Higgs potential, future colliders, and future GW interferometers

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M. Kakizaki, S. K., T. Matsui, arXiv: 1509.08394 Phy. Rev. D to appear.

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This talk

- **I would like to discuss fingerprinting Higgs models at future precision experiments**
- **At future colliders such as ILC, precision measurements of Higgs boson couplings will be performed**
- **We can fingerprint models if deviations are detected with a pattern**
- **The Higgs self-coupling can also be measured with 10% accuracy**

This talk

- I would like to discuss fingerprinting Higgs models at future precision experiments
- At future colliders such as ILC, precision measurements of Higgs boson couplings will be performed
- We can fingerprint models if deviations are detected with a pattern
- The Higgs self-coupling can also be measured with 10% accuracy

- But what if ILC is not approved??

ILC vs LISA/DECIGO

Can future gravitational interferometers work as a replacement of ILC ?

Introduction

Discovery of *h*(125) at LHC in 2012

- Existence of a scalar particle,
- Mass and measured couplings are consistent with the SM

Higgs sector remains unknown

- SM Higgs sector does not have a strong motivation/problematic …
- Most of extended Higgs sectors can also satisfy current data as well

Requirement of BSM

- Hierarchy Problem *SUSY, Dynamical Symmetry Breaking, Shift-Symmetry, …*
- BSM Phenomena *Baryon Asymmetry, Neutrino Masses, Dark Matter, …*

h(125) : a probe of the structure of the EWSB sector

- Shape of Higgs sector (multiplet structure, symmetry, scales, …) is related to BSM scenarios
- Essence of the Higgs particle is directly connected to a BSM paradigm

Extended Higgs models

Multiplet Structure (2nd simplest Higgs models)

Φ_{SM}+Singlet, Φ_{SM}+Doublet (2HDM),

Φ_{SM}+Triplet, …

Additional Symmetry

Discrete or Continuous?

Exact or Softly broken?

Interaction

Weakly coupled or Strongly Coupled ?

Decoupling or Non-decoupling?

Note: 2nd simplest Higgs models (HSM, 2HDMs, …) can be effective theories of more complicated Higgs sectors

How we test the Higgs sector

Direct searches of the 2nd Higgs boson

Clear evidence of non-minimal Higgs sectors

Indirect searches

- Mass generation mechanisms (Higgs mechanism, Yukawa interaction) has been confirmed
- By detailed measurements of *hVV* and *hff*, we can indirectly test extended Higgs sectors.

2HDM with softly broken Z_2

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)}$$
$$+\frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2$$
$$+\lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + (\text{h.c.})\right]$$

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

Φ_1 and Φ_2 $\Rightarrow$ $\underline{h, \quad H,}$ $\quad A^0$, $H^\pm$ $\oplus$ Goldstone bosons

↑ ↑ ↑charged

CPeven CPodd

Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix}$$

$$\begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix}$$

$$\begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan\beta$$

$$m_h^2 = v^2 \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2} \sin^2 2\beta\right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2\beta \cos^2\beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

M_{soft}: soft breaking scale

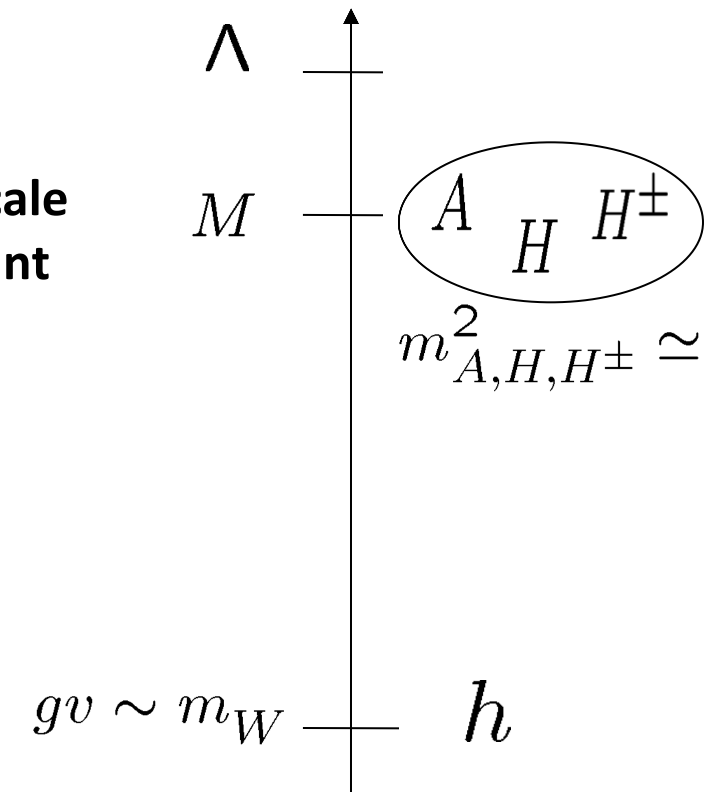
$M_{\text{soft}} \quad \left(= \frac{m_3}{\sqrt{\cos\beta \sin\beta}}\right)$:

soft-breaking scale
of the discrete symm.

Decoupling/Non-decoupling

Λ: Cutoff

***M*: Mass scale irrelevant to VEV**

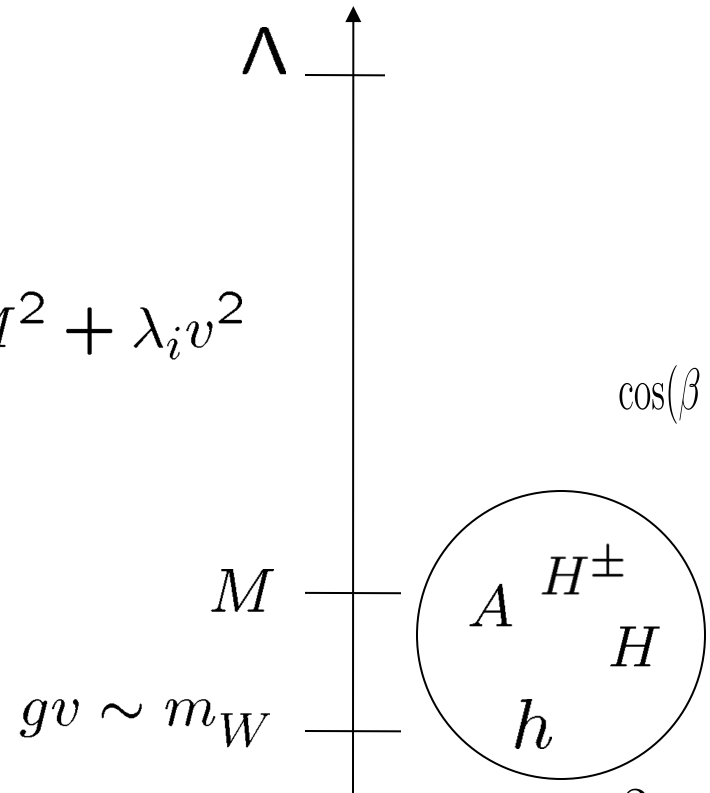


$$m^2_{A,H,H^{\pm}} \simeq M^2 + \lambda_i v^2$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{v^2}{M^2}\mathcal{O}^{(6)}$$

Effective Theory is the SM

$\cos(\beta - \alpha) \sim 0$



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{nonSM}} + \frac{v^2}{\Lambda^2}\mathcal{O}^{(6)}$$

Effective Theory is an extended Higgs sector

FCNC Suppression

In multi-doublet model, FCNC appears at tree via Higgs mediation

2 Higgs doublet model with a (softly broken) symmetry:

to avoid FCNC, give different charges to Φ_1 and Φ_2

ex) Discrete sym. $\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 = -\Phi_2$

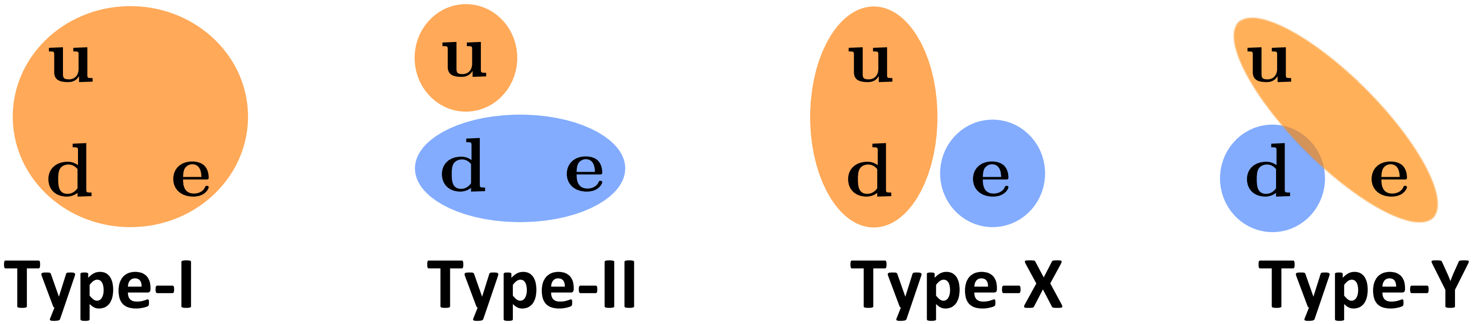
Each quark or lepton couples only one Higgs doublet

No FCNC at tree level

Barger, Hewett, Phillips

Classified by Z_2 charge assignment

Four Types of Yukawa coupling



Neutrino Philic etc

SUSY etc

Radiative seesaw etc

Fingerprinting the 2HDM

$$\kappa_V \equiv \frac{g_{hVV(2HDM)}}{g_{hVV(SM)}} = \sin(\beta - \alpha)$$

$x = \cos(\beta - \alpha)$ **SM-like case: $|x| \ll 1$**

$$\kappa_V = 1 - (1/2)\, x^2 + \ldots$$

When a Fermion couples to Φ_2

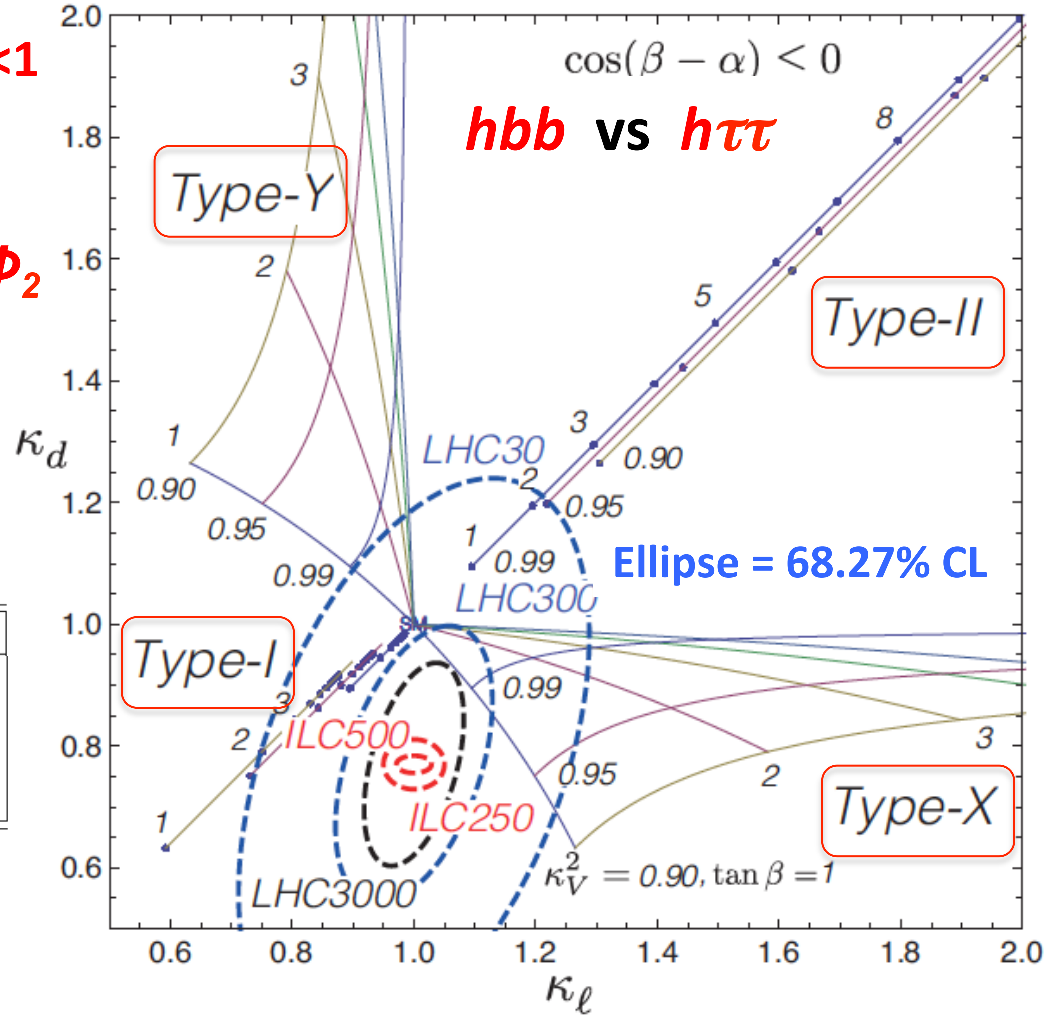
$$\kappa_f = 1 + \cot\beta\, x + \ldots$$

and if it couples to Φ_1

$$\kappa_f = 1 - \tan\beta\, x + \ldots$$

Model	μ	τ	b	c	t	g_V
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

SK, K. Tsumura, K. Yagyu, H. Yokoya 2014
ILC Higgs White Paper 2013



radiative corrections
=Mariko Kikuchi

Deviation in *hff*

If $\Delta\kappa_V = 1\ \%$

Singlet, Exotics,

$\Delta\kappa_u = -(1/2)\, x^2$, $\Delta\kappa_d = -(1/2)\, x^2$, $\Delta\kappa_\tau = -(1/2)\, x^2$ O(1) %

