The study of multi-Higgs-doublet model potentials

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3HDM example: $O(2) \times \mathbb{Z}_2$ model

- 3 Higgs-boson doublets to generate ν masses and mixing.

$$\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}, \qquad i = 1, 2, 3$$

W. Grimus, L. Lavoura and D. Neubauer, JHEP **0807**, 051 (2008)

- Symmetry $O(2) \times \mathbb{Z}_2 \cong \mathbb{Z}_2' \times U(1) \times \mathbb{Z}_2$

- Particles assigned to irreducible representations.

- Assignment to $\mathbb{Z}_2' \times U(1) \times \mathbb{Z}_2$

$$\mathbb{Z}_2': \quad D_{\mu L} \leftrightarrow D_{\tau L}, \quad \mu_R \leftrightarrow \tau_R, \quad \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \quad \varphi_1 \leftrightarrow \varphi_2$$

$$\mathbb{Z}_2: \quad \nu_{eR} \to -\nu_{eR}, \quad \nu_{\mu R} \to -\nu_{\mu R}, \quad \nu_{\tau R} \to -\nu_{\tau R},$$
$$e_R \to -e_R, \quad \varphi_3 \to -\varphi_3$$

$U(1)$:

$X \longrightarrow e^{i\theta} X$	$D_{\mu L}, \tau_R, \nu_{\mu R}$	$D_{\tau L}, \mu_R, \nu_{\tau R}$	φ_1	φ_2
θ	1	-1	2	-2

- Most general 3HDM potential

$$V_{O(2)\times\mathbb{Z}_2} = \mu_0\varphi_3^\dagger\varphi_3 + \mu_{12}\left(\varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2\right) + \mu_m\left(\varphi_1^\dagger\varphi_2 + \varphi_2^\dagger\varphi_1\right)$$
$$+a_1(\varphi_3^\dagger\varphi_3)^2 + a_2\varphi_3^\dagger\varphi_3\left(\varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2\right) + a_3\left(\varphi_3^\dagger\varphi_1\cdot\varphi_1^\dagger\varphi_3 + \varphi_3^\dagger\varphi_2\cdot\varphi_2^\dagger\varphi_3\right)$$
$$+ a_4\varphi_3^\dagger\varphi_1\cdot\varphi_3^\dagger\varphi_2 + a_4^*\varphi_1^\dagger\varphi_3\cdot\varphi_2^\dagger\varphi_3 + a_5\left((\varphi_1^\dagger\varphi_1)^2 + (\varphi_2^\dagger\varphi_2)^2\right)$$
$$+ a_6\varphi_1^\dagger\varphi_1\cdot\varphi_2^\dagger\varphi_2 + a_7\varphi_1^\dagger\varphi_2\cdot\varphi_2^\dagger\varphi_1$$

- μ_m term breaks the $U(1)$ symmetry explicitly but softly.
- One parameter a_4 may have imaginary part.

Bilinears

- Introduce matrix of Higgs doublets $\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}$, $i = 1, 2, 3$

$$\phi = \begin{pmatrix} \varphi_1^{\mathrm{T}} \\ \varphi_2^{\mathrm{T}} \\ \varphi_3^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \, \varphi_1^0 \\ \varphi_2^+ \, \varphi_2^0 \\ \varphi_3^+ \, \varphi_3^0 \end{pmatrix}$$

- Arrange all $SU(2)_L \times U(1)_Y$ invariants into hermitian 3×3 matrix

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 & \varphi_3^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 & \varphi_3^\dagger\varphi_2 \\ \varphi_1^\dagger\varphi_3 & \varphi_2^\dagger\varphi_3 & \varphi_3^\dagger\varphi_3 \end{pmatrix}$$

- $\underline{K} = \phi\phi^\dagger$ is hermitian, positive semidefinite with rank ≤ 2.
- Basis for $\underline{K}$ are Gell-Mann matrices λ_α, $\lambda_0 = \sqrt{\frac{2}{3}}\mathbb{1}_3$,

$$\underline{K} = \frac{1}{2}K_\alpha\lambda_\alpha, \qquad \alpha = 0, 1, \ldots, 8$$

- Invariant scalar products in terms of bilinears

$$
\begin{aligned}
\varphi_1^\dagger \varphi_1 &= \frac{K_0}{\sqrt{6}} + \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}}, & \varphi_1^\dagger \varphi_2 &= \frac{1}{2}\left(K_1 + iK_2\right),\\
\varphi_1^\dagger \varphi_3 &= \frac{1}{2}\left(K_4 + iK_5\right), & \varphi_2^\dagger \varphi_2 &= \frac{K_0}{\sqrt{6}} - \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}},\\
\varphi_2^\dagger \varphi_3 &= \frac{1}{2}\left(K_6 + iK_7\right), & \varphi_3^\dagger \varphi_3 &= \frac{K_0}{\sqrt{6}} - \frac{K_8}{\sqrt{3}}.
\end{aligned}
$$

- One-to-one correspondance between Higgs-boson doublets and Hermitean matrix $\underline{K}$ with rank ≤ 2.

