A Tale of Two Portals: Light, New Physics at Future e+e- Colliders

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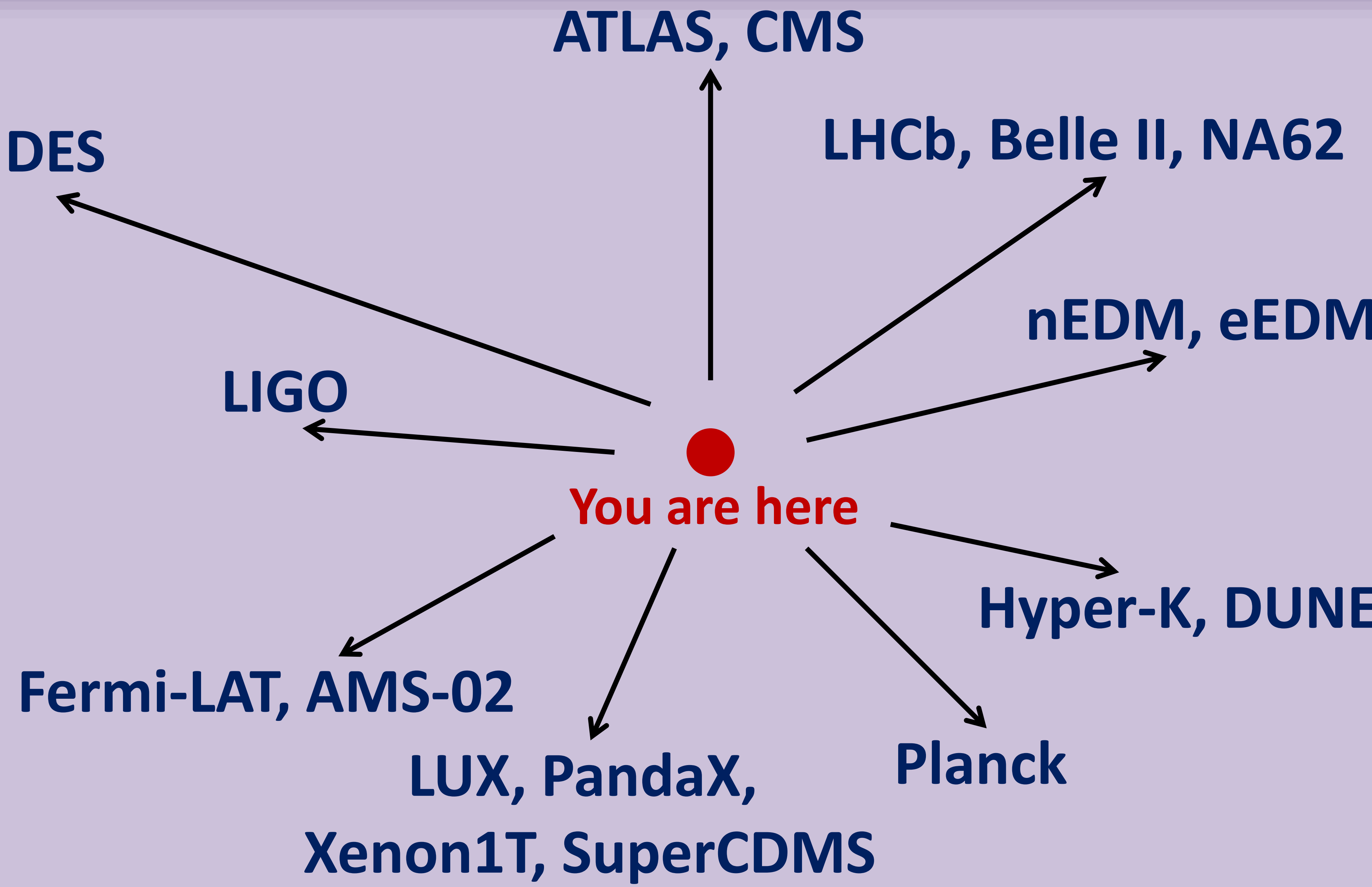
Naturalness
Cosmological constant
Flavor puzzle
Strong CP
Quantum gravity
You are here
ν masses, mixings
Baryogenesis
Dark matter
Inflation