Type I 2HDM

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = +\cot\beta\, x$, $\Delta\kappa_\tau = +\cot\beta\, x$ O(10) %

Type X (Lepton Specific) 2HDM

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = +\cot\beta\, x$, $\Delta\kappa_\tau = -\tan\beta\, x$ O(10) %

MSSM (Type II 2HDM)

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = -\tan\beta\, x$, $\Delta\kappa_\tau = -\tan\beta\, x$ O(10) %

MCHM4

$\Delta\kappa_u = -(1/2)\, x^2$, $\Delta\kappa_d = -(1/2)\, x^2$, $\Delta\kappa_\tau = -(1/2)\, x^2$ O(1) %

MCHM5

$\Delta\kappa_u = -(3/2)\, x^2$, $\Delta\kappa_d = -(3/2)\, x^2$, $\Delta\kappa_\tau = -(3/2)\, x^2$ O(1) %

Nature of EWSB

- **By detailed measurement of *hVV* and *hff* couplings at future collider experiments, we can obtain information of extended Higgs sectors or even new physics models**

- **However, in order to understand the nature of EWSB, we need to directly measure the Higgs potential**

Higgs potential

To understand the essence of EWSB, we must know the self-coupling in addition to the mass independently

$$\boxed{V_{\rm Higgs} = \frac{1}{2}\underline{m_h^2}h^2 + \frac{1}{3!}\underline{\lambda_{hhh}}h^3 + \frac{1}{4!}\lambda_{hhhh}h^4 + \cdots}$$

Higgs potential

To understand the essence of EWSB, we must know the self-coupling in addition to the mass independently

$$\boxed{V_{\text{Higgs}} = \frac{1}{2}\underline{m_h^2} h^2 + \frac{1}{3!}\underline{\lambda_{hhh}} h^3 + \frac{1}{4!}\lambda_{hhhh} h^4 + \cdots}$$

Effective potential $V_{\text{eff}}(\varphi) = -\frac{\mu_0^2}{2}\varphi^2 + \frac{\lambda_0}{4}\varphi^4 + \sum_f \frac{(-1)^{2s_f} N_{C_f} N_{S_f}}{64\pi^2} m_f(\varphi)^4 \left[\ln \frac{m_f(\varphi)^2}{Q^2} - \frac{3}{2}\right]$

Renormalization Conditions

$$\left.\frac{\partial V_{\text{eff}}}{\partial \varphi}\right|_{\varphi=v} = 0, \quad \left.\frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2}\right|_{\varphi=v} = m_h^2, \quad \left.\frac{\partial^3 V_{\text{eff}}}{\partial \varphi^3}\right|_{\varphi=v} = \lambda_{hhh}$$

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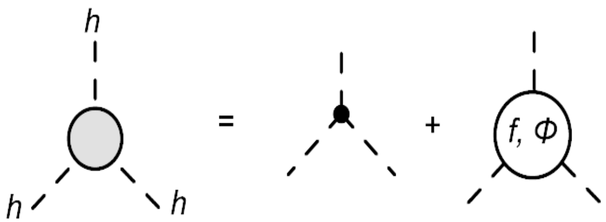
SM Case

$$\boxed{\lambda_{hhh}^{\mathrm{SMloop}} \sim \frac{3m_h^2}{v}\left(1 - \frac{N_c m_t^4}{3\pi^2 v^2 m_h^2} + \cdots\right)}$$

Non-decoupling effect

Case of Non-SUSY 2HDM

- **Consider when the lightest *h* is SM-like [sin(β−α)=1]**
- **At tree, the *hhh* coupling takes the same form as in the SM**
- **At 1-loop, non-decoupling effect $m_\Phi{}^4$ (If *M* < *v*)**



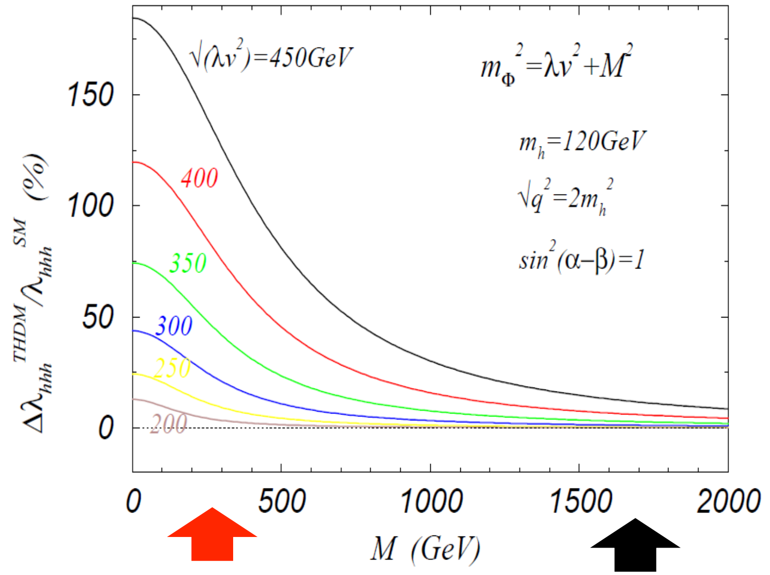
Φ = *H, A, H*$^\pm$

SK, Kiyoura, Okada, Senaha, Yuan, PLB558 (2003)

$$\lambda_{hhh}^{\rm 2HDM} \simeq \frac{3m_h^2}{v}\left[1+\underbrace{\frac{m_\Phi^4}{12\pi^2 m_h^2}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{Extra scalar loop}}-\underbrace{\frac{m_t^4}{\pi^2 v^2 m_h^2}}_{\text{Top loop}}\right]$$

$$m_\Phi^2 = M^2 + \lambda_i v^2$$

(Φ = *H, A, H*$^\pm$)



Non-decoupling effect

Decoupling

Correction can be large ～ 100%

V(ϕ) and new physics

In several BSM models, Higgs potential is drastically changed from the SM.

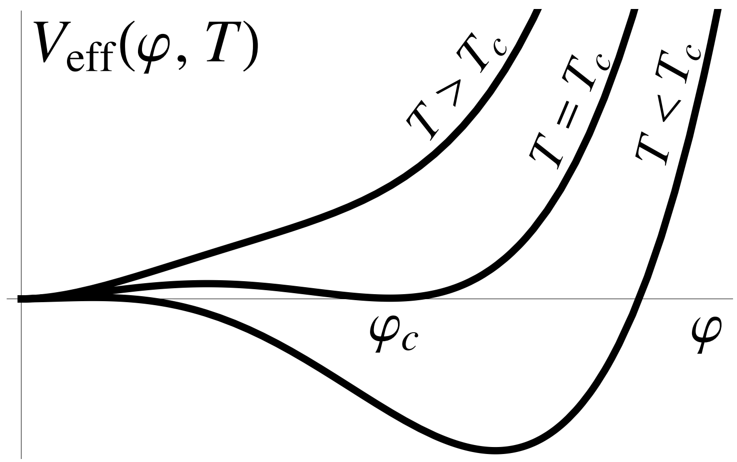
- **Extended Higgs models**
- **Classically scale invariant models**
- **Composite Higgs models**
- **Models with strong dynamics for EWSB**
- **Electroweak Baryogenesis (1^{st} OPT, CPV)**
- **Higgs Inflation**
- **…**

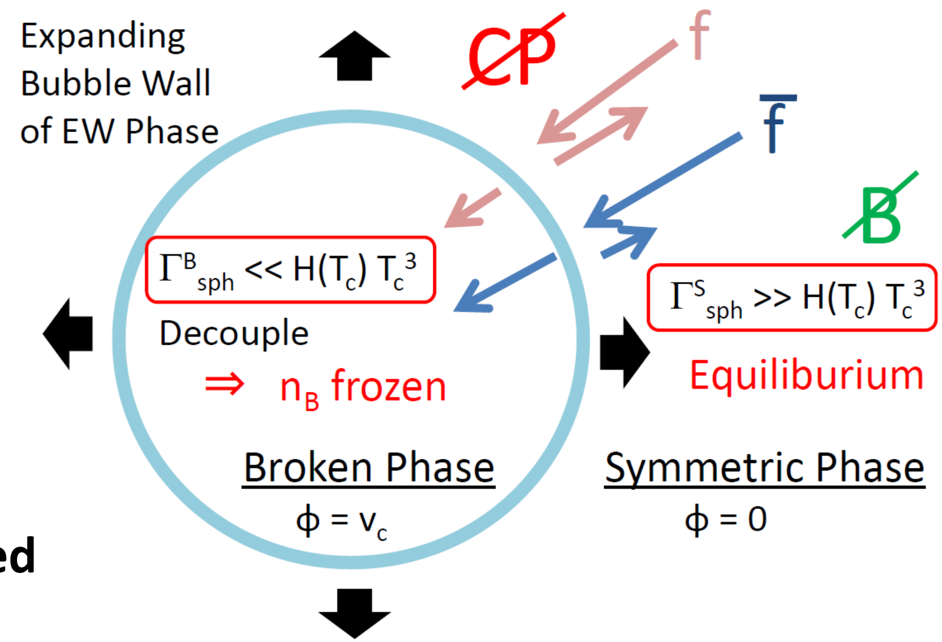
Electroweak Baryogenesis

$$\Gamma \sim e^{-E_{\rm sph}/T}\ (T < T_c)$$
$$\Gamma \sim \kappa(\alpha_W T)^4\ (T_c < T)$$

Sakharov's conditions:

- **B Violation** → **Sphaleron transition at high *T***
- **C and CP Violation** → **CP Phases in extended scalar sector**
- **Departure from Equilibrium** → **1st Order EW Phase Transition**





Quick sphaleron decoupling is required to retain sufficient baryon number in Broken Phase

(Sphaleron Rate) < (Expansion Rate) ➡ $\boldsymbol{\varphi_c / T_c > 1}$

Strongly 1st OPT

High Temperature Expansion (just for sketch)

$$V_{\rm eff}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \cdots$$

Condition of Strongly 1st OPT

$$\boxed{\frac{\varphi_C}{T_C} \simeq \frac{2E}{\lambda_{T_C}} > 1}$$

However, the SM cannot realize the strongly 1st OPT

$$E \simeq \frac{1}{12\pi v^3}\left(6m_W^3 + 3m_Z^3 + \cdots\right) \quad \lambda_{T_C} \sim \frac{m_h^2}{2v^2} + \cdots$$

$$\boxed{\frac{\varphi_C}{T_C} \simeq \frac{6m_W^3 + 3m_Z^3 + \cdots}{3\pi v m_h^2} \ll 1}$$

For m_h = 125 GeV

We need a mechanism to enlarge *E* to realize strongly 1st OPT

1^{st} OPT in extended Higgs sectors

High Temperature Expansion (just for sketch)

$$V_{\rm eff}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \cdots$$

Condition of Strongly 1^{st} OPT

$$\boxed{\frac{\varphi_C}{T_C} \simeq \frac{2E}{\lambda_{T_C}} > 1}$$

The condition can be satisfied by thermal loop effects of additional scalar bosons Φ (Φ = *H, A, H⁺*, …) $m_\Phi^2 \simeq M^2 + \lambda_i v^2$

$$\frac{\varphi_C}{T_C} \simeq \frac{1}{3\pi v m_h^2}\left\{6m_W^3 + 3m_Z^3 + \underline{\sum_\Phi m_\Phi^3 \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \left(1 + \frac{3M^2}{2m_\Phi^2}\right)}\right\} > 1$$

