Beyond the Coleman–Weinberg Effective Potential

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Based on A.P. and D. Teresi, Nucl. Phys. B **874** (2013) 594
`arXiv:1502.07986 & arXiv:1511.05347.`

Motivation:

New Era of High Precision QFT

- Improved Lattice Techniques
- Higher Order Loop Calculations in Perturbative 1PI QFT
- Improved Solutions to Schwinger–Dyson Equations
- On-Shell Amplitude Techniques
- Geometric S-Matrix Approach in $N = 4$ SYM Theories

$\vdots$

- Other Analytical Non-Perturbative Approaches?

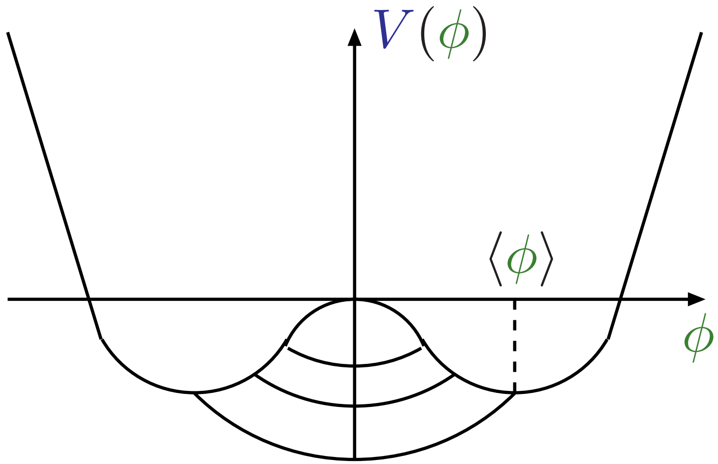
Outline:

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The 2PI Effective Action and Field-Theoretic Problems
- Symmetry-Improved 2PI (SI2PI) Formalism
- SI2PI Effective Higgs Potential
- IR Divergences in the Coleman–Weinberg Effective Potential
- SI2PI Approach to the Goldstone-Boson IR Problem
- Conclusions and Future Directions

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in SM: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{em}$

[P. W. Higgs '64; F. Englert, R. Brout '64; G. S. Guralnik, C. R. Hagen, T. W. B. Kibble '64]



Higgs potential $V(\phi)$

$$V(\phi) \;=\; -m^2\phi^\dagger\phi \;+\; \lambda(\phi^\dagger\phi)^2 \,.$$

Ground state:

$$\langle\phi\rangle \;=\; \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

carries weak charge, but no electric charge and colour.

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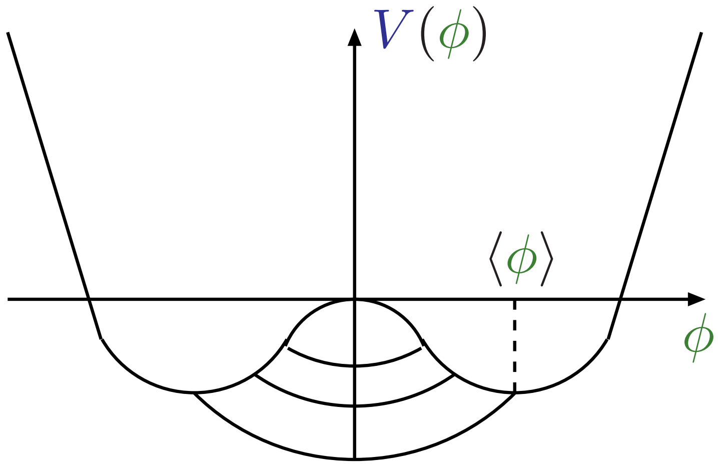

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Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

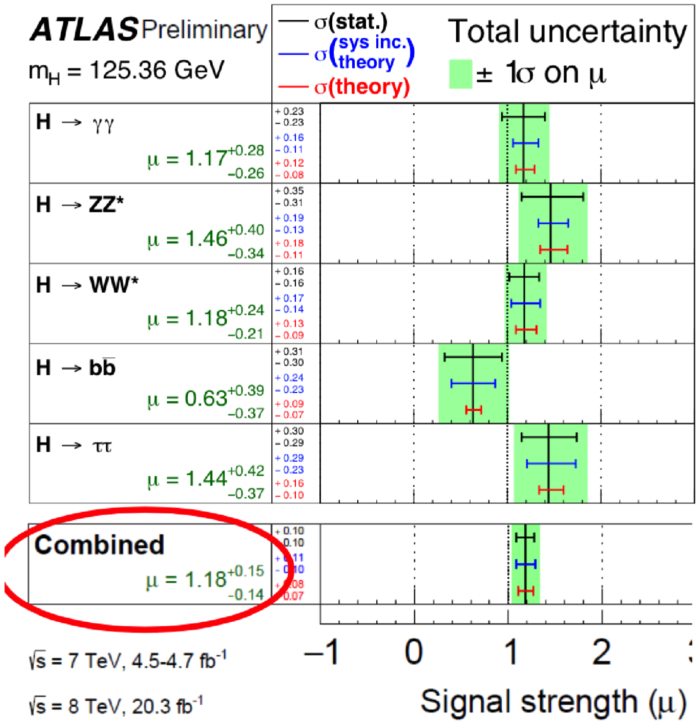
[e.g., P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

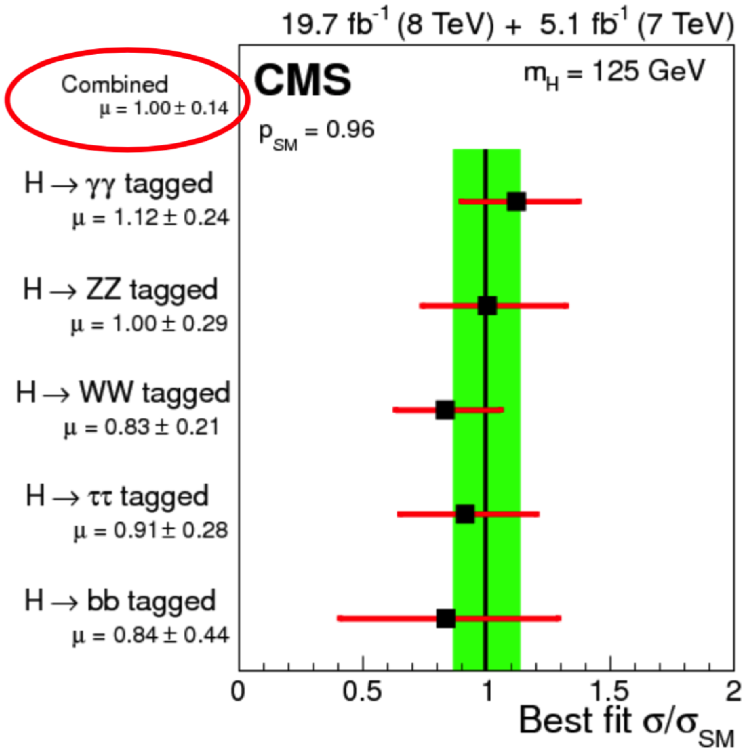
$$\mathbf{\Phi} \;\equiv\; (\phi\,,\; i\sigma^2\phi^*) \quad\mapsto\quad \mathbf{\Phi}' \;\equiv\; U_L\,\mathbf{\Phi}\,U_{\rm C}\,,$$

with $U_L \in \mathrm{SU}(2)_L$, $U_{\rm C} \in \mathrm{SU}(2)_{\rm C}$, and $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_{\rm C}/\mathbb{Z}_2 \simeq \mathrm{S}\mathbb{O}(4)$

Higgs Boson @ LHC: Signal Strength for Decay Modes

Signal strength: $\mu = \sigma_{observed}/\sigma_{SM}$





- Results consistent with SM

• The Coleman-Weinberg Effective Potential [S. Coleman and E. Weinberg '73; based on J. Goldstone, A. Salam, S. Weinberg '62, G. Jona-Lasinio, '64.]

Loopwise $\hbar$ Expansion of the Scalar Potential:

$$V_{\rm eff}(\phi) \;=\; V(\phi) \;-\; \frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\;\ln\left(\frac{k^2 - V''(\phi)}{k^2}\right) \;+\; \mathcal{O}(\hbar^2)$$

The Coleman–Weinberg Effective Potential

- is based on the One-Particle-Irreducible (1PI) formalism
- breaks Classical Scale Symmetries $\Longrightarrow$ Dimensional Transmutation

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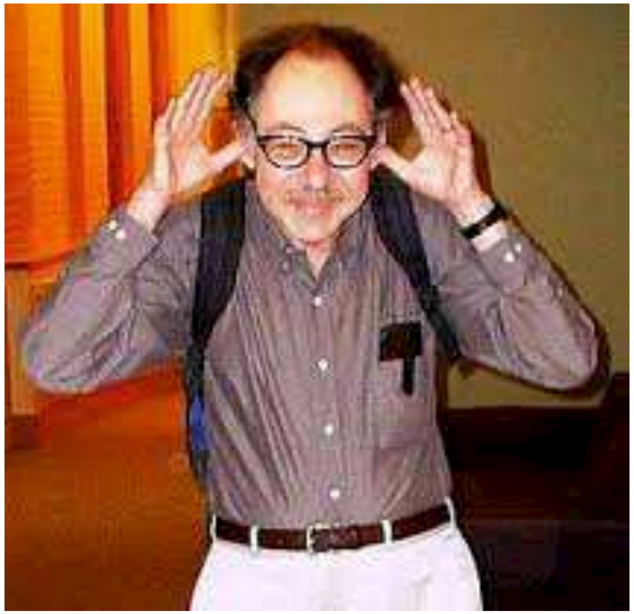
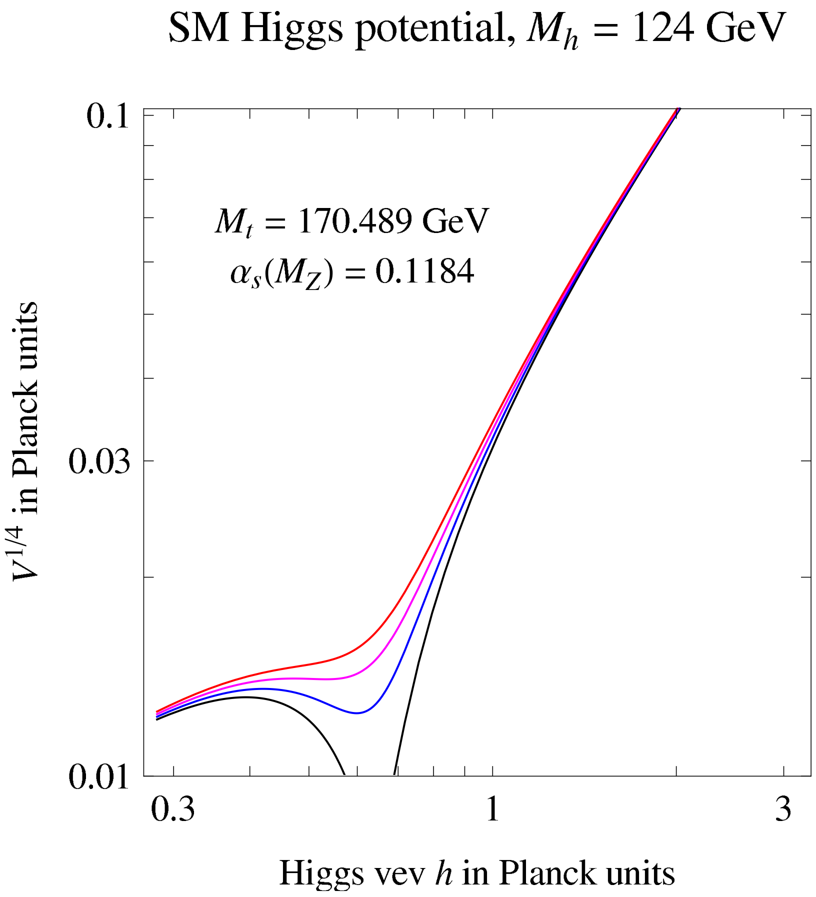
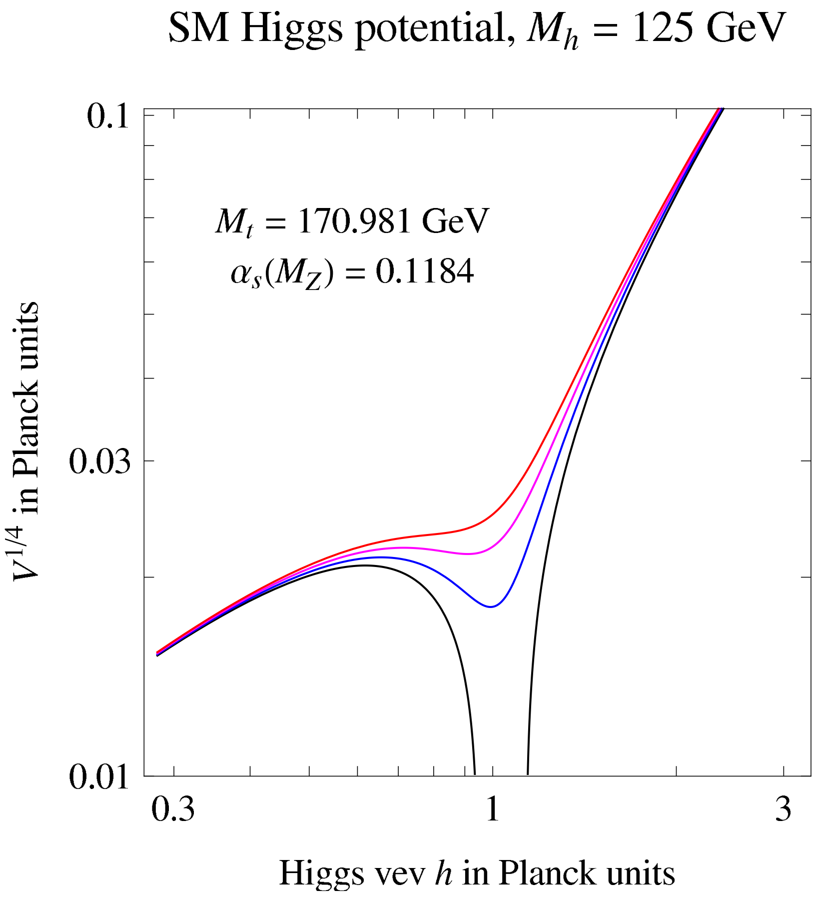


• The SM Coleman-Weinberg Effective Potential at NNLO

[G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia, JHEP1208 (2012) 098;
F. Bezrukov, M. Y. Kalmykov, B.A. Kniehl, M. Shaposhnikov, JHEP1210 (2012) 140.]



