How robust are particle physics predictions in asymptotic safety?

Daniele Rizzo

Based on

ArXiv:2304.08959

in collaboration with

K. Kowalska, W. Kotlarski & E.M. Sessolo

PLANCK 2023

The 25th International Conference From the Planck Scale to the Electroweak Scale

Warsaw – May 24th 2023

NARODOWE CENTRUM NAUKI

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Running of the Coupling Constants

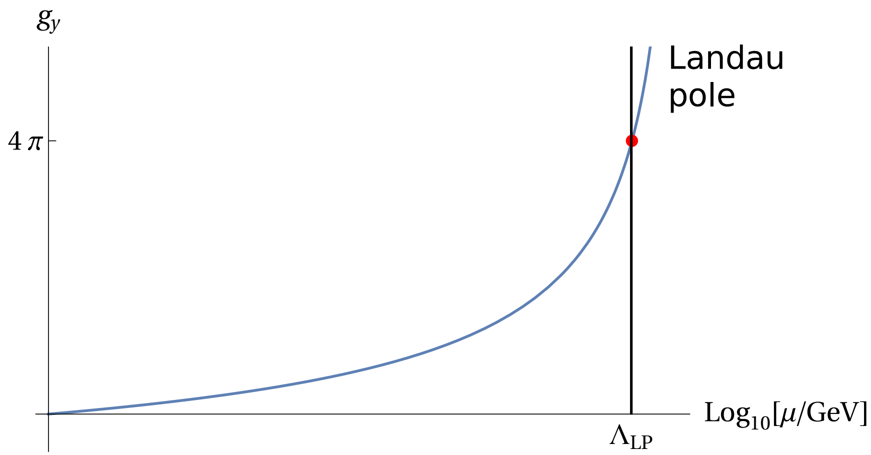
$$\beta(g_0) \equiv \frac{dg_0}{d\log\mu}$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Running of the Coupling Constants

In the Standard Model we see two possible asymptotic behavior:

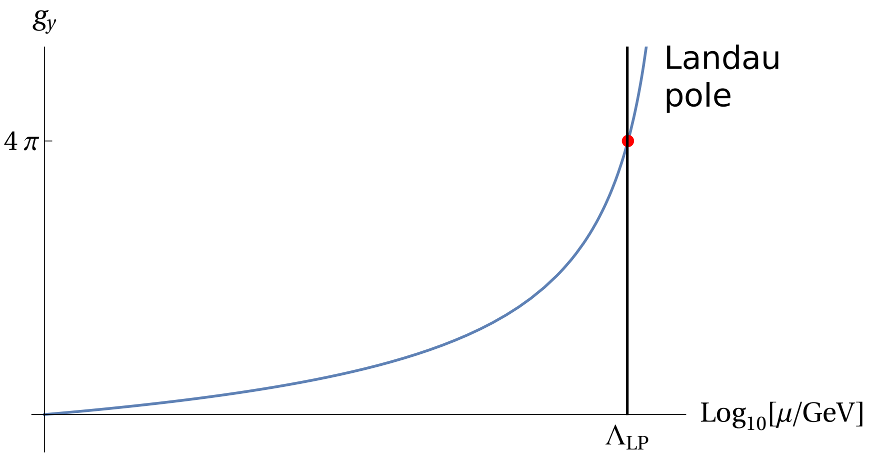


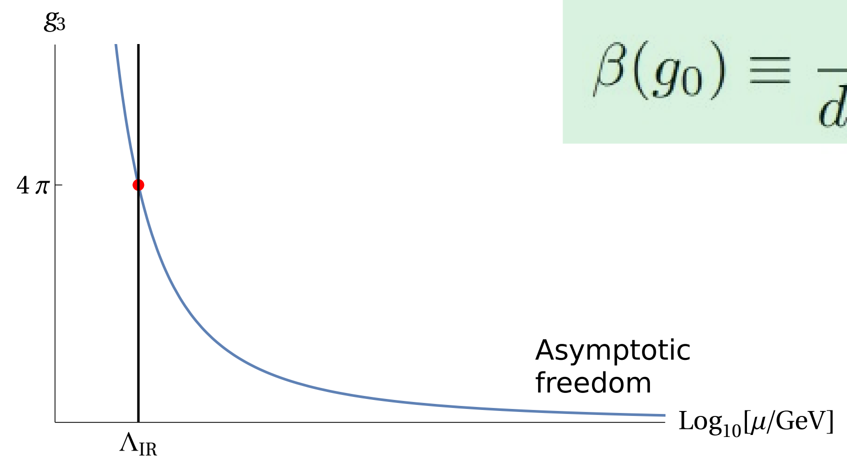
$$\beta(g_0) \equiv \frac{dg_0}{d\log\mu}$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

Running of the Coupling Constants

In the Standard Model we see two possible asymptotic behavior:



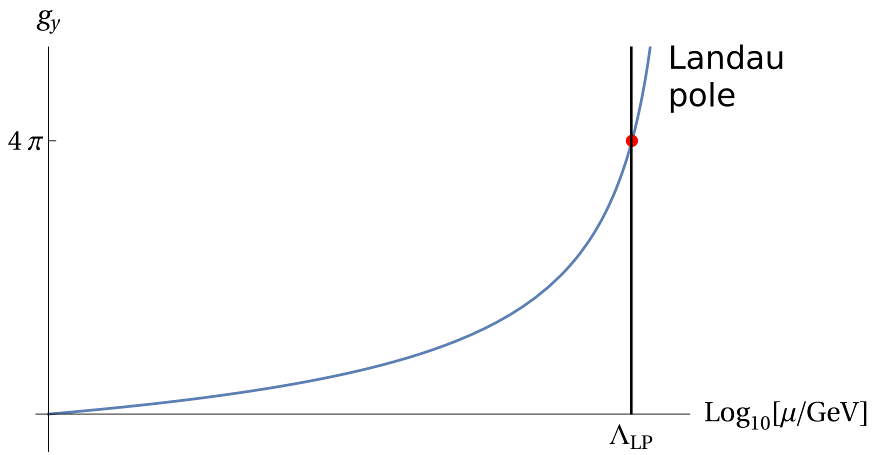


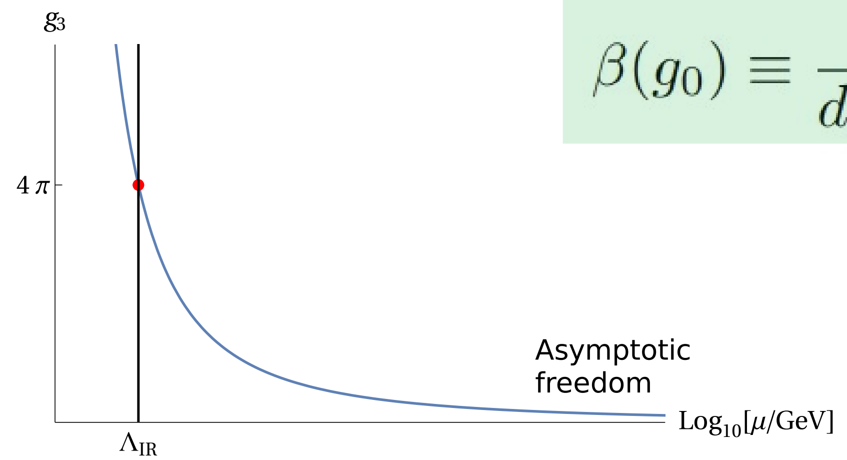
$$\beta(g_0) \equiv \frac{dg_0}{d\log\mu}$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Running of the Coupling Constants

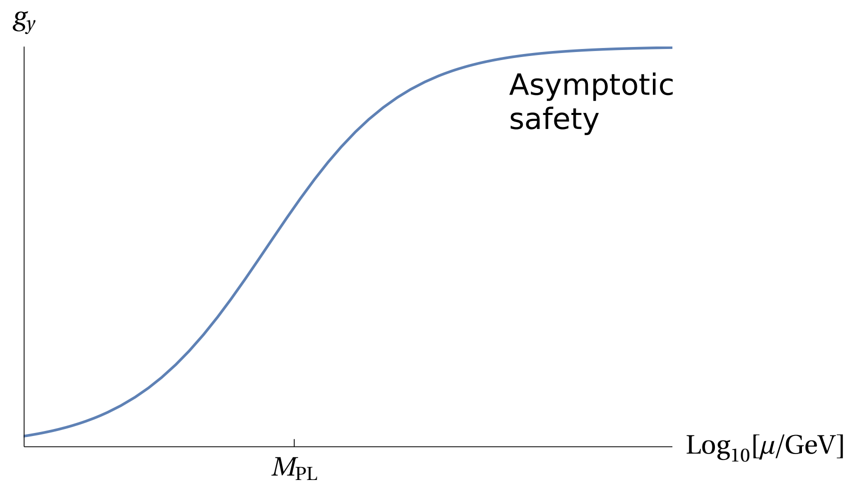
In the Standard Model we see two possible asymptotic behavior:





$$\beta(g_0) \equiv \frac{dg_0}{d\log\mu}$$

In a generic QFT a third asymptotic behaviour can be found:



Present in models of quantum gravity.

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g}\,(-R(g) + 2\Lambda)$$

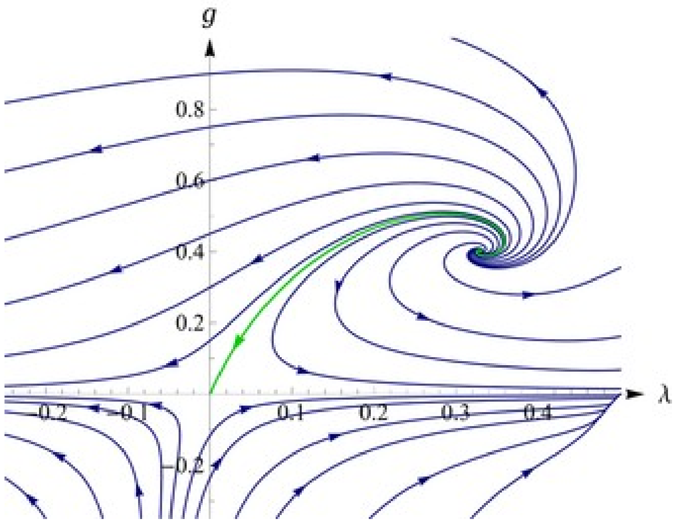
daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left(-R(g) + 2\Lambda\right)$$