1st OPT in extended Higgs sectors

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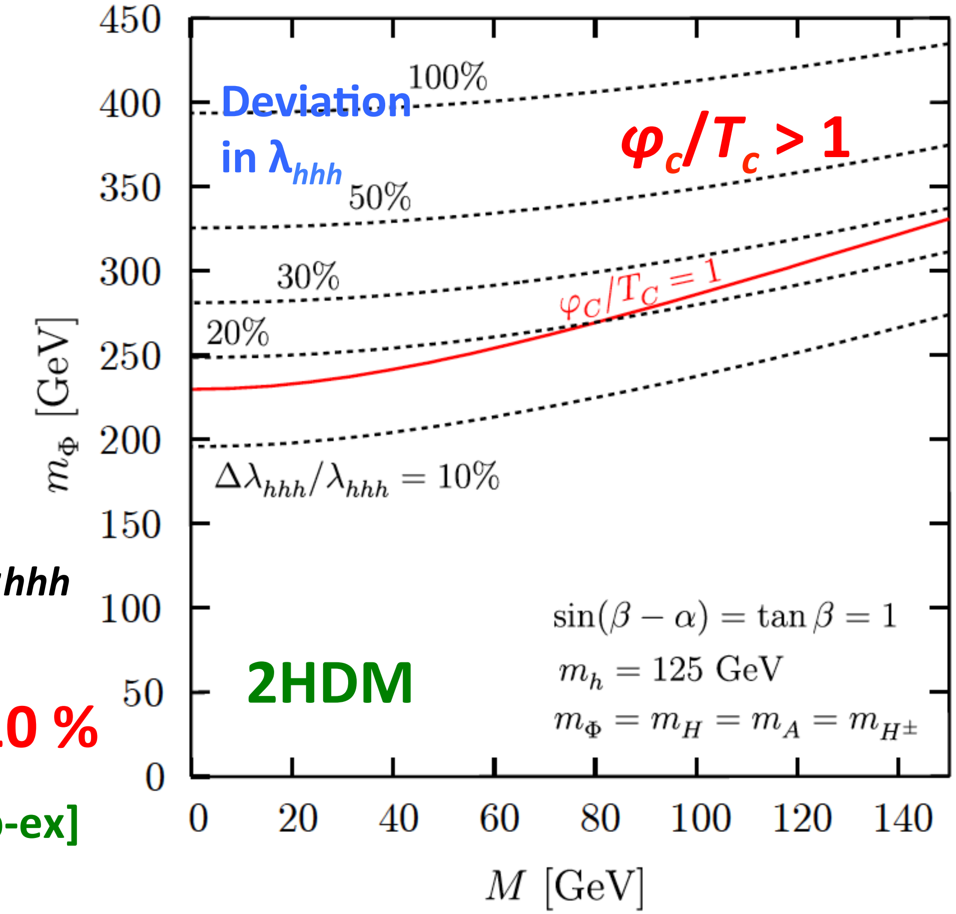
In this case, large quantum effects also appear in the *hhh* coupling

$$\lambda_{hhh} \simeq \frac{3m_h^2}{v^2}\left\{1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + \underline{\sum_\Phi \frac{m_\Phi^4}{12\pi^2 v^2 m_h^2}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}\right\} > \lambda_{hhh}^{SM}$$

Strong 1st OPT and the *hhh* coupling

SK, Y Okada, E Senaha (2005)

Strongly 1st OPT
⇔ Non-decoupling effect
⇔ large deviation in *hhh*



At LHC, challenging to measure λ_{hhh}

ILC (1 TeV) can measure λ_{hhh} by 10 %

K.Fujii et al., arXiv:1506.05992 [hep-ex]

Electroweak Baryogenesis can be tested at ILC!

GW : another probe of 1^{st} OPT?

Gravitational Wave Experiments

aLIGO (USA), KAGRA (JPN), aVIRGO (ITA), …

- **Trial for first discovery of GWs (Underway)**
- **GWs from astronomical phenomena (binary of neutron stars, …)**

Once, GW is found, era of GW astronomy will come ture

Future exp: eLISA [EUR], DESIGO [JPN], BBO [USA]…

- **GWs from very early Universe (Inflation, 1^{st} OPT, …)**

GWs may be used for exploration of the Higgs potential, as complementary mean with collider experiments.

Previous studies of relic abundance of GWs from 1st OPT

1. Model Independent Analyses [1]

2.. Higher Oder Operators [2]

3. Non-decoupling effects of sparticles ...

   Stop search results tell that strong 1st OPT cannot be realized in MSSM [3]

4. Non-thermal effect at the tree level (NMSSM [3], real singlet model [4])

[1] C. Grojean and G. Servant, PRD75, 043507 (2007);

K. Kohri et al., arXiv:1405.4166.

[2] C. Delaunay et al., JHEP0804, 029 (2008).

[3] R. Apreda et al., NPB631, 342 (2002).

[4] A. Ashoorioon and T. Konstandin, JCAP0809, 022 (2008).

Espinosa, et al (2010), No (2011), ...

GW from the EW bubble

Evaluation according to Grojean and Servant

$$\Omega_{\mathrm{GW}}(f)h^2 = \Omega_{\mathrm{coll}}(f)h^2 + \Omega_{\mathrm{turb}}(f)h^2$$

1 Collision of the bubbles

Kamionkowski, et al. (1994)

GW at the peak

$$\tilde{\Omega}_{\mathrm{coll}}h^2 \simeq c\kappa^2 \left(\frac{H_t}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{v_b^3}{0.24+v_b^3}\right)$$

$$c = 1.1 \times 10^{-6}$$

Frequency at the peak

$$\tilde{f}_{\mathrm{coll}} \simeq 5.2 \times 10^{-3}\mathrm{mHz} \left(\frac{\beta}{H_t}\right) \left(\frac{T_t}{100\mathrm{GeV}}\right)$$

2 Plasma Turbulence in the bubbles

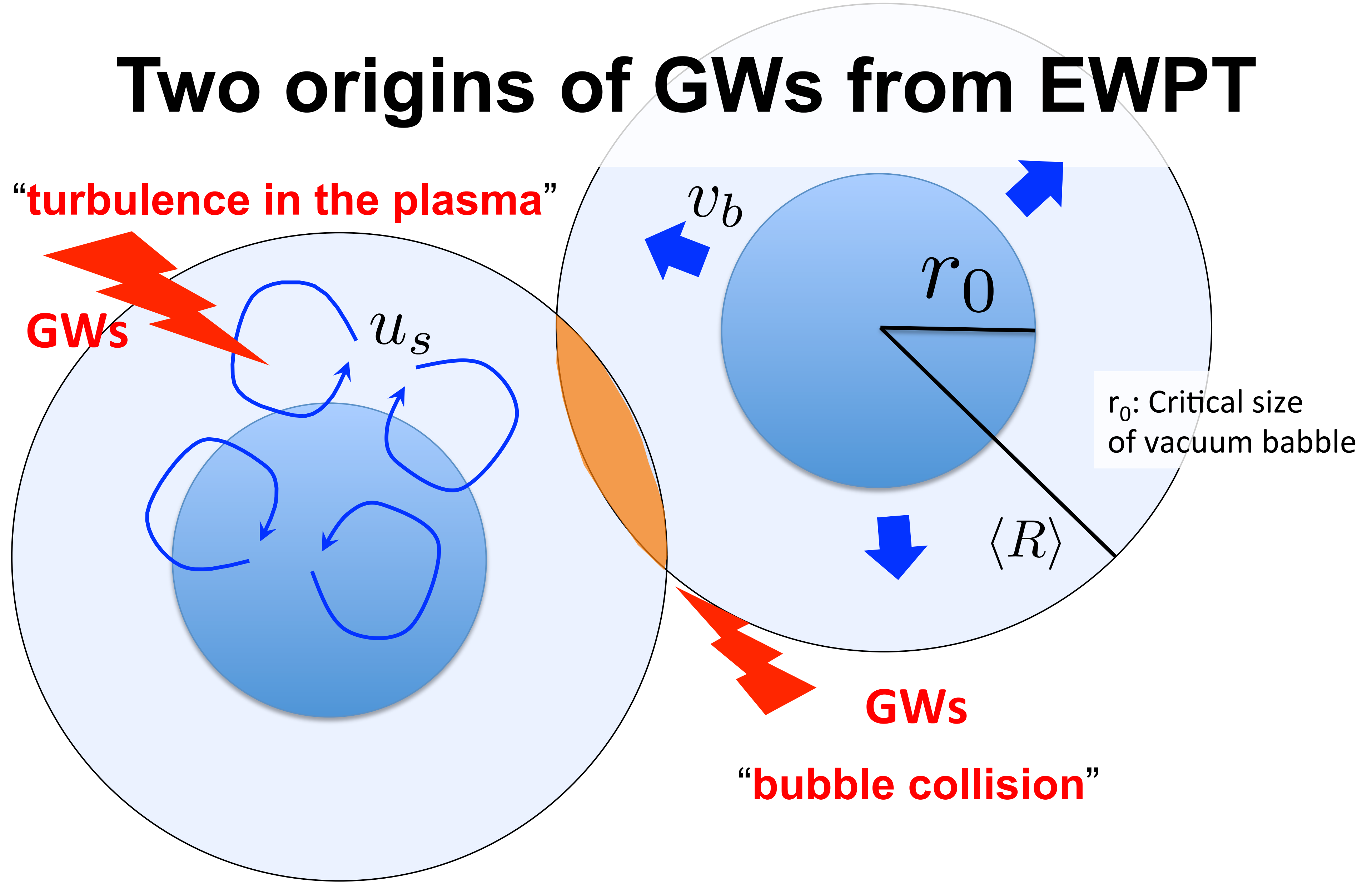
Nicolis (2004)

$$\tilde{\Omega}_{\mathrm{turb}}h^2 \simeq 1.4 \times 10^{-4} u_s^5 v_b^2 \left(\frac{H_t}{\beta}\right)^2$$

$$\tilde{f}_{\mathrm{turb}} \simeq 3.4 \times 10^{-3}\mathrm{mHz}\frac{u_s}{v_b} \left(\frac{\beta}{H_t}\right) \left(\frac{T_t}{100\mathrm{GeV}}\right)$$

The spectrum are evaluated by inputting the lattent heat α, variation of the bubble nuclearation rate β and transition temperature T_t

Two origins of GWs from EWPT



Higgs model with O(*N*) singlet fields

***N*-scalar singlets** $S^{\mathrm{T}} = (S_1, \cdots, S_N)$

$$V_0 = -\mu^2|\Phi|^2 + \frac{{\mu_S}^2}{2}|S|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{\lambda_S}{4}|S|^4 + \frac{c}{2}|\Phi|^2|S|^2$$

Mass of scalar fields: $\boxed{m_S^2 = \mu_S^2 + \frac{c}{2}v^2}$

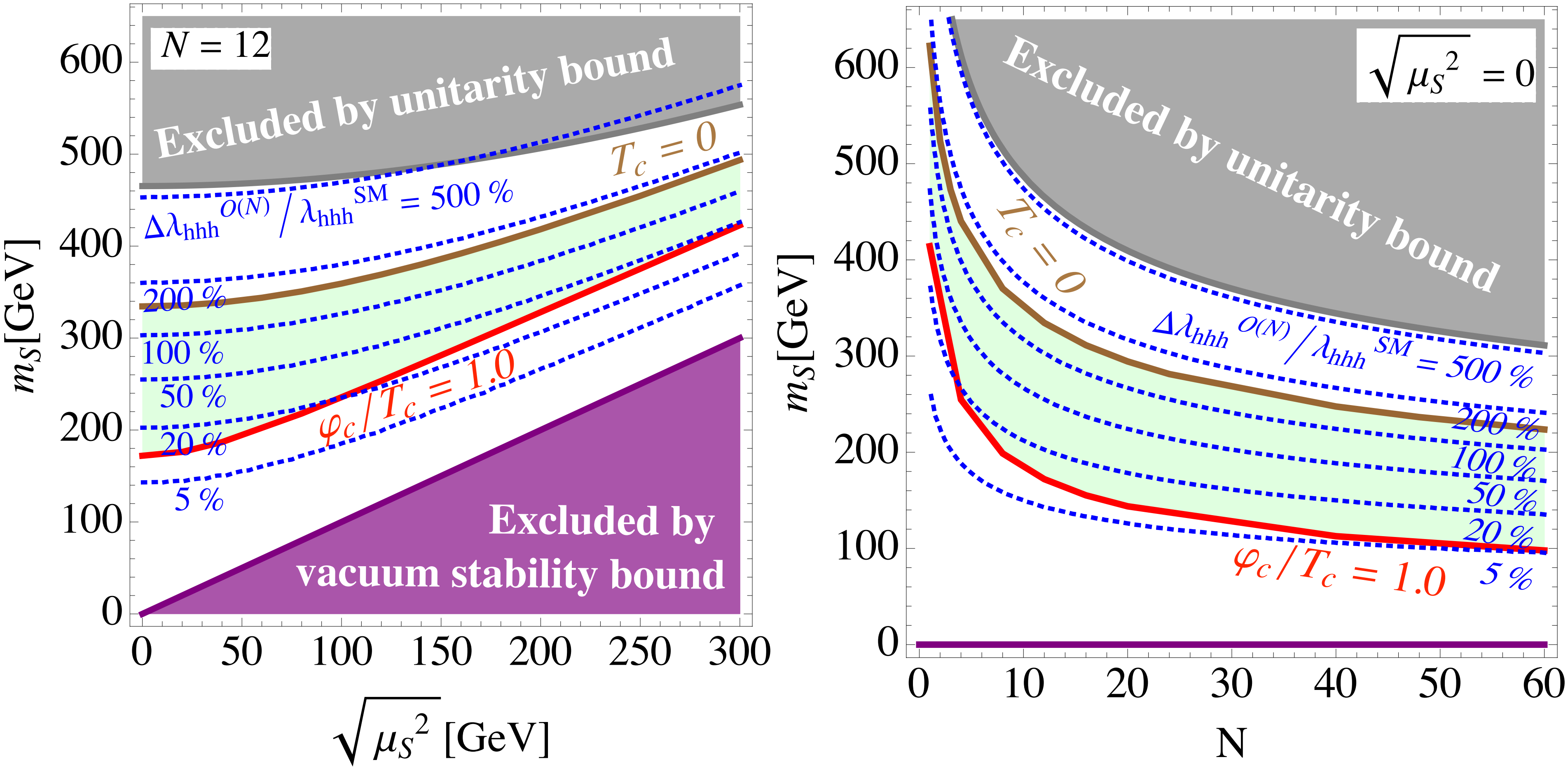
φ_c/T_c > 1 is satisfied by the nondecoupling effect of the singlet fields（compatible with m_h=125GeV）

$$\frac{\varphi_C}{T_C} \simeq \frac{1}{3\pi v m_h^2}\left\{6m_W^3 + 3m_Z^3 + \underline{N m_S^3\left(1 - \frac{\mu_S^2}{m_S^2}\right)^3\left(1 + \frac{3\mu_S^2}{2m_S^2}\right)}\right\}$$

$$\lambda_{hhh}^{O(N)} \simeq \frac{3m_h^2}{v^2}\left\{1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + N\underline{\frac{m_S^4}{12\pi^2 v^2 m_h^2}\left(1 - \frac{\mu_S^2}{m_S^2}\right)^3}\right\}$$