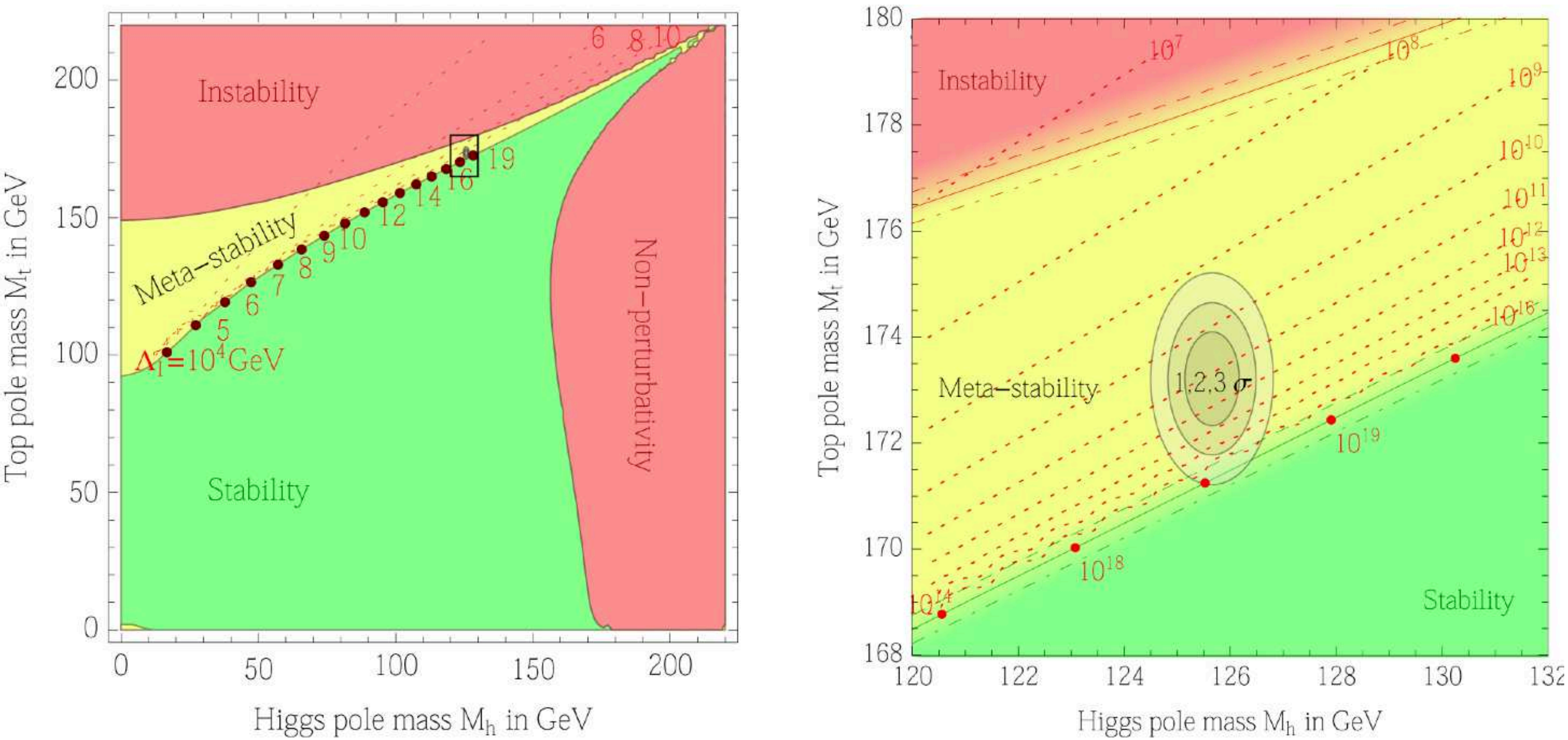


Higgs Potential versus Variations in Top Mass M_t by **0.1 MeV**

[Analysis includes the Multi-Critical scenario: D.L. Bennett, H.B. Nielsen, IJMA9 (1994) 5155]

• **Metastability** of the **SM Vacuum**

[D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio, and A. Strumia, JHEP1312 (2013) 089]



But, Planckian physics may modify stability predictions by many orders!

[V. Branchina, E. Messina, PRL111 (2013) 241801]

• The 2PI Effective Action and Field-Theoretic Problems

[J.M. Cornwall, R. Jackiw, E. Tomboulis, PRD10 (1974) 2428]

Connected Generating Functional of 2PI Effective Action:

$$W[J,K] \;=\; -i\,\ln \int \mathcal{D}\phi^i\,\exp\left[i\left(S[\phi]\;+\;J^i_x\,\phi^i_x\;+\;\frac{1}{2}\,K^{ij}_{xy}\,\phi^i_x\,\phi^j_y\right)\right]\;,$$

where $S[\phi] = \int_x \mathcal{L}[\phi]$ is the classical action of a $\mathbb{O}(N)$ theory.

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Legendre transform of $W[J,K]$ with respect to J and K:

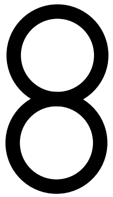
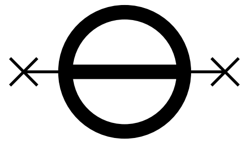
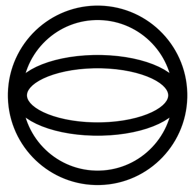
$$\frac{\delta W[J,K]}{\delta J^i_x}\;\equiv\;\phi^i_x\;,\qquad \frac{\delta W[J,K]}{\delta K^{ij}_{xy}}\;=\;\frac{1}{2}\,\left(i\Delta^{ij}_{xy}\;+\;\phi^i_x\,\phi^j_y\right)\;,$$

to get the 2PI effective action

$$\Gamma[\phi,\Delta]\;=\;W[J,K]\;-\;J^i_x\,\phi^i_x\;-\;\frac{1}{2}\,K^{ij}_{xy}\,\left(i\Delta^{ij}_{xy}\;+\;\phi^i_x\phi^j_y\right)\;.$$

The 2PI Effective Action:

$$\Gamma[\phi,\Delta] \;=\; S[\phi] \;+\; \frac{i}{2}\,\mathrm{Tr}\,\ln\Delta^{-1} \;+\; \frac{i}{2}\,\mathrm{Tr}\,(\Delta^{0^{-1}}\Delta) \;-\; i\,\Gamma^{(2)}_{\rm 2PI}[\phi,\Delta]\;,$$

where $\Gamma^{(2)}_{\rm 2PI}[\phi,\Delta]$ $=$  $+$  $+$  $+\;\ldots$

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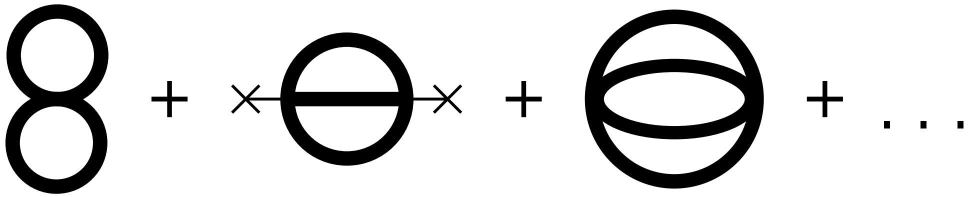
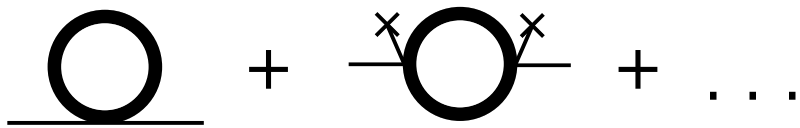
where $\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta] \;=$



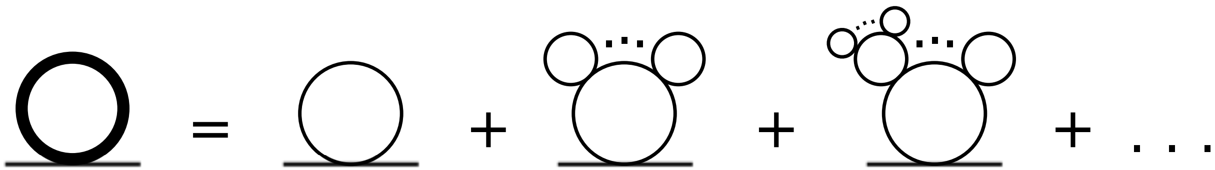
Equations of Motion:

- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi} \;=\; 0$$

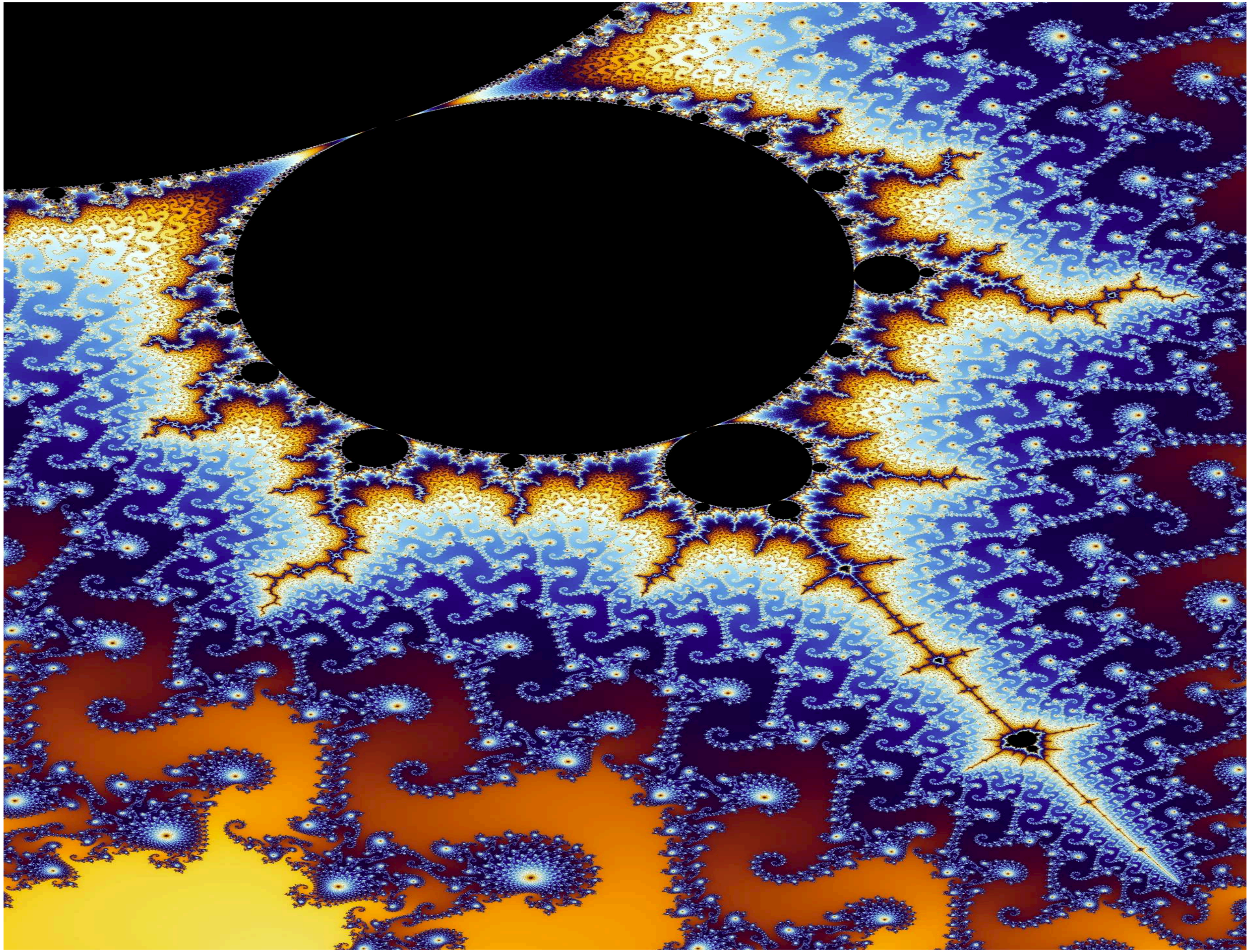
- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta} \;=\; 0 \;\Rightarrow\; \Delta^{-1} \;=\; {\Delta^0}^{-1} \;+$$



Hartree-Fock:



Artist's impression of the infinite HF-term!



– Field-Theoretic Problems addressed with 2PI

- Systematic formal resummation of high-order graphs:
  - Rigorous Derivation of Schwinger–Dyson Equations
  - Thermal Masses in the high-T Regime
  - Finite-Width Effects within Quantum Loops
  - The IR Problem of the Coleman-Weinberg Effective Potential

- Non-Equilibrium QFT, through Kadanoff–Baym equations.