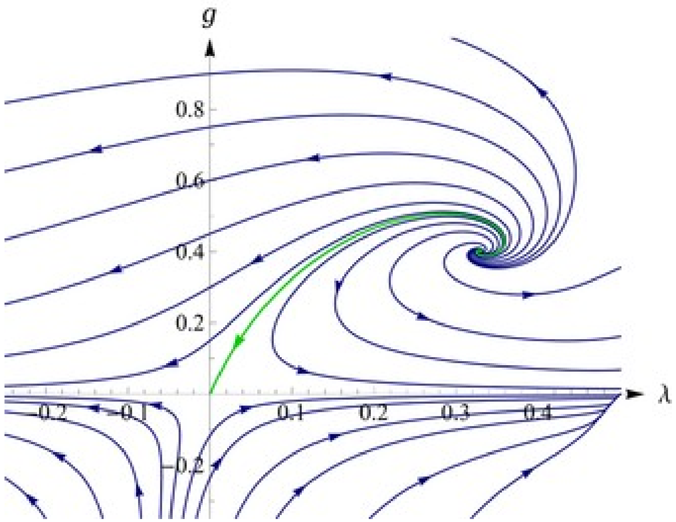
Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g}\, (-R(g) + 2\Lambda)$$



Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

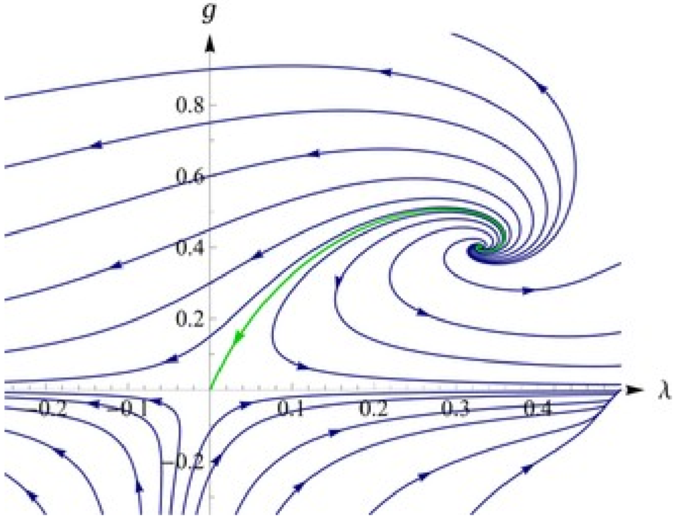
Large uncertainties when computed analytically.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, …]

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g)+2\Lambda)$$



Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

Large uncertainties when computed analytically.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, …]

Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

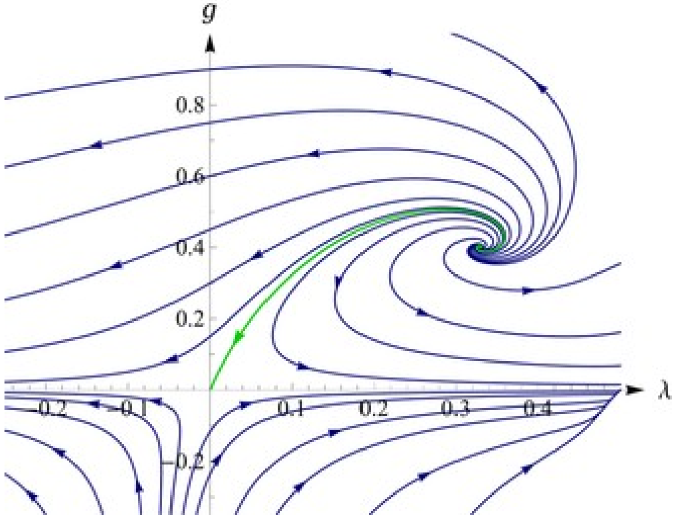
daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left(-R(g) + 2\Lambda\right)$$



Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

Large uncertainties when computed analytically.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, …]

Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

Renormalization Group Equations in the Trans-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}} - g\, f_g$$

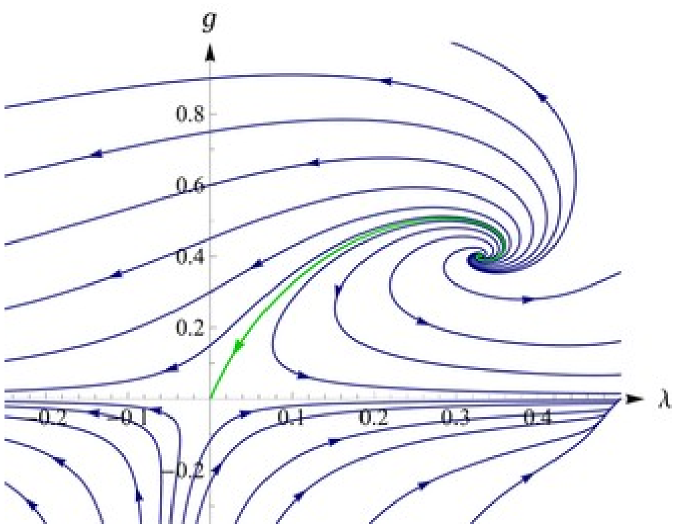
$$\beta_y = \beta_y^{\mathrm{SM+NP}} - y\, f_y$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Quantum Gravity contributions to the Running of the Coupling Constants

Einstein-Hilbert Gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g)+2\Lambda)$$



Reuter, Saueressig, hep-th/0110054
Picture: Wikipedia

Large uncertainties when computed analytically.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, …]

Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

Renormalization Group Equations in the Trans-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}} - g\,f_g$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}} - y\,f_y$$

In our project are determined by matching the low-energy data.

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Fixed Point Analysis

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Fixed Point Analysis

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Fixed Point Analysis

Stability Matrix

$$M_k^i \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Critical Exponents

$$\theta_m = -\lambda_m \quad \text{(eigen)}$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

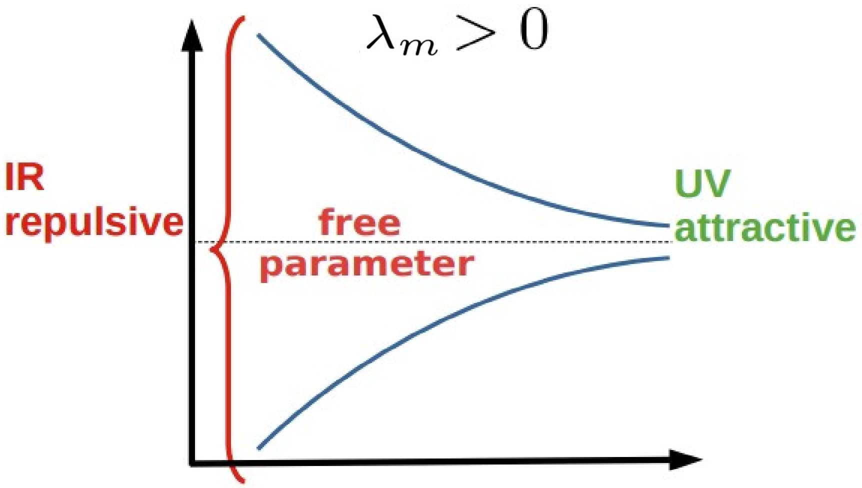
Fixed Point Analysis

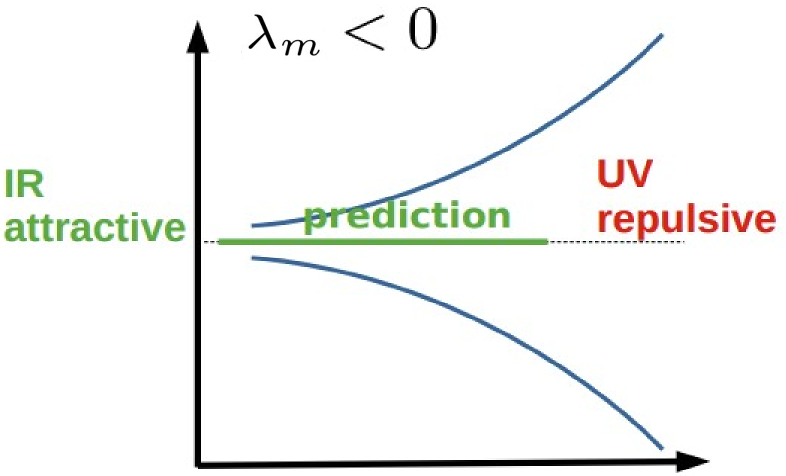
Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Critical Exponents

$$\theta_m = -\lambda_m \quad \text{(eigen)}$$





Relevant couplings are free parameters of the theory

Irrelevant couplings provide predictions

Source: Kamila Kowalska

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

The heuristic approach

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

The heuristic approach

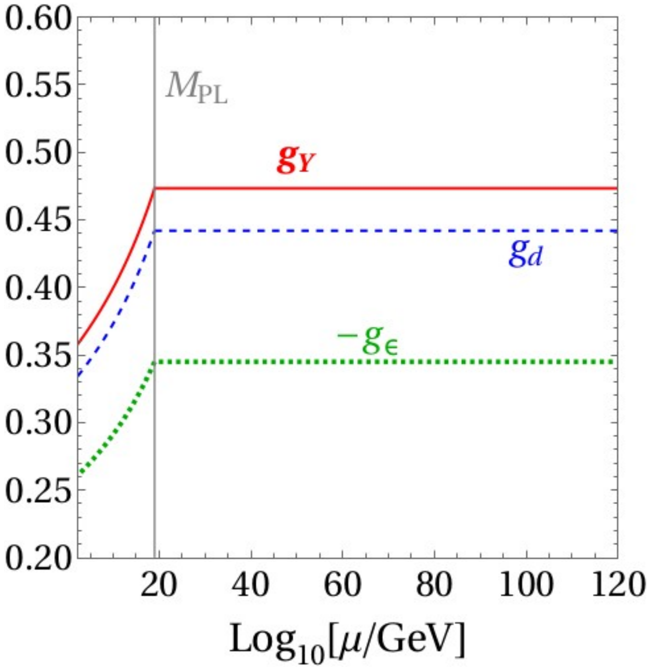
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

Gauge couplings



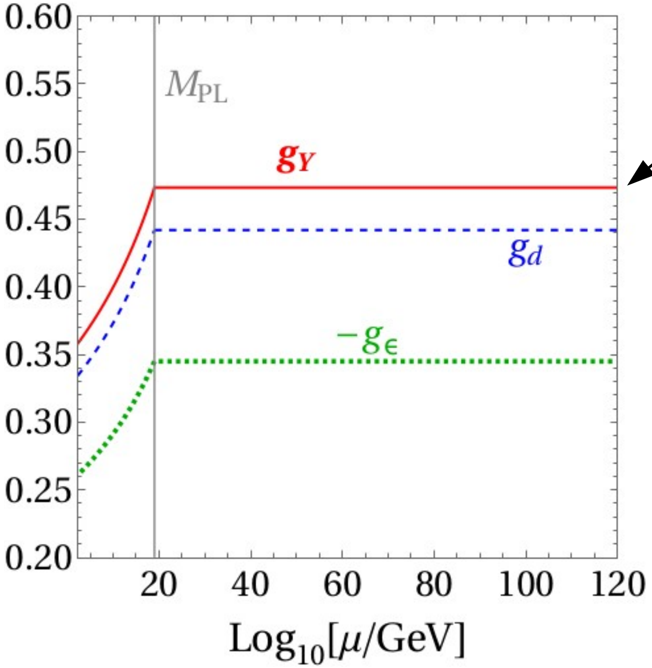
daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

