CP violation with an unbroken CP transformation

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w/ Michael Ratz.

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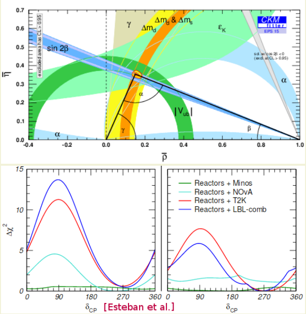
Motivation

- Standard Model flavor puzzle / CP violation in Nature.
- Origin of CP violation?
  - CP violation established
    in quark sector,
    consistent with SM (CKM). ✓
  - open question:

    CP violation in lepton sector ?
  - open question:

    Why $\overline{\theta} = (\theta + \arg\det y_u y_d) < 10^{-10}$?

    Why CPV *only* in FV processes?



$\Rightarrow$ Flavor and CP are intertwined in SM.

This talk: A "flavor"-symmetry reason for $\overline{\theta} = 0$ while $\delta_{\cancel{CP}} \neq 0$.

(...in a toy model)

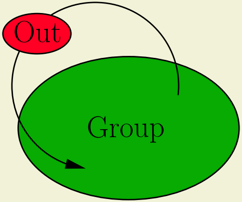
Outline

CP violation from a symmetry principle

CP violation in the breaking of $\mathbf{SU(3)} \rightarrow \mathbf{T_7}$

Summary

Two types of groups (without mathematical rigor)



List of representations: $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_k, {\boldsymbol{r}_k}^*, \ldots$

$$\mathrm{Out}\text{ in general }: \qquad \boldsymbol{r}_i \;\mapsto\; \boldsymbol{r}_j \quad \forall\text{ irreps } i{,}j\ (1:1)$$

Criterion:

Is there an (outer) automorphism transformation that maps

$$\boldsymbol{r}_i \;\mapsto\; {\boldsymbol{r}_i}^* \qquad \text{for all irreps } i\ ?$$

No	**Yes**
$\Rightarrow$ Group of "**type I**"	$\Rightarrow$ Group of "**type II**"

Why is this information important?

Physical CP transformation

In the Standard Model

$$\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1) \qquad \text{and} \qquad \mathrm{SO}(3,1)\ ,$$

physical CP is described by a *simultaneous* outer automorphism transformation of all symmetries which maps

$$\boldsymbol{r}_i \ \longleftrightarrow\ \boldsymbol{r}_i{}^*\ ,$$

$$\Big(\text{e.g. } (\mathbf{3},\mathbf{2})^{\mathrm{L}}_{1/6} \ \longleftrightarrow\ \left(\overline{\mathbf{3}},\overline{\mathbf{2}}\right)^{\mathrm{R}}_{-1/6}\Big)\ ,$$

for *all* representations of *all* symmetries.

[Grimus, Rebelo '95]
[Buchbinder et al. '01]
[AT '16]

Conservation of such a transformation warrants $\overline{\theta}$, $\delta_{\cancel{\mathrm{CP}}} = 0$.

Violation of such a transformation is implied by experiment, and necessary requirement for baryogenesis. [Sakharov '67]

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General answer: **No.**

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For example: Discrete groups of **type I**:

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\ldots$
SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$	

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- These are **inconsistent** with the trafo $\boldsymbol{r}_i \mapsto \boldsymbol{r}_i^* \; \forall\, i$.

$\Rightarrow$ CP transformation is inconsistent with a type I symmetry.
(assuming sufficient # of irreps are in the model)

There are models in which CP is violated *as a consequence* of another symmetry.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]

The corresponding CPV phases are calculable and quantized (e.g. $\delta_{\not{CP}} = 2\pi/3, \ldots$) stemming from the necessarily complex Clebsch-Gordan coefficients of the "type I" group. This has been termed "explicit geometrical" CP violation.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]
[Branco, '15], [de Medeiros Varzielas, '15]

An interesting observation

Observation:

Type I groups can arise as subgroups of type II groups.