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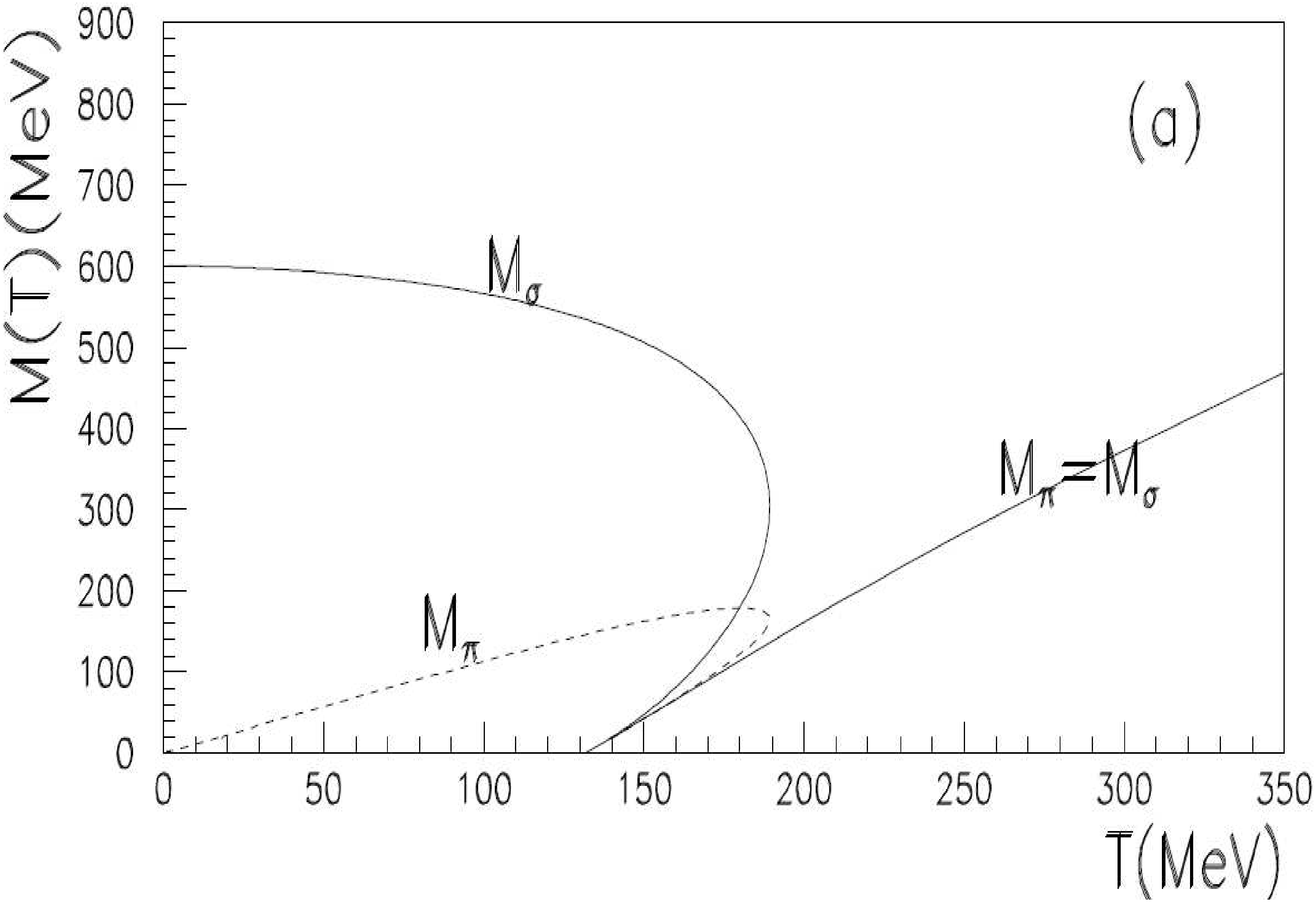
BUT

- Truncations of 2PI lead to residual violations of symmetries, e.g. global or local symmetries.

$\rightarrow$ Erroneous First-Order Phase Transition in $\mathbb{O}(N)$ Theories
$\rightarrow$ Goldstone Bosons become Massive
$\rightarrow$ Erroneous Thresholds for the Resummed Higgs-Boson Propagator.
$\rightarrow$ $\cdots$

• QCD Phase Transition in the Standard 2PI Formalism

[N. Petropoulos, J. Phys. G 25 (1999) 2225]



Pertinent Literature to the Goldstone-Symmetry Problem

- G. Baym, G. Grinstein, Phys. Rev. D **15** (1977) 2897.

⋮

- G. Amelino-Camelia, Phys. Lett. B **407** (1997) 268.
- N. Petropoulos, J. Phys. G **25** (1999) 2225.
- Y. Nemoto, K. Naito, M. Oka, Eur. Phys. J. A **9** (2000) 245.
- J. T. Lenaghan, D. H. Rischke, J. Phys. G **26** (2000) 431.
- H. van Hees, J. Knoll, Phys. Rev. D **66** (2002) 025028.
- J. Baacke, S. Michalski, Phys. Rev. D **67** (2003) 085006.
- Y. Ivanov, F. Riek, H. van Hees, J. Knoll, Phys. Rev. D **72** (2005) 036008.
- E. Seel, S. Struber, F. Giacosa, D. H. Rischke, Phys. Rev. D **86** (2012) 125010.
- G. Markó, U. Reinosa, Z. Szép, Phys. Rev. D **87** (2013) 105001.

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[AP, D. Teresi, NPB874 (2013) 594]

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Equivalence between 1PI and 2PI Effective Actions to All Orders:

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1PI Ward Identity (e.g. for $\mathbb{O}(2)$):

$$\frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta\phi^i_x}\,T^a_{ij}\,\phi^j_x \;=\; 0 \quad\Longrightarrow\quad v\int_x \frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta H} \;\to\; 0\,.$$

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Replace:

$$\frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \Delta^{-1,G}_{xy}\;,$$

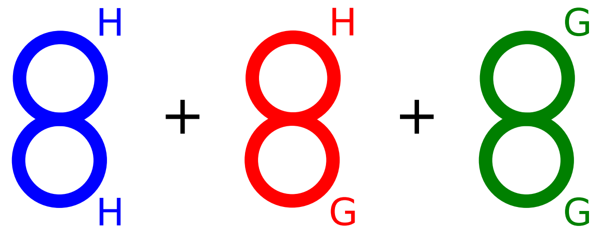
to obtain the Symmetry-Improved Equations of Motion:

$$\frac{\delta\Gamma[v\,,\,\Delta]}{\delta\Delta_{H/G}} \;=\; 0\;,$$

$$v\,\Delta^{-1}_G(k=0,v) \;=\; 0\;.$$

– $\mathbb{O}(2)$ Hartree–Fock Equations of Motion:

HF Approximation: $\Gamma^{(2)}_{\rm HF}[\Delta_H\,,\,\Delta_G] \;=$



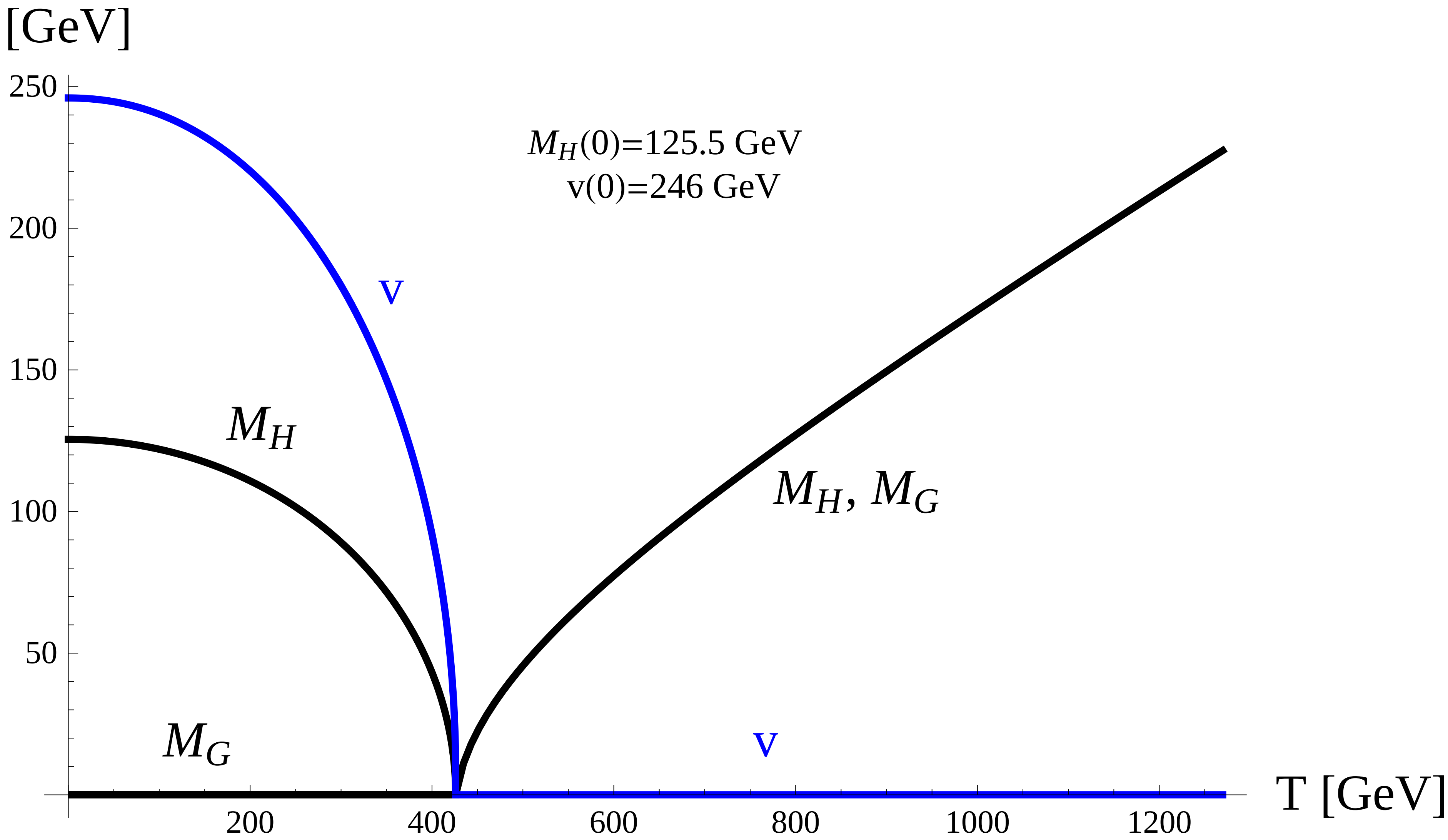
Ansatz: $\Delta^{-1}_{H/G}(k) \;=\; k^2 \;-\; M^2_{H/G} \;+\; i\varepsilon$

Equations of Motion:

$$
\begin{aligned}
M_H^2 \;&=\; 3\lambda v^2 - m^2 \;+\; (\delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;-\; \delta m_1^2 \\
&\quad +\; (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_H(k) \;+\; (\lambda + \delta\lambda_2^A)\int_k i\Delta_G(k)\;, \\
M_G^2 \;&=\; \lambda v^2 - m^2 \;+\; \delta\lambda_1^A v^2 \;-\; \delta m_1^2 \\
&\quad +\; (\lambda + \delta\lambda_2^A)\int_k i\Delta_H(k) \;+\; (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_G(k)\;, \\
v\,M_G^2 \;&=\; 0\;.
\end{aligned}
$$

– Second-Order Phase Transition in the HF Approximation

[AP, D. Teresi, NPB874 (2013) 594]



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Symmetry-Improved Effective Potential $\widetilde{V}_{\rm eff}(\phi)$ from 1PI Ward Identity:

$$\phi\,\Delta_G^{-1}(k=0;\phi) \;=\; -\,\frac{d\widetilde{V}_{\rm eff}(\phi)}{d\phi}\;.$$

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Solution:

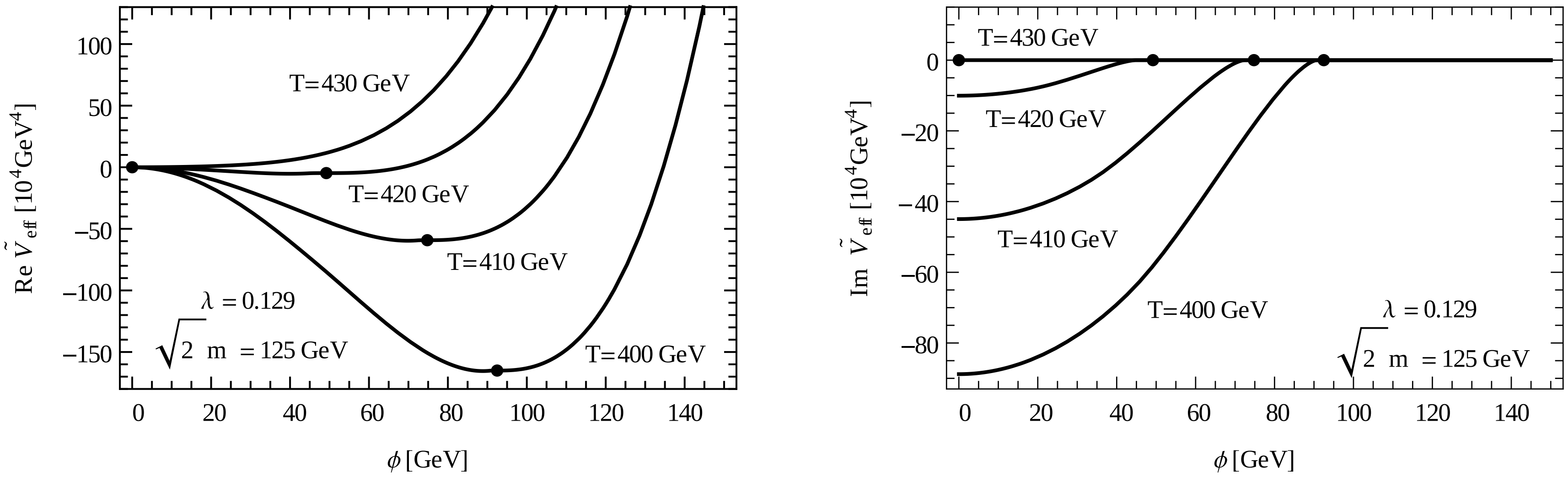
$$\begin{aligned}
\widetilde{V}_{\rm eff}(\phi) \;&=\; -\int_0^\phi d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi)\;+\;\widetilde{V}_{\rm eff}(\phi=0)\\
&=\; -\int_v^\phi d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi)\;+\;P(T,\mu)\ ,
\end{aligned}$$

where $P(T,\mu)$ is the thermodynamic pressure $=$ hydrostatic pressure, i.e. it satisfies Baym's thermodynamic consistency.

[G. Baym, PR127 (1962) 1391]

– SI2PI Effective Higgs Potential

[AP, D. Teresi, NPB874 (2013) 594]



Note: $\mathrm{Im}\,\widetilde{V}_{\rm eff}(\phi) < 0$, for $0 < \phi < v$

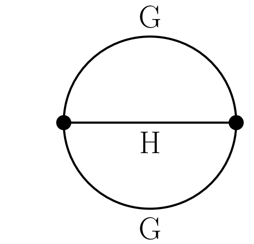
$\Longrightarrow$ Vacuum instability for the concave part of the potential.

