Invariants in Two Higgs Doublet Models

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Scalars 2015, Warsaw

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Gauge Invariance does not constrain the Flavour Structure of the SM

$$\Downarrow$$

Yukawa couplings in the quark sector consist of 2 arbitrary complex matrices

$$Y^u \longrightarrow 3\times 3 \text{ complex} \longrightarrow 18 \text{ par.}$$

$$Y^d \longrightarrow 3\times 3 \text{ complex} \longrightarrow 18 \text{ par.}$$

$$\overline{\qquad\qquad 36 \text{ parameters}}$$

To be compared with number of physical observables

6 Quark masses + 4 V^{CKM} parameters

Yukawa couplings have a large redundancy

Given a set of Yuk. couplings Y_u, Y_d one can make a WB transformation:

$$Q_L \equiv \begin{bmatrix} u \\ d \end{bmatrix}_L \longrightarrow Q_L = W_L Q'_L \, ; \quad \begin{array}{l} u_R \to u_R = W_R^u u'_R \\ d_R \to d_R = W_R^d d'_R \end{array}$$

Under these WB transformations

$$Y_u \to Y'_u = W_L^{\dagger} Y_u W_R^u$$

$$Y_d \to Y'_d = W_L^{\dagger} Y_d W_R^d$$

Physical quantities do not depend on the Weak Basis. The redundancy of Yukawa Couplings is a major obstacle in the search for an eventual Family Symmetry responsible for the observed pattern of fermion masses and mixing.

In what weak-basis would the pattern become "apparent"?

Texture zeros are WB dependent.

Some of them may have no physical meaning:

Example: It has been shown the following result (G.C.B, L.Lavoura, F.Mota)

Starting arbitrary quark mass matrices M_u, M_d, one can always make WB transformations such that:

$$M_u = \begin{bmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{bmatrix} \quad ; \quad M_d = \begin{bmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{bmatrix}$$

NNI basis

The "Fritzsch-type" texture zeros by themselves have no physical implications. They have physical implications only if one assumes these texture zeros and also Hermiticity.*

In this case it predicts:

$$V_{us} = \sqrt{\frac{m_d}{m_s}} - e^{i\varphi}\sqrt{\frac{m_u}{m_c}} \rightarrow \text{good}$$

$$V_{cb} = \sqrt{\frac{m_s}{m_b}} - e^{i\sigma}\sqrt{\frac{m_c}{m_t}} \rightarrow \text{ruled out}$$

In the SM, M_u, M_d are not Hermitian, in general.

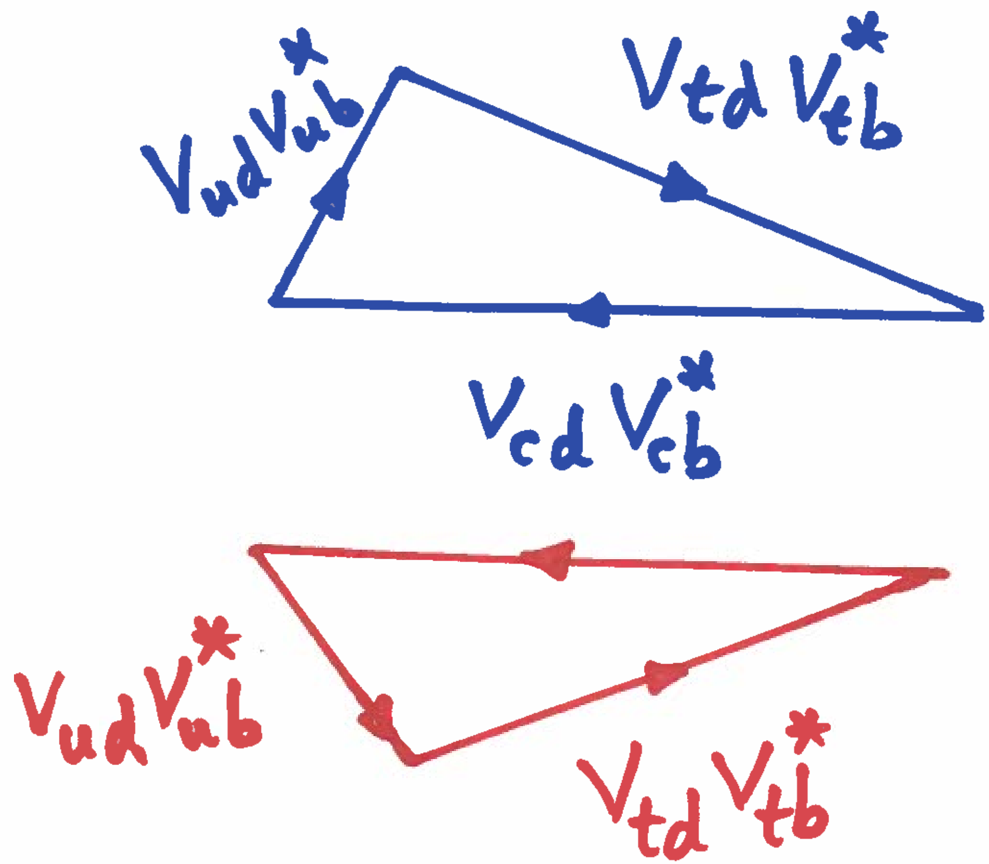
Question : Can one construct WB-invariants which fix the four parameters of V^{CKM}?