Predictions on the *hhh* coupling

M.Kakizaki, S.Kanemura, T.Matsui, arXiv:1509.08394 [hep-ph]



O(10)% deviations in *hhh* coupling

Relic abundance of GWs from EWPT

Numerical calculation
``Overshooting-undershooting method''

Model-independent analysis
C. Grojean and G. Servant, PRD**75,** 043507 (2007)

$$V_{\rm eff}(\varphi, T) \rightarrow \alpha,\ \tilde{\beta}\ \text{@}T=T_t \rightarrow \Omega_{\rm GW} h^2 (f)$$

Relic abundance of GWs is composed of two contributions.

$$\Omega_{\rm GW} h^2(f) \equiv \Omega_{\rm coll} h^2(f) + \Omega_{\rm turb} h^2(f)$$

"bubble collision"

$$\widetilde{\Omega}_{\rm coll} h^2 \simeq \frac{1.1\times 10^{-6}\kappa^2(\alpha) v_b^3(\alpha)}{0.24 + v_b^3(\alpha)} \times \left(\frac{\alpha}{1+\alpha}\right)^2 \tilde{\beta}^{-2}$$

$$\tilde{f}_{\rm coll} \simeq 5.2\times 10^{-6}{\rm Hz}\times (T_t/100{\rm GeV})\tilde{\beta}$$

"turbulence in the plasma"

$$\widetilde{\Omega}_{\rm turb} h^2 \simeq 1.4\times 10^{-4} u_s^5(\alpha) v_b^2(\alpha)\tilde{\beta}^{-2}$$

$$\tilde{f}_{\rm turb} \simeq 3.4\times 10^{-6}{\rm Hz}\times (u_s(\alpha)/v_b(\alpha))(T_t/100{\rm GeV})\tilde{\beta}$$

Electroweak Phase Transition

Numerical calculation

``Overshooting-undershooting method''

$$V_{\rm eff}(\varphi, T)$$

Electroweak Phase Transition

Numerical calculation

``Overshooting-undershooting method''

$$\boxed{V_{\text{eff}}(\varphi, T)}$$

``Spherical bubble configuration''

Eq. of motion: $\dfrac{d^2\varphi}{dr^2} + \dfrac{2}{r}\dfrac{d\varphi}{dr} - \dfrac{dV_{\text{eff}}}{d\varphi} = 0 \rightarrow \boxed{\varphi(r)}$

Electroweak Phase Transition

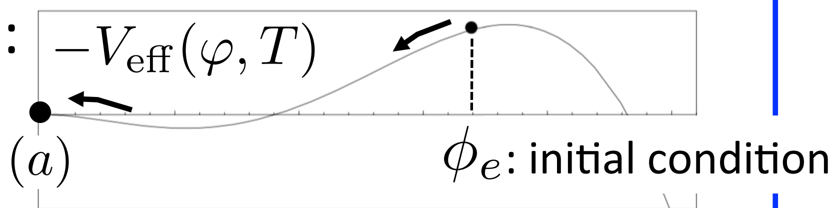
Numerical calculation

``Overshooting-undershooting method''

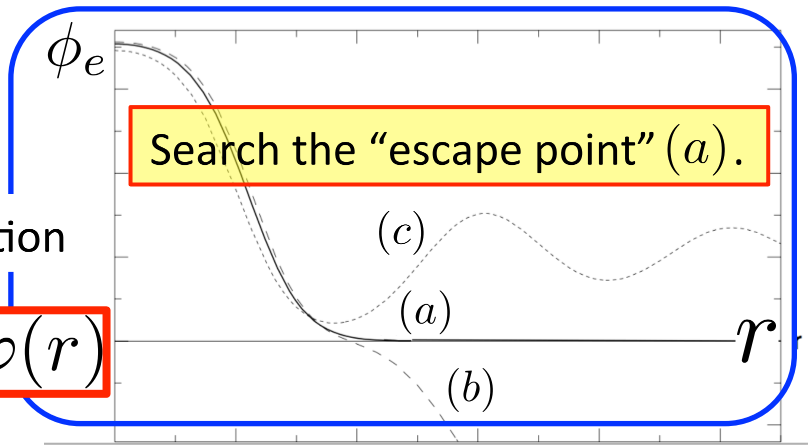
$$\boxed{V_{\rm eff}(\varphi, T)}$$

``Spherical bubble configuration''

Potential:



Eq. of motion: $\dfrac{d^2\varphi}{dr^2} + \dfrac{2}{r}\dfrac{d\varphi}{dr} - \dfrac{dV_{\rm eff}}{d\varphi} = 0 \rightarrow \boxed{\varphi(r)}$



Electroweak Phase Transition

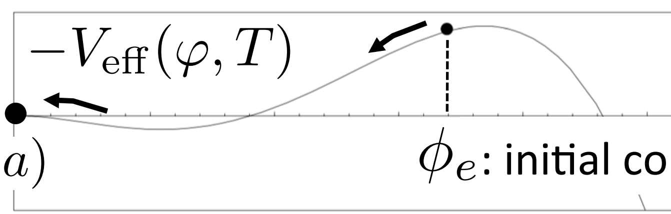
Numerical calculation

``Overshooting-undershooting method''

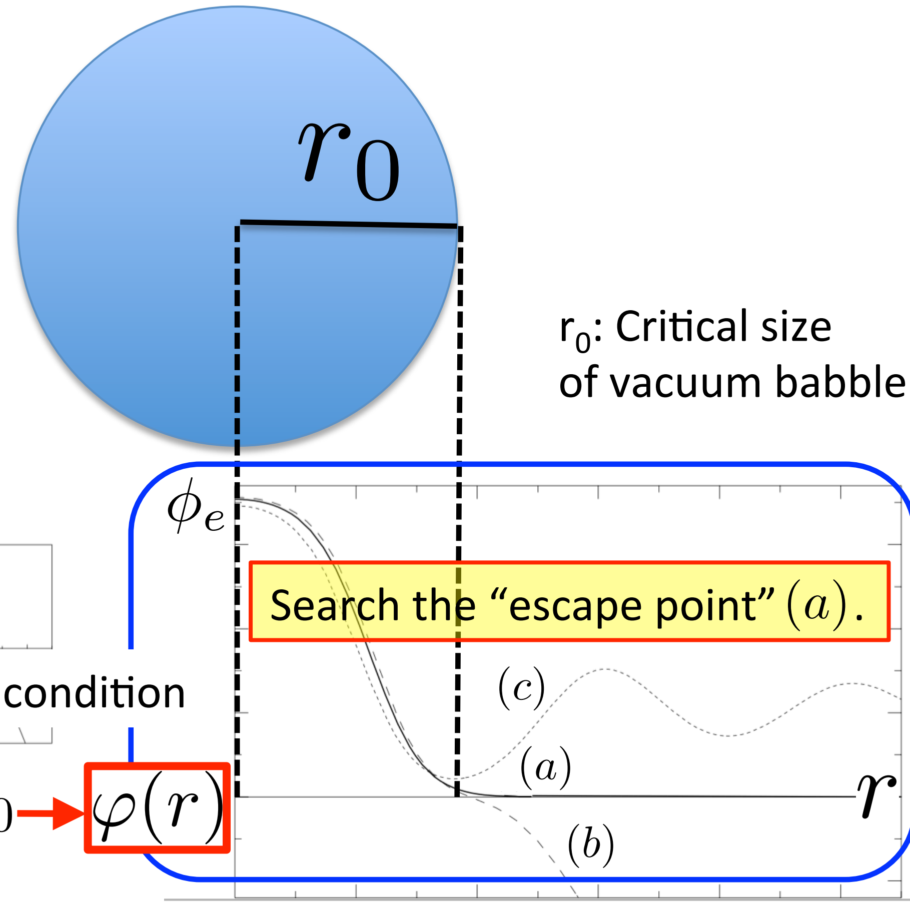
$$\boxed{V_{\mathrm{eff}}(\varphi, T)}$$

``Spherical bubble configuration''

Potential:



Eq. of motion: $\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} - \frac{dV_{\mathrm{eff}}}{d\varphi} = 0 \rightarrow \boxed{\varphi(r)}$



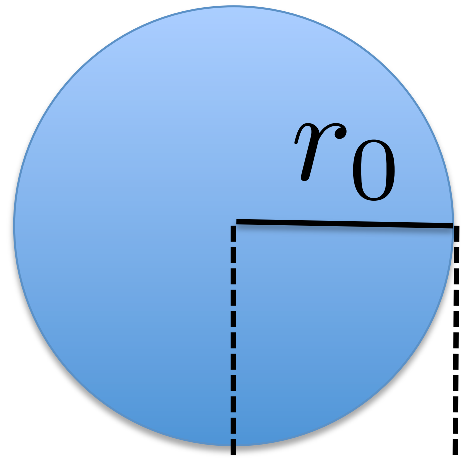
r_0: Critical size of vacuum babble

Electroweak Phase Transition

Numerical calculation

``Overshooting-undershooting method''

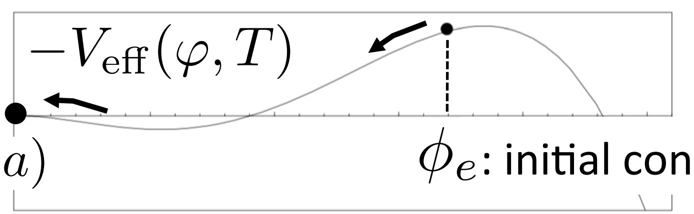
$$V_{\text{eff}}(\varphi, T)$$

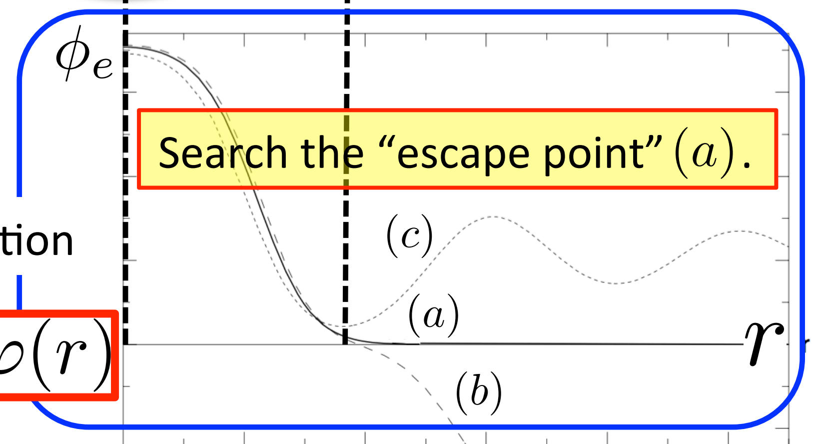


r_0: Critical size of vacuum babble

``Spherical bubble configuration''

Potential:





Eq. of motion: $\dfrac{d^2\varphi}{dr^2} + \dfrac{2}{r}\dfrac{d\varphi}{dr} - \dfrac{dV_{\text{eff}}}{d\varphi} = 0 \rightarrow \boxed{\varphi(r)}$

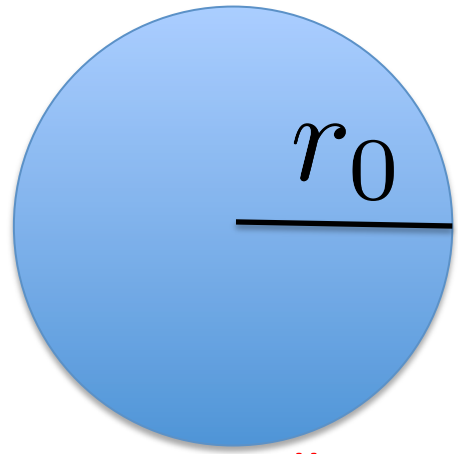
$\boxed{\varphi(r)} \xrightarrow{\text{each}T}$ 3-dim. Euclidean action: $S_3(T) = \int dr^3 \left\{ \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V_{\text{eff}}(\varphi, T) \right\}$

Electroweak Phase Transition

Numerical calculation

``Overshooting-undershooting method''

$$\boxed{V_{\text{eff}}(\varphi, T)}$$



r_0: Critical size of vacuum babble

``Definition of phase transition temperature T_t''

$$\boxed{\left.\frac{\Gamma}{H^4}\right|_{T=T_t} \simeq 1}$$ (H: Hubble parameter)

Phase transition completes when the probability for the nucleation of 1 bubble per 1 horizon volume and horizon time is of order 1.