Great challenges from all sides

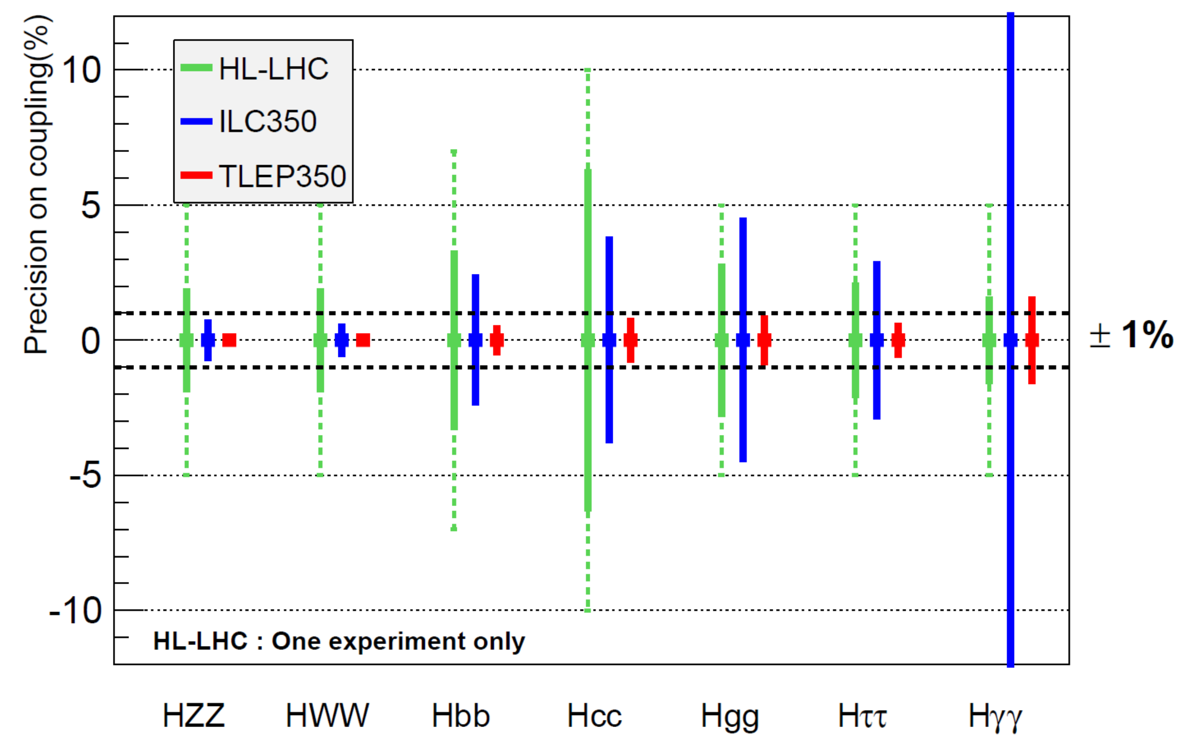


Pushing back in all directions

Motivation

- Era of exploratory particle physics
  - Possible NP models span decades in scale and couplings
  - Strong gains to come from e^+e^- precision Higgs program
  - ILC, FCC-ee, CEPC, CLIC machines under serious consideration

Precision Higgs physics requires measuring Γ_H: afforded by recoil mass method at e^+e^- machines



TLEP [1308.6176]

Motivation

- Era of exploratory particle physics
  - Possible NP models span decades in scale and couplings
  - Strong gains to come from e^+e^- precision Higgs program
  - ILC, FCC-ee, CEPC, CLIC machines under serious consideration
- Missing piece of story: e^+e^- collider production of new particles
  - More than a Higgs factory, but production of new, light states – especially when sensitivity exceeds possibilities at (HL-)LHC
  - Will discuss dark vector and dark scalar production and their SM and DM decays

