# Recognizing symmetries in 3HDM in a basis-invariant way

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based on: I. P. Ivanov, C. Nishi, J. P. Silva, A. Trautner, arXiv:1810.13396 and work in progress

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# Symmetries in 3HDM

The NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

may be invariant under global symmetries:

- family symmetries: $\phi_a \to U_{ab}\phi_b$, with $U \in U(N)$,
- GCP symmetries: $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, with $X \in U(N)$.

Each symmetry group $G$ and its breaking by vevs $G_v \subseteq G$ lead to a characteristic phenomenology (scalars, DM candidates, fermion masses, mixing, sources of CPV, etc).

# Symmetries in 3HDM

2HDM explored in detail; 3HDM gaining more attention.

Classification of symmetries in 3HDM:

- all abelian symmetries: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]
- non-abelian discrete symmetries: [Ivanov, Vdovin, 1206.7108, 1210.6553]
- listing all non-abelian continuous symmetries is straightforward
- mass-degenerate Higgses from $A_4$ or $S_4$ 3HDM [Degee, Ivanov, Keus, 1211.4989]
- symmetry breaking patterns $G \to G_v$: [Ivanov, Nishi, 1410.6139]
- various options of CP symmetries and their interplay with $G$ [classical works]
- higher order $CP$ symmetry CP4: [Ivanov, Keus, Vdovin, 1112.1660; Ivanov, Silva, 1512.09276].

# Basis invariants

With $N$ Higgs doublets, there is large freedom of basis changes.

A symmetry can be evident in one basis and hidden in another → challenge!

One needs basis-invariant criteria for various phenomena in NHDM.

Usual recipe [Botella, Silva, 1995]: construct basis invariants $J_k$ and link them to the feature you want to study.

It was applied, in particular, to the explicit $CP$-conservation in 2HDM [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$
$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$
$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

# Bilinear space formalism

Alternative road: geometric constructions in the bilinear space [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008].

$V$ depends on bilinears $\phi_a^\dagger \phi_b$. Organize them into combinations:

$$r_0 = \phi_a^\dagger \phi_a \equiv \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2\,, \quad r_i = \phi_a^\dagger \sigma^i_{ab} \phi_b \equiv \begin{pmatrix} 2\mathrm{Re}(\phi_1^\dagger \phi_2) \\ 2\mathrm{Im}(\phi_1^\dagger \phi_2) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix},$$

which satisfy $r_0 \geq 0$ and $r_0^2 - r_i^2 \geq 0$.

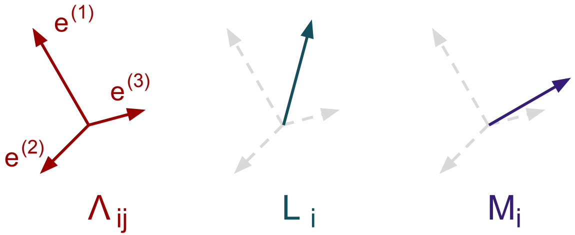
Basis change: an $SO(3)$ rotation; $CP$-transformation: a mirror reflection.

The general 2HDM Higgs potential is a quadratic form in $(r_0, r_i)$:

$$V = -M_0 r_0 - M_i r_i + \Lambda_0 r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j\,.$$

# Bilinear space formalism

2HDM scalar sector = $M_0$, $\Lambda_0$, 3-vectors $M_i$ and $L_i$, and $3 \times 3$ matrix $\Lambda_{ij}$.




Orientation of $M_i$ and $L_i$ with respect to eigenvectors of $\Lambda_{ij}$ $\Rightarrow$ symmetries.

Basis-independent conditions in terms of basis-covariant objects!

# Linking the two approaches

basis-invariants $\Leftrightarrow$ basis-invariant features of basis-covariant objects.

But it may be extremely challenging to explicitly establish the link!

Explicit CP2 conservation in 3HDM:

- invariants: attempted at in [Varzielas et al, 1603.06942; 1706.07606],
- bilinears: solved in [Nishi, hep-ph/0605153].

Explicit CP4 conservation in 3HDM:

- invariants: none,
- bilinears: solved in [Ivanov, Nishi, Silva, Trautner, 1810.13396],
- (for a related question see [Haber, Ogreid, Osland, Rebelo, 1808.08629]).

# Bilinears for 3HDM

Bilinear approach for 3HDM:

$$r_0 = \frac{1}{\sqrt{3}}\phi_a^\dagger\phi_a\,, \quad r_i = \phi_a^\dagger(t^i)_{ab}\phi_b\,, \quad i = 1,\dots,8\,,$$

where $t_i = \lambda_i/2$ are $SU(3)$ generators satisfying

$$[t_i, t_j] = if_{ijk}t_k\,, \quad \{t_i, t_j\} = \frac{1}{3}\delta_{ij}\mathbf{1}_3 + d_{ijk}t_k\,.$$

The potential takes the same form

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j\,,$$

with vectors $M_i, L_i \in \mathbb{R}^8$ and an $8\times 8$ matrix $\Lambda_{ij}$.

Basis changes $\to SO(8)$ rotations. However, $SU(3) \subset SO(8) \Rightarrow$ matrix $\Lambda_{ij}$ is not in general diagonalizable by a basis change!

# Constructions in the adjoint space

Suppose vectors $a_i, b_i \in \mathbb{R}^8$. One can define new products:

$$F_i \equiv 2f_{ijk} a_j b_k \,, \quad D_i \equiv \sqrt{3} d_{ijk} a_j b_k \,.$$

One can also define non-linear action $a_i \mapsto \sqrt{3} d_{ijk} a_j a_k$.

