac{\sqrt{2}H^+}{v}\bar{u}\left(VN_d\gamma_R - N_u^\dagger\, V\gamma_L\right) d + \text{h.c.} - \frac{H^0}{v}\left(\bar{u}D_u u + \bar{d}D_d\, d\right) -$$

$$- \frac{R}{v}\left[\bar{u}(N_u\gamma_R + N_u^\dagger\gamma_L)u + \bar{d}(N_d\gamma_R + N_d^\dagger\gamma_L)\, d\right] +$$

$$+ i\frac{I}{v}\left[\bar{u}(N_u\gamma_R - N_u^\dagger\gamma_L)u - \bar{d}(N_d\gamma_R - N_d^\dagger\gamma_L)\, d\right]$$

$$\gamma_L = (1-\gamma_5)/2 \qquad\qquad \gamma_R = (1+\gamma_5)/2$$

$$V = V_{CKM}$$

FCNC controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For general 2HDM, N_d, N_u are are arbitrary complex 3×3 matrices.

One can rewrite N_d as:

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(t + \frac{1}{t} \right) \underbrace{U_{dL}^\dagger e^{i\alpha} \Gamma_2 U_{dR}}_{\text{leads to FCNC}}$$

$t \equiv \tan\beta = \frac{v_2}{v_1}$

In general N_d, N_u depend on (U_{dL}, U_{dR}), (U_{uL}, U_{uR}) respectively.

Our initial aim was to prove that it is "impossible" to have N_d, N_u to depend only on V_{CKM}.

While looking for the "proof" we (Gérard, Grimus and I) discovered the so called BGL models

Example of a BGL-type model: Impose the following discrete symmetry:

$$Q^0_{Lj} \to \exp(i\tau)\,Q^0_{Lj} \;;\quad u^0_{Rj} \to \exp(2i\tau)\,u^0_{Rj}\;;$$

$$\phi_2 \to \exp(i\tau)\,\phi_2$$

$\tau \neq 0, \pi$ { Z_2 excluded

Γ_j, Δ_j have the form:

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix};\quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix};\quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix};\quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

FCNC only in the down sector

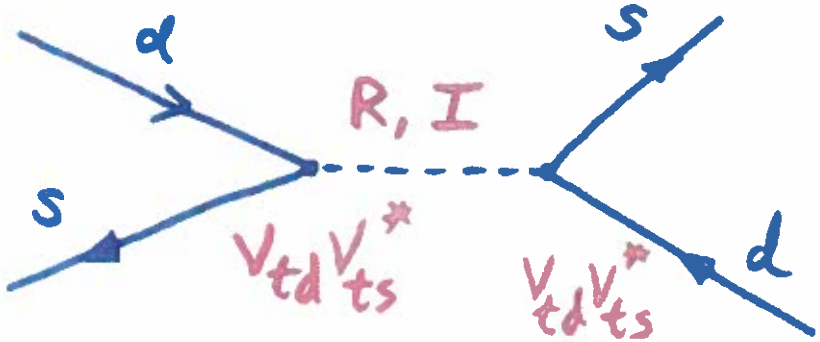
If one imposes $d^0_{Rj} \to \exp(2i\tau)\,d^0_{Rj}$ instead of $u^0_{Rj} \to \exp(2i\tau)\,u^0_{Rj}$ only FCNC in up sector

Considering only the Quark sector there are 6 different BGL type models. In the example considered, one has:

$$(N_d)_{rs} = \frac{v_2}{v_1}(D_d)_{rs} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM}^{\dagger})_{r3}(V_{CKM})_{3s}(D_d)_{ss}$$

$$N_u = -\frac{v_1}{v_2}\,\mathrm{diag}\,(0, 0, m_t) + \frac{v_2}{v_1}\,\mathrm{diag.}\,(m_u, m_c, 0)$$

Strong and natural suppression of $K^0 - \overline{K}^0$ transitions



$\longrightarrow \lambda^{10}$ suppression

Neutral couplings in BGL models

$$N_u = -\frac{v_1}{v_2}\text{diag}(0,0,m_t) + \frac{v_2}{v_1}\text{diag}(m_u,m_c,0)$$

Explicitely

$$N_d = \frac{v_2}{v_1}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} -$$

$$\left(\frac{v_1}{v_2}+\frac{v_2}{v_1}\right)\begin{pmatrix} m_d|V_{31}|^2 & m_s V_{31}^* V_{32} & m_b V_{31}^* V_{33} \\ m_d V_{32}^* V_{31} & m_s|V_{32}|^2 & m_b V_{32}^* V_{33} \\ m_d V_{33}^* V_{31} & m_s V_{33}^* V_{32} & m_b|V_{33}|^2 \end{pmatrix}$$

<u>It all comes from the symmetry</u>

suggested in 1996!