Gauge couplings



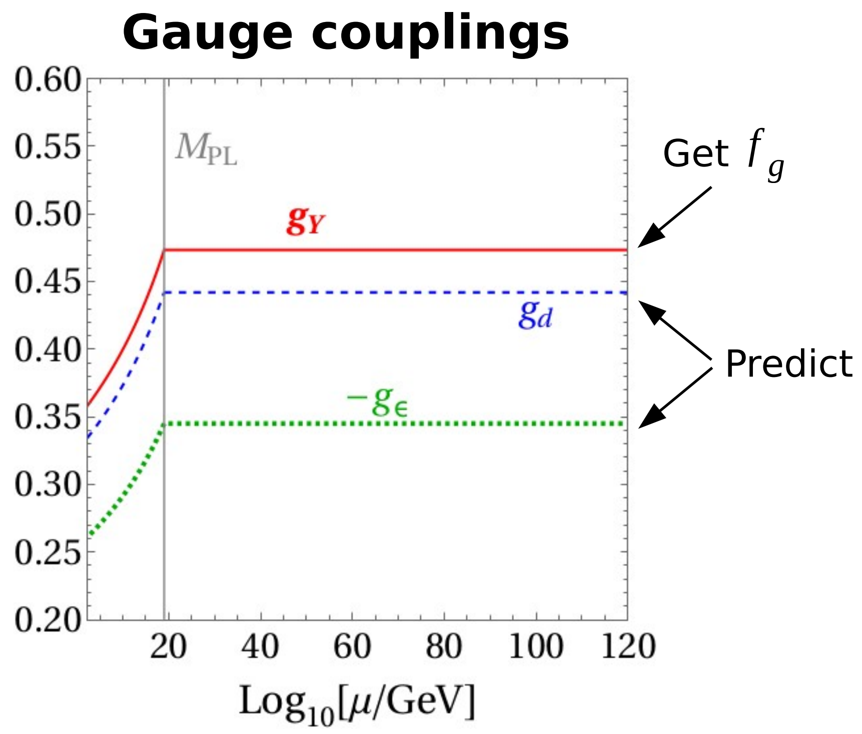
Get f_g

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$



daniele.rizzo@ncbj.gov.pl

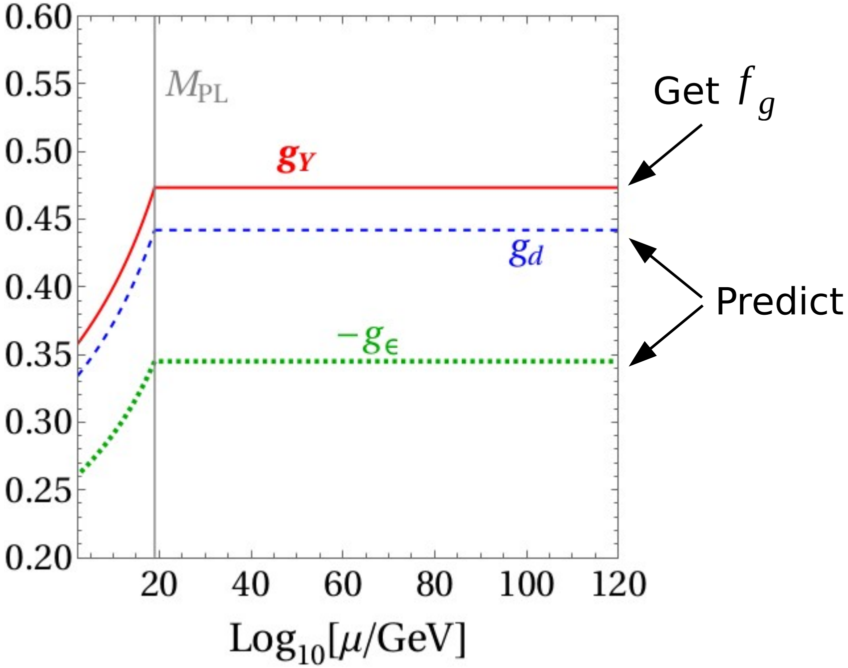
Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N SNN + \mathrm{H.c.}$$

Gauge couplings



daniele.rizzo@ncbj.gov.pl

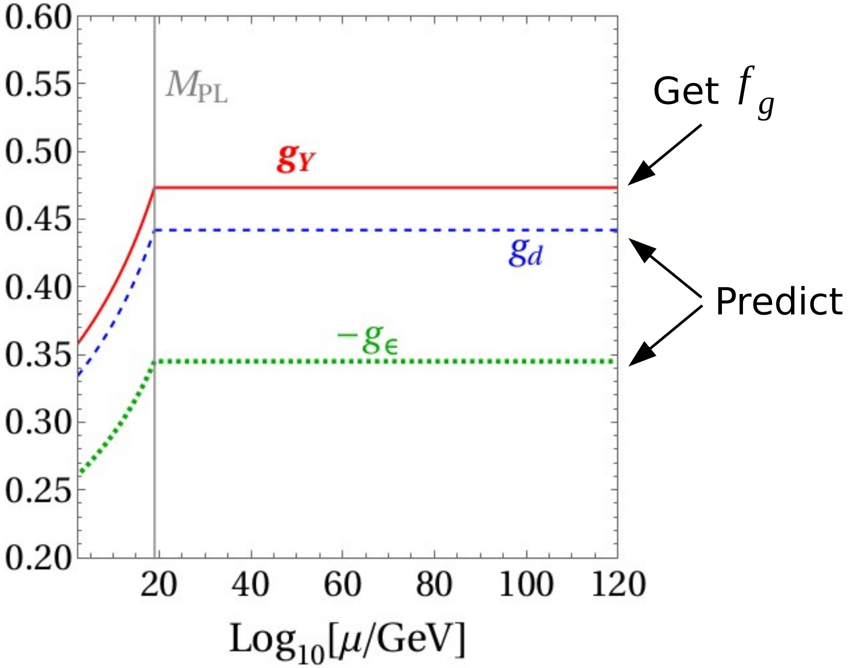
Daniele Rizzo

The heuristic approach

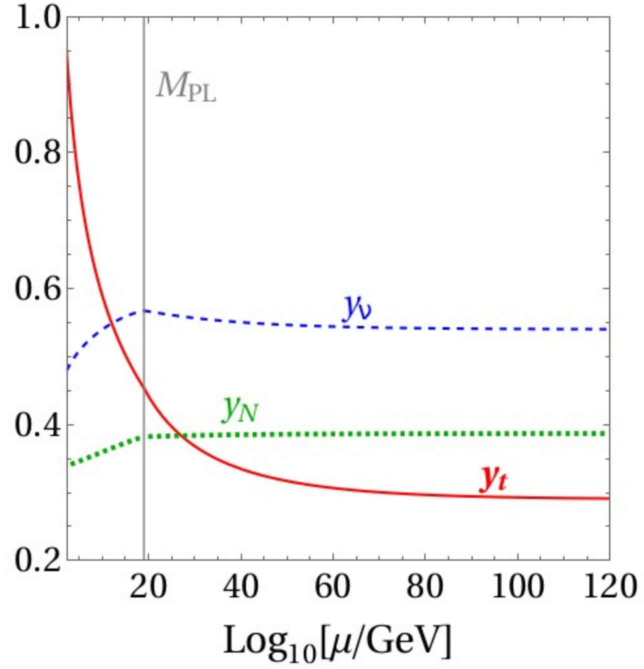
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\,(\tilde{\epsilon}H^*)^\dagger\, L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

Gauge couplings



Yukawa couplings



daniele.rizzo@ncbj.gov.pl

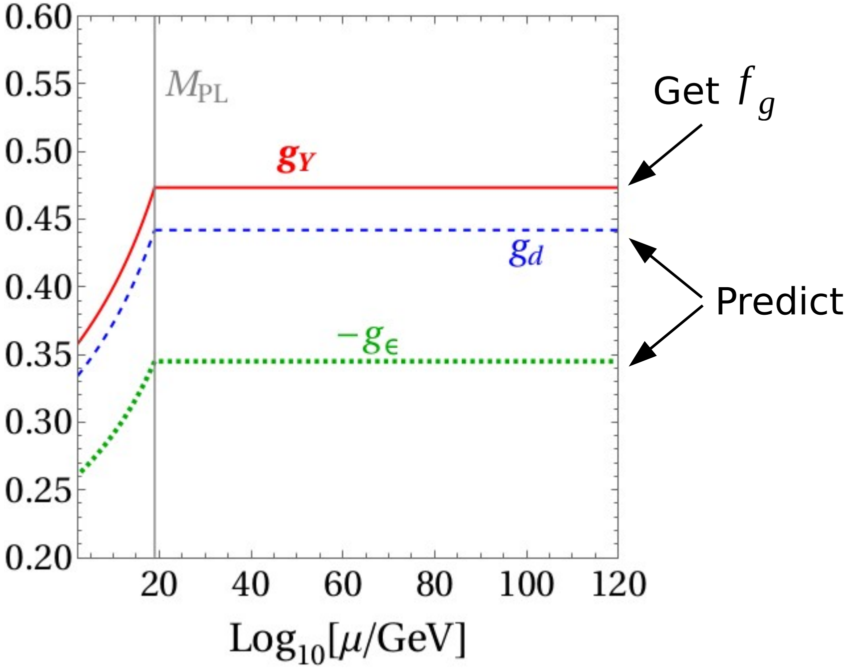
Daniele Rizzo

The heuristic approach

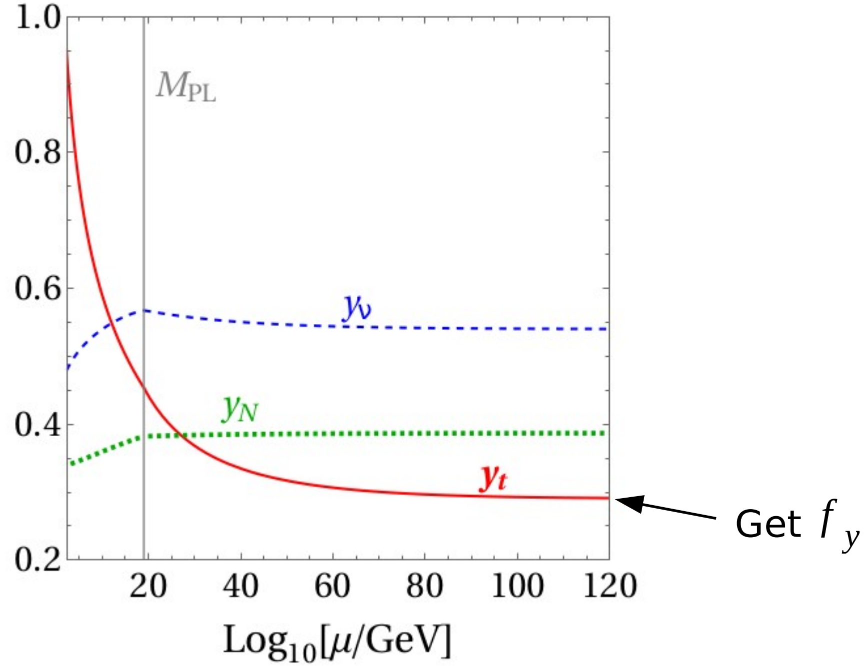
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N S N N + \text{H.c.}$$