For example: small finite subgroups of simple Lie groups.

$$\mathrm{SU}(3) \supset \mathrm{T}_7$$

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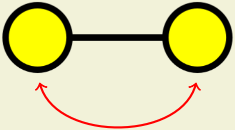
Structure of outer automorphisms:

$$\mathrm{Out}(\,\mathfrak{su}(3)\,) \cong \mathbb{Z}_2$$



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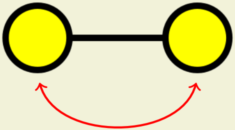

$$\boldsymbol{r}_i \leftrightarrow \boldsymbol{r}_i^{\,*} \quad \forall i \;✓$$

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
$\overline{\mathbf{3}}_1$	3	0	0	η^*	η

$$\boldsymbol{r}_i \cancel{\leftrightarrow} \boldsymbol{r}_i^{\,*} \quad \forall i \;✗$$

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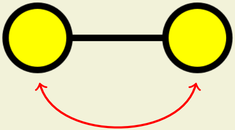

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Note: $\mathrm{Out}(\,\mathfrak{su}(3)\,)$ ***is*** $\mathrm{Out}(\,\mathrm{T}_7\,)$ of the $\mathrm{T}_7 \subset \mathrm{SU}(3)$ subgroup.

Toy model overview

Facts:

- $\mathrm{SU}(3)$ is **consistent** with a physical CP transformation.
- The T_7 subgroup of $\mathrm{SU}(3)$ is **inconsistent** with a physical CP transformation.

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Toy model: gauged $\mathrm{SU}(3)$ + complex scalar $\mathrm{SU}(3)$ $\mathbf{15}$-plet ϕ. [Ratz, AT '16]

$$\mathscr{L} \;=\; (D_\mu \phi)^\dagger \, (D^\mu \, \phi) - \frac{1}{4} \, G^a_{\mu\nu} \, G^{\mu\nu,a} - V(\phi) \;,$$

$$V(\phi) \;=\; -\,\mu^2 \phi^\dagger \phi + \sum_{i=1}^{5} \lambda_i \, \mathcal{I}_i^{(4)}(\phi) \;. \qquad \text{with } \lambda_i \in \mathbb{R}$$

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- VEV of the $\mathbf{15}$-plet $\langle \phi \rangle$ breaks $\mathrm{SU}(3) \to \mathrm{T}_7$. [Luhn, '11], [Merle, Zwicky '11]
- $\mathrm{Out}(\,\mathfrak{su}(3)\,) \cong \mathbb{Z}_2 \;\to\; \mathrm{Out}(\,\mathrm{T}_7\,) \cong \mathbb{Z}_2$; Out **unbroken** by VEV.

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \;\xrightarrow{\langle \phi \rangle}\; \mathrm{T}_7 \rtimes \mathbb{Z}_2 ; .$$

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

[Ratz, AT '16]

Name	$\mathrm{SU}(3)$	$\xrightarrow{\langle\phi\rangle}$	Name	T_7	mass
A_μ	$\mathbf{8}$		Z_μ	$\mathbf{1_1}$	$m_Z^2 = 7/3\, g^2\, v^2$
			W_μ	$\mathbf{3}$	$m_W^2 = g^2\, v^2$
ϕ	$\mathbf{15}$		$\mathrm{Re}\,\sigma_0$	$\mathbf{1_0}$	$m^2_{\mathrm{Re}\,\sigma_0} = 2\,\mu^2$
			$\mathrm{Im}\,\sigma_0$	$\mathbf{1_0}$	$m^2_{\mathrm{Im}\,\sigma_0} = 0$
			σ_1	$\mathbf{1_1}$	$m^2_{\sigma_1} = -\mu^2 + \sqrt{15}\,\lambda_5\, v^2$
			τ_1	$\mathbf{3}$	$m^2_{\tau_1} = m^2_{\tau_1}(\mu, \lambda_i)$
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		τ_3	$\mathbf{3}$	$m_{\tau_3}^2 = m_{\tau_3}^2(\mu, \lambda_i)$

The action is invariant under the $\mathbb{Z}_2 - \mathrm{Out}$ transformation:

SU(3)	T_7
$A_\mu^a(x) \mapsto R^{ab}\,\mathcal{P}_\mu^{\ \nu}\,A_\nu^b(\mathcal{P}\,x)\,,$ $\phi_i(x) \mapsto U_{ij}\,\phi_j^*(\mathcal{P}\,x)\,.$	$W_\mu(x) \mapsto \mathcal{P}_\mu^{\ \nu}\,W_\nu^*(\mathcal{P}\,x)\,,$ $\sigma_0(x) \mapsto \sigma_0(\mathcal{P}\,x)\,,$ $\tau_i(x) \mapsto \tau_i^*(\mathcal{P}\,x)\,,$ $Z_\mu(x) \mapsto -\mathcal{P}_\mu^{\ \nu}\,Z_\nu(\mathcal{P}\,x)\,,$ $\sigma_1(x) \mapsto \sigma_1(\mathcal{P}\,x)\,.$
physical CP ✓	physical CP ✗