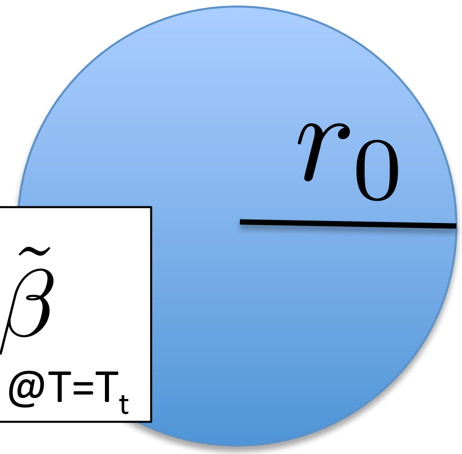
- Bubble nucleation rate: $\Gamma(T) \simeq T^4 e^{-\frac{S_3(T)}{T}}$
- 3-dim. Euclidean action: $S_3(T) = \int dr^3 \left\{ \frac{1}{2} \left(\vec{\nabla}\varphi \right)^2 + V_{\text{eff}}(\varphi, T) \right\}$

Electroweak Phase Transition

Numerical calculation

``Overshooting-undershooting method''

$V_{\mathrm{eff}}(\varphi, T)$ → $\alpha,\ \tilde{\beta}$ @T=T_t



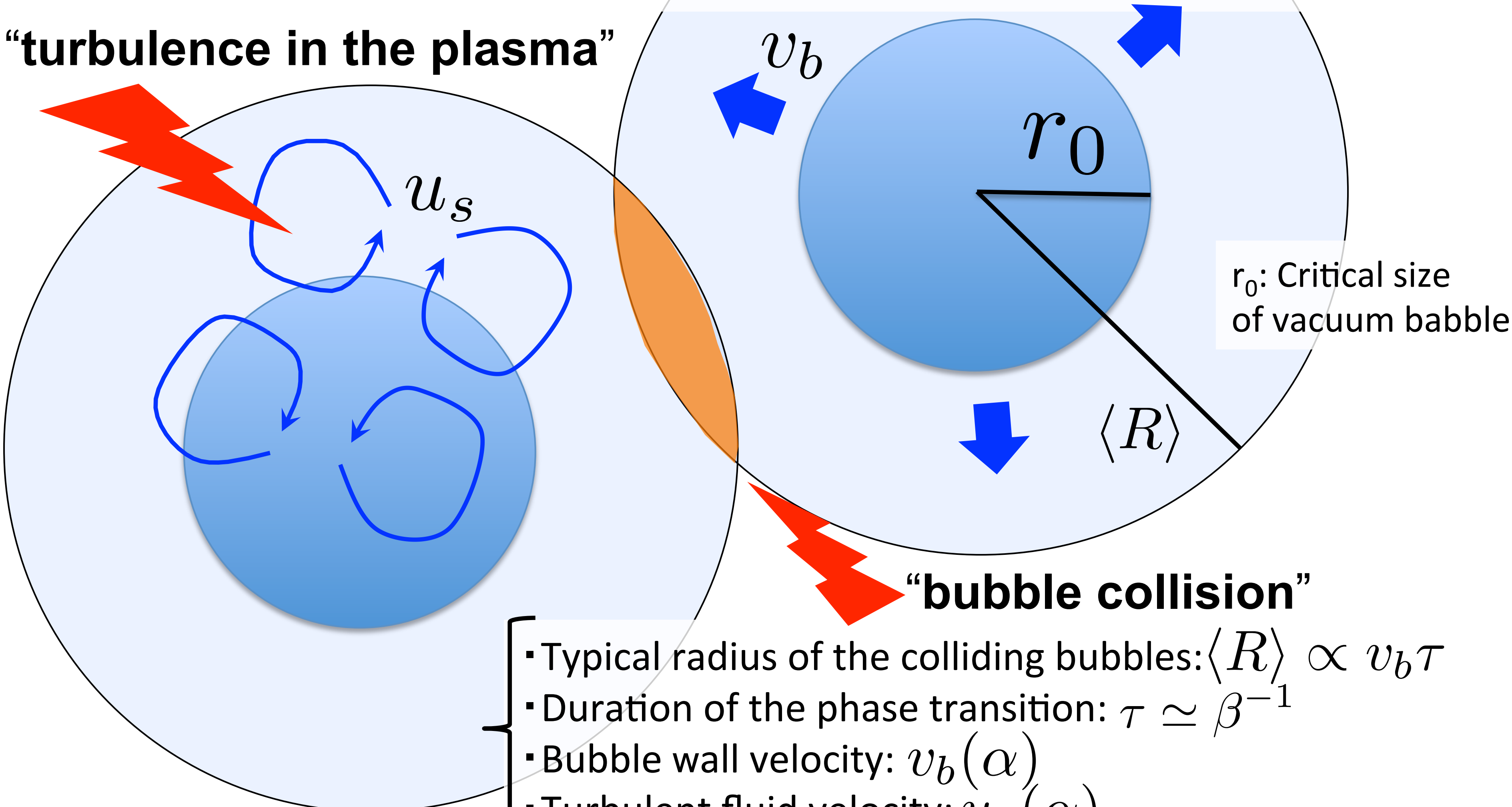
r_0: Critical size of vacuum babble

``Characteristic parameters of GWs''

- α is defined as $\alpha \equiv \left.\dfrac{\epsilon}{\rho_{\mathrm{rad}}}\right|_{T=T_t}$. ($\rho_{\mathrm{rad}}$ is energy density of rad.)
  - Latent heat: $\epsilon(T) \equiv -\Delta V_{\mathrm{eff}}(\varphi_B(T), T) + T\dfrac{\partial \Delta V_{\mathrm{eff}}(\varphi_B(T))}{\partial T}$ cf. U=-F+T(dF/dT)
- β is defined as $\beta \equiv \left.\dfrac{1}{\Gamma}\dfrac{d\Gamma}{dt}\right|_{t=t_t}$. $\rightarrow$ $\tilde{\beta}\left(\equiv \dfrac{\beta}{H_t}\right) = T_t \left.\dfrac{d(S_3(T)/T)}{dT}\right|_{T=T_t}$

(H_t : Hubble parameter @T_t)

Two origins of GWs from EWPT



- Typical radius of the colliding bubbles: $\langle R \rangle \propto v_b \tau$
- Duration of the phase transition: $\tau \simeq \beta^{-1}$
- Bubble wall velocity: $v_b(\alpha)$
- Turbulent fluid velocity: $u_s(\alpha)$

Relic abundance of GWs from EWPT

Numerical calculation
``Overshooting-undershooting method''

Model-independent analysis
C. Grojean and G. Servant, PRD**75,** 043507 (2007)

$$V_{\rm eff}(\varphi, T) \rightarrow \alpha,\ \tilde{\beta}\ \text{@T=T}_t \rightarrow \Omega_{GW} h^2 (f)$$

Relic abundance of GWs is composed of two contributions.

$$\Omega_{\rm GW} h^2(f) \equiv \Omega_{\rm coll} h^2(f) + \Omega_{\rm turb} h^2(f)$$

"bubble collision"

$$\widetilde{\Omega}_{\rm coll} h^2 \simeq \frac{1.1 \times 10^{-6} \kappa^2(\alpha) v_b^3(\alpha)}{0.24 + v_b^3(\alpha)} \times \left(\frac{\alpha}{1+\alpha} \right)^2 \tilde{\beta}^{-2}$$

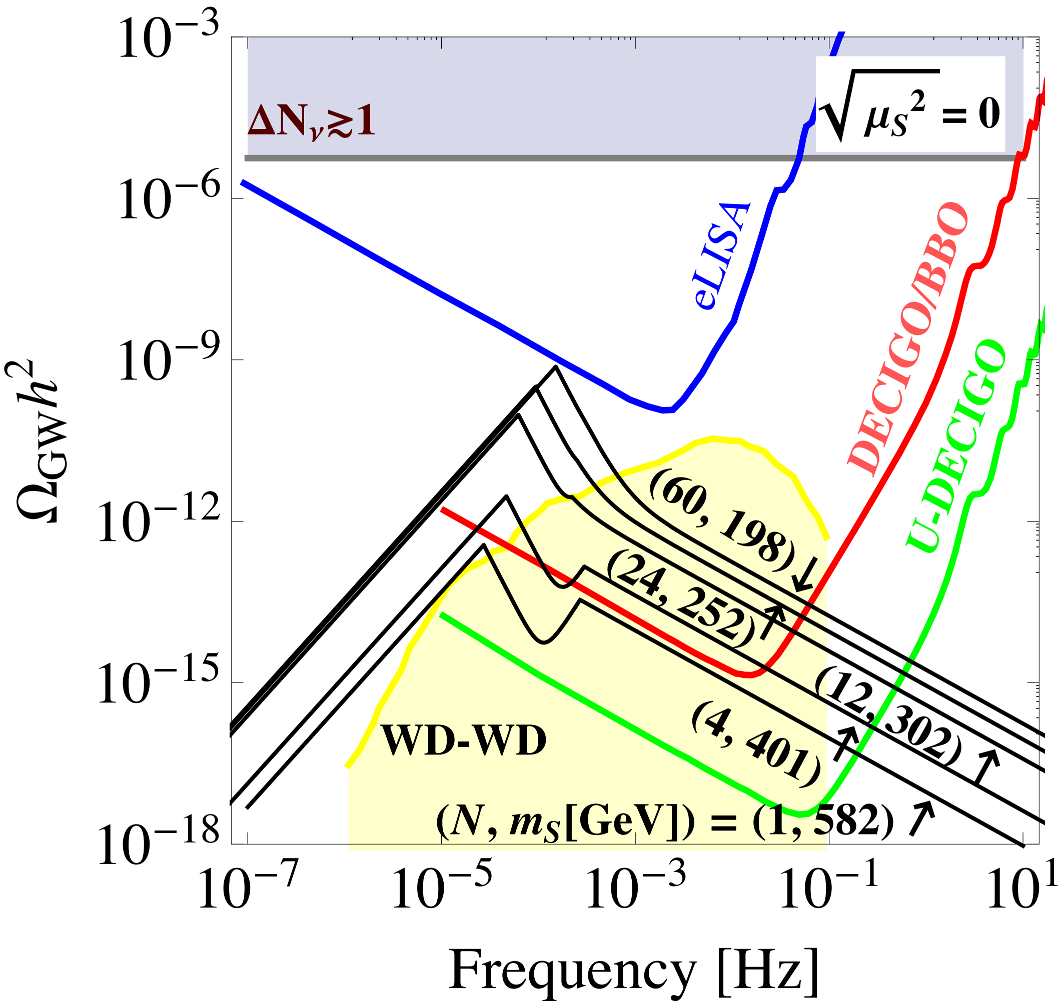
$$\tilde{f}_{\rm coll} \simeq 5.2 \times 10^{-6} {\rm Hz} \times (T_t/100{\rm GeV}) \tilde{\beta}$$

"turbulence in the plasma"

$$\widetilde{\Omega}_{\rm turb} h^2 \simeq 1.4 \times 10^{-4} u_s^5(\alpha) v_b^2(\alpha) \tilde{\beta}^{-2}$$

$$\tilde{f}_{\rm turb} \simeq 3.4 \times 10^{-6} {\rm Hz} \times (u_s(\alpha)/v_b(\alpha))(T_t/100{\rm GeV}) \tilde{\beta}$$

GW spectrum from 1st OPT



Sensitivities

H.Kudoh, *et al.*, PRD73, 064006 (2006)

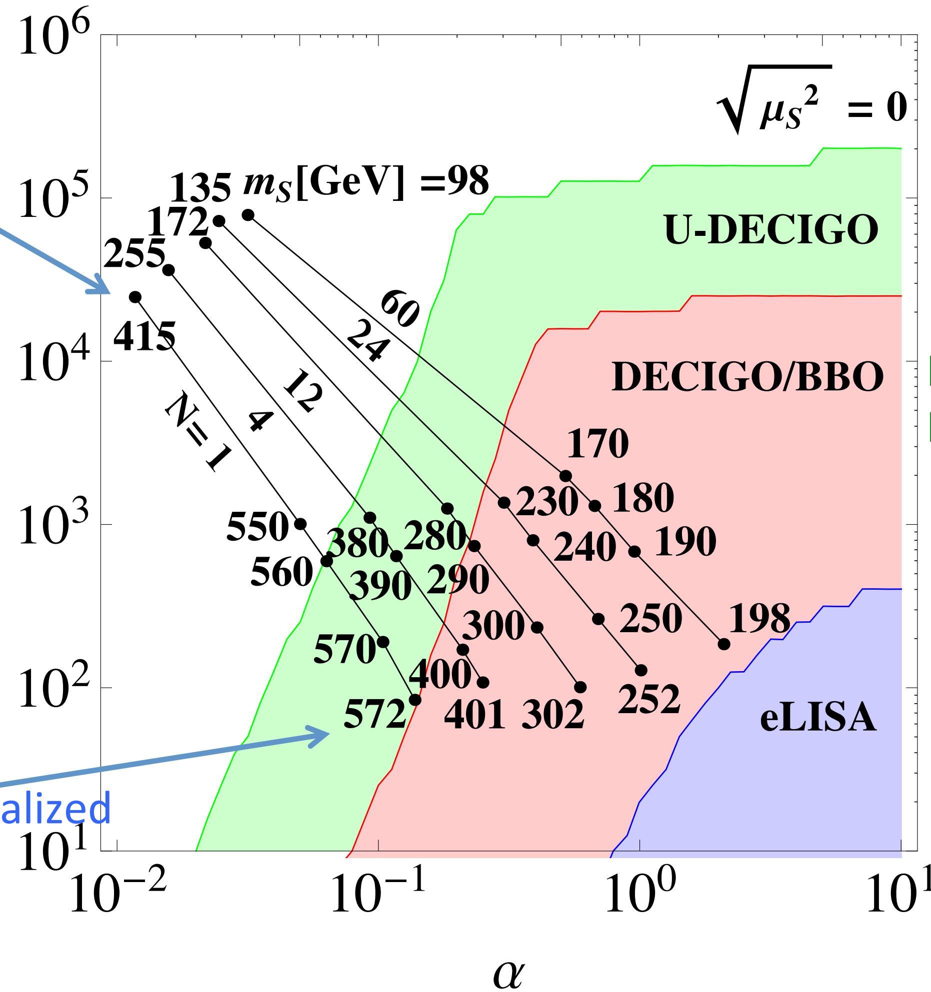
GW from WD-WD

R. Schineider, *et al.*, Class. Quant. Grav. 27, 194007 (2010)

M.Kakizaki, S.Kanemura, T.Matsui, arXiv:1509.08394 [hep-ph]

Dependences on (N, m_s)

For smaller $\boldsymbol{m_s}$
$\boldsymbol{\varphi_c / T_c}$>1
cannot be satisfied



For larger $\boldsymbol{m_s}$
Γ/H^4 = 1 cannot be realized

Sensitivities

H.Kudoh, *et al.*,
PRD73, 064006 (2006)

M.Kakizaki, S.Kanemura, T.Matsui, arXiv:1509.08394 [hep-ph]

Future improvements

There are uncertainties in evaluation of GW from 1st OPT (babble dynamics, formulas of GW spectrum, …)

Recent detailed analysis of bubble collision

Efficiency factor (rate of GW from latent heat)

Espinosa, et al. (2010), No (2011) $\kappa(\alpha) \rightarrow \kappa(\alpha, v_w)$

Which model of plasma turbulence to be used?