[E.J. Weinberg, A.-Q. Wu, PRD36 (1987) 2474]

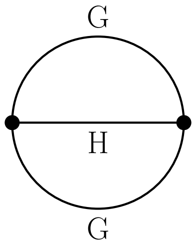
• IR Divergences in the Coleman–Weinberg Effective Potential

[S.P. Martin, PRD89 (2014) 013003]

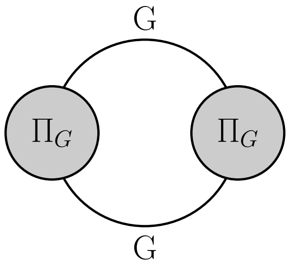
Contributions to the SM effective potential:



$\sim \quad m_G^2 \log m_G^2$ ✓

$\dfrac{d}{d\phi}$ 

$\sim \quad \log m_G^2$ ✗



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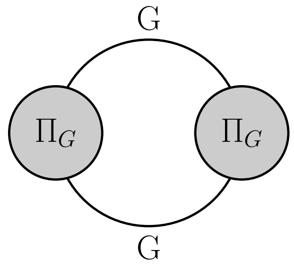
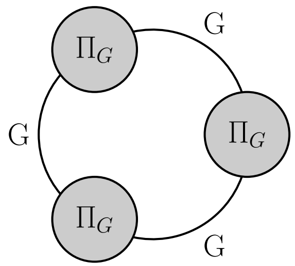
$\dfrac{d}{d\phi}$ 

$\sim \quad \dfrac{1}{m_G^2}$ ✗



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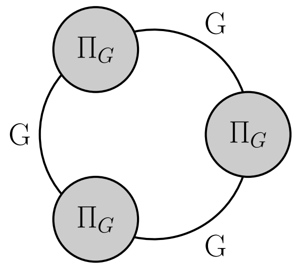
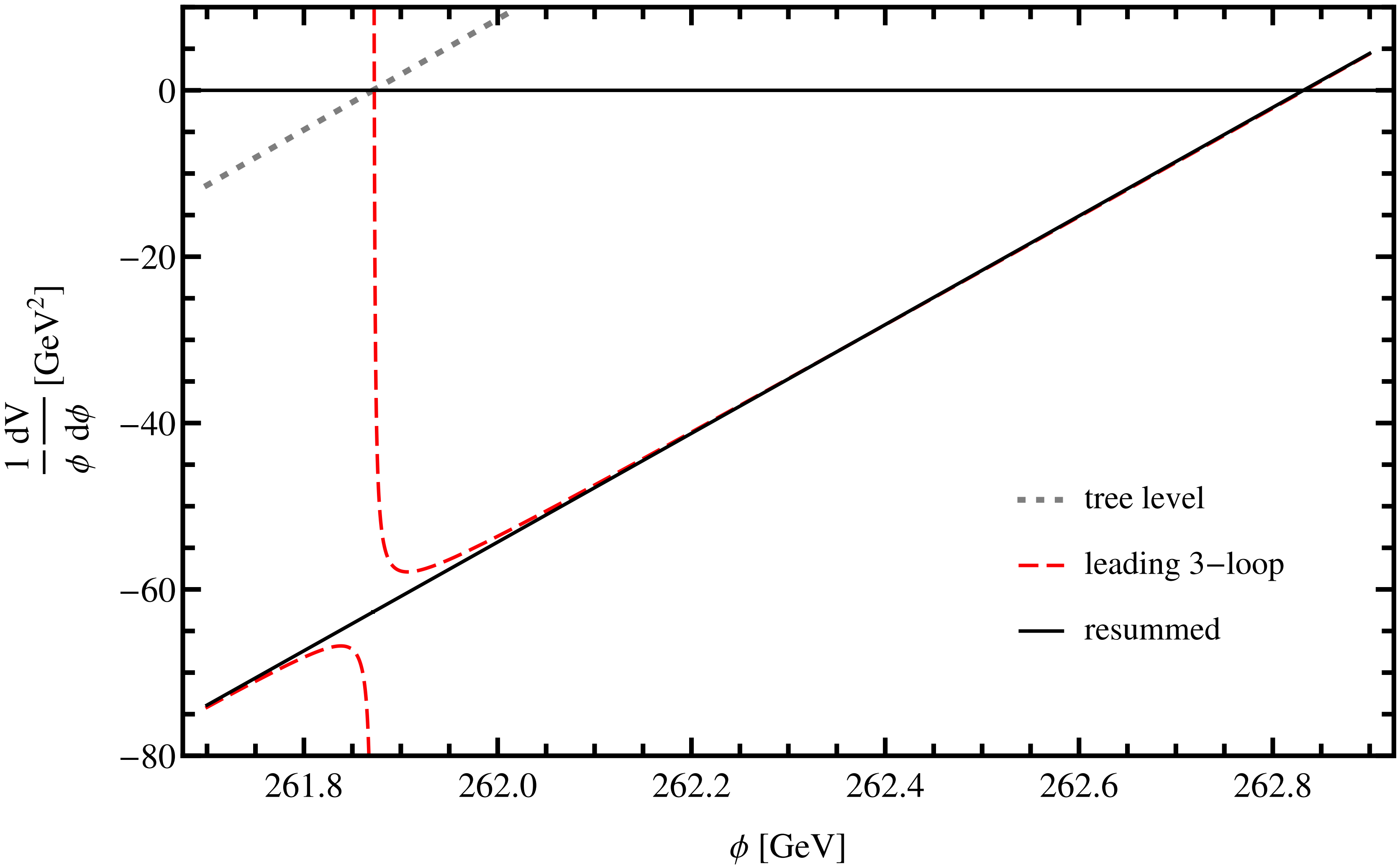
$\dfrac{d}{d\phi}$ 

$\sim \quad \left(\dfrac{1}{m_G^2}\right)^2$ ✗

$m_G^2 = \lambda\phi^2 - m^2$ $\qquad$ $M_G^2 = m_G^2 + \Pi_G^{(1)}|_{k=0} + \ldots = 0$ at $\phi = v$

– IR Divergence in the Coleman–Weinberg Potential



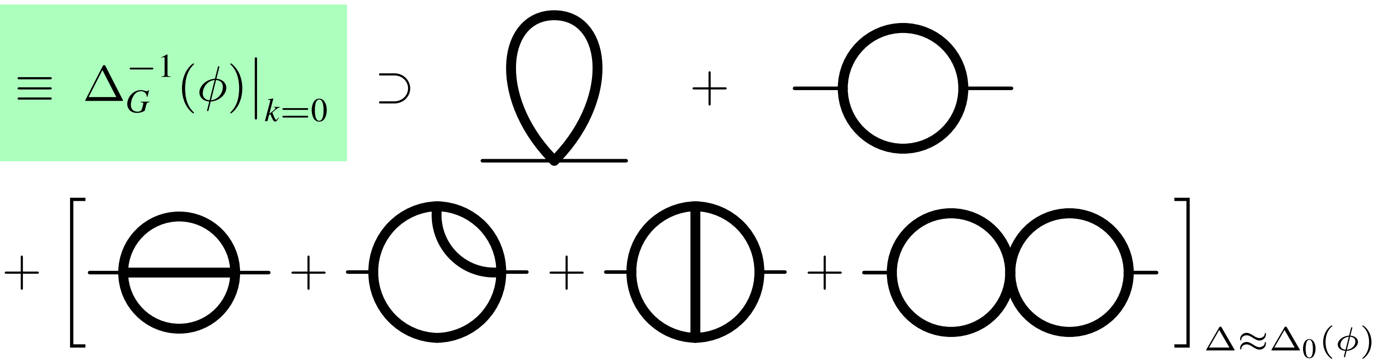
• SI2PI Approach to the Goldstone-Boson IR Problem

[A.P., D. Teresi, arXiv:1511.05347.]

- More complete resummation:
  - first-principle approach
  - it takes into account the momentum-dependence of self-energy insertions
  - more topologies
  - no ad-hoc subtraction of $m_G^2 \log m_G^2$ contributions in Π_G

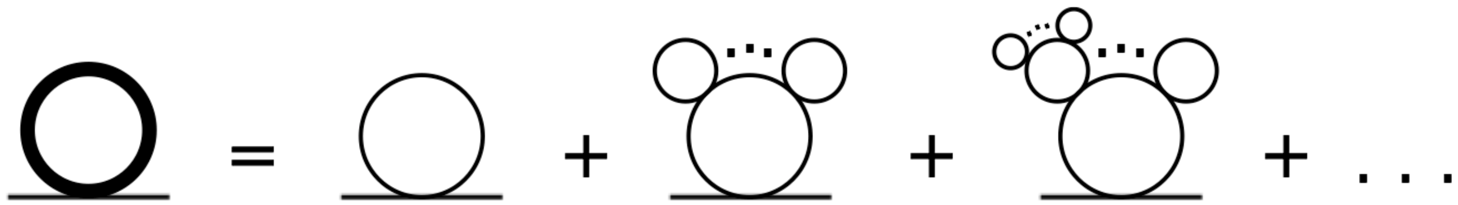
[S.P. Martin, PRD90 (2014) 016013;
J. Elias-Miro, J. Espinosa, T. Konstandin, JHEP1408 (2014) 034.]

- $$-\frac{1}{\phi}\frac{d\widetilde{V}_{\rm eff}}{d\phi} \equiv \Delta_G^{-1}(\phi)\Big|_{k=0} \supset$$



- No IR divergences: the would-be divergent self-energies are 2PR

– 2PI *Fractal* Resummation



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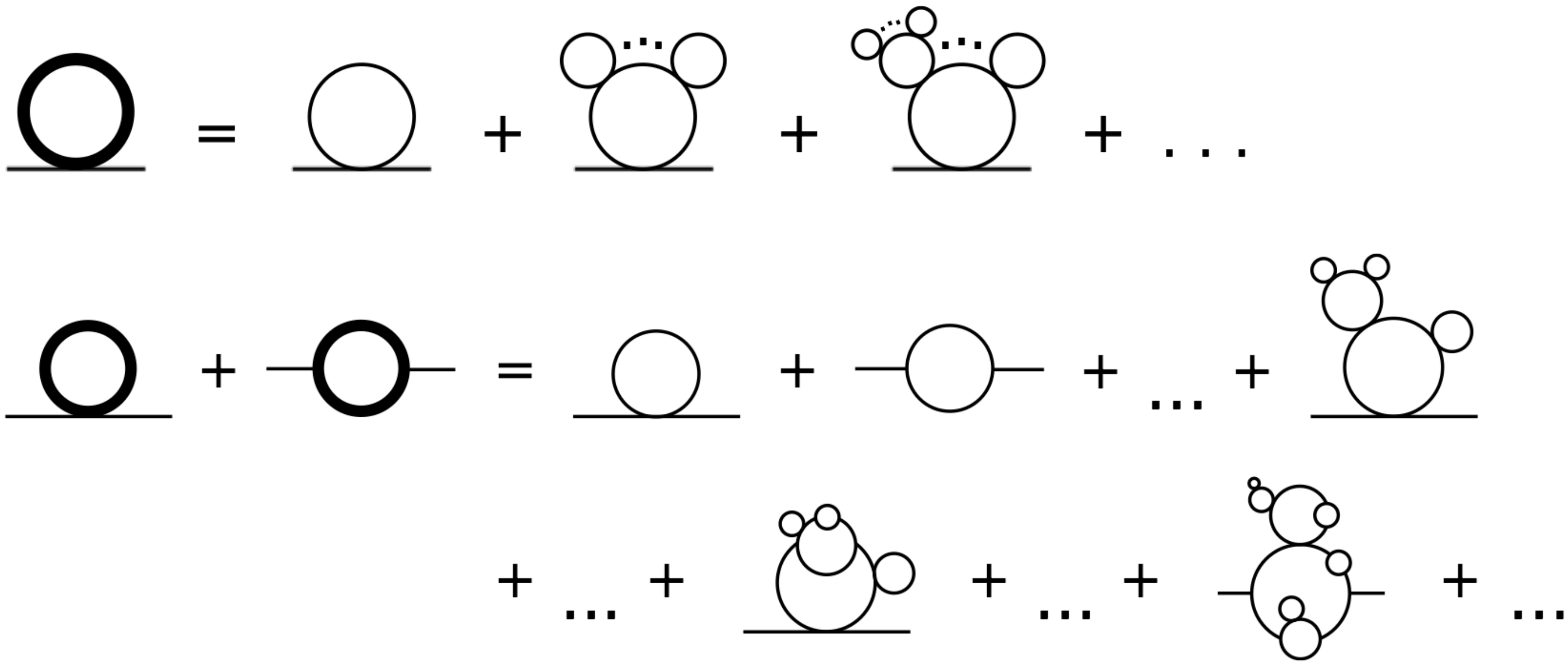