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

- The VEV does **not** break the CP transformation, $U\langle\phi\rangle^* = \langle\phi\rangle$.
- However, at the level of T_7, the $\mathrm{SU}(3)$-CP transformation merges to $\mathrm{Out}(\,\mathrm{T}_7\,)$:

$$
\begin{array}{lccccccccccccccccc}
 & \mathbf{15} & \rightarrow & \mathbf{1_0} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{1}}\mathbf{_1} & \oplus & \mathbf{3} & \oplus & \mathbf{3} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} \\
\mathbb{Z}_2 - \mathrm{Out}: & \big\downarrow & & & & & & & & & & & & & & \\
 & \overline{\mathbf{15}} & \rightarrow & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}\mathbf{_1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3}
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$$\mathbb{Z}_2 - \mathrm{Out}: \quad \begin{tikzcd}
\mathbf{15} \arrow[d] & \to & \mathbf{1_0} \arrow[d] & \oplus & \mathbf{1_1} \arrow[drr] & \oplus & \overline{\mathbf{1}}_\mathbf{1} \arrow[dll] & \oplus & \mathbf{3} \arrow[d] & \oplus & \mathbf{3} \arrow[d] & \oplus & \overline{\mathbf{3}} \arrow[d] & \oplus & \overline{\mathbf{3}} \arrow[d] \\
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\end{array}
$$

$\Rightarrow$ The $\mathbb{Z}_2$-Out is conserved at the level of T_7, but it is **not** interpreted as a physical CP trafo,

$$
\mathrm{SU}(3) \rtimes \mathbb{Z}_2^{(\mathrm{CP})} \xrightarrow{\langle\phi\rangle} \mathrm{T}_7 \rtimes \mathbb{Z}_2^{\cancel{(\mathrm{CP})}} .
$$

- There is no other possible allowed CP transformation at the level of T_7 (type I).
- Imposing a transformation $\boldsymbol{r}_{\mathrm{T}_7,i} \leftrightarrow \boldsymbol{r}_{\mathrm{T}_7,i}{}^*$ enforces decoupling, $g = \lambda_i = 0$.

CP violation in $\mathbf{SU(3)} \rightarrow \mathbf{T_7}$ toy model

Explicit crosscheck: compute decay asymmetry.

$$\varepsilon_{\sigma_1 \rightarrow W\,W^*} := \frac{\left|\mathscr{M}(\sigma_1 \rightarrow W\,W^*)\right|^2 - \left|\mathscr{M}(\sigma_1^* \rightarrow W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1 \rightarrow W\,W^*)\right|^2 + \left|\mathscr{M}(\sigma_1^* \rightarrow W\,W^*)\right|^2}\,.$$

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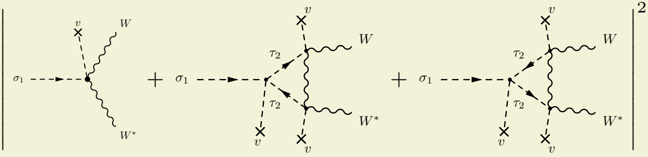
Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ from interference terms, e.g.



,

corresponding to non-vanishing CP-odd basis invariants

$$\mathcal{I}_1 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{imk} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jm\ell}\;,$$

$$\mathcal{I}_2 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{i\ell m} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jkm}\;.$$

- ✓ Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ is proportional to $\operatorname{Im}\mathcal{I}_{1,2} \neq 0$.
- ✓ All CP odd phases are geometrical, $\mathcal{I}_1 = \mathrm{e}^{2\,\pi\,\mathrm{i}/3}\,\mathcal{I}_2$.
- ✓ $\left(\varepsilon_{\sigma_1 \to W\,W^*}\right) \to 0$ for $v \to 0$, i.e. CP is restored in limit of vanishing VEV.

Natural protection of $\theta = 0$

Topological vacuum term of the gauge group

$$\mathscr{L}_\theta \;=\; \theta\,\frac{g^2}{32\pi^2}\,G^a_{\mu\nu}\,\widetilde{G}^{\mu\nu,a}\;,$$

is forbidden by $\mathbb{Z}_2-\mathrm{Out}$ (the $\mathrm{SU}(3)$-CP transformation).