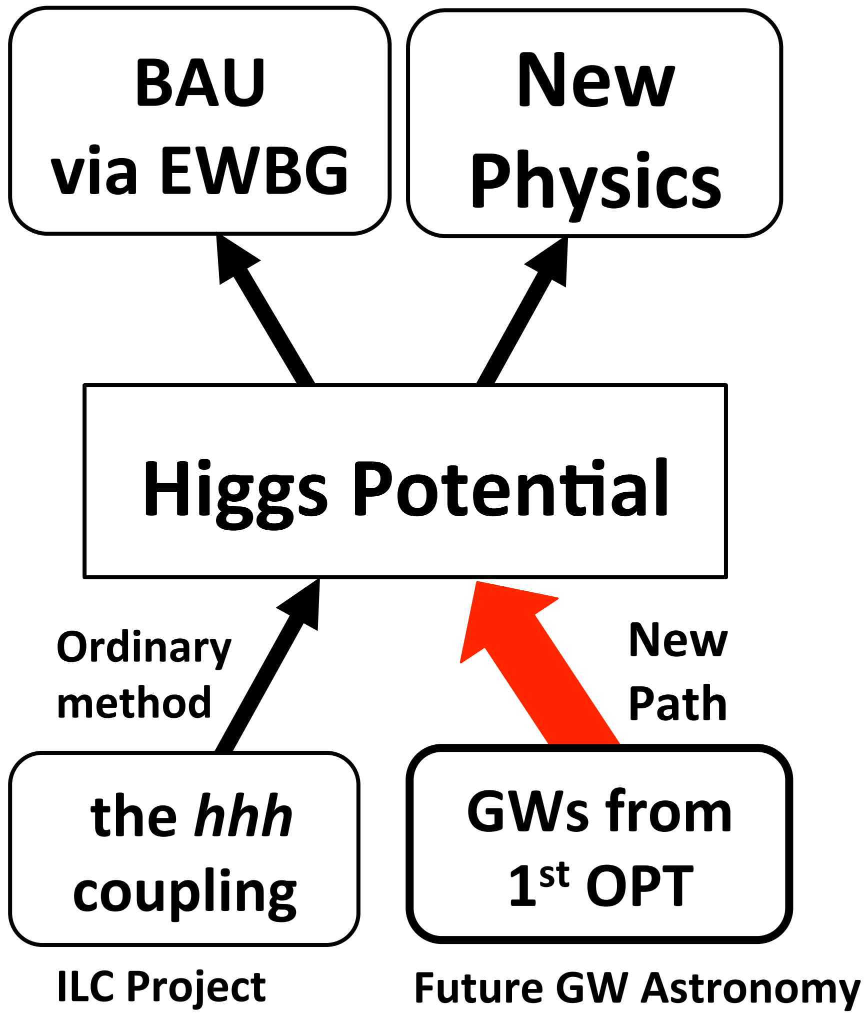
Nicolis (2004)

various fluid modes

Understanding of Foregrouds (ex: WD-WD)

Requirement future GW interferometers

ILC vs LISA/DECIGO



Summary

Multi-plet strucures etc of the Higgs boson can be tested by using the precision measurement of the *hVV* and *hff* couplings at LHC, LH-LHC, ILC, …

The nature of the Higgs potential (with 1st OPT) can only be tested by

measuring the *hhh* coupling by 10% at ILC, CLIC
measuring spectra of GWs at eLISA, DECIGO, …

Future GW Astronomy may provide a good probe of the Higgs potential with 1st OPT

More detained study will be done in future

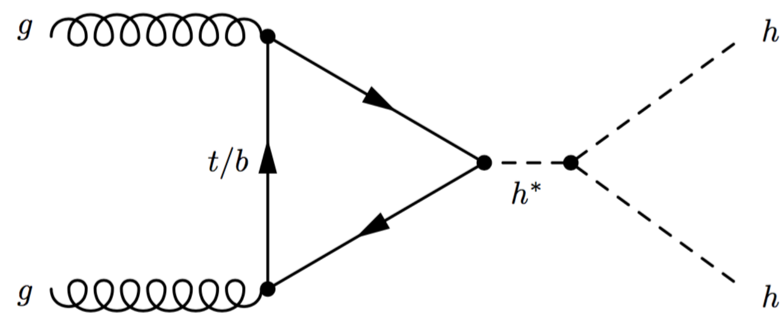
Buck up slides

Triple Higgs boson coupling measurements

- HL-LHC (14TeV, 3000fb^{-1})

$\Delta\lambda_{hhh}/\lambda_{hhh}$~50%(gg→hh)

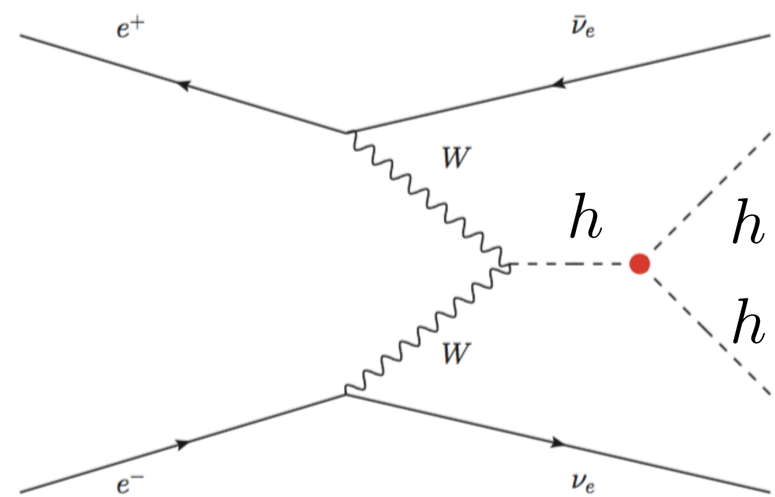
Snowmass Higgs working group, arXiv:1310.8361 [hep-ex]



- ILC1000-up (500/1000GeV, 1600+2500fb^{-1})

$\Delta\lambda_{hhh}/\lambda_{hhh}$~10%(ee→ννhh)

K.Fujii *et al.*, arXiv:1506.05992 [hep-ex]

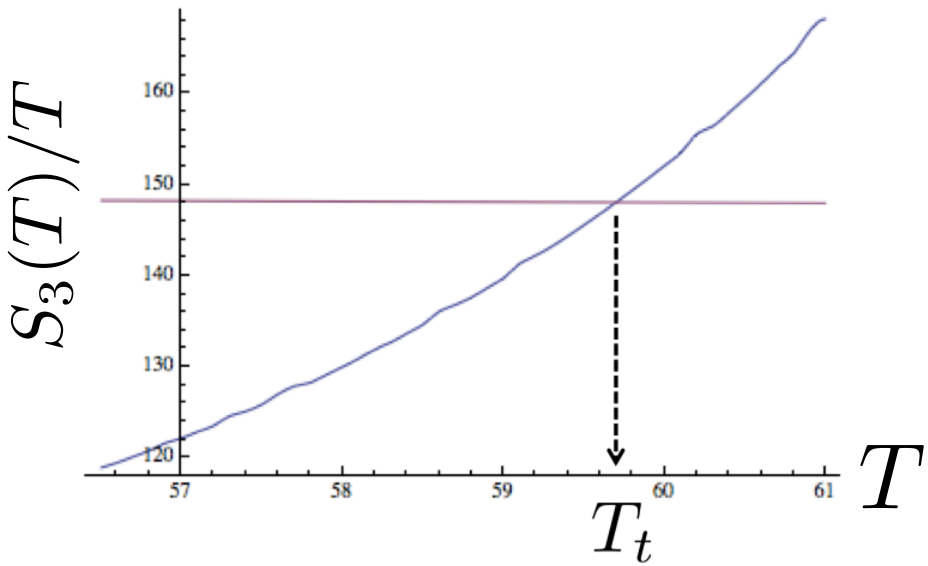


Definition of PT temp.

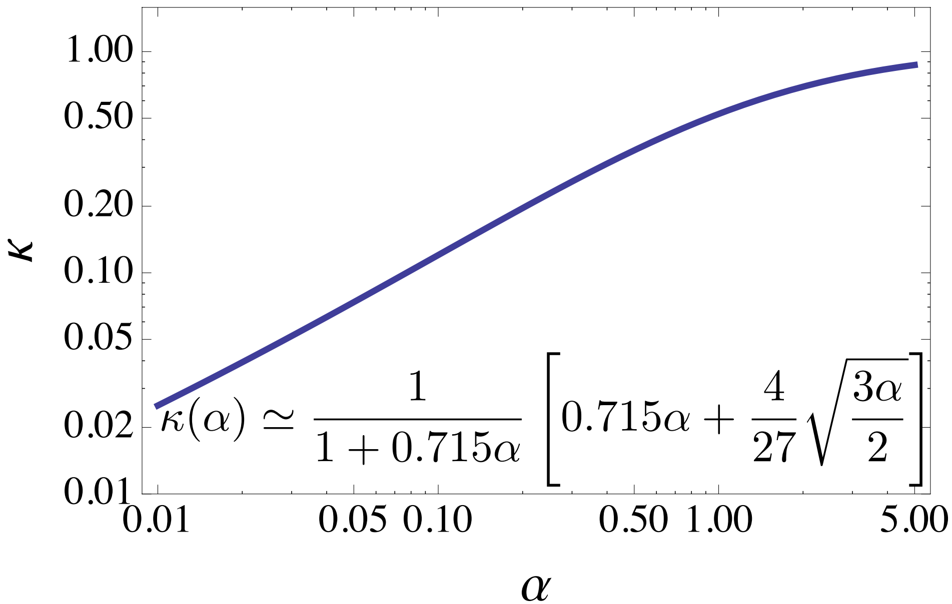
$$\boxed{\left.\frac{\Gamma}{H^4}\right|_{T=T_t} \simeq 1}$$

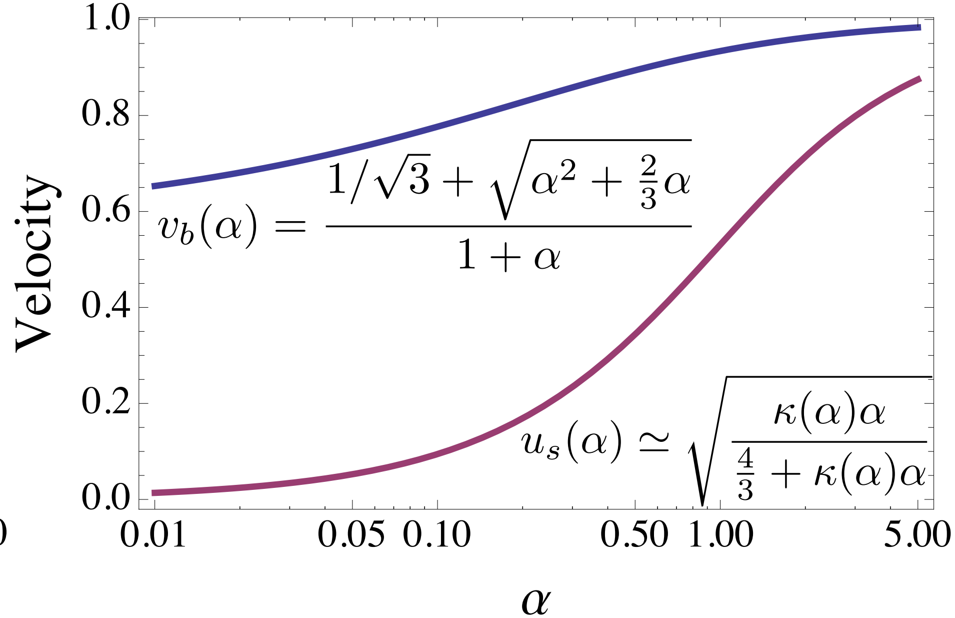
PT completes when the probability for the nucleation of 1 bubble per 1 horizon volume and horizon time is of order 1.

$$\leftrightarrows \left.\frac{S_3(T)}{T}\right|_{T=T_t} = 4\log\left[\frac{T_t}{H(T_t)}\right] \qquad (\simeq 140-150)$$