Gauge couplings



Yukawa couplings



daniele.rizzo@ncbj.gov.pl

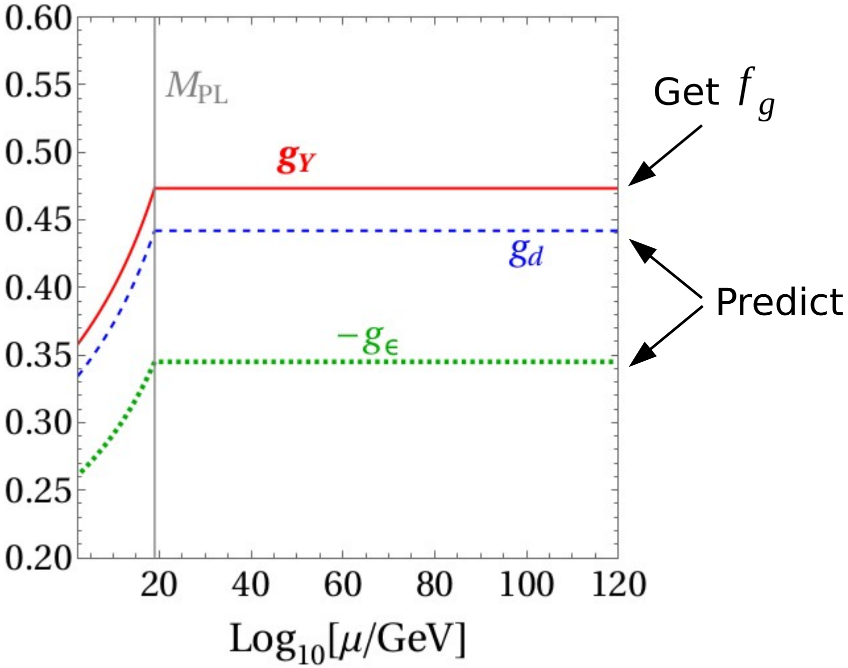
Daniele Rizzo

The heuristic approach

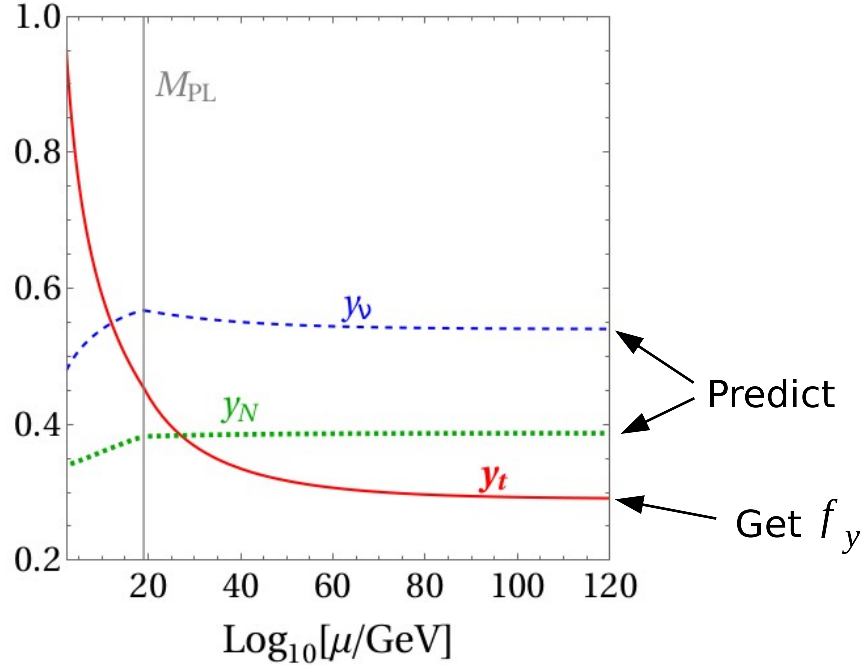
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N S N N + \text{H.c.}$$

Gauge couplings



Yukawa couplings



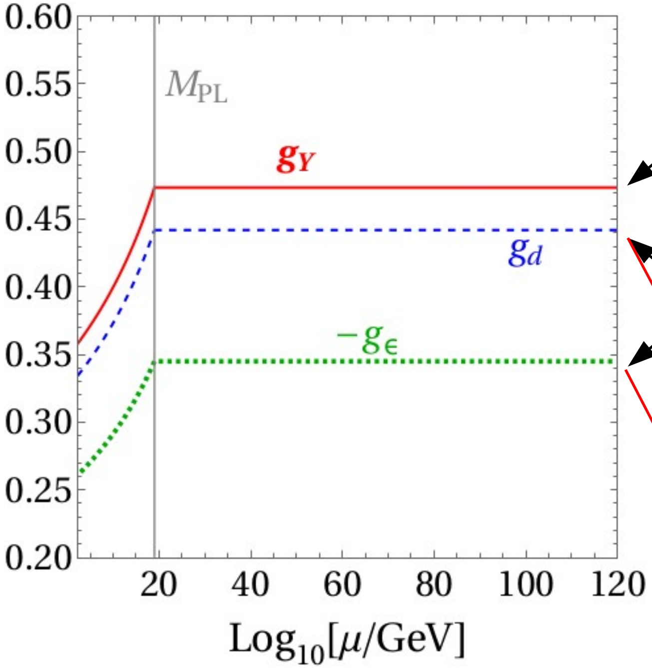
daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N S N N + \text{H.c.}$$

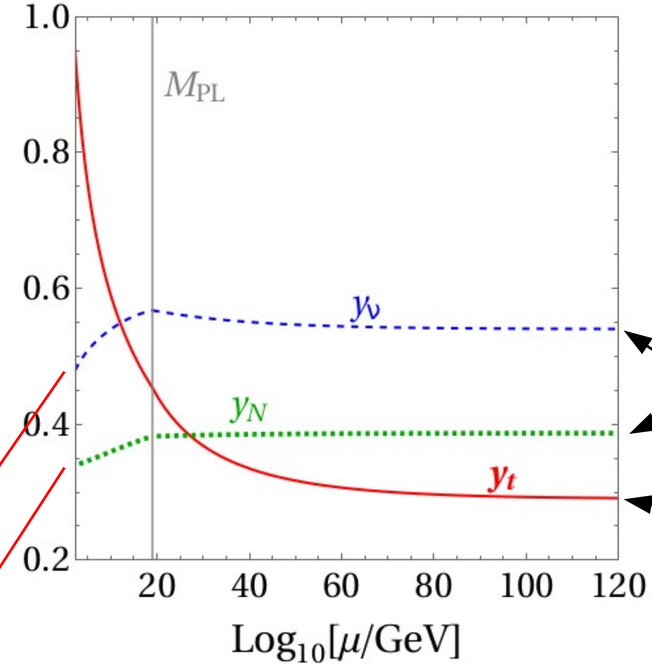
Gauge couplings



Get f_g

Predict

Yukawa couplings



Predict

Get f_y

Phenomenology!

cf. *e.g.* Chikkaballi, Kotlarski, Kowalska, **DR**, Sessolo JHEP (2023).

daniele.rizzo@ncbj.gov.pl

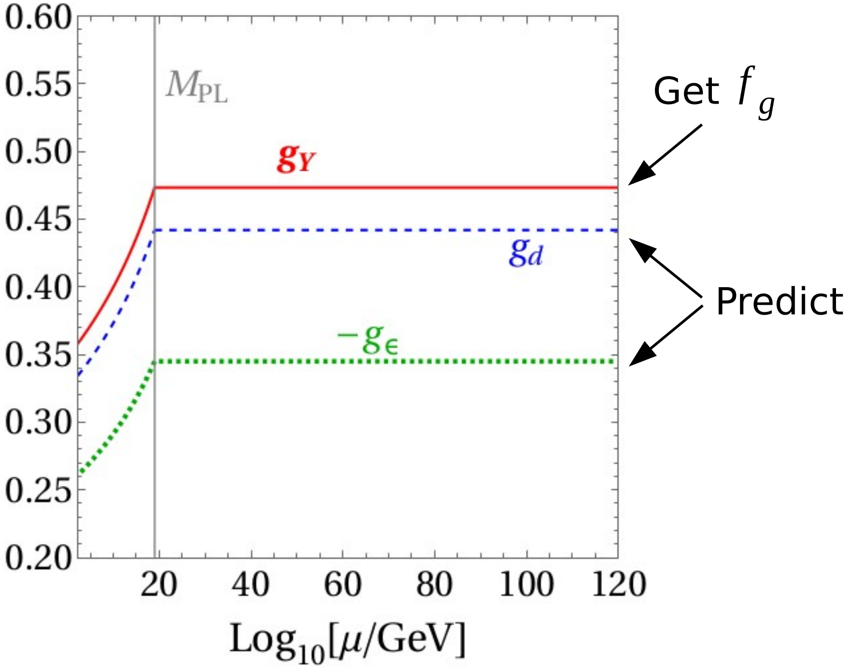
Daniele Rizzo

The heuristic approach

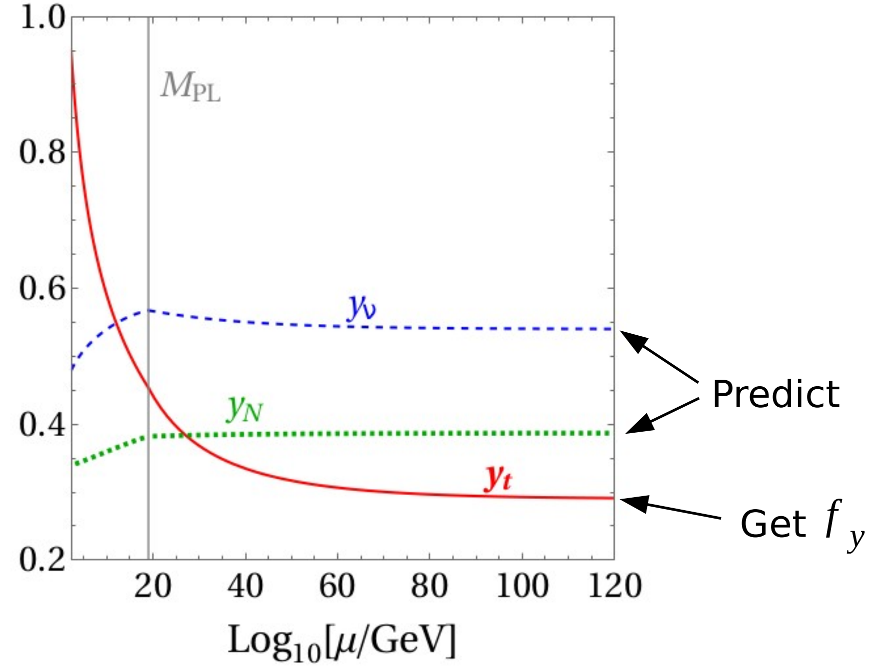
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N \left(\tilde{\epsilon} H^*\right)^\dagger L - \frac{1}{2}Y_N S N N + \text{H.c.}$$