Answer – It has been shown (GCB, L. Lavoura 1988) that the 4-independent WB invariants:

$$\mathrm{tr}(H_u H_d)\,;\ \mathrm{tr}(H_u^2 H_d)\,;\ \mathrm{tr}(H_u H_d^2)\,;\ \mathrm{tr}(H_u^2 H_d^2)$$

completely fix 4-independent moduli of V^{CKM}. Convenient choice : $|V_{us}|, |V_{cb}|, |V_{ub}|$, $|V_{td}|$

The moduli of V^{CKM} completely fix V^{CKM} with a 2-fold ambiguity : The sign of CP violation



The two triangles have a different sign of $\mathrm{Im}\, Q$. $Q = \left(V_{us} V_{cb} V_{cs}^* V_{ub}^*\right)$

The sign of $\mathrm{Im}\left(V_{us} V_{cb} V_{cs}^{*} V_{ub}^{*}\right)$ can be fixed by :

$$\mathrm{tr}\left[H_u, H_d\right]^3$$

CP-odd
WB invariant

Question : Can one derive, from first principles, a WB invariant which controls CP violation in the SM?

Answer : Yes ! J. Bernabeu, GCB, M. Gronau

- Given a Lagrangian $\mathcal{L}$, separate it in two parts:

$$\mathcal{L} = \mathcal{L}^{CP} + \mathcal{L}^{\text{remaining}}$$

$\mathcal{L}^{CP}$ is the part of the Lagrangian which we know that conserves CP, like the gauge interactions. (M.N. Rebelo, W. Grimus)

- Look for restrictions on $\mathcal{L}^{\text{remaining}}$, from CP invariance.

- Construct the most general CP transformation which leaves $\mathcal{L}^{CP}$ invariant.

In the SM, this transformation is:

$$u_L \to W_L C \bar{u}_L^T \qquad u_R \to W_R^u C \bar{u}_R^T$$

$$d_L \to W_L C \bar{d}_L^T \qquad d_R \to W_R^d C \bar{d}_R^T$$

W_L, W_R^u, W_R^d are unitary matrices acting in family space. Recall that gauge interactions do not distinguish the various families.

CP invariance constrains the Yukawa matrices to satisfy the relations

$$W_L^\dagger Y_d W_R^d = Y_d^* \; ; \; W_L^\dagger Y_u W_R^u = Y_u^* ,$$

which imply:

$$W_L^\dagger \left(Y_{\substack{d\\u}} Y_{\substack{d\\u}}^\dagger\right) W_L = \left(Y_{\substack{d\\u}} Y_{\substack{d\\u}}^\dagger\right)^* = \left(Y_{\substack{d\\u}} Y_{\substack{d\\u}}^\dagger\right)^T$$

$$W_L^\dagger \left[\left(Y_d Y_d^\dagger\right), \left(Y_u Y_u^\dagger\right)\right] W_L = -\left[Y_d Y_d^\dagger, Y_u Y_u^\dagger\right]^T$$

$$\Rightarrow \quad \mathrm{tr}\left(\left[Y_d Y_d^\dagger, Y_u Y_u^\dagger\right]^3\right) = 0$$

For 3 or more generations, this implies CP violation

The minimal non-trivial case corresponds to $r = 3$.