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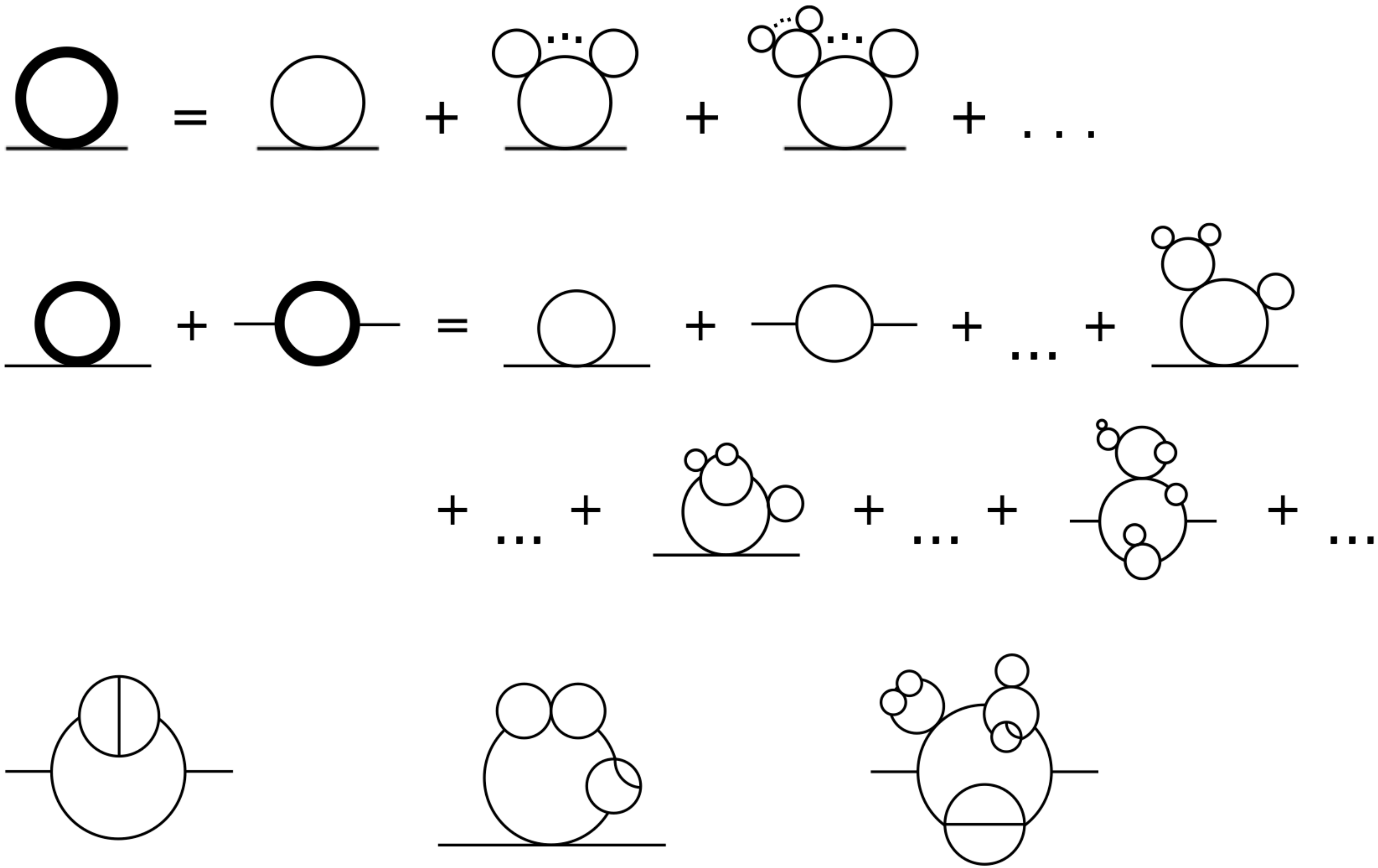
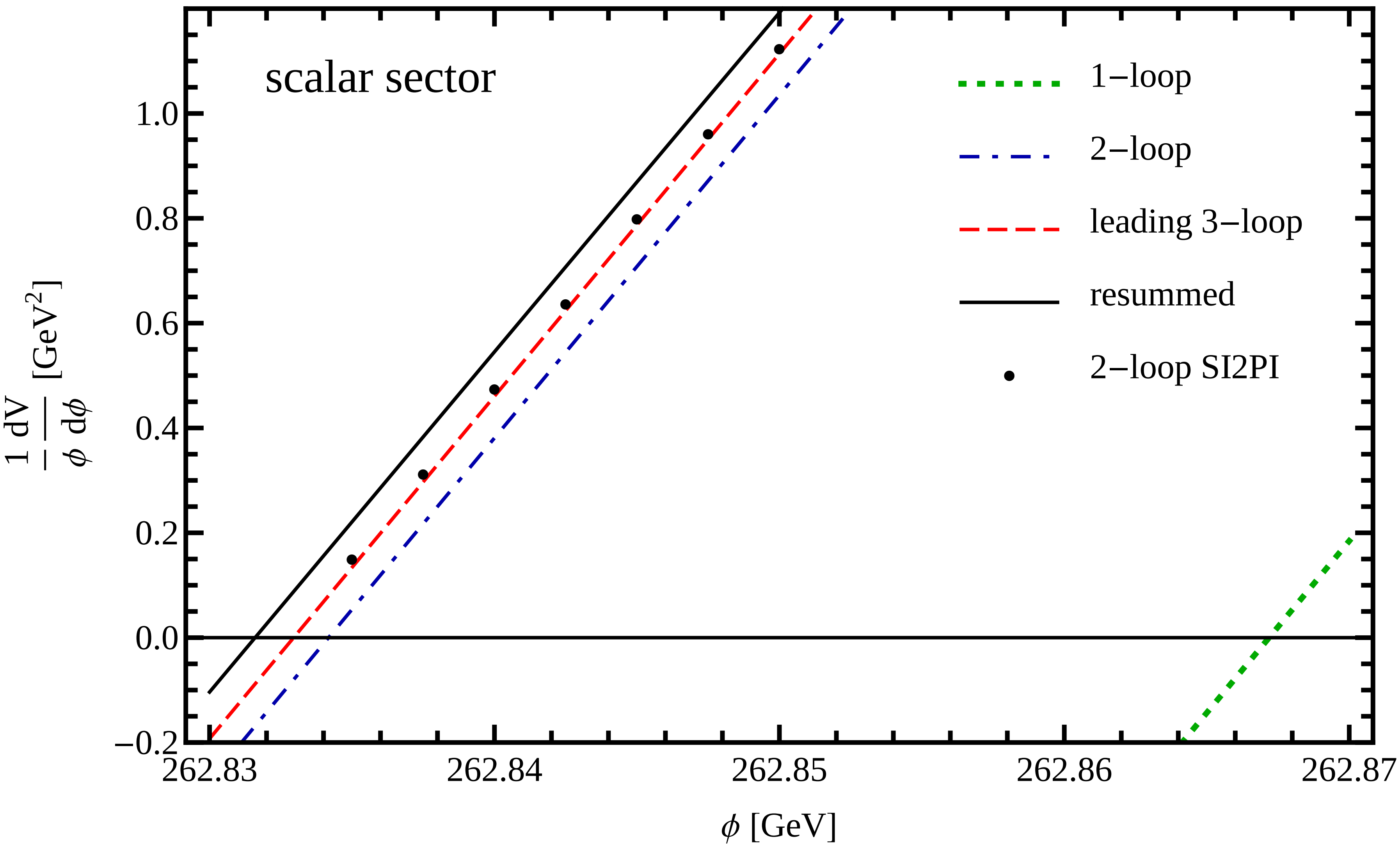


– Numerical Estimates

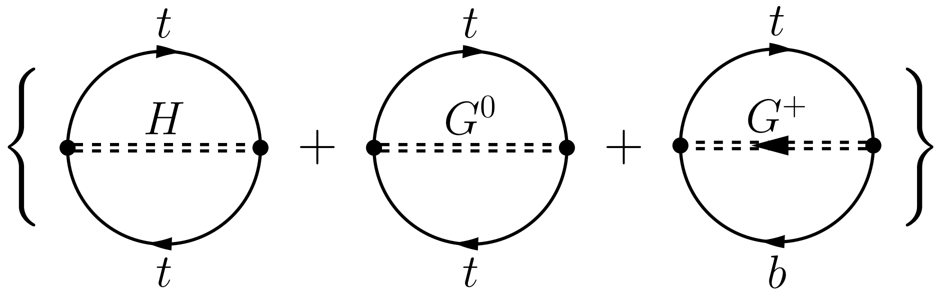
[A.P., D. Teresi, arXiv:1511.05347.]



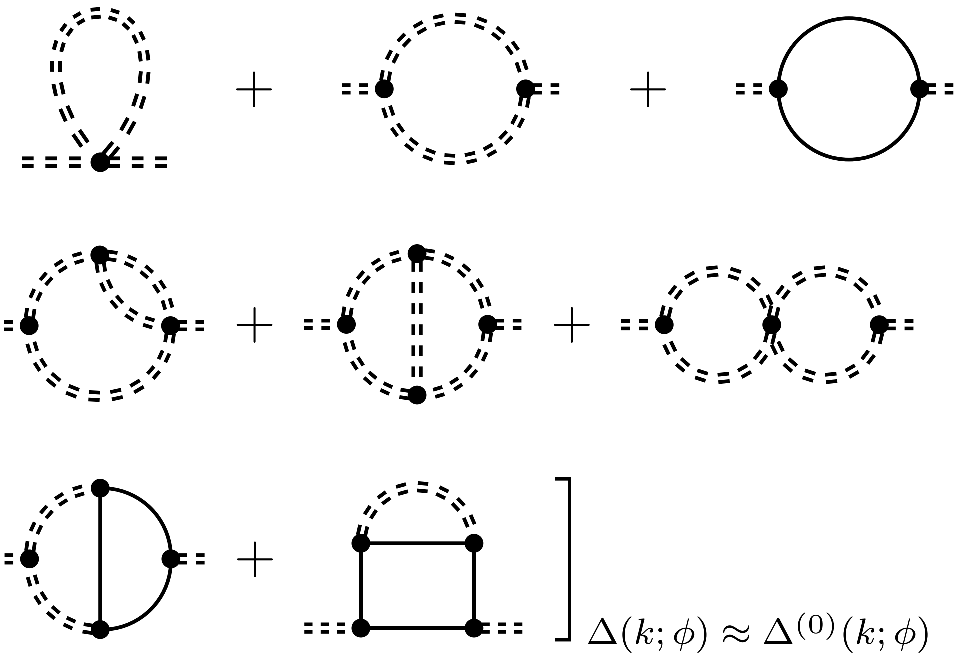
– Quantum Effects from Chiral Fermions

$$\Gamma^{(2)}[\phi, \Delta^H, \Delta^G, \Delta^+] \;=\; \Gamma^{(2)}_{\rm scalar}[\phi, \Delta^H, \Delta^G, \Delta^+] \;+\; 3\,i\; {\rm Tr}\ln S^{\alpha\,(0)}[\phi]$$

$$-\; i$$



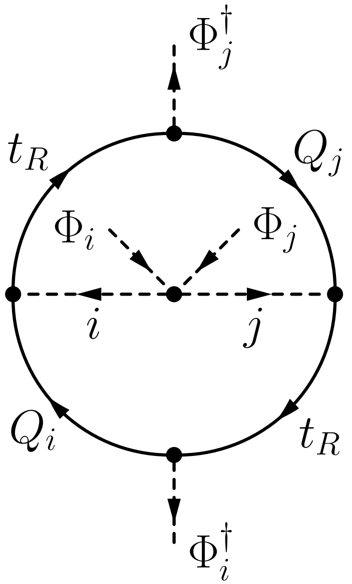
$$\Delta^{-1}(k;\,\phi) \quad = \quad \Delta^{(0)\,-1}(k;\,\phi) \quad +$$



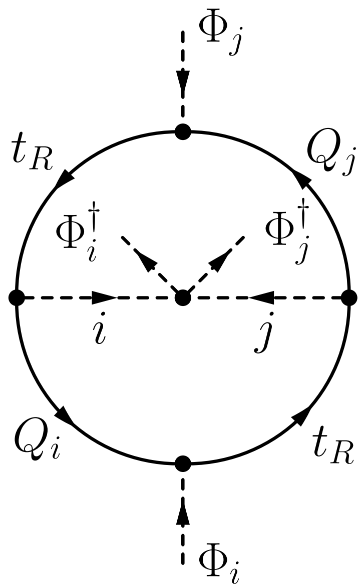
$$\Delta(k;\phi) \approx \Delta^{(0)}(k;\phi)$$

– Renormalization of Spurious Custodially Breaking Effects

[A.P., D. Teresi, arXiv:1511.05347.]

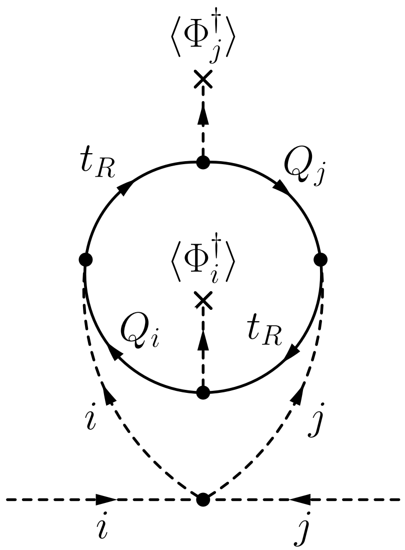


$+$

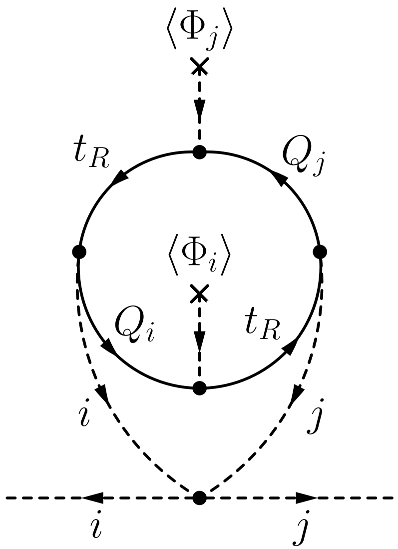


$$\sim \quad \lambda\, h_t^4\, (\Phi^\dagger\Phi)^2$$

• Graphs contained in the 2PI resummation:



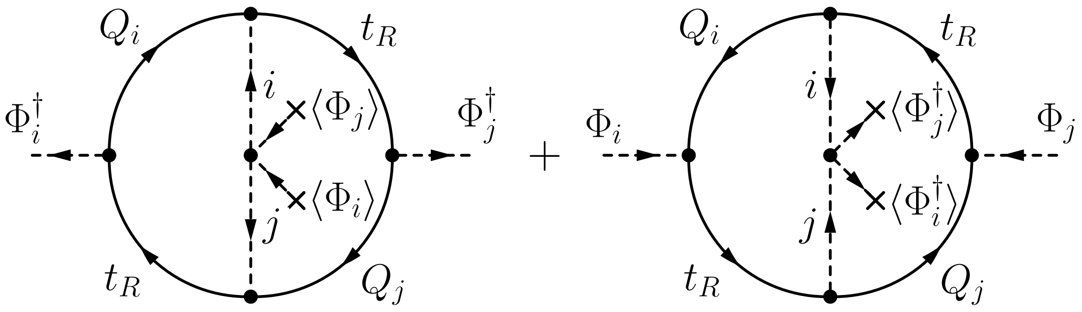
$+$



$$= C\,\Phi_i\Phi_j\langle\Phi_i^\dagger\rangle\langle\Phi_j^\dagger\rangle \;+\; C\,\Phi_i^\dagger\Phi_j^\dagger\langle\Phi_i\rangle\langle\Phi_j\rangle$$