Gauge couplings



Yukawa couplings



Sources of uncertainties

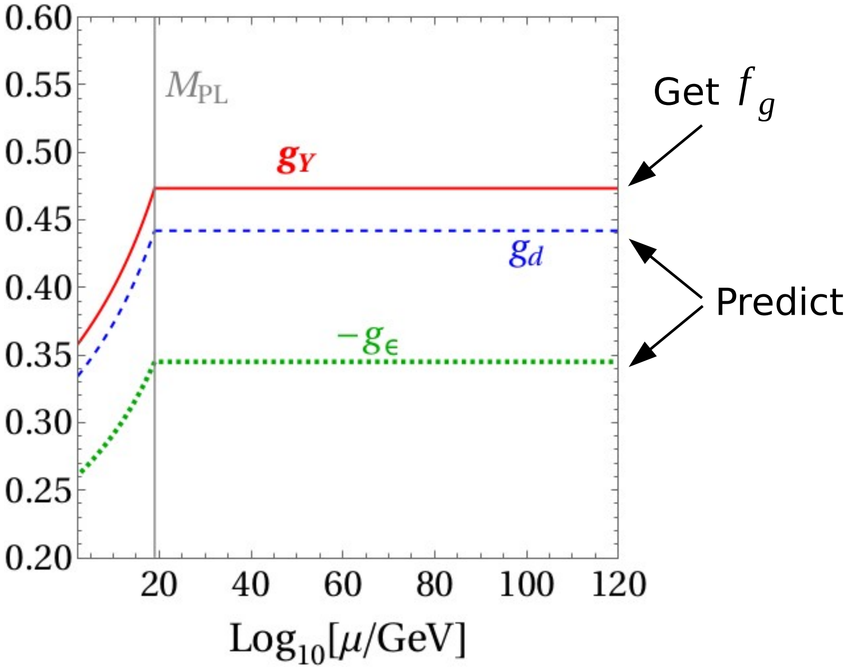
daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

The heuristic approach

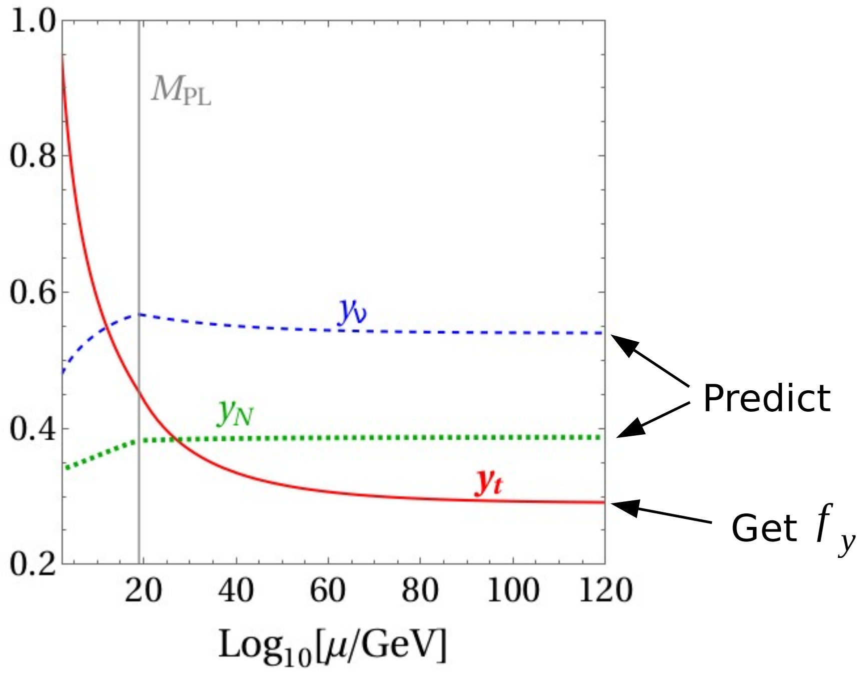
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N SNN + \mathrm{H.c.}$$

Gauge couplings



Yukawa couplings



Sources of uncertainties →

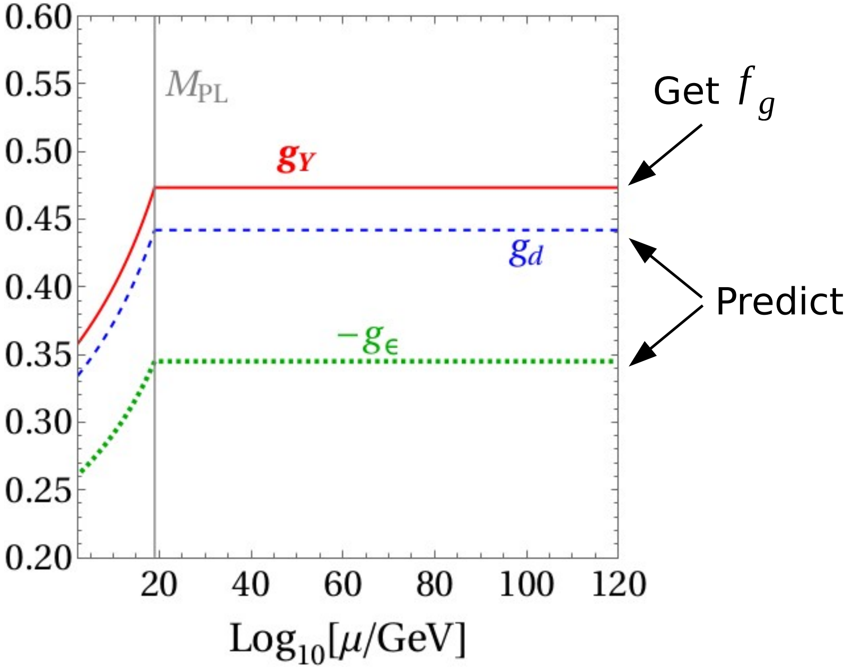
1 - Computations of the beta functions are performed at 1-loop level.

The heuristic approach

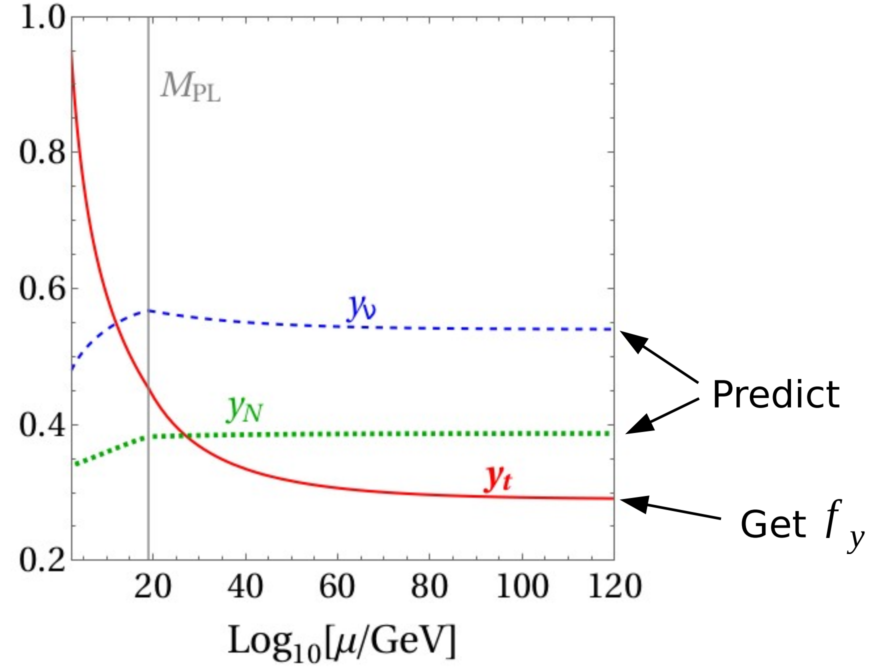
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

Gauge couplings



Yukawa couplings



Sources of uncertainties →

1 - Computations of the beta functions are performed at 1-loop level.

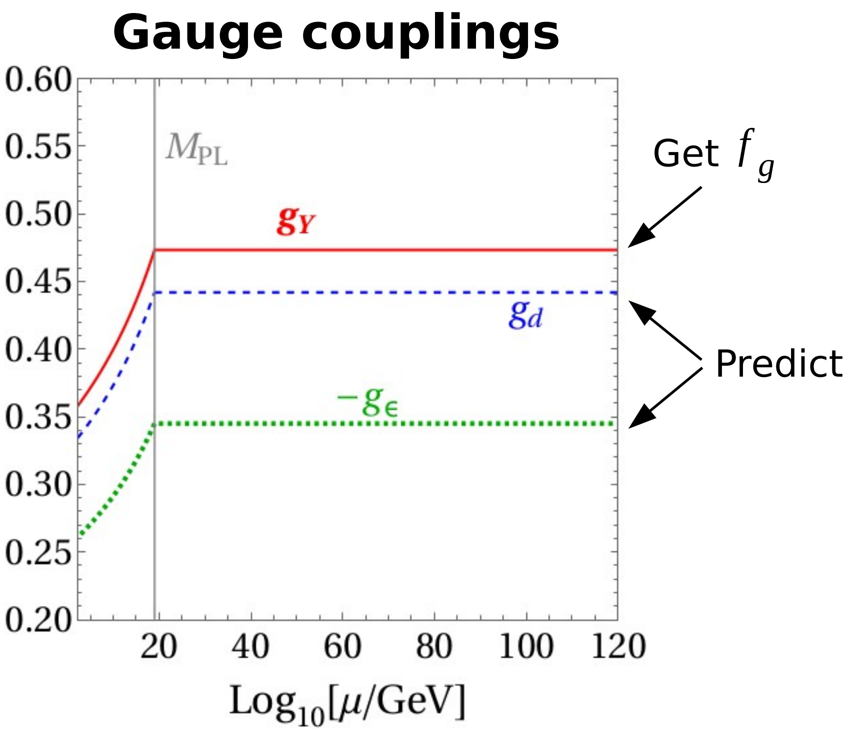
2 - Planck scale is set arbitrarily at 10^{19} GeV.