Outline

- Theory review: Double Dark Portal
  - Simultaneous kinetic mixing and scalar Higgs portal
- Phenomenology: dark matter probes
  - Direct detection and indirect detection probes
- Phenomenology: collider signatures
  - Unique capabilities of e^+e^- machine for probing dark vector, dark scalar production
- Conclusions

Double Dark Portal model

Kinetic mixing of *K* with hypercharge gauge boson *B*

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\,\mu\nu} - \frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{\epsilon}{2\cos\theta_W}B_{\mu\nu}K^{\mu\nu}$$
$$+ |D_\mu H|^2 + |D_\mu \Phi|^2 + \mu_H^2|H|^2 - \lambda_H|H|^4 + \mu_D^2|\Phi|^2 - \lambda_D|\Phi|^4 - \lambda_{HP}|H|^2|\Phi|^2$$
$$+ \bar{\chi}(i\not{D} - m_\chi)\chi$$

$U(1)_D$ charges

$$\Phi \sim +1\ ,\ \chi \sim +1$$

Scalar Higgs portal between dark Higgs Φ and SM *H*

- Two marginal operators: simultaneous vector portal and scalar portal couplings
  - Constraints driven by searches, not known from first principles (possible in UV completions)

Double Dark Portal model

Recipe for solving the neutral vector Lagrangian

1. Diagonalize gauge boson mass matrix
2. Remove kinetic mixing and canonically normalize
3. Rediagonalize mass matrix (and can expand in ε if desired)

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\,\mu\nu} - \frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{\epsilon}{2\cos\theta_W}B_{\mu\nu}K^{\mu\nu}$$
$$+ |D_\mu H|^2 + |D_\mu \Phi|^2 + \mu_H^2|H|^2 - \lambda_H|H|^4 + \mu_D^2|\Phi|^2 - \lambda_D|\Phi|^4 - \lambda_{HP}|H|^2|\Phi|^2$$
$$+ \bar{\chi}(i\not{D} - m_\chi)\chi$$

Double Dark Portal model

- Fermion bilinears experience the new currents

$$
\begin{aligned}
\mathcal{L} \supset\, & g Z_{\mu,\,\mathrm{SM}} J_Z^\mu + e A_{\mu,\,\mathrm{SM}} J_{\mathrm{em}}^\mu + g_D K_\mu J_D^\mu \\
&= \tilde{Z}_\mu \left(g J_Z^\mu - g_D \frac{m_{Z,\,\mathrm{SM}}^2 t_W}{m_{Z,\,\mathrm{SM}}^2 - m_K^2} \epsilon J_D^\mu + g \frac{m_{Z,\,\mathrm{SM}}^2 (m_{Z,\,\mathrm{SM}}^2 - 2 m_K^2) t_W^2}{2(m_K^2 - m_{Z,\,\mathrm{SM}}^2)^2} \epsilon^2 J_Z^\mu - e \frac{m_{Z,\,\mathrm{SM}}^2 t_W}{m_{Z,\,\mathrm{SM}}^2 - m_K^2} \epsilon^2 J_{\mathrm{em}}^\mu \right) \\
&+ \tilde{K}_\mu \left(g_D J_D^\mu + g \frac{m_K^2 t_W}{m_{Z,\,\mathrm{SM}}^2 - m_K^2} \epsilon J_Z^\mu + e \epsilon J_{\mathrm{em}}^\mu + g_D \frac{(m_{Z,\,\mathrm{SM}}^4 c_W^2 - 2 m_K^2 m_{Z,\,\mathrm{SM}}^2 + m_K^4) c_W^{-2}}{2(m_{Z,\,\mathrm{SM}}^2 - m_K^2)^2} \epsilon^2 J_D^\mu \right) \\
&+ \tilde{A}_\mu e J_{\mathrm{em}}^\mu
\end{aligned}
$$