> Applied to the eigenvectors of $\Lambda_{ij}$, these products help detect basis-invariant structures in $\Lambda_{ij} \Rightarrow$ detecting symmetries in 3HDM.

I will show below two examples:

- basis-invariant recognition of explicit CP2 conservation in 3HDM.
- basis-invariant recognition of explicit CP4 conservation in 3HDM.

But the method is general and can be developed for all symmetries in 3HDM.

# Explicit CP2 conservation

CP2: there exists a basis in which it takes the standard form: $\phi_a \to \phi_a^*$.

In the bilinear space, the standard *CP* is the following reflection:

- vectors from $V_+ = (r_3,\, r_8,\, r_1,\, r_4,\, r_6)$ stay unchanged,
- vectors from $V_- = (r_2,\, r_5,\, r_7)$ flip signs.

3HDM potential is explicitly CP2-invariant if there exists a basis in which

- vectors $M_i, L_i \in V_+$,
- $\Lambda_{ij}$ has the block-diagonal form: $\Lambda_{ij} = \begin{pmatrix} \square_{3\times 3} & 0 \\ 0 & \square_{5\times 5} \end{pmatrix}$

  with arbitrary blocks: $\square_{3\times 3}$ in $V_-$ and $\square_{5\times 5}$ in $V_+$.

# Explicit CP2 conservation

Detecting $\square_{3\times 3}$ in $(r_2,\ r_5,\ r_7)$:

- There exist three mutually orthogonal eigenvectors of $\Lambda_{ij}$ denoted $e$, $e'$, $e''$, which are closed under $f$-product: $f_{ijk} e_j e'_k \propto e''_i$, etc.
- Compute $\mathcal{I} = 2|f_{ijk} e_i e'_j e''_k|$. There exist only two options:
  - $\mathcal{I} = 1$: there exists a basis in which $(e, e', e'') = (r_2, r_5, r_7)$;
  - $\mathcal{I} = 2$: there exists a basis in which $(e, e', e'') = (r_1, r_2, r_3)$.

Together with the condition that $M, L$ are orthogonal to $e, e', e''$, the value $\mathcal{I} = 1$ leads to the explicit CP2 conservation [Nishi, hep-ph/0605153].

# Explicit CP4 conservation

CP4 leads in a certain basis in the bilinear space to

$$r_8 \to r_8\,, \quad (r_1, r_2, r_3) \to -(r_1, r_2, r_3)$$
$$r_4 \to r_6\,, \quad r_6 \to -r_4\,, \quad r_5 \to -r_7\,, \quad r_7 \to r_5\,.$$

3HDM potential is explicitly CP4-invariant if there exists a basis in which

- all possible vectors $M_i$, $L_i$, $(\Lambda^n)_{ij} L_j$, $K_i \equiv d_{ijk}\Lambda_{jk}, \ldots$ are all parallel to $r_8$ (complete alignment),
- the matrix $\Lambda_{ij}$ is

$$\Lambda_{ij} = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$

with an arbitrary $3 \times 3$ block in the subspace $(r_1, r_2, r_3)$ and a specific pattern in the $4 \times 4$ block.

# Detecting $r_8$

- Consider $a_i \in \mathbb{R}^8$. If $\sqrt{3} d_{ijk} a_j a_k$ is parallel to $a_i$, we say that $a_i$ is self-aligned.
- $a_i$ is self-aligned $\Leftrightarrow$ there is a basis in which $a_i$ is along $r_8$.
- Thus, if there exists an eigenvector of $\Lambda_{ij}$ which is self-aligned, it can be rotated along direction $r_8$. We denote it as $e_i^{(8)}$.

# Detecting block in $(r_1, r_2, r_3)$

The defining feature of CP4 3HDM is complete alignment and the block-diagonal structure

$$\Lambda_{ij} = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$

That is, three eigenvectors of $\Lambda_{ij}$ belong to the $(r_1, r_2, r_3)$ subspace.

Vectors in $(r_1, r_2, r_3)$ can be recognized in the basis-invariant way:

$$a_i \in (r_1, r_2, r_3) \quad \Leftrightarrow \quad f_{ijk}\, a_j\, e_k^{(8)} = 0\,.$$

That is, $a_i$ is $f$-orthogonal to $e_i^{(8)}$.

# Necessary and sufficient conditions for CP4 in 3HDM

A basis-invariant algorithm for recognizing the presence of CP4 in 3HDM.

Write down $M_i$, $L_i$, $\Lambda_{ij}$. Calculate eigenvectors of $\Lambda_{ij}$.

The model possesses an explicit CP4 if and only if

- there exists a self-aligned eigenvector: $d_{ijk} e_j^{(8)} e_k^{(8)}$ is parallel to $e_i^{(8)}$;
- there exist three eigenvectors $e, e', e''$ which are $f$-orthogonal to $e_i^{(8)}$.
- $M_i$, $L_i$, $K_i = d_{ijk}\Lambda_{jk}$, and $K_i^{(2)} = d_{ijk}(\Lambda^2)_{jk}$ are aligned with $e_i^{(8)}$.

See more details in [Ivanov, Nishi, Silva, Trautner, 1810.13396].

# Conclusions

**Work to do**

All 3HDMs with symmetries can be detected in this way!