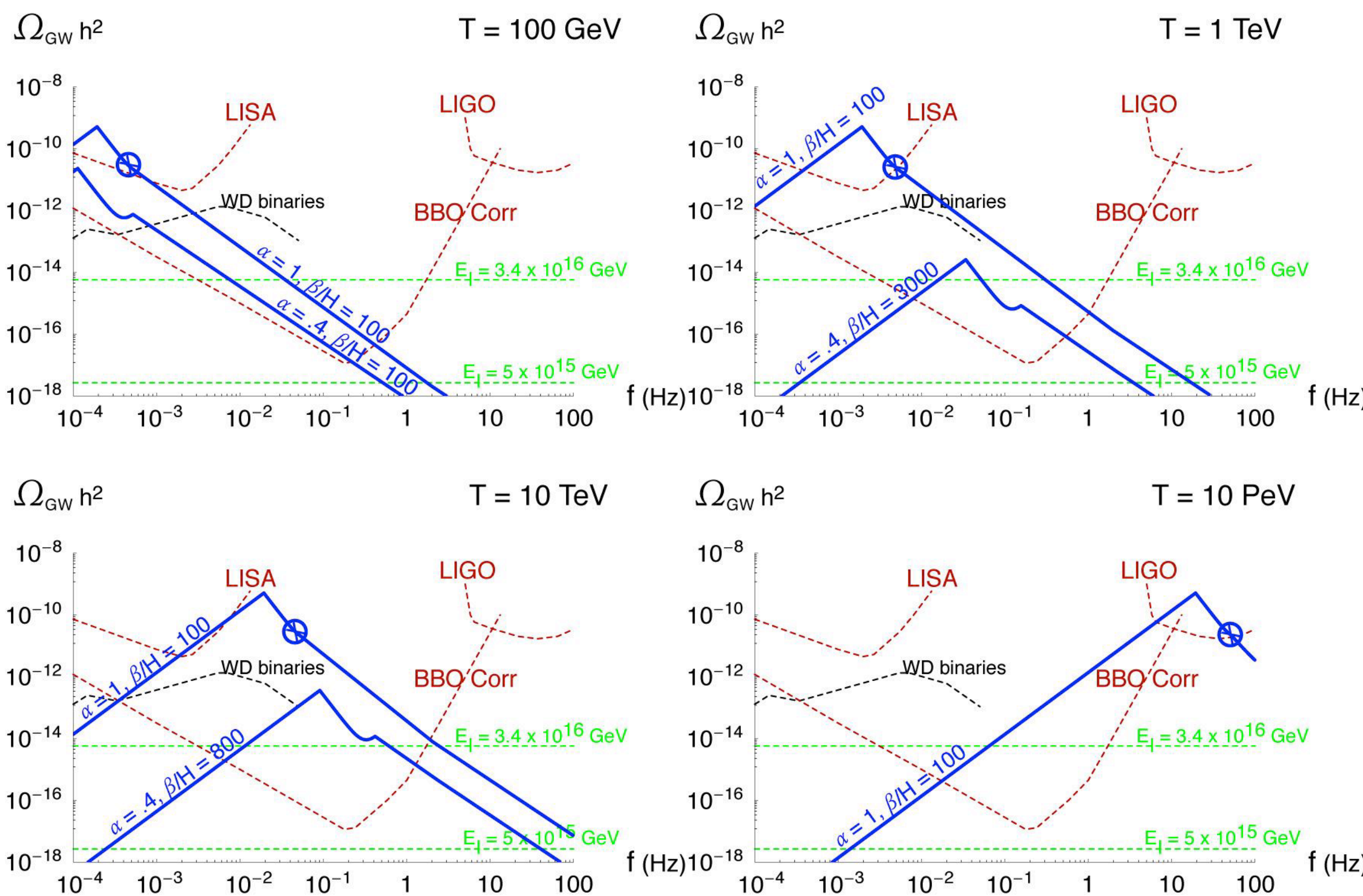
Efficiency factor κ(α)
Bubble wall velocity $v_b(\alpha)$/Turbulent fluid velocity $u_s(\alpha)$





Model independent analysis

C. Grojean and G. Servant, PRD**75,** 043507 (2007)



Recent work of other souse of GW "sound wave"

M.Hindmarsh, *et al*., PRL **112**, 041301 (2014); arXiv:1504.03291 [astro-ph.CO].

Numerical simulations of acoustically generated gravitational waves at a first order phase transition

Mark Hindmarsh,[1,2,*] Stephan J. Huber,[1,†] Kari Rummukainen,[2,‡] and David J. Weir[3,§]

[1] *Department of Physics and Astronomy, University of Sussex, Falmer, Brighton BN1 9QH, U.K.*
[2] *Department of Physics and Helsinki Institute of Physics, PL 64, FI-00014 University of Helsinki, Finland*
[3] *Institute of Mathematics and Natural Sciences, University of Stavanger, 4036 Stavanger, Norway*

(Dated: April 14, 2015)

We present details of numerical simulations of the gravitational radiation produced by a first order thermal phase transition in the early universe. We confirm that the dominant source of gravitational waves is sound waves generated by the expanding bubbles of the low-temperature phase. We demonstrate that the sound waves have a power spectrum with power-law form between the scales set by the average bubble separation (which sets the length scale of the fluid flow L_f) and the bubble wall width. The sound waves generate gravitational waves whose power spectrum also has a power-law form, at a rate proportional to L_f and the square of the fluid kinetic energy density. We identify a dimensionless parameter $\tilde{\Omega}_{GW}$ characterising the efficiency of this "acoustic" gravitational wave production whose value is $8\pi\tilde{\Omega}_{GW} \simeq 0.8 \pm 0.1$ across all our simulations. We compare the acoustic gravitational waves with the standard prediction from the envelope approximation. Not only is the power spectrum steeper (apart from an initial transient) but the gravitational wave energy density is generically two orders of magnitude or more larger.

Scaling Factors

LHC current data of *h*(125) couplings

Data at LHC ($\sqrt{s}$ = 7 and 8 TeV)

ATLAS-CONF-2014-009, 1412.8662

$$\kappa_V = 1.15 \pm 0.08, \quad \kappa_F = 0.99^{+0.08}_{-0.15}, \quad \text{ATLAS}$$

$$\kappa_V = 1.01 \pm 0.07, \quad \kappa_F = 0.87^{+0.14}_{-0.13}, \quad \text{CMS}$$

(Assumption; $\kappa_F = \kappa_t = \kappa_b = \kappa_\tau$, $\kappa_V = \kappa_Z = \kappa_W$)

$$\kappa_g = 1.08^{+0.15}_{-0.13}, \quad \kappa_\gamma = 1.19^{+0.15}_{-0.12}, \quad \text{ATLAS}$$

$$\kappa_g = 0.89^{+0.11}_{-0.10}, \quad \kappa_\gamma = 1.14^{+0.12}_{-0.13}, \quad \text{CMS}$$

(Assumption; $\kappa_F = \kappa_V$)

Scaling factors are in agreement with those of the SM within the 2-sigma uncertainties of the current data.

Future *h*(125)-coupling measurements

Facility	LHC	HL-LHC	ILC500	ILC500-up
$\sqrt{s}$ (GeV)	14,000	14,000	250/500	250/500
$\int \mathcal{L}dt$ (fb^{-1})	300/expt	3000/expt	250+500	1150+1600
κ_γ	$5-7\%$	$2-5\%$	8.3%	4.4%
κ_g	$6-8\%$	$3-5\%$	2.0%	1.1%
κ_W	$4-6\%$	$2-5\%$	0.39%	0.21%
κ_Z	$4-6\%$	$2-4\%$	0.49%	0.24%
κ_ℓ	$6-8\%$	$2-5\%$	1.9%	0.98%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%	0.60%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%	1.3%

Snowmass Higgs Working Group Report 1310.8361

Landau Pole and UV theory

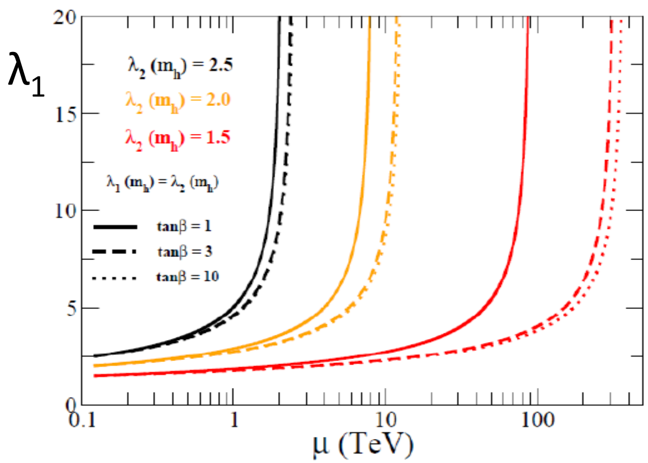
EW Phase Transition and Landau Pole

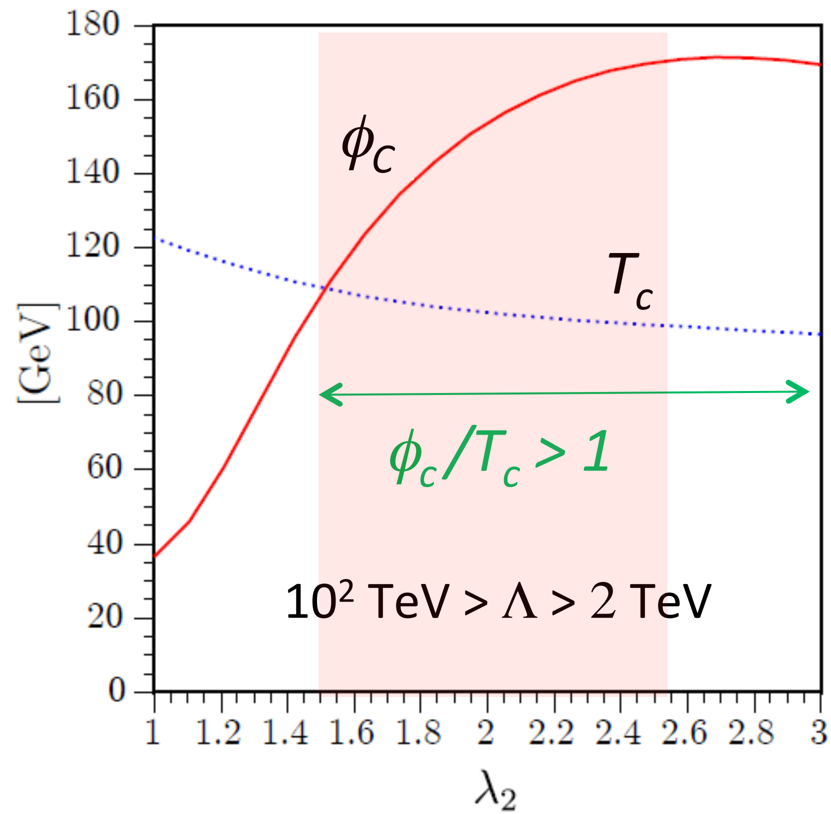
Strong 1st OPT → large λ' at EW
→ Landau pole

S.K., E. Senaha, T. Shindou 2011

Ex) 4HDM+Ω

$$W = \lambda_1 \; H_u H_u' \Omega_1 + \lambda_2 \; H_d H_d' \Omega_2$$





$$\varphi_c/T_c > 1 \quad \Rightarrow \quad \Lambda_{cutoff} = 2 - 100 \text{ TeV}$$

What is the UV theory?

Ex) **Minimal SUSY Fat Higgs Model**

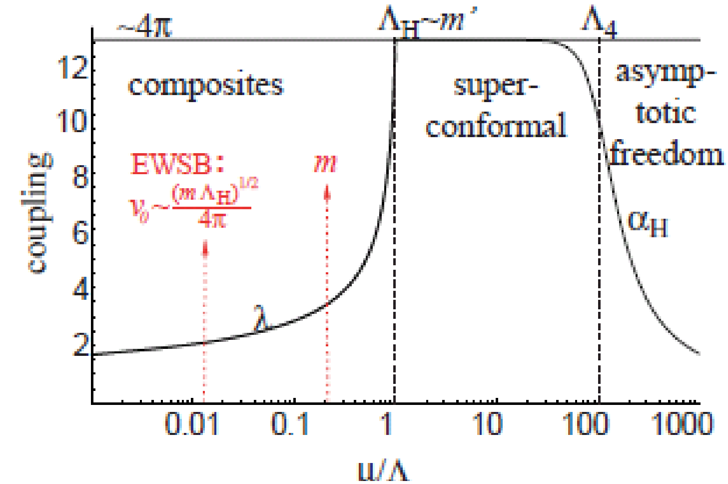
Harnik, Kribs, Larson, Murayama, 2004

- **SU(2)$_H$ gauge theory with N_f=4→3**
- **Confinement at the cutoff Λ_H**
- **Below Λ_H, Higgs fields appear as composite states**

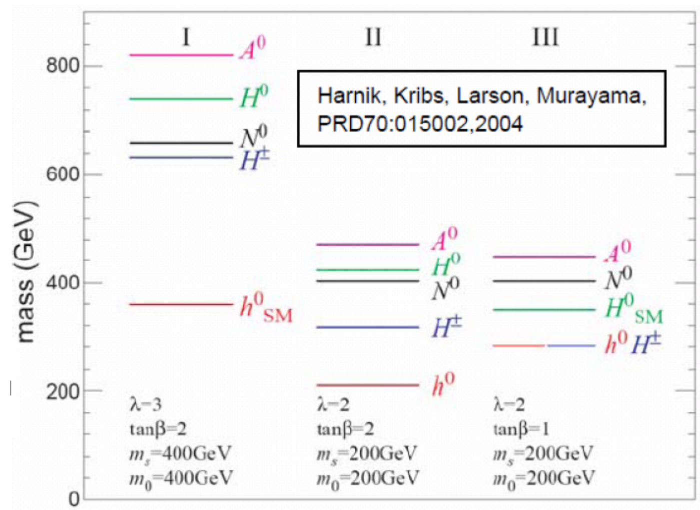
  H_1, H_2, N

- **Low energy effective theory is minimized to be the *nMSSM***
- **SM-like Higgs boson is heavy (fat)**

$$\boxed{m_h^2 \simeq \lambda^2 v^2 + \mathcal{O}(m_Z^2)}$$



$$W = \lambda(NH_1H_2 - v_0^2)$$



Revisit the minimal SUSY Fat Higgs

H_1, H_2, N

- Particles are minimal at low energy (nMSSM)
  - In $SU(2)_H$ with N_f=3 model, 15 composite states appear
  - Unnecessary 10 composite superfields are made heavy in an artificial way by introducing newly additional heavy fields

- A 125 GeV can be easily possible with λ＝O(1) :

  Fat Higgs (tanβ~1) ⇔ Light Higgs (tanβ > 10)

$$\boxed{m_{h\text{tree}}^2 < M_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2}{g^2+g'^2} \sin^2 2\beta \right)}$$

- Neutrino Masses, Baryon Asymmetry and DM are not really discussed

We reconsider the $SU(2)_H$ gauge theory with N_f=3 in order to explain these BSM problems.