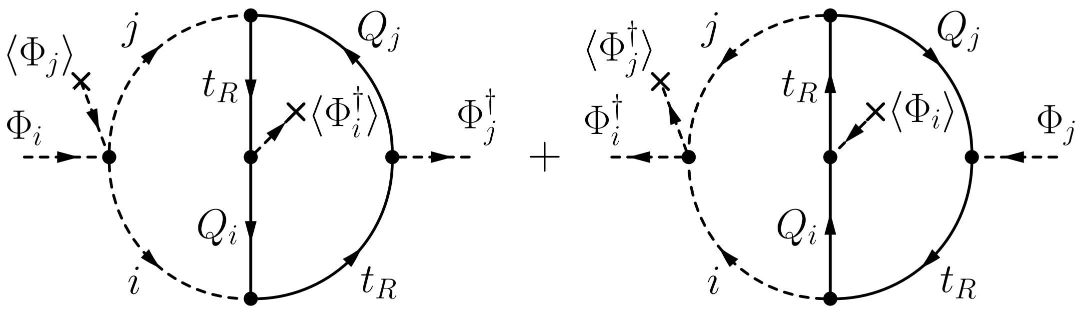
$$\supset\; -C\,\phi^2\,G^0G^0$$

- **Graphs NOT contained in the 2PI resummation:**



$$= C\,\langle\Phi_i\rangle\langle\Phi_j\rangle\Phi_i^\dagger\Phi_j^\dagger$$

$$+\ C\,\langle\Phi_i^\dagger\rangle\langle\Phi_j^\dagger\rangle\Phi_i\Phi_j\ \supset\ -C\,\phi^2\,G^0G^0$$



$$+\quad (i\leftrightarrow j)$$

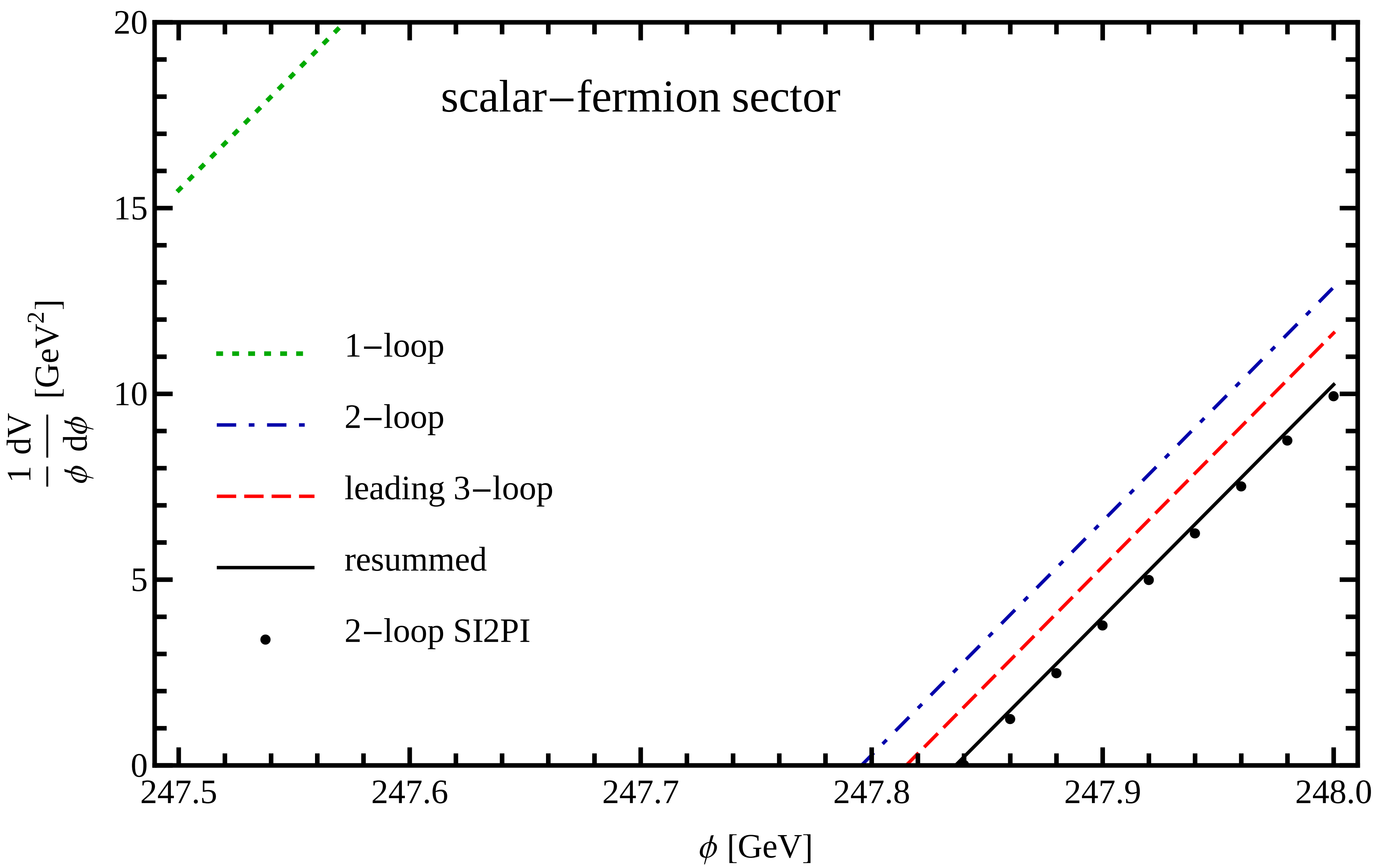
$$= C\,\Phi_i\langle\Phi_j\rangle\langle\Phi_i^\dagger\rangle\Phi_j^\dagger\ +\ C\,\Phi_i^\dagger\langle\Phi_j^\dagger\rangle\langle\Phi_i\rangle\Phi_j\ +\ (i\leftrightarrow j)\ \supset\ 2\,C\,\phi^2\,G^0G^0$$

- **Renormalization condition for determining $\delta\lambda^{\rm cb}$:**

$$\Delta^{-1,G^0}(k=0;v)\ \stackrel{!}{=}\ \Delta^{-1,G^+}(k=0;v)\ \stackrel{!}{=}\ 0\,.$$

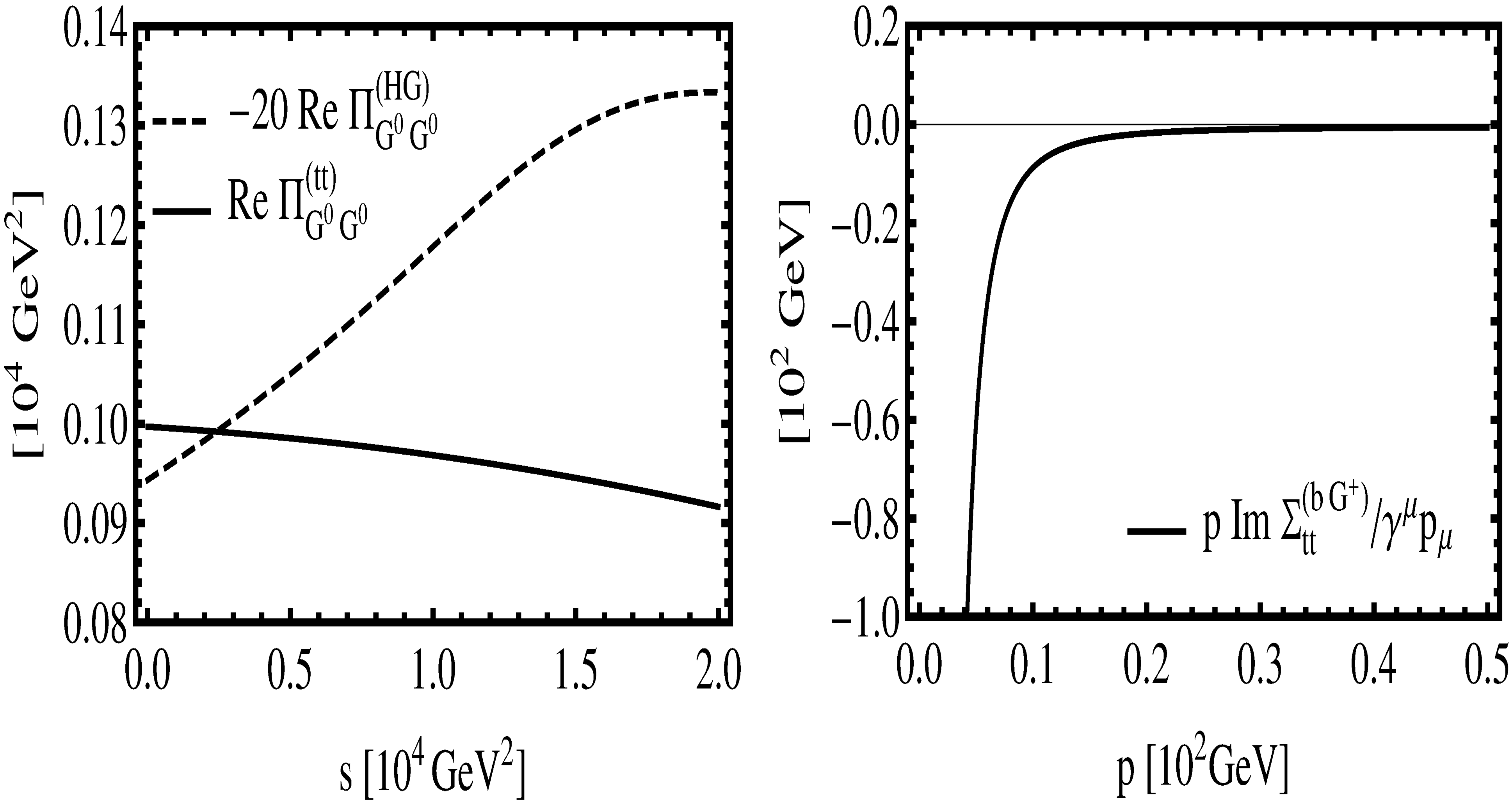
– Numerical Estimates (with fermion effects included)

[A.P., D. Teresi, arXiv:1511.05347.]



– Scalar versus Fermion Contributions to $\Pi_{GG}(s)$

[A.P., D. Teresi, arXiv:1511.05347.]



• Conclusions

- **Preserving symmetries in loop-truncated 2PI actions is a long-standing problem**

- **Novel Approach to Global Symmetries:**

  $\Longrightarrow$ **Symmetry-Improved 2PI Effective Action**

$\rightarrow$ Massless Goldstone Bosons
$\rightarrow$ 2nd-Order Phase Transition in the HF Approximation

- $\overline{\rm MS}$ **Renormalization with T-independent Resummed Counterterms**

- **Absorptive Effects properly described:**

$\rightarrow$ Smooth thresholds consistent with massless Goldstone bosons
$\rightarrow$ Consistent resummation within Quantum Loops.

- **The SI2PI Effective Potential is IR Safe by construction**

• Future Directions

- **Extension: 2PI $\rightarrow$ nPI Effective Actions**

  [3PI: M.J. Brown and I.B. Whittingham, PRD91 (2015) 085020.]