BGL models have some features in Common with the Minimal Flavour Violation Framework

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)

D'Ambrosio, Giudice, Isidori, Strumia (2002)

Namely, Flavour dependence of New Physics is completely controlled by V^{CKM}, with no other flavour parameters.

Note: MFV is an "Hypothesis" not a model!!!

An important question:

Can one introduce other discrete symmetries leading to other models with FCNC, completely controlled by V_{CKM}?

Answer: In the framework of 2HDM with Abelian symmetries and the constraint that FCNC only depend on V_{CKM}, BGL models are unique!

Ferreira and Silva 2010.

The explicit example of a BGL model was written in a weak-basis chosen by the symmetry. How to recognize a BGL model when written in a different WB?

The following relations

$$\Delta_1^\dagger \Delta_2 = 0 \;;\quad \Delta_1 \Delta_2^\dagger = 0 \;;\quad \Gamma_1^\dagger \Delta_2 = 0 \;;\quad \Gamma_2^\dagger \Delta_1 = 0$$

are necessary and sufficient conditions for a set of Yukawa matrices Γ_i, Δ_i to be of the BGL type, with FCNC in the down sector.

- In a certain sense, BGL models are rather unique. They have FCNC either in the up or the down sectors but not in both.
- If one restricts oneself to abelian symmetries, BGL models are the only 2HDM with FCNC at tree level, but no new flavour parameters, apart from V_{CKM}.
- Question – Can one generalize BGL models and construct a 2HDM with non-vanishing but controlled FCNC in both the up and down sectors? These gBGL models would contain BGL models as special cases, corresponding to specific values of the parameters of gBGL

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Answer: Yes!!

gBGL allowing for HFCNC both in up and down sectors

Symmetry: (not flavour blind!!)

$$Q_{L_3} \mapsto -Q_{L_3},$$

$$d_R \mapsto d_R, \qquad \Phi_1 \mapsto \Phi_1,$$

$$u_R \mapsto u_R, \qquad \Phi_2 \mapsto -\Phi_2.$$

- drastic reduction in number of free parameters

-no NFC

one may say that *the principle leading to gBGL constrains the Yukawa couplings so that each line of* Γ_j, Δ_j *couples only to one Higgs doublet.*

$$\Gamma_1 = \begin{pmatrix} \times & \times & \gamma_{13} \\ \times & \times & \gamma_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \times \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{pmatrix},$$

- renormalisable;

- FCNC both in up and down sectors;

- no longer of MFV type, four additional flavour parameters;

- both up and down type BGL appear as special limits;

gBGL verify:

$$\Gamma_2^\dagger \Gamma_1 = 0, \qquad \Gamma_2^\dagger \Delta_1 = 0,$$

$$\Delta_2^\dagger \Delta_1 = 0, \qquad \Delta_2^\dagger \Gamma_1 = 0.$$

Structure of Yukawa Couplings

$$\Gamma_1 = \begin{bmatrix} \times & \times & \gamma_{13} \\ \times & \times & \gamma_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{bmatrix}$$

For $\delta_{ij} = 0$ one obtains uBGL models, with

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

Similarly for $\gamma_{ij} = 0$, one obtains dBGL

It can be shown that N_d, N_u can be parametrized as:

$$N_d = \left[t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1}) V^\dagger U P_3 U^\dagger V \right] M_d$$

$$N_u = \left[t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1}) U P_3 U^\dagger \right] M_u$$

$V \equiv V^{CKM}$. It is clear that in gBGL one has more freedom, due to the presence of the arbitrary matrix U. Nevertheless, there is much less freedom than one might expect, since the only quantities involving U are:

$$\left[U P_3 U^\dagger \right]_{ij} = U_{i3} U_{j3}^*$$

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Mass matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Mass matrices

$$M_d^0 = \frac{v e^{i\theta_1}}{\sqrt{2}} (c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2), \quad M_u^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}} (c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2)$$

- **Important:** with $\hat{M}_d^0$ and $\hat{M}_u^0$ *real*

$$M_d^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0, \qquad M_u^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \hat{M}_u^0$$

$$M_d^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{i\theta} \mathrm{P}_3\right] \hat{M}_d^0, \quad M_u^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{-i\theta} \mathrm{P}_3\right] \hat{M}_u^0$$

$$\mathrm{P}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Bidiagonalisation of M_d^0, M_u^0

$$\mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{U}_{d_R} = \mathrm{diag}(m_{d_i}), \quad \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{U}_{u_R} = \mathrm{diag}(m_{u_i})$$