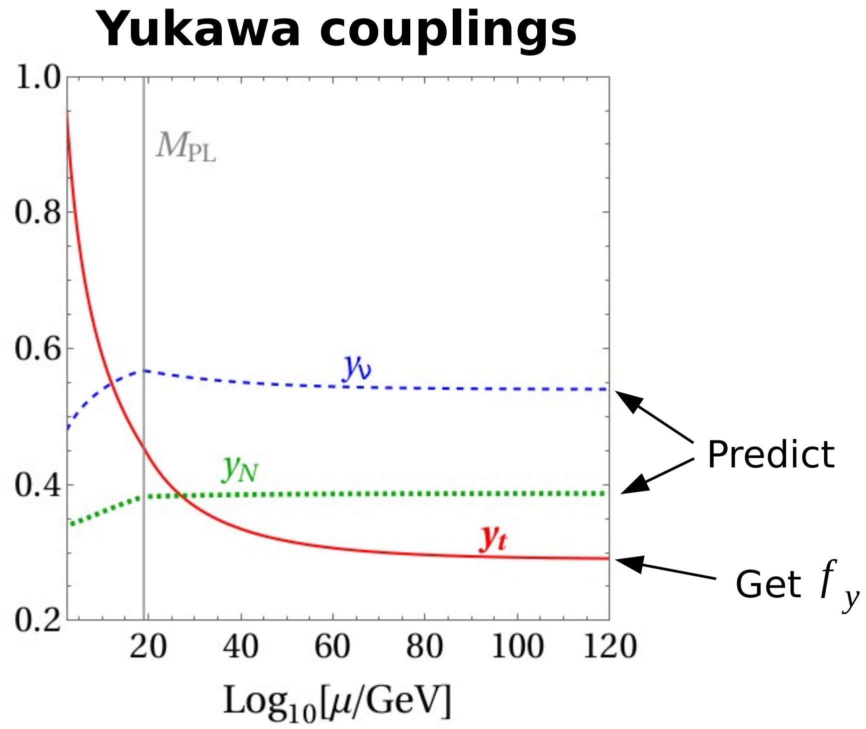
daniele.rizzo@ncbj.gov.pl Daniele Rizzo

The heuristic approach

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$





Sources of uncertainties →

1 - Computations of the beta functions are performed at 1-loop level.
2 - Planck scale is set arbitrarily at 10^{19} GeV.
3 - Gravity decouples instantaneously at the Planck scale.

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Gauge Sector

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Gauge Sector

Renormalization Group Equations:

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\tilde{b}_Y g_Y^3 - f_g\, g_Y$$

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y g_d g_\epsilon^2 + \tilde{b}_d g_d^3 + \tilde{b}_\epsilon g_d^2 g_\epsilon\right] - f_g\, g_d$$

$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \tilde{b}_d g_d^2 g_\epsilon + \tilde{b}_\epsilon\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

$$\tilde{b}_y = \left(b_Y + \Pi_{n\geq 2}^{(Y)}\right)$$

$$\tilde{b}_d = \left(b_d + \Pi_{n\geq 2}^{(Y)}\right)$$

$$\tilde{b}_\epsilon = \left(b_\epsilon + \Pi_{n\geq 2}^{(Y)}\right)$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Gauge Sector

Renormalization Group Equations:

Known from experiments

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\tilde{b}_Y g_Y^3 - f_g\, g_Y$$

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y g_d g_\epsilon^2 + \tilde{b}_d g_d^3 + \tilde{b}_\epsilon g_d^2 g_\epsilon\right] - f_g\, g_d$$

$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \tilde{b}_d g_d^2 g_\epsilon + \tilde{b}_\epsilon\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

$$\tilde{b}_y = \left(b_Y + \Pi_{n\geq 2}^{(Y)}\right)$$

$$\tilde{b}_d = \left(b_d + \Pi_{n\geq 2}^{(Y)}\right)$$

$$\tilde{b}_\epsilon = \left(b_\epsilon + \Pi_{n\geq 2}^{(Y)}\right)$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

Higher loops computations: Gauge Sector

Renormalization Group Equations:

Known from experiments

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\tilde{b}_Y g_Y^3 - f_g\, g_Y$$
$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y g_d g_\epsilon^2 + \tilde{b}_d g_d^3 + \tilde{b}_\epsilon g_d^2 g_\epsilon\right] - f_g\, g_d$$
$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \tilde{b}_d g_d^2 g_\epsilon + \tilde{b}_\epsilon\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

$$\tilde{b}_y = \left(b_Y + \Pi^{(Y)}_{n\geq 2}\right)$$
$$\tilde{b}_d = \left(b_d + \Pi^{(Y)}_{n\geq 2}\right)$$
$$\tilde{b}_\epsilon = \left(b_\epsilon + \Pi^{(Y)}_{n\geq 2}\right)$$

At the fixed point, the ratio of gauge couplings does not depend on f_g :

$$r^*_{g,d}(n\text{ loops}) \equiv \frac{g_d^*}{g_Y^*}(n\text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$r^*_{g,\epsilon}(n\text{ loops}) \equiv \frac{g_\epsilon^*}{g_Y^*}(n\text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

Higher loops computations: Gauge Sector

Renormalization Group Equations:

Known from experiments

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\tilde{b}_Y g_Y^3 - f_g\, g_Y$$
$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y g_d g_\epsilon^2 + \tilde{b}_d g_d^3 + \tilde{b}_\epsilon g_d^2 g_\epsilon\right] - f_g\, g_d$$
$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \tilde{b}_d g_d^2 g_\epsilon + \tilde{b}_\epsilon\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

$$\tilde{b}_y = \left(b_Y + \Pi_{n\geq 2}^{(Y)}\right)$$
$$\tilde{b}_d = \left(b_d + \Pi_{n\geq 2}^{(Y)}\right)$$
$$\tilde{b}_\epsilon = \left(b_\epsilon + \Pi_{n\geq 2}^{(Y)}\right)$$

At the fixed point, the ratio of gauge couplings does not depend on f_g :

$$r_{g,d}^*(n\text{ loops}) \equiv \frac{g_d^*}{g_Y^*}(n\text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$r_{g,\epsilon}^*(n\text{ loops}) \equiv \frac{g_\epsilon^*}{g_Y^*}(n\text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

Uncertainties:

$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
0.3%	-0.1%	-0.1%	-0.4%	-0.5%

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations:
Yukawa Sector

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Yukawa Sector

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Yukawa Sector

Known from experiments

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Yukawa Sector

Known from experiments

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)}y_1^2 + a_2^{(1)}y_2^2 - a'^{(1)}g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)}y_1^2 + a_2^{(2)}y_2^2 - a'^{(2)}g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

At the fixed point, solve the first equation for the gravity parameter f_y and insert it in the second equation to get:

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Yukawa Sector

Known from experiments

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)}y_1^2 + a_2^{(1)}y_2^2 - a'^{(1)}g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)}y_1^2 + a_2^{(2)}y_2^2 - a'^{(2)}g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

At the fixed point, solve the first equation for the gravity parameter f_y and insert it in the second equation to get:

$$y_2^*(2\ \text{loops}) \approx \left[\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\ \text{loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}} + \frac{\left(a_1^{(2)} - a_1^{(1)}\right)\delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Yukawa Sector

Known from experiments

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n\geq 2}\widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

At the fixed point, solve the first equation for the gravity parameter f_y and insert it in the second equation to get:

$$y_2^*(2\text{ loops}) \approx \left[\frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}^{\text{1 loop}}}{a_2^{(1)} - a_2^{(2)}} + \frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}^{\text{Higher loops}}}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations:
Yukawa Sector - Large Yukawa

$$y_2^*(2 \text{ loops}) \approx \left[\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1 \text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}} + \frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}} \right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations: Yukawa Sector - Large Yukawa

$$y_2^*(2\ \text{loops}) \approx \left[\overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\ \text{loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}^{O(1)} + \overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}^{\text{Small}} \right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations:
Yukawa Sector - Large Yukawa

$$y_2^*(2\text{ loops}) \approx \left[\overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}^{O(1)} + \overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}^{\text{Small}} \right]^{1/2}$$

Higher loops are negligible → predictions are very stable.

$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$
-6.0%	-3.3%	-1.4%

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

Higher loops computations: Yukawa Sector - Large Yukawa

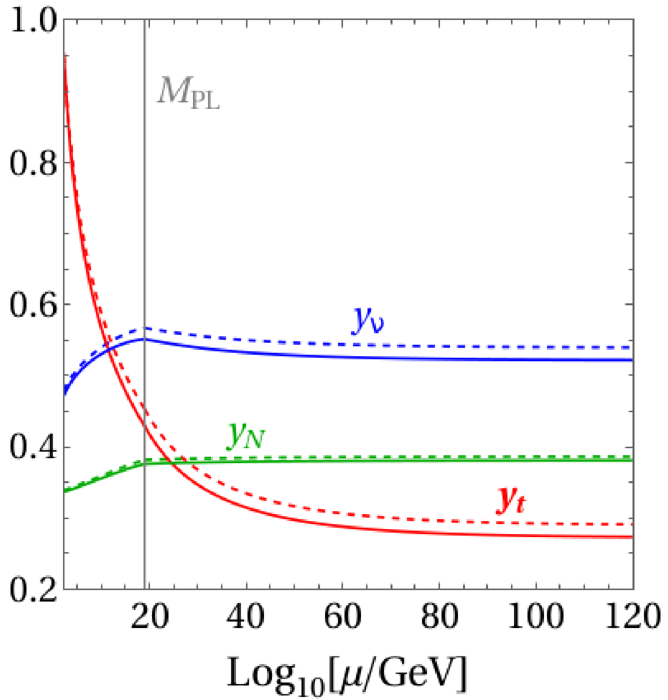
$$y_2^*(2\text{ loops}) \approx \left[\overbrace{\frac{\left(a_1^{(2)}-a_1^{(1)}\right)y_1^{*2}(1\text{ loop})+\left(a'^{(1)}-a'^{(2)}\right)g_1^{*2}}{a_2^{(1)}-a_2^{(2)}}}^{O(1)}+\overbrace{\frac{\left(a_1^{(2)}-a_1^{(1)}\right)\delta y_1^{*2}+\left(\widetilde{\Pi}_2^{(2)*}-\widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)}-a_2^{(2)}}}^{\text{Small}}\right]^{1/2}$$

Higher loops are negligible → predictions are very stable.

$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$
-6.0%	-3.3%	-1.4%

Focusing in the infrared → uncertainties are reduced.

$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
-1.4%	-0.8%



daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Yukawa Sector - Small Yukawa

$$y_2^*(2\text{ loops}) \approx \left[\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}} + \frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations:
Yukawa Sector - Small Yukawa

$$y_2^*(2\text{ loops}) \approx \left[\underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}_{\text{Cancel out}} + \underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}_{\text{Main contribution}} \right]^{1/2}$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Yukawa Sector - Small Yukawa

$$y_2^*(2\text{ loops}) \approx \left[\underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}_{\text{Cancel out}} + \underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}_{\text{Main contribution}} \right]^{1/2}$$

Higher loops are important → predictions are unstable.