The unbroken Out

$$\mathbb{Z}_2-\mathrm{Out}\;:\;W_\mu(x)\;\mapsto\;\mathcal{P}_\mu^{\;\nu}\,W^*_\nu(\mathcal{P}\,x)\;,\quad Z_\mu(x)\;\mapsto\;-\,\mathcal{P}_\mu^{\;\nu}\,Z_\nu(\mathcal{P}\,x)\;,$$

still enforces $\theta = 0$ even though CP is violated for the physical T_7 states.

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Physical scalars (T_7 singlets and triplets):

$$\begin{aligned}\mathrm{Re}\,\sigma_0 \;&=\; \frac{1}{\sqrt{2}}\,(\phi_1+\phi_1^*)\;, \qquad\qquad \mathrm{Im}\,\sigma_0 \;=\; -\,\frac{\mathrm{i}}{\sqrt{2}}\,(\phi_1-\phi_1^*)\;,\\ \sigma_1 \;&=\; \phi_2\;,\end{aligned}$$

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} \;=\; \begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}\;.$$

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Possible application to strong CP problem?

- Starting point: CP conserving theory based on

$$[G_{\mathrm{SM}} \times G_{\mathrm{F}}] \rtimes \mathrm{CP}\;.$$

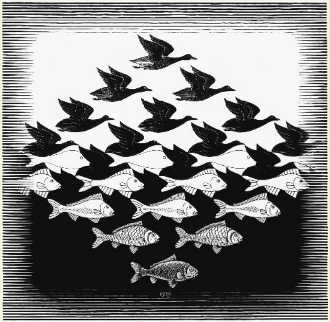
- break $G_{\mathrm{F}} \rtimes \mathrm{CP} \;\longrightarrow\; \mathrm{Type\,I} \rtimes \mathrm{Out}.$

↷ CP broken in flavor sector but not in strong interactions.

- Main problem: finding realistic model based on Type I group allowing for outer automorphism.

Summary

- There are certain (discrete) groups which are inconsistent with physical CP transformations.
- These groups allow for models with (explicit and/or spontaneous) CP violation with calculable (quantized) “geometrical” phases.
- Physical interpretation of one and the same transformation (namely the $\mathbb{Z}_2$-Out) changes depending on the symmetries of the ground state of a model.
- We have found an explicit toy model example, $\mathrm{SU}(3) \to \mathrm{T}_7$, in which CP is spontaneously violated for the physical states of the theory (with geometrical phases) while an unbroken outer automorphism protects $\theta = 0$.



Thank You!

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Additional slides

"Physical" CP transformation

Recall: e.g. complex scalar field σ, with field operator

$$\widehat{\boldsymbol{\sigma}}(x) \;=\; \int \widetilde{\mathrm{d}p}\, \left\{ \widehat{\boldsymbol{a}}(\vec{p})\, \mathrm{e}^{-\mathrm{i}\, p\, x} + \widehat{\boldsymbol{b}}^{\dagger}(\vec{p})\, \mathrm{e}^{\mathrm{i}\, p\, x} \right\} \;.$$

Physical CP transformation of the complex scalar field

$$\mathrm{CP} \;:\quad \sigma(x) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \sigma^{*}(\mathcal{P}\, x) \;,$$

corresponds to

$$\mathrm{CP} \;:\quad \widehat{\boldsymbol{a}}(\vec{p}) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \widehat{\boldsymbol{b}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^{\dagger}(\vec{p}) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \widehat{\boldsymbol{a}}^{\dagger}(-\vec{p}) \;.$$

Note:

"matter": $\widehat{\boldsymbol{a}}^{(\dagger)}$ "anti-matter": $\widehat{\boldsymbol{b}}^{(\dagger)}$.