- Potential can be written with K_0, $\boldsymbol{K} = \begin{pmatrix} K_1 \\ \vdots \\ K_8 \end{pmatrix}$

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

with real parameters

$$\xi_0,\ \eta_{00},\ \boldsymbol{\xi},\ \boldsymbol{\eta},\ E = E^{\mathrm{T}}$$

C. Nishi **PRD 74** (2006),

MM, A. Manteuffel, O. Nachtmann, F. Nagel **EPJC 48** (2006),

MM, O. Nachtmann JHEP 1502 (2015),

MM, O. Nachtmann PRD 92 (2015)

- $O(2) \times \mathbb{Z}_2$ potential with parameters

$$\xi_0 = \frac{\mu_0 + 2\mu_{12}}{\sqrt{6}}, \quad \boldsymbol{\xi} = \left(\mu_m, 0, 0, 0, 0, 0, 0, \frac{\mu_{12} - \mu_0}{\sqrt{3}}\right)^{\mathrm{T}},$$

$$\eta_{00} = \frac{1}{6}(a_1 + 2a_2 + 2a_5 + a_6), \quad \boldsymbol{\eta} = \left(0, 0, 0, 0, 0, 0, 0, \frac{1}{3\sqrt{2}}(-a_1 - \frac{a_2}{2} + a_5 + \frac{a_6}{2})\right)^{\mathrm{T}},$$

$$E = \frac{1}{4}\begin{pmatrix} a_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2a_5 - a_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & \mathrm{Re}\, a_4 & \mathrm{Im}\, a_4 & 0 \\ 0 & 0 & 0 & 0 & a_3 & \mathrm{Im}\, a_4 & -\mathrm{Re}\, a_4 & 0 \\ 0 & 0 & 0 & \mathrm{Re}\, a_4 & \mathrm{Im}\, a_4 & a_3 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{Im}\, a_4 & -\mathrm{Re}\, a_4 & 0 & a_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4}{3}a_1 - \frac{4}{3}a_2 + \frac{2}{3}a_5 + \frac{1}{3}a_6 \end{pmatrix}$$

- All parameters are real in terms of bilinears.

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi'_1(x)^{\mathrm{T}} \\ \varphi'_2(x)^{\mathrm{T}} \\ \varphi'_3(x)^{\mathrm{T}} \end{pmatrix} = U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \varphi_2(x)^{\mathrm{T}} \\ \varphi_3(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$K'_0 = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

with $U^\dagger \lambda_a U = R_{ab}(U)\lambda_b$, $R \in SO(8)$, proper rotations in $\boldsymbol{K}$-space.

- Under a change of basis $K'_0 = K_0$, $\boldsymbol{K}' = R(U)\boldsymbol{K}$ potential remains invariant if

$$\xi'_0 = \xi_0, \quad \eta'_{00} = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

Symmetries

- Symmetry desirable to restrict nHDM.
- Symmetries easily formulated in terms of bilinears.

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to \bar{R}\boldsymbol{K}$, $\bar{R} \in O(8)$ is symmetry of potential iff

$$\boldsymbol{\xi} = \bar{R}\, \boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R}\, \boldsymbol{\eta}, \qquad E = \bar{R}\, E\, \bar{R}^{\mathrm{T}}$$

- $\bar{R} \in O(8)$, keeping kinetic terms invariant.

I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
MM, O. Nachtmann, JHEP 11 151 (2011)
V. Keus, S.F. King, S. Moretti, JHEP 1401 (2014)
B. Grzadkowski, MM, J. Wudka, JHEP 1111 (2011)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

CP symmetry

- CP transformation of the doublet fields

$$\varphi_i(x) \longrightarrow \varphi_i^*(x'), \qquad i = 1, \ldots n, \quad x = (t, \boldsymbol{x})^{\mathrm{T}}, \quad x' = (t, -\boldsymbol{x})^{\mathrm{T}}$$

- In terms of bilinears

$$K_0(x) \longrightarrow K_0(x'), \qquad \boldsymbol{K}(x) \longrightarrow \bar{R}\, \boldsymbol{K}(x')$$

- $\bar{R}$ is defined by the (generalized) Gell-Mann matrices

$$\lambda_a^{\mathrm{T}} = \bar{R}_{ab}\lambda_b, \qquad a, b \in \{1, \ldots, n^2 - 1\}.$$

THDM: $\bar{R} = \mathrm{diag}(1, -1, 1)$,

3HDM: $\bar{R} = \mathrm{diag}(1, -1, 1, 1, -1, 1, -1, 1)$

- CP symmetry conditions

$$\boldsymbol{\xi} = \bar{R}\,\boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R}\,\boldsymbol{\eta}, \qquad E = \bar{R}\,E\,\bar{R}^{\mathrm{T}}.$$

- $O(2) \otimes \mathbb{Z}_2$ model: conditions fulfilled for $\mathrm{Im}\, a_4 = 0$.