$\delta y_t^* / y_t^*$	$\delta y_{\mathrm{LQ}}^* / y_{\mathrm{LQ}}^*$
-8.8%	-24.5%

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Higher loops computations: Yukawa Sector - Small Yukawa

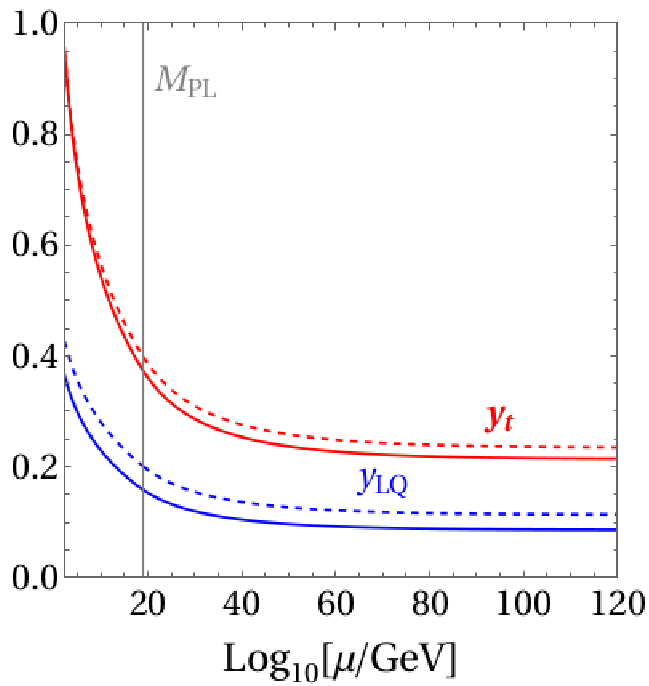
$$y_2^*(2\ \text{loops}) \approx \left[\overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\ \text{loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}^{\text{Cancel out}} + \overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}^{\text{Main contribution}} \right]^{1/2}$$

Higher loops are important → predictions are unstable.

$\delta y_t^* / y_t^*$	$\delta y_{\rm LQ}^* / y_{\rm LQ}^*$
-8.8%	-24.5%

Focusing in the infrared → uncertainties are reduced.

$\delta y_{\rm LQ} / y_{\rm LQ}(M_t)$
-14.3%



daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Dependence on the position of the Planck scale

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Dependence on the position of the Planck scale

Gauge sector

$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g^*_d}{g^*_Y}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}}$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g^*_\epsilon}{g^*_Y}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}}$$

The ratio does not depend on f_g :

$\delta g^*_Y/g^*_Y$	$\delta g^*_d/g^*_d$	$\delta g^*_\epsilon/g^*_\epsilon$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
-6.1%	-6.1%	-6.1%	0.0%	0.0%

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Dependence on the position of the Planck scale

Gauge sector

$$r^*_{g,d}(n\text{ loops}) \equiv \frac{g^*_d}{g^*_Y}(n\text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$r^*_{g,\epsilon}(n\text{ loops}) \equiv \frac{g^*_\epsilon}{g^*_Y}(n\text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

The ratio does not depend on f_g :

$\delta g^*_Y/g^*_Y$	$\delta g^*_d/g^*_d$	$\delta g^*_\epsilon/g^*_\epsilon$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
-6.1%	-6.1%	-6.1%	0.0%	0.0%

Yukawa sector

$$r^*_{g,k} = \frac{g^*_k}{y^*_1} \qquad r^*_{y,2} = \frac{y^*_2}{y^*_1}$$

$$\delta r^*_{y,2} \propto \frac{r^*_{g,1}}{r^*_{y,2}} \cdot \delta r^*_{g,1}$$

The uncertainty depends on the size of the couplings:

$\delta r^*_{y,2} \longrightarrow$ 3-18%

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Scale-dependence of the gravitational corrections

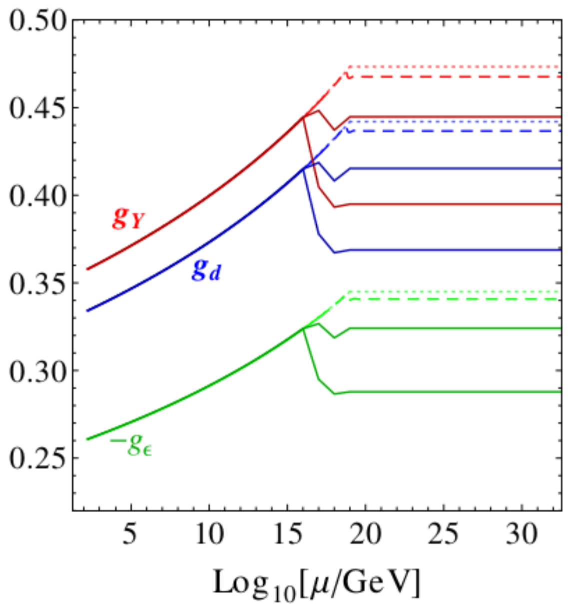
daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Scale-dependence of the gravitational corrections

Gauge sector

$$r^*_{g,d}(n\text{ loops}) \equiv \frac{g^*_d}{g^*_Y}(n\text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}}$$

$$r^*_{g,\epsilon}(n\text{ loops}) \equiv \frac{g^*_\epsilon}{g^*_Y}(n\text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}^2_\epsilon}}$$



daniele.rizzo@ncbj.gov.pl

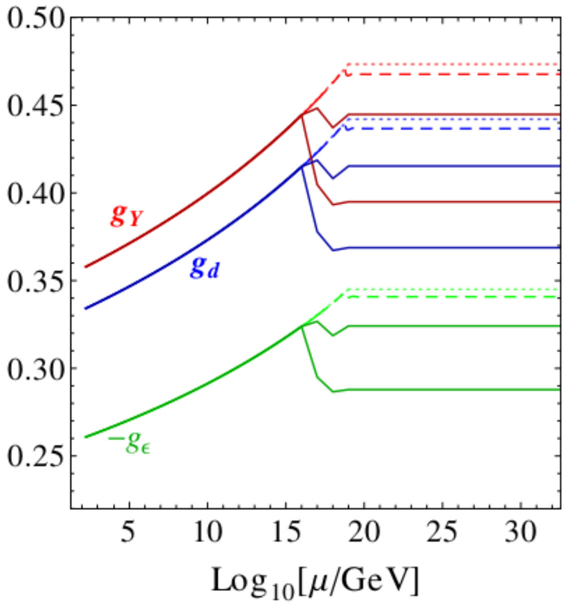
Daniele Rizzo

Scale-dependence of the gravitational corrections

Gauge sector

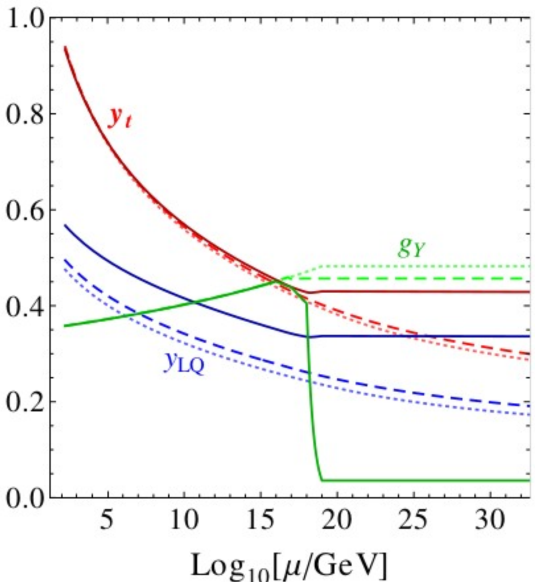
$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g^*_d}{g^*_Y}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g^*_\epsilon}{g^*_Y}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$



Yukawa sector

$$r^*_{y,2} = \sqrt{\frac{A' f_g + B' f_y}{A f_g + B f_y}}$$



daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Conclusions

- The heuristic approach to asymptotic safety is able to give predictions for the coupling constants of BSM models.

- Predictions might be affected by the simplifying assumptions: loop approximation, position of the Planck scale, functional dependence of the gravitational parameters.

- We have relaxed such assumptions to understand the robustness of the predictions in both the high and the low energy scale regime.

- Our main findings are that the gauge sector is extremely robust, while in the Yukawa sector the magnitude of the coupling we want to predict plays an important role in the determination of the robustness of the predictions.

- The infrared focusing of the RGEs reduces the uncertainties in the predictions, so that the analytical formula obtained at the fixed point can be read as an upper bound.

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

Backup

Models

Gauged *B - L*

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu} + i\bar{f}\left(\partial^\mu - ig_Y Q_Y \tilde{B}^\mu - ig_{B-L}Q_{B-L}\tilde{X}^\mu\right)\gamma_\mu f$$

$$\mathcal{L} \supset -Y_\nu N\,(\tilde{\epsilon}H^*)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

By matching the SM at 1-loop we get:

$g_Y^*\,(1\text{ loop}) = 0.4734$ $\quad$ $y_t^*\,(1\text{ loop}) = 0.2901$

The predictions for the BSM couplings are:

$g_d^*\,(1\text{ loop}) = 0.4420\,,$ $\quad$ $g_\epsilon^*\,(1\text{ loop}) = -0.3450\,,$

$y_\nu^*\,(1\text{ loop}) = 0.5398\,,$ $\quad$ $y_N^*\,(1\text{ loop}) = 0.3868\,.$

Leptoquark

$$S_3 : \left(\bar{\mathbf{3}}, \mathbf{3}, 1/3\right).$$

$$\mathcal{L} \supset -Y_{\mathrm{LQ}}\,Q^T\tilde{\epsilon}S_3\,L + \text{H.c.}$$

By matching the SM at 1-loop we get:

$g_Y^*\,(1\text{ loop}) = 0.4823\,,$ $\quad$ $y_t^*\,(1\text{ loop}) = 0.2340$

The predictions for the BSM coupling are:

$$y_{\mathrm{LQ}}^*\,(1\text{ loop}) = 0.1132$$

daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Gauged *B – L* - Gauge Sector