Toy model details

Complex scalar ϕ in T_7-diagonal basis of $\mathrm{SU}(3)$: (in unitary gauge)

$$\phi \;=\; \left(v+\phi_1,\,\frac{\phi_2}{\sqrt{2}},\,\frac{\phi_2^*}{\sqrt{2}},\,\phi_4,\,\phi_5,\,\phi_6,\,\frac{\phi_7}{\sqrt{2}},\,\frac{\phi_8}{\sqrt{2}},\,\frac{\phi_9}{\sqrt{2}},\,\phi_{10},\,\phi_{11},\,\phi_{12},\,\frac{\phi_7^*}{\sqrt{2}},\,\frac{\phi_8^*}{\sqrt{2}},\,\frac{\phi_9^*}{\sqrt{2}}\right)\;.$$

T_7 representations of the components:

$$\phi_1 \;\widehat{=}\; \mathbf{1}_0\;, \qquad \phi_2 \;\widehat{=}\; \mathbf{1}_1\;,$$

$$T_1 \;:=\; (\phi_4,\,\phi_5,\,\phi_6)\;\widehat{=}\;\mathbf{3}\;, \qquad T_2 \;:=\; (\phi_7,\,\phi_8,\,\phi_9)\;\widehat{=}\;\mathbf{3}\;,$$

$$\overline{T}_3 \;:=\; (\phi_{10},\,\phi_{11},\,\phi_{12})\;\widehat{=}\;\overline{\mathbf{3}}\;.$$

The physical scalars are

$$\mathrm{Re}\,\sigma_0 \;=\; \frac{1}{\sqrt{2}}\left(\phi_1+\phi_1^*\right)\;, \qquad \mathrm{Im}\,\sigma_0 \;=\; -\,\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1-\phi_1^*\right)\;,$$

$$\sigma_1 \;=\; \phi_2\;,$$

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} \;=\; \begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}\;.$$

The physical vectors are

$$Z^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_7^\mu - \mathrm{i}\,A_8^\mu\right)\;, \qquad W_1^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_4^\mu - \mathrm{i}\,A_1^\mu\right)\;,$$

$$W_2^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_5^\mu - \mathrm{i}\,A_2^\mu\right)\;, \qquad W_3^\mu \;=\; \frac{\mathrm{i}}{\sqrt{2}}\left(A_6^\mu - \mathrm{i}\,A_3^\mu\right)\;.$$

Toy model details

The VEV in this basis is simply

$$\langle\phi\rangle_1 \;=\; v \quad \text{and} \quad \langle\phi\rangle_i \;=\; 0 \quad \text{for} \quad i=2,\dots,15\;,$$

where

$$|v| \;=\; \mu\times 3\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1+14\sqrt{15}\,\lambda_2+20\sqrt{6}\,\lambda_4+13\sqrt{15}\,\lambda_5\right)^{-1/2}\;.$$

The masses of the physical states are

$$m_Z^2 \;=\; \frac{7}{3}\,g^2\,v^2 \quad \text{and} \quad m_W^2 \;=\; g^2\,v^2\;.$$

$$\begin{aligned} m^2_{\mathrm{Re}\,\sigma_0} \;&=\; 2\,\mu^2\;, \qquad m^2_{\mathrm{Im}\,\sigma_0} \;=\; 0\;,\\ m^2_{\sigma_1} \;&=\; -\,\mu^2+\sqrt{15}\,\lambda_5\,v^2\;. \end{aligned}$$

The massless mode is the goldstone boson of an additional $\mathrm{U}(1)$ symmetry of the potential. It can be avoided by either

- gauging the additional $\mathrm{U}(1)$,
- or breaking it softly by a cubic coupling of ϕ.

Toy model details

T_7 invariant couplings ($\omega := \mathrm{e}^{2\pi\mathrm{i}/3}$)

$$Y_{\sigma_1 W W^*} \;=\; \frac{v\,g^2}{\sqrt{6}}\,\mathrm{e}^{-\pi\,\mathrm{i}/6}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;, \quad Y_{\sigma_1\tau_2\tau_2^*} \;=\; v\,y_{\sigma_1\tau_2\tau_2^*}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;,$$

$$\begin{aligned}\left[Y_{\tau_2^* W W^*}\right]_{121} \;&=\; \left[Y_{\tau_2^* W W^*}\right]_{232} \;=\; \left[Y_{\tau_2^* W W^*}\right]_{313} \;=\; v\,g^2\,y_{\tau_2^* W W^*}\;,\\ \left[Y_{\tau_2^* W W^*}\right]_{ijk} \;&=\; 0 \qquad \text{(else)}\;.\end{aligned}$$