Neutrino Masses in the Strong-But-Light Scenario

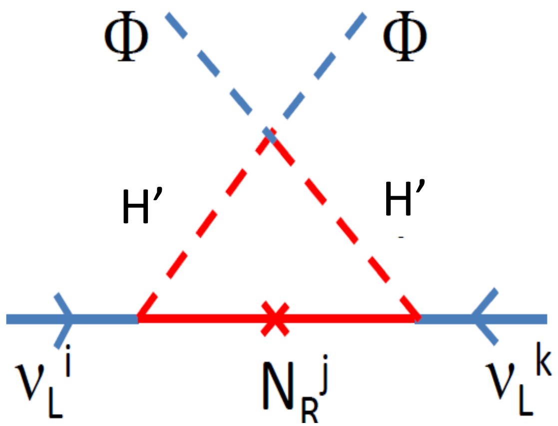
- EW Baryogenesis requires a relatively large coupling in a extended Higgs sector, which causes Landau Pole at O(10) TeV
- In such a case, we may consider the scenario where dim-5 operators ($\nu\nu\Phi\Phi$) appears below the Landau pole
- Neutrino masses are generated at O(1) TeV in the radiative seesaw scenario

Radiative seesaw with Z_2

Z_2-parity plays roles:
1. No tree-level seesaw (Radiative neutrino mass)
2. Stability of the lightest Z_2-odd particle (WIMP)

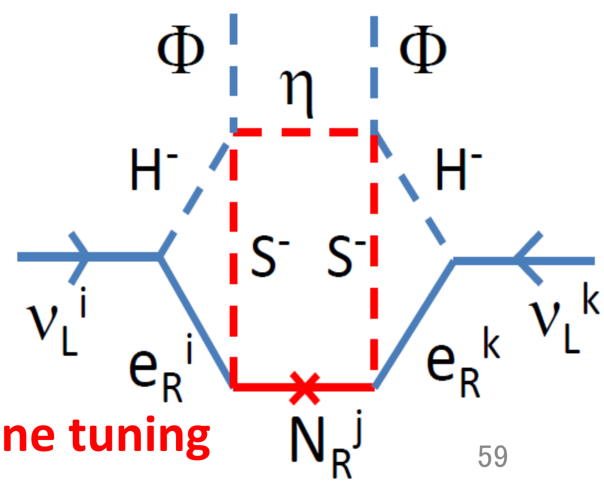
Ex1) 1-loop Ma (2006)

- Simplest model
- SM + N_R + Inert doublet (H')
- DM candidate [H' or NR]



Ex2) 3-loop Aoki-SK-Seto (2008)

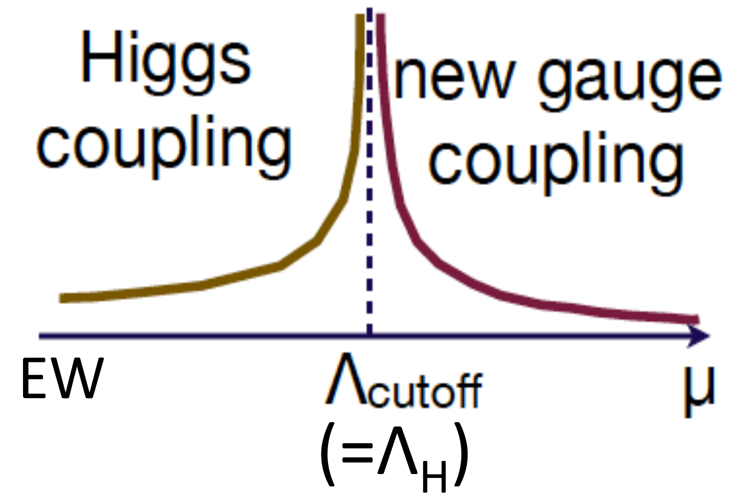
- Neutrino mass from O(1) coupling
- 2HDM + η^0 + S^+ + N_R
- DM candidate [η^0 (or NR)]
- Electroweak Baryogenesis



All 3 problems may be solved by TeV physics w/o fine tuning

Outline of the Model

- Origin of the Higgs force (λ) is the SU(N_c) gauge symmetry (N_c=2, N_f=3)
  [Same as Minimal SUSY Fat Higgs model]
  Harnik, et al
- Confinement ($N_f = N_c+1$) at Λ_H
  (∼ Landau Pole) Intriligator and Seiberg
- At low energy 4HDM+Singlets appears with a coupling λ (Higgses as Mesons)
- λ(EW) is set by ϕc/Tc > 1 (strong) but within perturbative ⇒ Λ_H = O(10) TeV



By the extended Higgs sector with additional Z_2 and RH Neutrinos, radiative seesaw scenario is realized at TeV scale

SUSY $SU(2)_H$ gauge theory

Minimal model for confinement (N_f=3)
→ 3 pairs of $SU(2)_H$ fundamental rep.

Put current mass terms to give masses of T_i

Six $SU(2)_H$ doublets *T_i* charged under the SM gauge groups and a new Z_2-parity

Field	$SU(2)_L$	$U(1)_Y$	Z_2
$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$	2	0	$+$
T_3	1	$+1/2$	$+$
T_4	1	$-1/2$	$+$
T_5	1	$+1/2$	$-$
T_6	1	$-1/2$	$-$

SK, T. Shindou, T. Yamada, 2012

Current mass term $W_m = m_1 T_1 T_2 + m_3 T_3 T_4 + m_5 T_5 T_6$

Effective Theory

- **The theory becomes strongly coupled at Λ_H, and T_i (i=1-6) are confined**

K. Intriligator and N. Seiberg (1996)

- **Below Λ_H the theory is described by Meson superfields**

$$\boxed{M_{ij} = T_i T_j}$$

- **Effective Superpotential**

$$W_{eff} = \frac{1}{\Lambda^3}\epsilon_{ijklmn} M_{ij} M_{kl} M_{mn} + m_1 M_{12} + m_3 M_{34} + m_5 M_{56}$$

- **By using Naïve Dimensional Analysis, it is rewritten by canonically normalized fields**

$$W_{eff} \simeq \lambda\, \epsilon_{ijklmn} \hat{M}_{ij} \hat{M}_{kl} \hat{M}_{mn} + \frac{m_1 \Lambda_H}{4\pi} \hat{M}_{12} + \frac{m_3 \Lambda_H}{4\pi} \hat{M}_{34} + \frac{m_5 \Lambda_H}{4\pi} \hat{M}_{56}$$

- **The coupling λ becomes non-perturbative at Λ_H**

$$\lambda(\mu = \Lambda_H) \simeq 4\pi$$

Naïve Dimesional Analysis

Higgses as Mesons

Fifteen mesons $M_{ij} = T_i T_j$ can be identified as the MSSM Higgses and extra superfields

MSSM Higgs doublets

Exotic Superfields

Extra Higgs doublets

Charged Higgs singlets

Z_2-even Higgs singlets

Z_2-odd Higgs singlets

Field	$SU(2)_L$	$U(1)_Y$	Z_2
H_u	2	$+1/2$	$+$
H_d	2	$-1/2$	$+$
Φ_u	2	$+1/2$	$-$
Φ_d	2	$-1/2$	$-$
Ω^+	1	$+1$	$-$
Ω^-	1	-1	$-$
$N,\ N_\Phi,\ N_\Omega$	1	0	$+$
$\zeta,\ \eta$	1	0	$-$

Superpotential is rewritten as

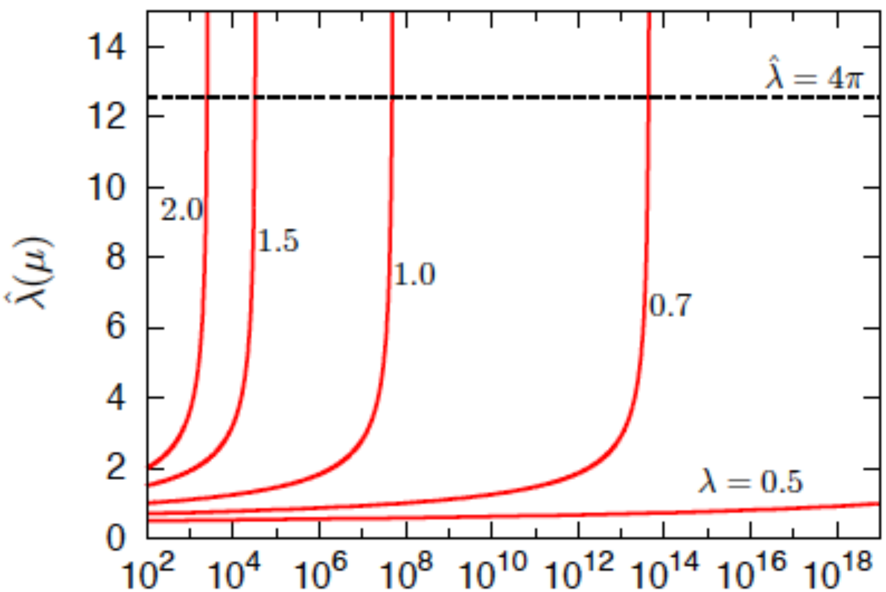
$$\begin{aligned} W_{eff} \;=\; & \lambda\ \{\ N(H_u H_d + v_0^2)\ +\ N_\Phi(\Phi_u \Phi_d + v_\Phi^2)\ +\ N_\Omega(\Omega^+\Omega^- + v_\Omega^2) \\ & -\ N N_\Phi N_\Omega\ -\ N_\Omega \zeta\eta\ +\ \zeta H_d \Phi_u\ +\ \eta H_u \Phi_d\ -\ \Omega^+ H_d \Phi_d\ -\ \Omega^- H_u \Phi_u\ \} \end{aligned}$$

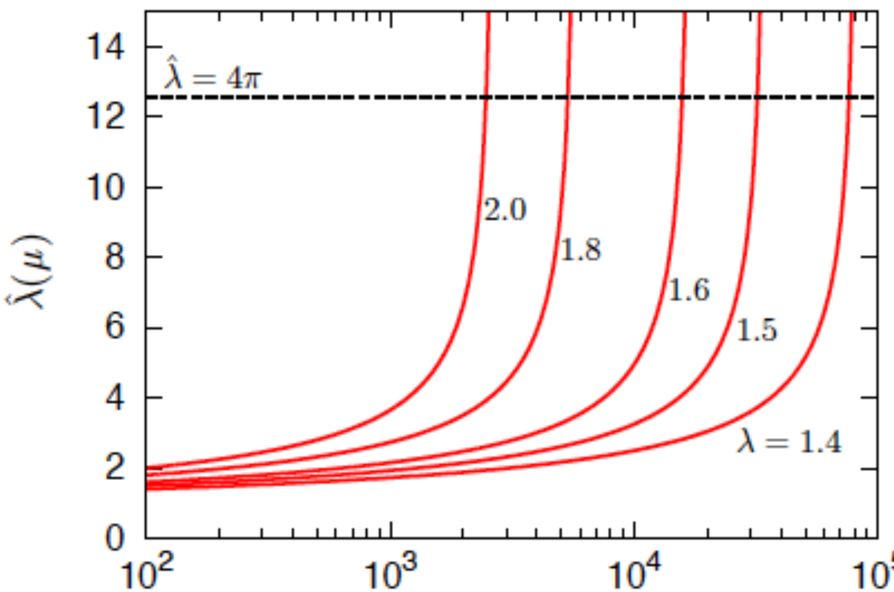
The low energy theory is 4HDM+Singlets but with a common λ !

MSSM-like Higgs doublets

$$W = -\mu H_u H_d - \mu_\Phi \Phi_u \Phi_d - \mu_\Omega(\Omega_+ \Omega_- - \zeta\eta)$$
$$+ \hat{\lambda}\{H_d \Phi_u \zeta + H_u \Phi_d \eta - H_u \Phi_u \Omega_- - H_d \Phi_d \Omega_+\}$$

$\hat{\lambda}(\Lambda_H) \simeq 4\pi$ (Naive dimensional analysis)





RG running of λ

λ=λ(μ_{EW}) determines the cutoff scale