- $U(1)_D$- charged fermions pick up ε weak charge mediated by *Z*
- SM charged fermions pick up ε weak charge and ε electric charge mediated by dark photon
- Photon remains massless, long-range
  - (Singular behavior at $m_K = m_{Z,\,SM}$ is maximal mixing limit)

Double Dark Portal model

- Scalar boson mixing
  - Higgs portal coupling leads to mass mixing between dark Higgs and SM Higgs
    - Mixing angle

$$\tan 2\alpha = \frac{\lambda_{HP} v_H v_D}{\lambda_D v_D^2 - \lambda_H v_H^2}$$

    - Masses

$$m_{S,\,H_0}^2 = \lambda_H v_H^2 + \lambda_D v_D^2 \pm \sqrt{(\lambda_H v_H^2 - \lambda_D v_D^2)^2 + \lambda_{HP} v_H^2 v_D^2}$$

  - Dominant effect is cos α-suppression of Higgs couplings to fermions, dark Higgs mass eigenstate *S* picks up sin α-suppressed couplings to SM fermions

Double Dark Portal model

- Scalar-vector-vector interactions
  - Plays a key role in e^+e^- studies
  - To O(ε)

$$\mathcal{L} \supset m_{Z,\text{SM}}^2 \left(\frac{\cos\alpha}{v_H}\right) \tilde{Z}_\mu \tilde{Z}^\mu H_0$$
$$+ 2\epsilon t_W \frac{m_K^2 m_{Z,\text{ SM}}^2}{(m_{Z,\text{ SM}}^2 - m_K^2)} \left(\frac{\cos\alpha}{v_H} + \frac{\sin\alpha}{v_D}\right) \tilde{Z}_\mu \tilde{K}^\mu H_0$$
$$+ m_K^2 \left(-\frac{\sin\alpha}{v_D}\right) \tilde{K}_\mu \tilde{K}^\mu H_0$$
$$+ m_{Z,\text{SM}}^2 \left(\frac{\sin\alpha}{v_H}\right) \tilde{Z}_\mu \tilde{Z}^\mu S$$
$$+ 2\epsilon t_W \frac{m_K^2 m_{Z,\text{ SM}}^2}{(m_{Z,\text{ SM}}^2 - m_K^2)} \left(-\frac{\cos\alpha}{v_D} + \frac{\sin\alpha}{v_H}\right) \tilde{Z}_\mu \tilde{K}^\mu S$$
$$+ m_K^2 \left(\frac{\cos\alpha}{v_D}\right) \tilde{K}_\mu \tilde{K}^\mu S$$

Phenomenology

- Three new states $\tilde{K}$, S , χ
- Many new interactions
  - Deviations in *Z* couplings
  - Deviations in Higgs couplings
  - New 125-GeV Higgs decays
    - Invisible, semi-visible, fully visible
  - Interactions with dark matter mediated by dark photon
- Rich phenomenology with signatures in direct detection, indirect detection, astrophysics, and colliders
  - Double Dark Portal model ties together two marginal couplings simultaneously