Toy model details

$$
\begin{aligned}
y_{\sigma_1 \tau_2 \tau_2^*} \;=\; \frac{1}{504\sqrt{3}} \Big\{ & V_{21}^2 \, \Big[-14\sqrt{10}\, \left(17 + 5\sqrt{3}\,\mathrm{i}\right) \lambda_1 + 84\sqrt{30} \left(\sqrt{3} - \mathrm{i}\right) \lambda_2 \\
& \qquad - 240 \left(1 + \sqrt{3}\,\mathrm{i}\right) \lambda_4 - \sqrt{10} \left(197 - 55\sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \\
& + 8\, V_{22}^2 \, \Big[28\sqrt{10} \left(1 - \sqrt{3}\,\mathrm{i}\right) \lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 + 112\sqrt{3}\,\mathrm{i}\,\lambda_3 \\
& \qquad - \left(30 - 26\sqrt{3}\,\mathrm{i}\right) \lambda_4 + \sqrt{10} \left(20 - \sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \\
& + 8\, V_{23}^2 \, \Big[28\sqrt{10} \left(1 + \sqrt{3}\,\mathrm{i}\right) \lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 - 168\,\lambda_3 \\
& \qquad + \left(6 + 65\sqrt{3}\,\mathrm{i}\right) \lambda_4 - 4\sqrt{10} \left(1 - 2\sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \\
& + 8\, V_{21}\, V_{22} \, \Big[-35\sqrt{10} \left(1 - \sqrt{3}\,\mathrm{i}\right) \lambda_1 + 21\sqrt{30} \left(\sqrt{3} + \mathrm{i}\right) \lambda_2 \\
& \qquad - 56 \left(3 + \sqrt{3}\,\mathrm{i}\right) \lambda_3 + 6 \left(1 + 17\sqrt{3}\,\mathrm{i}\right) \lambda_4 - \sqrt{10} \left(67 + 19\sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \\
& + 4\, V_{21}\, V_{23} \, \Big[-28\sqrt{10} \left(2 + \sqrt{3}\,\mathrm{i}\right) \lambda_1 - 42\sqrt{30} \left(\sqrt{3} + \mathrm{i}\right) \lambda_2 \\
& \qquad + 30 \left(11 + 3\sqrt{3}\,\mathrm{i}\right) \lambda_4 - \sqrt{10} \left(31 + 11\sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \\
& - 8\, V_{22}\, V_{23} \, \Big[14\sqrt{10}\,\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 \\
& \qquad + 10 \left(3 + 5\sqrt{3}\,\mathrm{i}\right) \lambda_4 + \sqrt{10} \left(1 - 3\sqrt{3}\,\mathrm{i}\right) \lambda_5 \Big] \Big\}
\end{aligned}
$$

and

$$
y_{\tau_2^* W W^*} \;=\; -\,\frac{\sqrt{2}}{3} \left(2\, V_{21} + V_{22} + 2\, V_{23}\right) \;.
$$

Physical CP transformation

We *extrapolate* from the SM to possible symmetries in BSM.

$\Rightarrow$ “Definition” of CP in words:

CP is **a** special outer automorphism transformation which maps *all present* symmetry representations (global, local, space-time) to their complex conjugate representations.

[AT ’16]

This definition is consistent with the definitions in [Buchbinder et al. ’01] & [Grimus, Rebelo ’95]

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Any such transformation:

- warrants *physical* CP conservation (if conserved),

$\Rightarrow$ must be broken (by observation).

Note that a physical CP transformation

- does not have to be unique,
- does not have to be of order 2, e.g. [Grimus et al. '87], [Weinberg '05], [Ivanov, Silva '15]
- is, in general, not guaranteed to exist for a given symmetry group. (It *does* exist for G_{SM}).

What is an outer automorphism?

Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id},\, \mathsf{a},\, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \left(\text{think: } \mathsf{u}\,\mathsf{a}\,\mathsf{u}^{-1} \,=\, \mathsf{a}^2\right)$$

$\mathbb{Z}_3$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3})$

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In words: Out is a **"symmetry of the symmetry"**.

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In words: Out is a "**symmetry of the symmetry**".

Concrete: Out is a 1:1 mapping of representations $\boldsymbol{r} \mapsto \boldsymbol{r}'$.
Comes with a transformation matrix U, which is given by

$$U\rho_{\boldsymbol{r}'}(\mathrm{g})U^{-1} = \rho_{\boldsymbol{r}}(u(\mathrm{g}))\ , \qquad \forall \mathrm{g} \in G\ .$$

(consistency condition)

[Fallbacher, AT, '15]
[Holthausen, Lindner, Schmidt, '13]

- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$
- $u : g \mapsto u(g)$: **outer** automorphism

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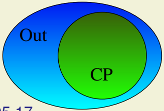
- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$
- $u : g \mapsto u(g)$: **outer** automorphism

Note: Physical CP trafo $\boldsymbol{r}' = \boldsymbol{r}^*$ is a special case of this.