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu} + i\bar{f}\left(\partial^\mu - i g_Y Q_Y \tilde{B}^\mu - i g_{B-L} Q_{B-L} \tilde{X}^\mu\right)\gamma_\mu f$$

$$\begin{pmatrix} \tilde{B}^\mu \\ \tilde{X}^\mu \end{pmatrix} = \begin{pmatrix} 1 & -\epsilon/\sqrt{(1-\epsilon^2)} \\ 0 & 1/\sqrt{(1-\epsilon^2)} \end{pmatrix} \begin{pmatrix} V^\mu \\ D^\mu \end{pmatrix}$$

$$(Q_Y\,, Q_{B-L}) \begin{pmatrix} g_Y & 0 \\ 0 & g_{B-L} \end{pmatrix} \begin{pmatrix} \tilde{B}^\mu \\ \tilde{X}^\mu \end{pmatrix} \quad \rightarrow \quad (Q_Y\,, Q_{B-L}) \begin{pmatrix} g_Y & g_\epsilon \\ 0 & g_d \end{pmatrix} \begin{pmatrix} V^\mu \\ D^\mu \end{pmatrix}.$$

$$g_Y \rightarrow g_Y\,, \quad g_d = \frac{g_{B-L}}{\sqrt{1-\epsilon^2}}\,, \quad g_\epsilon = -\frac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}\,.$$

$$g_Y^* \text{ (1 loop)} = 0.4734$$

$$g_d^* \text{ (1 loop)} = 0.4420$$

$$g_\epsilon^* \text{ (1 loop)} = -0.3450$$

daniele.rizzo@ncbj.gov.pl Daniele Rizzo

Renormalization group equations
Gauged *B – L* - 1 Loop

$$\beta^{(1)}(g_2) = -\frac{19}{6}g_2^3 \qquad \beta^{(1)}(g_3) = -7g_3^3$$

$$\beta^{(1)}(g_Y) = \frac{41}{6}g_Y^3 \qquad \beta^{(1)}(g_d) = +12g_d^3 + \frac{41}{6}g_d g_\epsilon^2 + \frac{32}{3}g_d^2 g_\epsilon$$

$$\beta^{(1)}(g_\epsilon) = +\frac{32}{3}g_Y^2 g_d + \frac{32}{3}g_d g_\epsilon^2 + \frac{41}{3}g_Y^2 g_\epsilon + 12g_d^2 g_\epsilon + \frac{41}{6}g_\epsilon^3$$

$$\beta^{(1)}(Y_u) = +\frac{3}{2}Y_u Y_u^\dagger Y_u + 3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_u + \mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right)Y_u - \frac{17}{12}g_Y^2 Y_u - \frac{2}{3}g_d^2 Y_u - \frac{5}{3}g_d g_\epsilon Y_u - \frac{17}{12}g_\epsilon^2 Y_u - \frac{9}{4}g_2^2 Y_u - 8g_3^2 Y_u$$

$$\beta^{(1)}(Y_\nu) = +\frac{3}{2}Y_\nu Y_\nu^\dagger Y_\nu + 2Y_\nu Y_N^* Y_N + 3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_\nu + \mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right)Y_\nu - \frac{3}{4}g_Y^2 Y_\nu - 6g_d^2 Y_\nu - 3g_d g_\epsilon Y_\nu - \frac{3}{4}g_\epsilon^2 Y_\nu - \frac{9}{4}g_2^2 Y_\nu$$

$$\beta^{(1)}(Y_N) = +Y_\nu^{\mathrm{T}} Y_\nu^* Y_N + Y_N Y_\nu^\dagger Y_\nu + 4Y_N Y_N^* Y_N + 2\mathrm{Tr}\left(Y_N^* Y_N\right)Y_N - 6g_d^2 Y_N$$

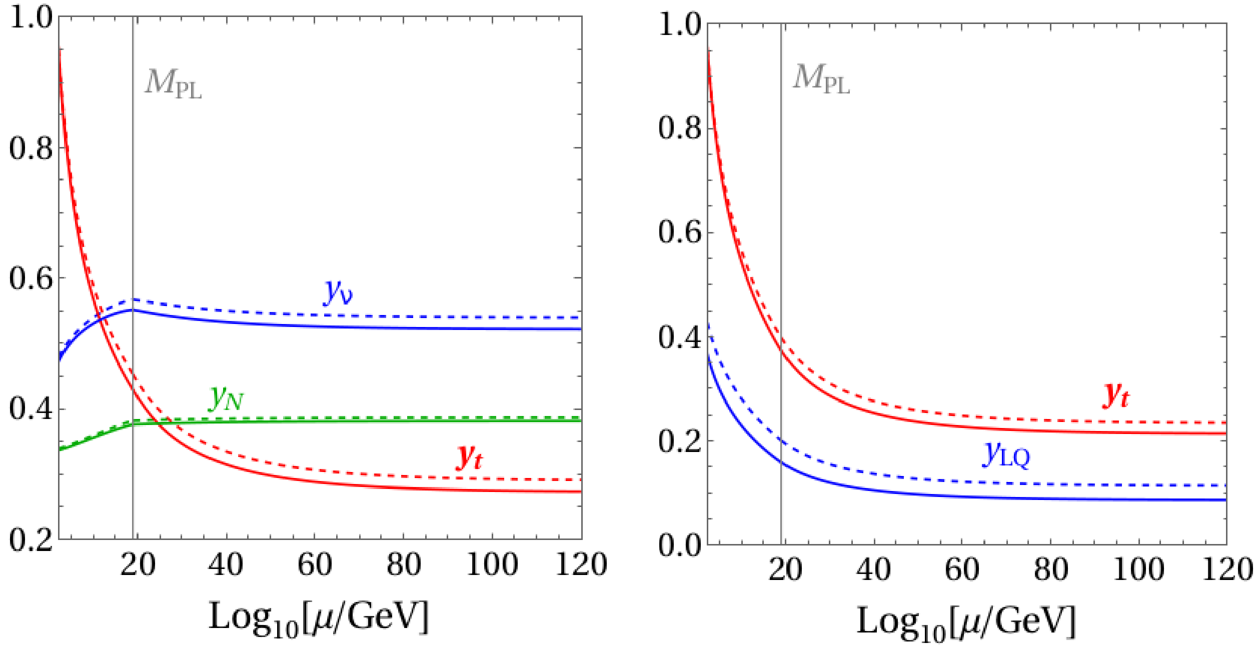
daniele.rizzo@ncbj.gov.pl

Daniele Rizzo

Higher loops computations

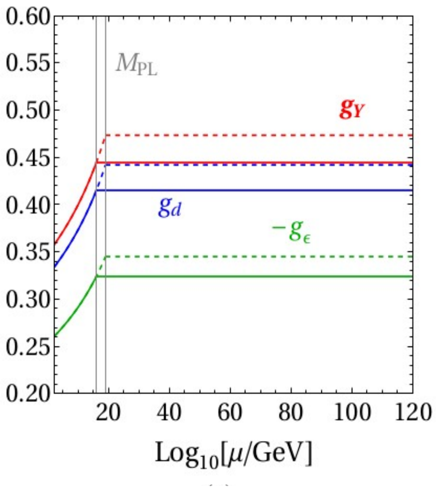
$\boldsymbol{B-L}$	f_g	g_Y^*	g_d^*	g_ϵ^*	$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
	0.0098	0.4748	0.4415	-0.3445	0.3%	-0.1%	-0.1%	-0.4%	-0.5%
	f_y	y_t^*	y_ν^*	y_N^*	$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$	$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
	0.0016	0.2727	0.5220	0.3813	-6.0%	-3.3%	-1.4%	-1.4%	-0.8%

$\boldsymbol{S_3}$ **LQ**	f_y	y_t^*	$y_{\rm LQ}^*$	$\delta y_t^*/y_t^*$	$\delta y_{\rm LQ}^*/y_{\rm LQ}^*$	$\delta y_{\rm LQ}/y_{\rm LQ}(M_t)$
	-0.0007	0.2133	0.0855	-8.8%	-24.5%	-14.3%



daniele.rizzo@ncbj.gov.pl
Daniele Rizzo

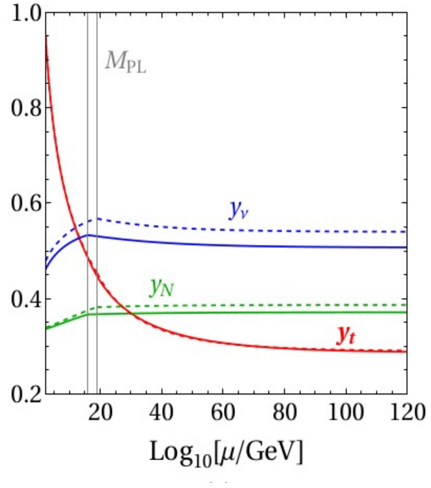
Dependence on the position of the Planck scale

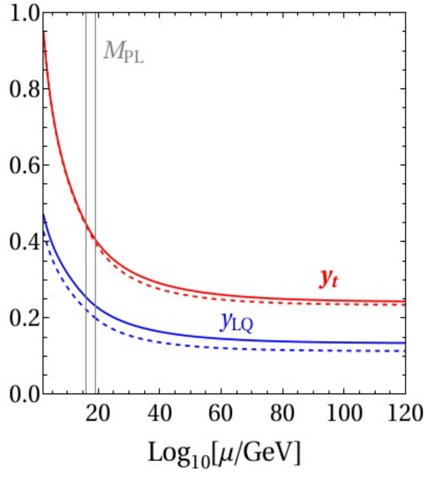


$B-L$	f_g	g_Y^*	g_d^*	g_ϵ^*	$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
10^{20} GeV	0.0102	0.4843	0.4522	−0.3530	2.3%	2.3%	2.3%	0.0%	0.0%
10^{16} GeV	0.0086	0.4445	0.4151	−0.3240	−6.1%	−6.1%	−6.1%	0.0%	0.0%

	f_y	y_t^*	y_ν^*	y_N^*	$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$	$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
10^{20} GeV	0.0020	0.2914	0.5523	0.3927	0.4%	2.3%	1.5%	1.3%	0.3%
10^{16} GeV	0.0020	0.2869	0.5069	0.3715	−1.1%	−6.1%	−4.0%	−3.7%	−0.9%

S_3 LQ	f_y	y_t^*	y_{LQ}^*	$\delta y_t^*/y_t^*$	$\delta y_{LQ}^*/y_{LQ}^*$	$\delta y_{LQ}/y_{LQ}(M_t)$
10^{20} GeV	−0.0006	0.2309	0.1043	−1.3%	−7.8%	−5.1%
10^{16} GeV	0.00002	0.2422	0.1337	3.5%	18.1%	10.1%





$$\frac{\delta r^*_{g(y),i}}{r^*_{g(y),i}} = \frac{r^*_{g(y),i}(M_{\rm Pl} \neq 10^{19}\,\text{GeV}) - r^*_{g(y),i}(M_{\rm Pl} = 10^{19}\,\text{GeV})}{r^*_{g(y),i}(M_{\rm Pl} = 10^{19}\,\text{GeV})}$$

$$\frac{\delta r^*_{y,2}}{r^*_{y,2}} = \frac{1}{r^{*2}_{y,2}} G_1\left(\sum_{l,k} a'^{(r)}_{lk}\, r^*_{g,l} r^*_{g,k} \cdot \frac{1}{2}\left[\frac{\delta r^*_{g,l}}{r^*_{g,l}} + \frac{\delta r^*_{g,k}}{r^*_{g,k}}\right];\, a^{(r)}_{j\neq 1}\right)$$

daniele.rizzo@ncbj.gov.pl
Daniele Rizzo