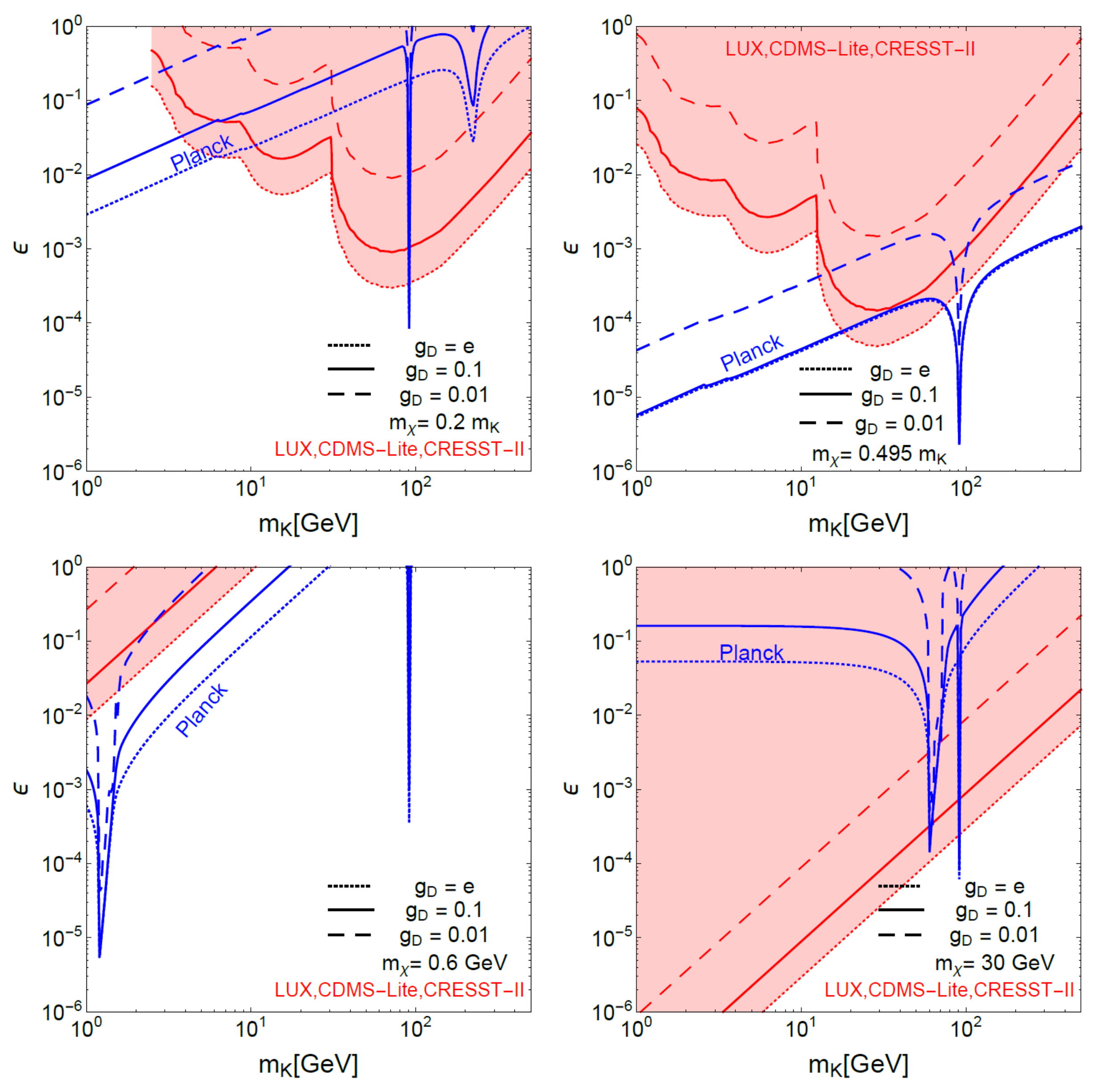
Dark matter direct detection

- Dark matter scattering off protons dominantly from dark photon exchange, suppressed by $(\varepsilon e)^2$
  - Intrinsic cancellation between weak charged currents mediated by massive Z and K vectors (at this order in ε)
  - Dark matter does not interact with photon, hence only protons contribute to direct detection

$$\sigma_p \simeq \frac{\epsilon^2 g_D^2 e^2}{\pi} \frac{\mu_{\chi p}^2}{m_{\tilde{K}}^4} \approx 10^{-44}\ \mathrm{cm}^2 \left(\frac{g_D}{e}\right)^2 \left(\frac{\epsilon}{10^{-5}}\right)^2 \left(\frac{10\ \mathrm{GeV}}{m_{\tilde{K}}}\right)^2$$