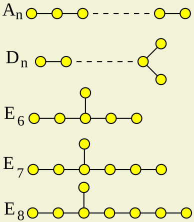
Outer automorphisms of groups

Outer automorphisms exist for continuous & discrete groups.

There are easy ways to depict this:

Continuous groups:

Outer automorphisms of a simple Lie algebra are the symmetries of the corresponding Dynkin diagram.



	Lie Group	Out	Action on reps
$A_{n>1}$	$\mathrm{SU}(N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
$D_{n=4}$	$\mathrm{SO}(8)$	S_3	$\boldsymbol{r}_i \rightarrow \boldsymbol{r}_j$
$D_{n>4}$	$\mathrm{SO}(2N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
E_6	E_6	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
all others		/	/

Outer automorphisms of groups

Discrete groups:

Outer automorphisms of a discrete group are symmetries of the character table (not 1:1).

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
$\overline{\mathbf{3}}_1$	3	0	0	η^*	η

$\mathbb{Z}_2$

The outer automorphisms group of any ("small") discrete group can easily be found with `GAP` [GAP].

Group	Out	Action on reps
$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$A_{n\neq 6}$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$S_{n\neq 6}$	/	/
$\Delta(27)$	$\mathrm{GL}(2,3)$	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
$\Delta(54)$	S_4	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
$\ldots$		

CP violation in $\mathbf{SU(3)} \rightarrow \mathbf{T_7}$ toy model

The conserved $\mathbb{Z}_2$-Out acts on the physical states as

$$
\mathbb{Z}_2 - \mathrm{Out} \;: \quad
\begin{array}{lll}
W_\mu(x) & \mapsto & \mathcal{P}_\mu^{\;\nu}\, W_\nu^*(\mathcal{P}\,x)\,, \qquad Z_\mu(x) \;\mapsto\; -\,\mathcal{P}_\mu^{\;\nu}\, Z_\nu(\mathcal{P}\,x)\,, \\
\sigma_0(x) & \mapsto & \sigma_0(\mathcal{P}\,x)\,, \qquad \sigma_1(x) \;\mapsto\; \sigma_1(\mathcal{P}\,x)\,, \\
\tau_i(x) & \mapsto & \tau_i^*(\mathcal{P}\,x)\,.
\end{array}
$$

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This does **not** correspond to a physical CP transformation. Mode expansion

$$\widehat{\boldsymbol{\sigma}}_1(x) \;=\; \int \widetilde{\mathrm{d}p}\, \left\{\widehat{\boldsymbol{a}}(\vec{p})\,\mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^\dagger(\vec{p})\,\mathrm{e}^{\mathrm{i}\,p\,x}\right\}\,.$$

The $\mathbb{Z}_2$-Out transformation corresponds to a map

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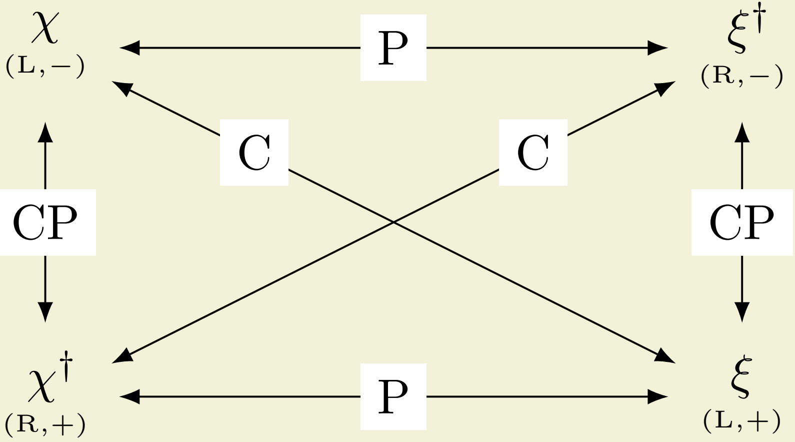
which does not correspond to a physical CP transformation.
The QFT CP transformation

$$\mathrm{CP} \;:\quad \widehat{\boldsymbol{a}}(\vec{p}) \;\mapsto\; \widehat{\boldsymbol{b}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^\dagger(\vec{p}) \;\mapsto\; \widehat{\boldsymbol{a}}^\dagger(-\vec{p})\;,$$

is not a symmetry of the action (imposing it enforces decoupling, $g = \lambda_i = 0$).