For two generations the invariant automatically vanishes

For 3 generations one obtains :

$$\mathrm{tr}\,[H_u, H_d]^3 = 6i\,(m_t^2 - m_c^2)(m_t^2 - m_u^2) \times (m_c^2 - m_u^2) \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\ \mathrm{Im}\,Q$$

$\mathrm{tr}\,[H_u, H_d]^3$ is a necessary condition for CP invariance for any number of generations. For $n_g = 3$, it is a necessary and sufficient condition for CP invariance. For $n_g = 3$ one has also :

$$\mathrm{tr}\,[H_u, H_d]^3 = 3 \det\,[H_u, H_d]$$ Jarlskog

Invariants and BAU

Generation of the Baryon-Asymmetry of the Universe (BAU)

The ingredients to dynamically generate BAU from an initial state with zero BA, were formulated by Sakharov a longtime ago (1967)

(i) Baryon number Violation

(ii) C and CP Violation

(iii) Departure from thermal equilibrium

All these ingredients exist in the SM but it has been established that in the SM one cannot generate the observed BAU :

$$\boxed{\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}}$$

n_B , $n_{\bar{B}}$, n_γ number density of baryons anti-baryons and photons at present time

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small:

$$\frac{\mathrm{tr}\,[H_u, H_d]^3}{T_{ew}^{12}} \approx 10^{-20}$$

(ii) Successful baryogenesis needs a strongly first order phase transition which would require a light Higgs mass:

$$m_H \lesssim 70\,\mathrm{GeV}$$

Two Higgs Doublet Models (THDM) (No extra symmetries)

Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q_L^0}\ \Gamma_1 \Phi_1 d_R^0 - \overline{Q_L^0}\ \Gamma_2 \Phi_2 d_R^0 - \overline{Q_L^0}\ \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\ \Delta_2 \tilde{\Phi}_2 u_R^0 + \text{h.c.}$$

$$\tilde{\Phi}_i = -i\tau_2 \Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}(v_1 \Gamma_1 + v_2 e^{i\theta} \Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1 \Delta_1 + v_2 e^{-i\theta} \Delta_2),$$

Diagonalised by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag}\ (m_d, m_s, m_b)\,,$$
$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag}\ (m_u, m_c, m_t)\,.$$

Expansion around the vev's

$$\Phi_j = \begin{bmatrix} \phi_j^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}\left(v_j + \rho_j + i\eta_j\right) \end{bmatrix} \qquad j = 1,2$$

Let us perform the following transformation:

$$\begin{bmatrix} H^0 \\ R \end{bmatrix} = U \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} \; ; \; \begin{bmatrix} G^0 \\ I \end{bmatrix} = U \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \; ; \; \begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = U \begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix}$$

$$U = \frac{1}{v} \begin{bmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{bmatrix} ;$$

$H^0 \rightarrow$ coupling to quarks proportional to mass matrices

Physical neutral scalars are combinations of H^0, R and I.

Neutral and charged Higgs Interactions for the quark sector

$$\begin{aligned}\mathcal{L}_Y(\text{quark, Higgs}) = & -\overline{d_L^0}\frac{1}{v}\left[M_d H^0 + N_d^0 R + i N_d^0 I\right] d_R^0 \\ & -\overline{u_L^0}\frac{1}{v}\left[M_u H^0 + N_u^0 R + i N_u^0 I\right] u_R^0 \\ & -\frac{\sqrt{2}H^+}{v}\left(\overline{u_L^0} N_d^0\, d_R^0 - \overline{u_R^0} N_u^{0\dagger}\, d_L^0\right) + \text{h.c.}\end{aligned}$$

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\theta}\Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1 e^{-i\theta}\Delta_2).$$

Flavour structure of quark sector of 2HDM characterised by:

four matrices $M_d,\ M_u,\ N_d^0,\ N_u^0.$

Likewise for Leptonic sector, Dirac neutrinos:

$$M_\ell,\ M_\nu,\ N_\ell^0,\ N_\nu^0.$$

Yukawa Couplings in terms of quark mass eigenstates

for H^+, H^0, R, I

$$\mathcal{L}_Y(\text{quark, Higgs}) =$$

$$-\frac{\sqrt{2}H^+}{v}\bar{u}\left(V N_d \gamma_R - N_u^\dagger\, V \gamma_L\right) d + \text{h.c.} - \frac{H^0}{v}\left(\bar{u} D_u u + \bar{d} D_d\, d\right) -$$

$$-\frac{R}{v}\left[\bar{u}(N_u \gamma_R + N_u^\dagger \gamma_L) u + \bar{d}(N_d \gamma_R + N_d^\dagger \gamma_L)\, d\right] +$$

$$+ i\frac{I}{v}\left[\bar{u}(N_u \gamma_R - N_u^\dagger \gamma_L) u - \bar{d}(N_d \gamma_R - N_d^\dagger \gamma_L)\, d\right]$$

$$\gamma_L = (1 - \gamma_5)/2 \qquad \gamma_R = (1 + \gamma_5)/2$$

$$V = V_{CKM}$$

- If $H^\pm$, H^0, R, I are discovered one will be able to check whether they arise from a THDM. There are more physical observables than free parameters!