- $M_d^0 M_d^{0\dagger}$

$$M_d^0 M_d^{0\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0 \hat{M}_d^{0\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$\hat{M}_d^0 \hat{M}_d^{0\,T}$ *real* and *symmetric*

$$\mathcal{O}_L^{d\,T}\, \hat{M}_d^0 \hat{M}_d^{0\,T}\, \mathcal{O}_L^d = \mathrm{diag}(m_{d_i}^2) \qquad \text{with real orthogonal } \mathcal{O}_L^d$$

$$\mathcal{U}_{d_L}^\dagger M_d^0 M_d^{0\dagger} \mathcal{U}_{d_L} = \mathrm{diag}(m_{d_i}^2)\,, \text{ with } \mathcal{U}_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \mathcal{O}_L^d$$

- Similarly

$$\mathcal{U}_{u_L}^\dagger M_u^0 M_u^{0\dagger} \mathcal{U}_{u_L} = \mathrm{diag}(m_{u_i}^2)\,, \text{ with } \mathcal{U}_{u_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \mathcal{O}_L^u$$

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- Right-handed transformations

$$M_d^{0\dagger} M_d^0 = \hat{M}_d^{0\,T} \hat{M}_d^0, \qquad M_u^{0\dagger} M_u^0 = \hat{M}_u^{0\,T} \hat{M}_u^0$$

$$\mathcal{O}_R^{d\,T} M_d^{0\dagger} M_d^0 \mathcal{O}_R^d = \mathrm{diag}(m_{d_i}^2), \quad \mathcal{O}_R^{u\,T} M_u^{0\dagger} M_u^0 \mathcal{O}_R^u = \mathrm{diag}(m_{u_i}^2)$$

with real orthogonal $\mathcal{O}_R^d$ and $\mathcal{O}_R^u$

- Finally

$$M_d = \mathrm{diag}(m_{d_i}) = \mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{O}_R^d, \quad M_u = \mathrm{diag}(m_{u_i}) = \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{O}_R^u$$

- The CKM matrix $V \equiv \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$ is

$$V = \mathcal{O}_L^{u\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} \mathcal{O}_L^d$$

requires $e^{i2\theta} \neq \pm 1$ for CP violation!

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The fact that it is necessary to have $\boxed{e^{2i\theta} \neq \pm 1}$ in order to have CP violation, was to be expected, as it can be seen from a close analysis of the scalar potential.

One can show that for $\theta = \pm \pi/2$ the vacuum is CP conserving!!

Scalar sector

- 2HDM potential
  - CP invariant (all couplings are real)
  - $\mathbb{Z}_2$ symmetry, softly broken by $\mu_{12}^2 \neq 0$

$$\mathcal{V}(\Phi_1, \Phi_2) = \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)$$
$$+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$
$$+ \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]$$

$\cos\theta = \dfrac{-\mu_{12}^2}{2\lambda_5 v_1 v_2}$; For $\theta = \pm \pi/2$ the vacuum is CP invariant!!

Phenommology implications

We have shown that there a region of the parameters of the model where:

- One can obtain a correct CKM matrix: reproduce the moduli of the first and second ~~[illegible]~~ lines of V_{CKM}; obtain correct γ
- Satisfy the stringent constraints from experiment
  K^0-$\bar{K}^0$, B_d-$\bar{B}_d$, B_s-$\bar{B}_s$, rare top decays etc

- We point out that there is a deep connection between the generation of a complex CKM matrix from a vacuum phase and the appearance of SFCNC.
- The new scalars are necessarily lighter than 1 TeV
- Possibility of observing New Physics, such as:

  $t \to hc\,,\ hu \to$ relevant for LHC

  $h \to bs\,,\ bd \to$ relevant for ILC

Conclusions

- It is possible to have a realistic 2HDM with spontaneous CP violation and controlled SFCNC
  The crucial point is the use of a flavoured Z_2 symmetry where the 3^{rd} family is odd and the first two families are even.
- The model predicts the existence of new scalars lighter than 1TeV

There is no scientific reason to believe in the {dogma / myth} that flavour can only be understood at the Planck scale

- FCC, ILC, etc should be constructed. We should try to preserve "continuity"

Construct the tunnel !!!