C, P, and CP transformations and spinor representations



CP as a special outer automorphism

The most general possible CP transformation for (SM) gauge and *one generation* of (chiral) fermion fields:

$$
\begin{aligned}
W_\mu^a(x) \;&\mapsto\; R^{ab}\,\mathcal{P}_\mu^{\;\nu}\,W_\nu^b(\mathcal{P}\,x)\;, \\
\Psi_\alpha^i(x) \;&\mapsto\; \eta_{\mathsf{CP}}\,U^{ij}\,\mathcal{C}_{\alpha\beta}\,\Psi^{*}{}_\beta^{j}(\mathcal{P}\,x)\;.
\end{aligned}
$$

cf. e.g. [Grimus, Rebelo,'95]

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cf. e.g. [Grimus, Rebelo,'95]

This is a conserved symmetry of the kinetic terms iff $\qquad [T_a, T_b] = \mathrm{i} f_{abc}\,T_c$

$$\begin{aligned}
\text{(i)} &: & R_{aa'}\,R_{bb'}\,f_{a'b'c} &= f_{abc'}\,R_{c'c}\;,\\
\text{(ii)} &: & U\,(-T_a^{\mathrm{T}})\,U^{-1} &= R_{ab}\,T_b\;,\\
\text{(iii)} &: & \mathcal{C}\,(-\gamma^{\mu\,\mathrm{T}})\,\mathcal{C}^{-1} &= \gamma^\mu\;.
\end{aligned}$$

Note: These are precisely the **consistency conditions** for a mapping of $\boldsymbol{r} \mapsto \boldsymbol{r}^*$ for *both*, the gauge and space-time symmetries of a model.

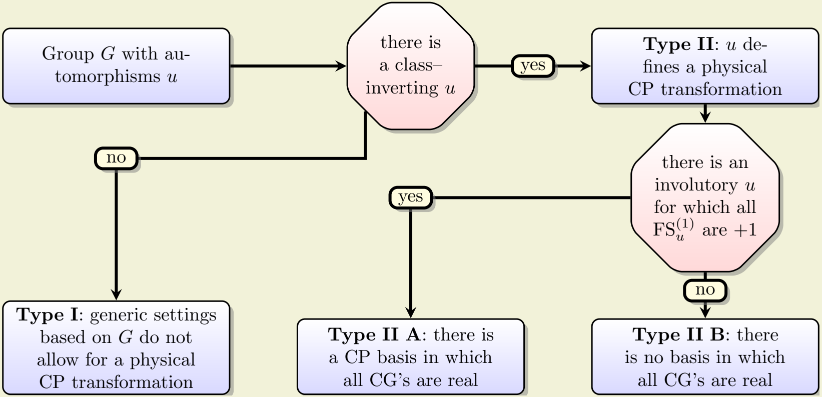
$\Rightarrow$ "Definition" of CP in words:

CP is **a** special outer automorphism transformation which maps *all present* symmetry representations (global, local, space-time) to their complex conjugates.

(This transformation does not have to be unique nor is it guaranteed to exist at all)

(This is consistent with the transformations considered in [Buchbinder et al.'01] & [Grimus, Rebelo,'95])

CP symmetries in settings with discrete G



(For details see [Chen, Fallbacher, Mahanthappa, Ratz, AT, '14])

Mathematical tool to decide: Twisted Frobenius-Schur indicator FS_u(Backup slides)

Twisted Frobenius–Schur indicator

Criterion to decide: existence of a CP outer automorphism.

$\curvearrowright$ can be probed by computing the

“twisted Frobenius–Schur indicator” $\mathbf{FS}_{\boldsymbol{u}}$

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|} \sum_{g\in G} \chi_{\boldsymbol{r}_i}(g\,u(g))$$

($\chi_{\boldsymbol{r}_i(g)}$: Character)

[Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014]

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \text{ or } -1 \quad \forall\, i, & \Rightarrow\; u \text{ is good for CP,} \\ \text{different from } \pm 1, & \Rightarrow\; u \text{ is no good for CP.} \end{cases}$$

In analogy to the Frobenius–Schur indicator

$\mathrm{FS}_{\cancel{u}}(\boldsymbol{r}_i) = +1, -1, 0$ for real / pseudo–real / complex irrep.