- In order to construct WB invariants in the framework of THDM one has to recall how M_d, M_u, N_d^0, N_u^0 transform under a WB transformation:

$$\begin{cases} d_L^0 = W_L \, d_L^{0\prime} \; ; & d_R^0 = W_R^d \, d_R^{0\prime} \\ u_L^0 = W_L \, u_L^{0\prime} \; ; & u_R^0 = W_R^u \, u_R^{0\prime} \end{cases}$$

Under these WB transformations:

$$M_d \longrightarrow M_d' = W_L^\dagger M_d W_R^d \ ; \quad M_u \rightarrow M_u' = W_L^\dagger M_u W_R^u$$

$$N_d^0 \rightarrow N_d^{0\prime} = W_L^\dagger N_d^0 W_R^d \ ; \quad N_u^0 \rightarrow N_u^{0\prime} = W_L^\dagger N_u^0 W_R^u$$

The simplest WB invariants

$$\mathrm{tr}\left(M_d N_d^{0\dagger}\right) \ ; \quad \mathrm{tr}\left(M_u N_u^{0\dagger}\right)$$

In the basis where M_d is diagonal, real:

$$\mathrm{tr}\left(M_d N_d^{0\dagger}\right) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33}$$

Similarly for up quarks. Not sensitive for FCNC but important to <u>edm</u>!

Example of invariant sensitive to FCNC:

$$I_2 \equiv \mathrm{tr}\left[M_d N_d^{0\dagger}, M_d M_d^{\dagger}\right] = -2 m_d m_s (m_s^2 - m_d^2) \cdot (N_d^*)_{12} (N_d^*)_{21} - 2 m_d m_b (m_b^2 - m_d^2)^2 (N_d^*)_{13} (N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2)^2 (N_d^*)_{23} (N_d^*)_{32}$$

Very useful to estimate the size of FCNC in a specific model.

A convenient Parametrization of N_u, N_d

Without loss of generality, one can write:

$$N_d^0 = K_L \hat{V}_L^{N_d} D^{N_d} \bar{K} (\hat{V}_R^{N_d})^\dagger K_R^\dagger \quad , \text{ where}$$

$$K_{L,R} \equiv \text{diag}\left[1, \exp i\varphi_{1L,R}, \exp(i\varphi_{2L,R})\right]$$

$\hat{V}^{N_d}_{L,R} \rightarrow$ unitary matrices with one physical non-factorizable phase, each

$$\bar{K} = \text{diag.}\left[\exp(i\sigma_1), \exp(i\sigma_2), \exp(i\sigma_3)\right]$$

$D^{N_d} \rightarrow$ real diagonal matrix

Similarly for N_u^0.

Counting Parameters :

phases : $2(K_L) + 2(K_R) + 1(\hat{V}_L^{Nd}) + 1(\hat{V}_R^{Nd}) + 3(\bar{K}) = 9$

real parameters : $3(\hat{V}_L^{Nd}) + 3(\hat{V}_R^{Nd}) + 3(D^{Nd}) = 9$

New CP-odd invariants:

$$I_2^{CP} \equiv \mathrm{tr}\left[H_u, H_{N_d^0}\right]^3 = 6i\,\Delta_u\,\Delta_{N_d}\,\mathrm{Im}\,Q_2$$

$Q_2 \to$ rephasing invariant quartet of $V_2 = U_L^{u\dagger} U_{N_d^0 L}$

$$\Delta_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$\Delta_{N_d} \to$ analogous for N_d

V_2 measures the misalignment between H_u and $(N_d^0)(N_d^{0\dagger})$ in flavour space

One can construct many other CP odd invariants. For example one may have CP odd invariants sensitive to **right-handed mixing**:

$$I_7^{CP} \equiv \operatorname{tr}\left[H'_d, H'_{N_d^0}\right]^3 = 6i\,\Delta_d\,\Delta_{N_d}\,\operatorname{Im} Q_7$$

$$H'_d = M_d^\dagger M_d \qquad H'_{N_d^0} = \left(N_d^{0\dagger} N_d^0\right)$$

$Q_7 \rightarrow$ rephasing invariant quartet of $\left(U_{dR}\, U^\dagger_{N^0_{dR}}\right)$

- N_d^0, N_u^0 have a large number of parameters
- In general THDM lead to dangerous FCNC
- It is possible to have models where N_d^0, N_u^0 are only functions of V_{CKM}!

BGL models (GCB, L. Lavoura, W. Grimus)

See next talks by Gui Rebelo and Francisco Botella

Conclusions

- Invariants are very useful in the study of flavour and CP Violation in THDM.
- Let us hope that nature chooses THDM !!
- It would be very exciting both for experimentalists and theoreticians!!