- **Extension to Local Symmetries: $\mathbb{U}(1)$, $\mathbb{SU}(N)$**

- **Spontaneous Breaking of Local Gauge Symmetries**

- **Precision Computations of SM & BSM Effective Potentials**

$\vdots$

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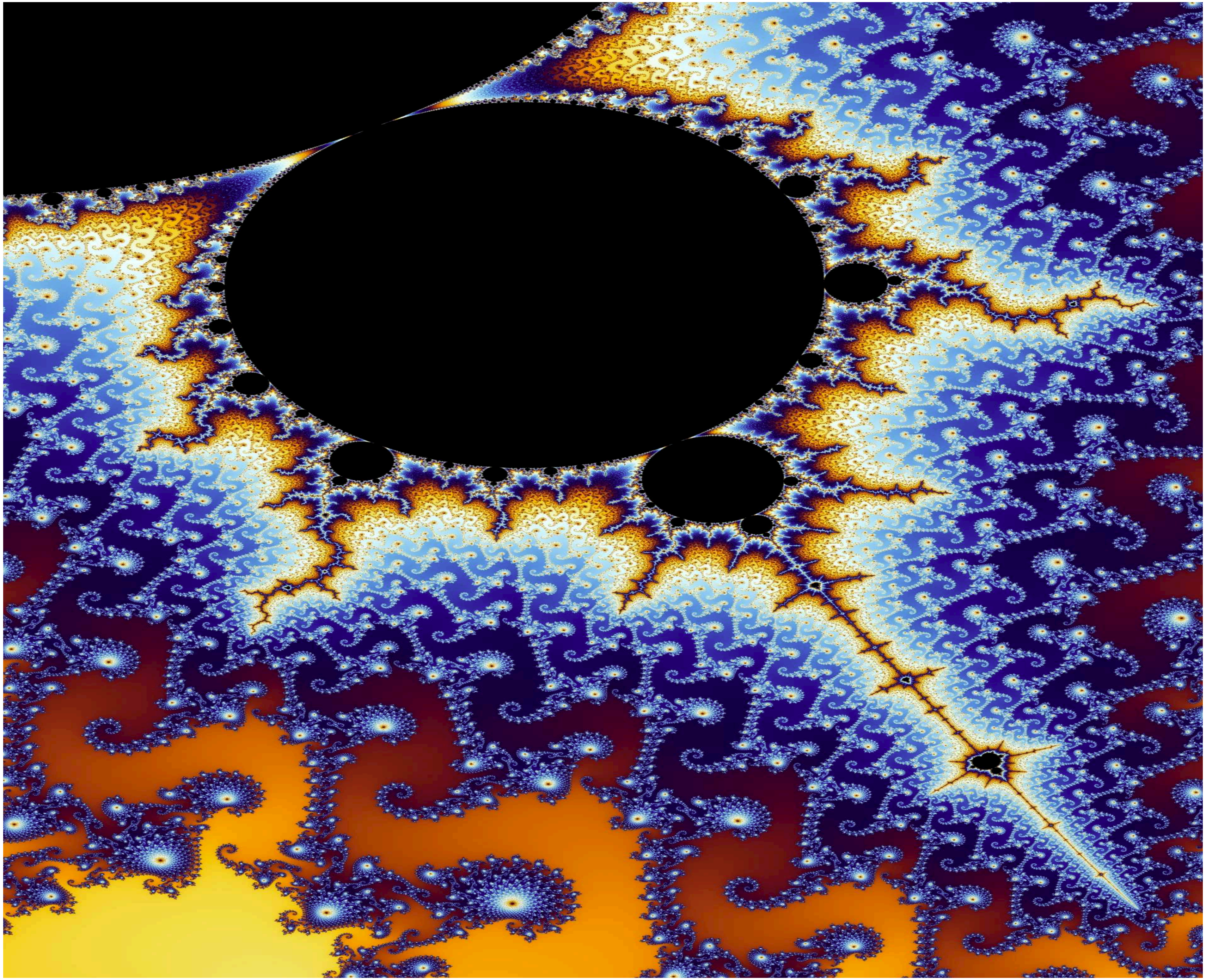
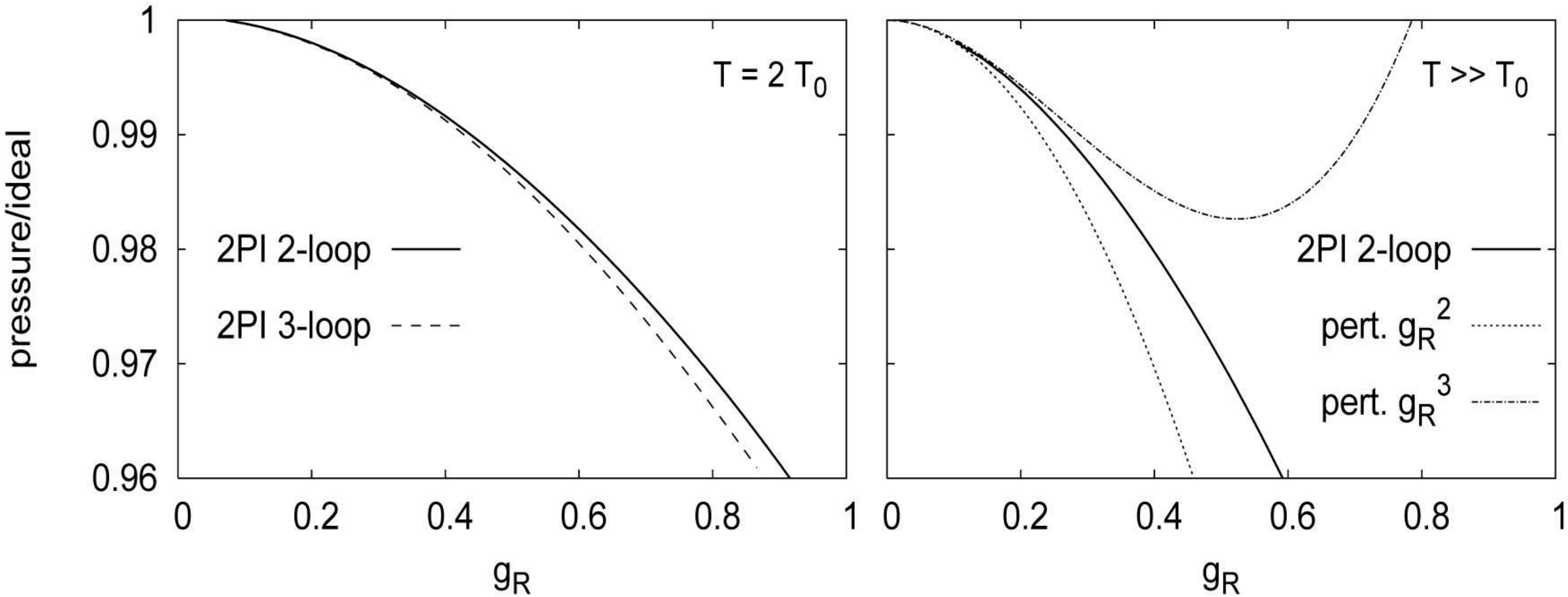
New Era of Analytical Non-Perturbative QFT



Back-Up Slides

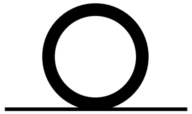
• Improved Loopwise Convergence in the 2PI Formalism

[J. Berges, S. Borsanyi, U. Reinosa, J. Serreau, PRD71 (2005) 105004]



• $\overline{\mathrm{MS}}$ Renormalization in 2PI

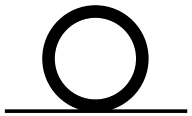
Naive Renormalization:

 $= \int_k i\Delta(k) \ \sim \ M^2 \, \frac{1}{\epsilon}\,,$ Naive CT: $\delta m^2 \stackrel{?}{=} M^2 \, \frac{1}{\epsilon}\ .$

But, $M^2 = M^2(T) \quad \Longrightarrow \quad T$emperature-dependent CT?!

• $\overline{\mathrm{MS}}$ Renormalization in 2PI

Naive Renormalization:

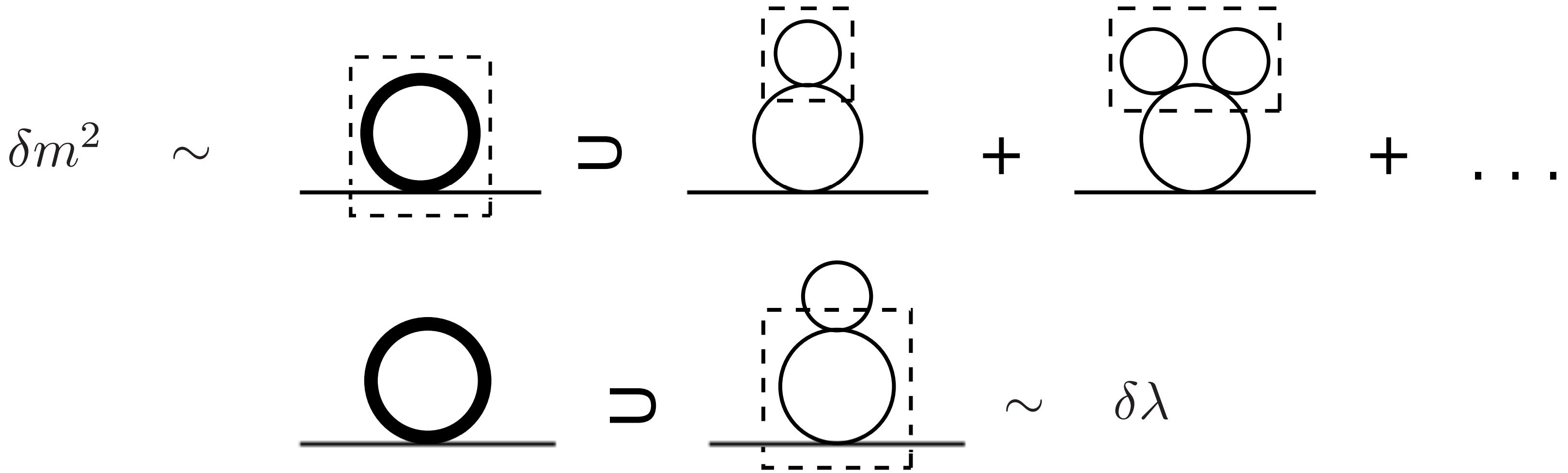


$$= \int_k i\Delta(k) \sim M^2 \frac{1}{\epsilon}\,, \qquad \text{Naive CT:} \quad \delta m^2 \overset{?}{=} M^2 \frac{1}{\epsilon}\,.$$

But, $M^2 = M^2(T) \quad \Longrightarrow \quad T$emperature-dependent CT?!

What has gone wrong?

[J.-P. Blaizot, E. Iancu, U. Reinosa, NPA736 (2004) 149;
J. Berges *et al*, AP320 (2005) 344]



Is there any systematic renormalization, e.g. in the $\overline{\mathrm{MS}}$ scheme?

[W.A. Bardeen, A. Buras, D. Duke, T. Muta, PRD18 (1978) 3998]

– $\overline{\mathrm{MS}}$ Renormalization in the 2PI Formalism

Procedure:

- Isolate UV infinities in EoMs, e.g. by Dimensional Regularization.

- Require that the UV-finite part of EoMs **be** UV finite:

$$(\ldots)_{\mathrm{UV}}\,\mathcal{T}_H^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\,\mathcal{T}_G^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\,v^2 \;+\; (\ldots)_{\mathrm{UV}}\,1 \;\stackrel{!}{=}\; 0$$

- Cancel separately the UV infinities $\propto$ $\mathcal{T}_H^{\mathrm{fin}}(T)$, $\mathcal{T}_G^{\mathrm{fin}}(T)$, $v^2(T)$, 1.

- Check UV consistency:

$4 \times 2 = 8$ Constraints, for 5 CTs: $\delta m_1^2\,,\ \delta\lambda_1^A\,,\ \delta\lambda_1^B\,,\ \delta\lambda_2^A\,,\ \delta\lambda_2^B\,.$

This is a non-trivial check!

– T-independent Resummed Counterterms in the HF approximation:

[AP, D. Teresi, NPB874 (2013) 594]

$$\delta\lambda_1^A \;=\; \delta\lambda_2^A \;=\; \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{3-\dfrac{4\lambda}{16\pi^2\epsilon}}{1-\dfrac{6\lambda}{16\pi^2\epsilon}+\dfrac{8\lambda^2}{(16\pi^2\epsilon)^2}}$$

$$\;=\; -\,\lambda\;+\;\frac{(16\pi^2\epsilon)^2}{8\lambda}\;+\;O(\epsilon^3)\;,$$

$$\delta\lambda_1^B \;=\; \delta\lambda_2^B \;=\; \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{2\lambda}{16\pi^2\epsilon}}$$

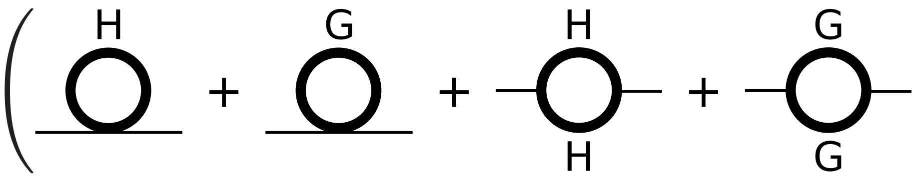
$$\;=\; -\,\lambda\;-\;\frac{16\pi^2\epsilon}{2}\;-\;\frac{(16\pi^2\epsilon)^2}{4\lambda}\;+\;O(\epsilon^3)\;,$$

$$\delta m_1^2 \;=\; \frac{4\lambda m^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{4\lambda}{16\pi^2\epsilon}} \;=\; -\,m^2\;-\;m^2\frac{16\pi^2\epsilon}{4\lambda}\;+\;O(\epsilon^2)\;.$$

• Finite-Width Effects within Quantum Loops

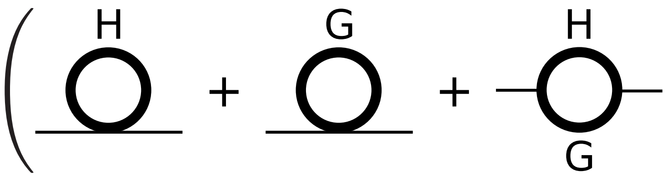
Equations of Motion including Sunset Diagrams:

- $$\Delta_H^{-1}(p) \;=\; p^2 \;-\; (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

$$-i\,\Bigg($$ 

$$\Bigg)\,,$$

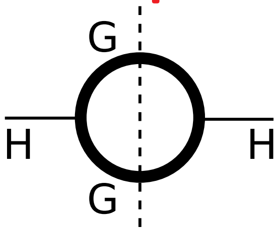
- $$\Delta_G^{-1}(p) \;=\; p^2 \;-\; (\lambda + \delta\lambda_1^A)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

$$-i\,\Bigg($$ 

$$\Bigg)\,,$$

- $$v\,\Delta_G^{-1}(0) \;=\; 0\,.$$

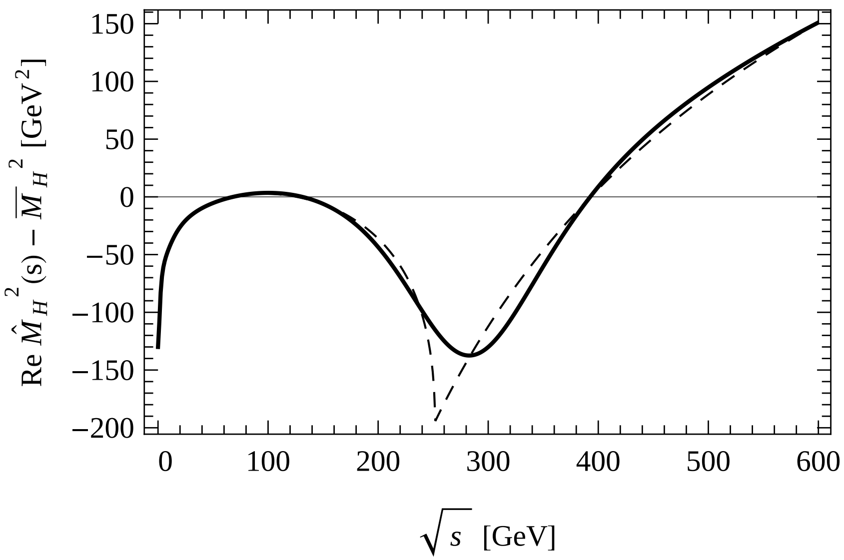
Absorptive Effects:

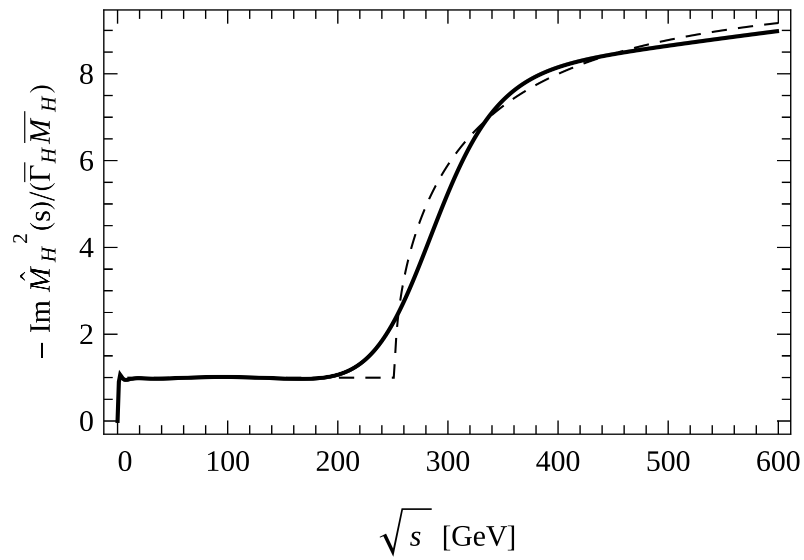


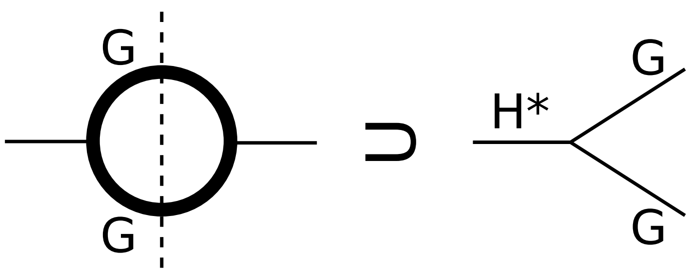
G consistently massless $\Longleftrightarrow$ threshold at $s \equiv p^2 = 0$

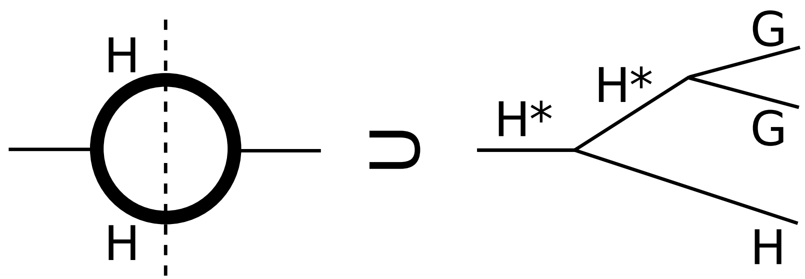
– Higgs Selfenergy in 2PI

[AP, D. Teresi, NPB874 (2013) 594]



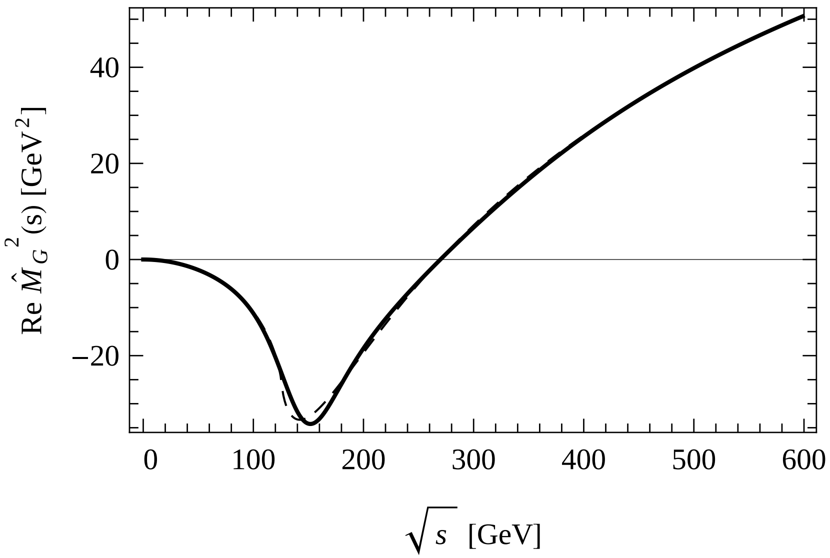


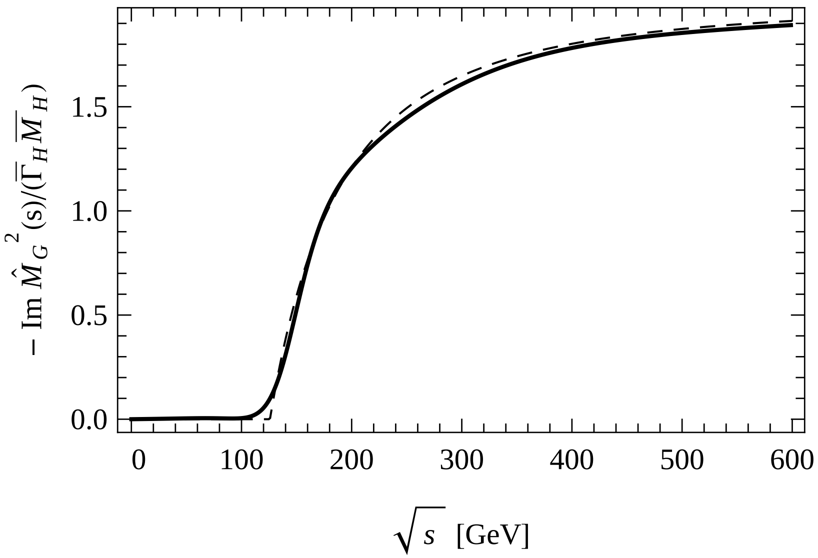


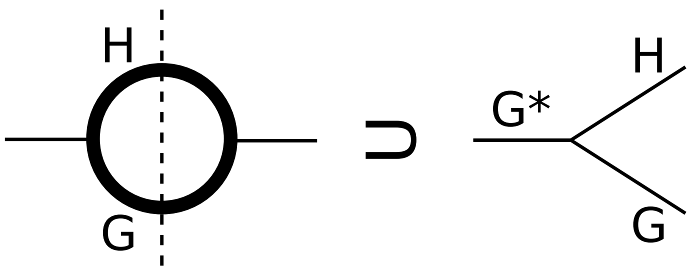


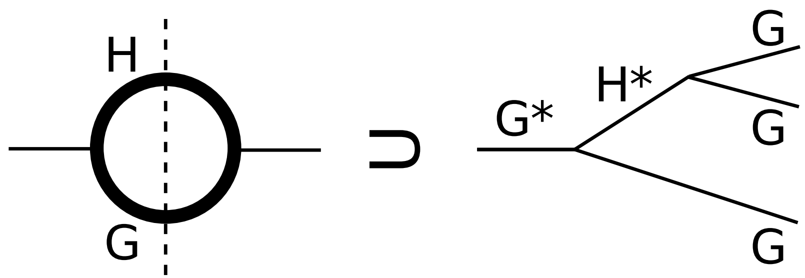
– Goldstone Selfenergy in 2PI

[AP, D. Teresi, NPB874 (2013) 594]



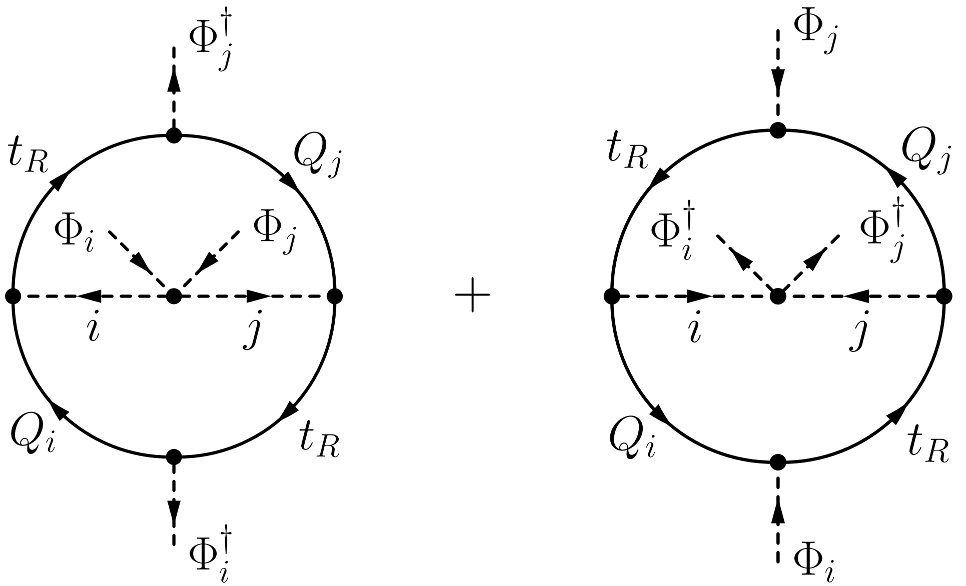






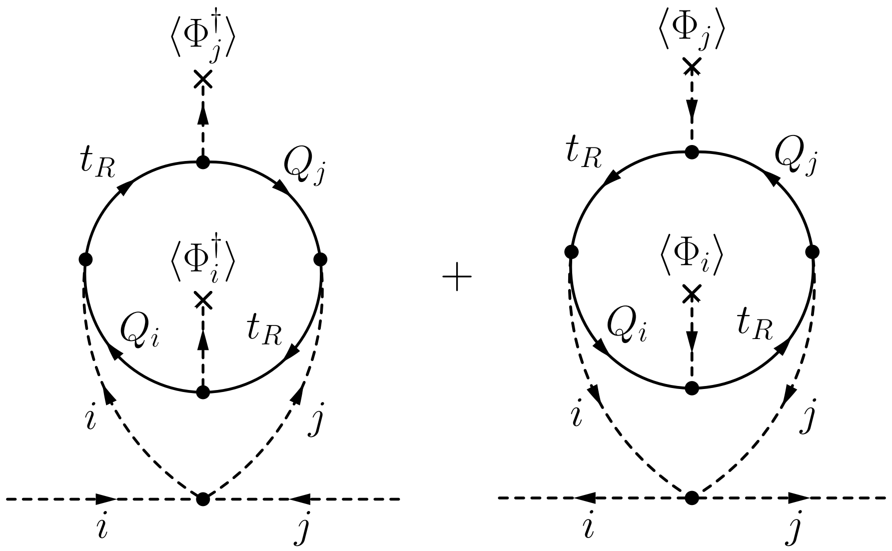
– Renormalization of Spurious Custodially Breaking Effects

[A.P., D. Teresi, arXiv:1511.05347.]



$$\sim \quad \lambda\, h_t^4\, (\Phi^\dagger \Phi)^2$$

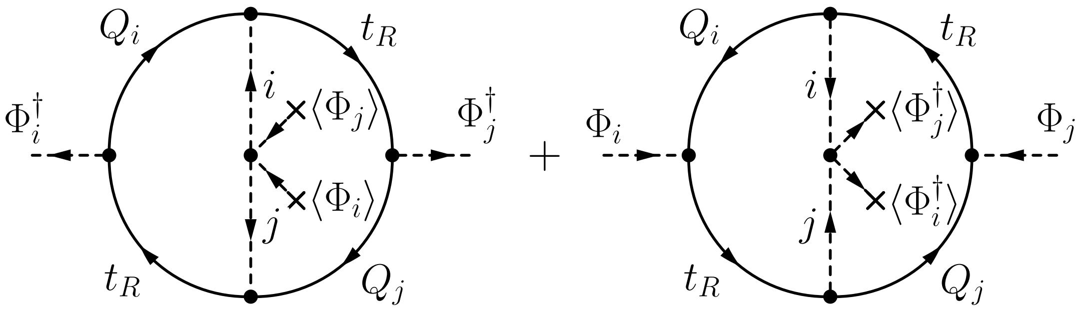
• Graphs contained in the 2PI resummation:



$$= \; C\,\Phi_i \Phi_j \langle \Phi_i^\dagger \rangle \langle \Phi_j^\dagger \rangle \;+\; C\,\Phi_i^\dagger \Phi_j^\dagger \langle \Phi_i \rangle \langle \Phi_j \rangle$$

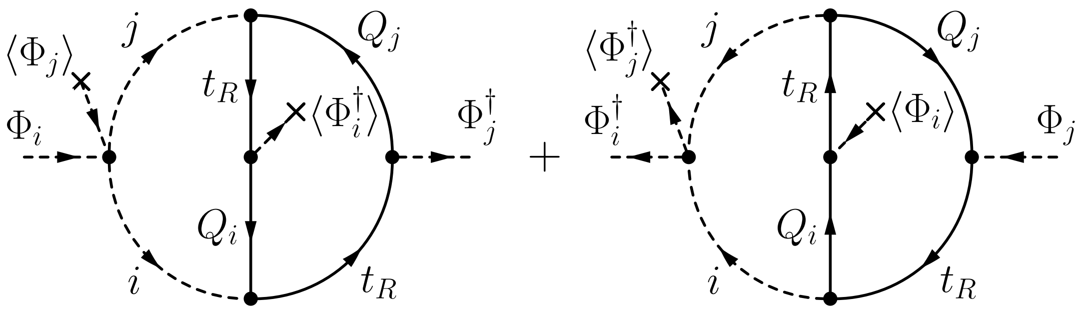
$$\supset \; -\,C\,\phi^2\, G^0 G^0$$

- **Graphs NOT contained in the 2PI resummation:**



$$= C\,\langle\Phi_i\rangle\langle\Phi_j\rangle\Phi_i^\dagger\Phi_j^\dagger$$

$$+\ C\,\langle\Phi_i^\dagger\rangle\langle\Phi_j^\dagger\rangle\Phi_i\Phi_j \ \supset\ -C\,\phi^2\,G^0G^0$$



$$+\quad (i \leftrightarrow j)$$

$$= C\,\Phi_i\langle\Phi_j\rangle\langle\Phi_i^\dagger\rangle\Phi_j^\dagger \ +\ C\,\Phi_i^\dagger\langle\Phi_j^\dagger\rangle\langle\Phi_i\rangle\Phi_j \ +\ (i\leftrightarrow j)\ \supset\ 2\,C\,\phi^2\,G^0G^0$$

- **Renormalization condition for determining $\delta\lambda^{\rm cb}$:**

$$\Delta^{-1,G^0}(k=0;v)\ \stackrel{!}{=}\ \Delta^{-1,G^+}(k=0;v)\ \stackrel{!}{=}\ 0\,.$$