- $O(2) \otimes \mathbb{Z}_2$ model is CP symmetric for $\mathrm{Im}\, a_4 = 0$

- CP symmetry respected by vacuum if

$$\bar{R}\langle \boldsymbol{K} \rangle = \langle \boldsymbol{K} \rangle$$

- Basis invariant formulation given in THDM.
  C. Nishi **PRD 74** (2006),
  MM, A. Manteuffel, O. Nachtmann, EPJC57 (2008)
  P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

Stability

- Stability: potential to be bounded from below.
- Formulate stability in terms of the biliners.
- For $K_0 = \sqrt{\frac{2}{3}}(\varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2 + \varphi_3^\dagger\varphi_3) = 0$, $V = 0$.
- For $K_0 > 0$ we define

$$\boldsymbol{k} = \frac{\boldsymbol{K}}{K_0}$$

- $\boldsymbol{k}$ is defined on the domain

$$2 - \boldsymbol{k}^2 \geq 0, \qquad \det(\sqrt{2/3}\mathbb{1}_3 + k_a \lambda_a) = 0.$$

- Potential V reads

$$V = K_0 \underbrace{\left(\xi_0 + \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{k}\right)}_{J_2(\boldsymbol{k})} + {K_0}^2 \underbrace{\left(\eta_{00} + 2\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{k} + \boldsymbol{k}^{\mathrm{T}}E\boldsymbol{k}\right)}_{J_4(\boldsymbol{k})}$$

- Stability determined by behavior of V in the limit $K_0 \to \infty$.

- Stability requires $J_4(\boldsymbol{k}) \geq 0$ and $J_2(\boldsymbol{k}) \geq 0$ where $J_4(\boldsymbol{k}) = 0$ in all possible directions $\boldsymbol{k}$.

- Look for global minimum of $J_4(\boldsymbol{k})$, gradient equations.
MM, O. Nachtmann JHEP 1502 (2015),
MM, O. Nachtmann PRD 92 075017 (2015)

Electroweak symmetry breaking

- EW symmetry breaking given by global minimum

$$\langle \phi \rangle = \left\langle \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \varphi_2^+ & \varphi_2^0 \\ \varphi_3^+ & \varphi_3^0 \end{pmatrix} \right\rangle = \begin{pmatrix} v_1^+ & v_1^0 \\ v_2^+ & v_2^0 \\ v_3^+ & v_3^0 \end{pmatrix}, \quad \underline{K} = \langle \phi \rangle \langle \phi \rangle^\dagger$$

- Fully broken EW symmetry corresponds to $\langle \phi \rangle$ rank 2

$$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) > 0, \qquad \det(\underline{K}) = 0.$$

- $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ corresponds to $\langle \phi \rangle$ rank 1,

$$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) = 0, \qquad \det(\underline{K}) = 0.$$

- Unbroken $SU(2)_L \times U(1)_Y$ corresponds to $\langle \phi \rangle$ rank 0,

$$\underline{K} = 0, \qquad \varphi_i = 0, \qquad V = 0$$

Stationarity

- Suppose the potential V is stable.
- Stationarity equations given by gradient of potential V.
- Different sets w.r.t electroweak symmetry breaking behavior.
- Groebner basis approach, homotopy continuation.
- Deepest solution is global minimum.

Mass matrices

- Suppose stability, breaking $SU(2)_L \times U(1)_Y \ \to U(1)_{em}$
- Unitary gauge

$$\varphi_{1/2}(x) = \begin{pmatrix} H^+_{1/2}(x) \\ \frac{1}{\sqrt{2}} \left(H^0_{1/2}(x) + iA^0_{1/2}(x) \right) \end{pmatrix}, \quad \varphi_3(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h_0(x) \end{pmatrix}$$

- Quadratic part at minimum

$$V_{\text{quad.}} = \frac{1}{2} \left(H^0_1 \, A^0_1 \, H^0_2 \, A^0_2 \, h_0 \right) \mathcal{M}^2_{\text{neutral}} \begin{pmatrix} H^0_1 \\ A^0_1 \\ H^0_2 \\ A^0_2 \\ h_0 \end{pmatrix} + \left(H^-_1 \; H^-_2 \right) \mathcal{M}^2_{\text{charged}} \begin{pmatrix} H^+_1 \\ H^+_2 \end{pmatrix}$$

- Mass matrices $\mathcal{M}^2_{\text{neutral}}$, $\mathcal{M}^2_{\text{charged}}$ in general nHDM found.
- Constraints on parameter space.
- Conditions for alignment of $h_0(x)$ with vev given.

$O(2) \times \mathbb{Z}_2$ model

- Back to the $O(2) \times \mathbb{Z}_2$ model

- Parameters found for stable potential, stationarity, correct E/W symmetry breaking:
  MM, D. Mehta, C. M. Reyes, PRD93 (2015)

$$a_1 = 3,\ a_2 = 1,\ a_3 = -5,\ a_4 = -0.0474,\ a_5 = 1.5,\ a_6 = 2,\ a_7 = 3,$$
$$v_1/v_2 = m_\tau/m_\mu,\ v_3 = v_0/\sqrt{2}$$

Conclusion

- Bilinears are powerful tool in nHDM.
- Basis-transformations, stability, stationarity, EWSB, symmetries easily formulated.
- Note, minimizing with vanishing gradient of potential in general not sufficient.
- Thank you for your attention!