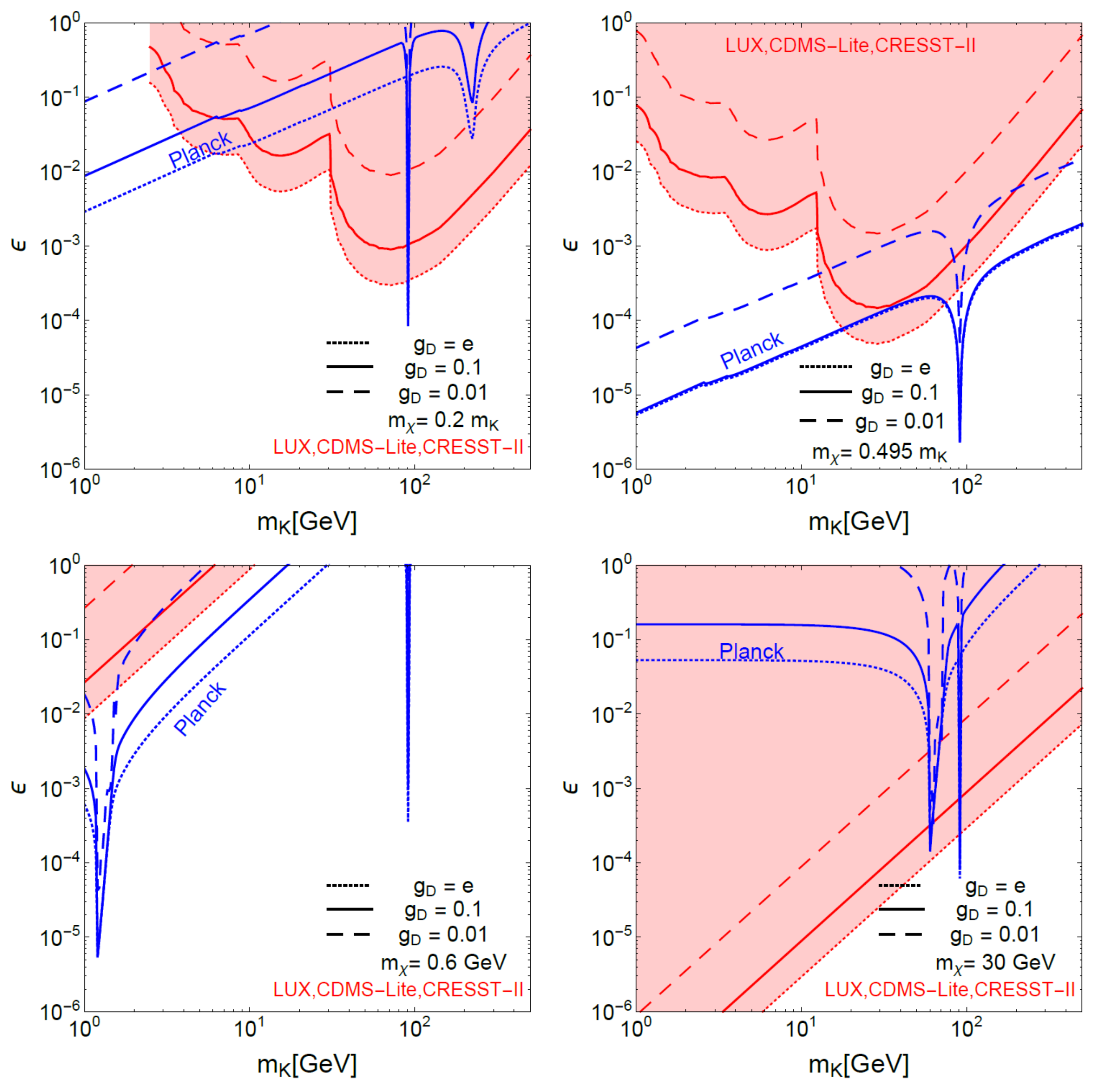
Dark matter direct detection

- Exclusion limits are highly sensitive to the dark matter mass
  - Nuclear recoil energy threshold becomes too soft for light dark matter (about 5 GeV)



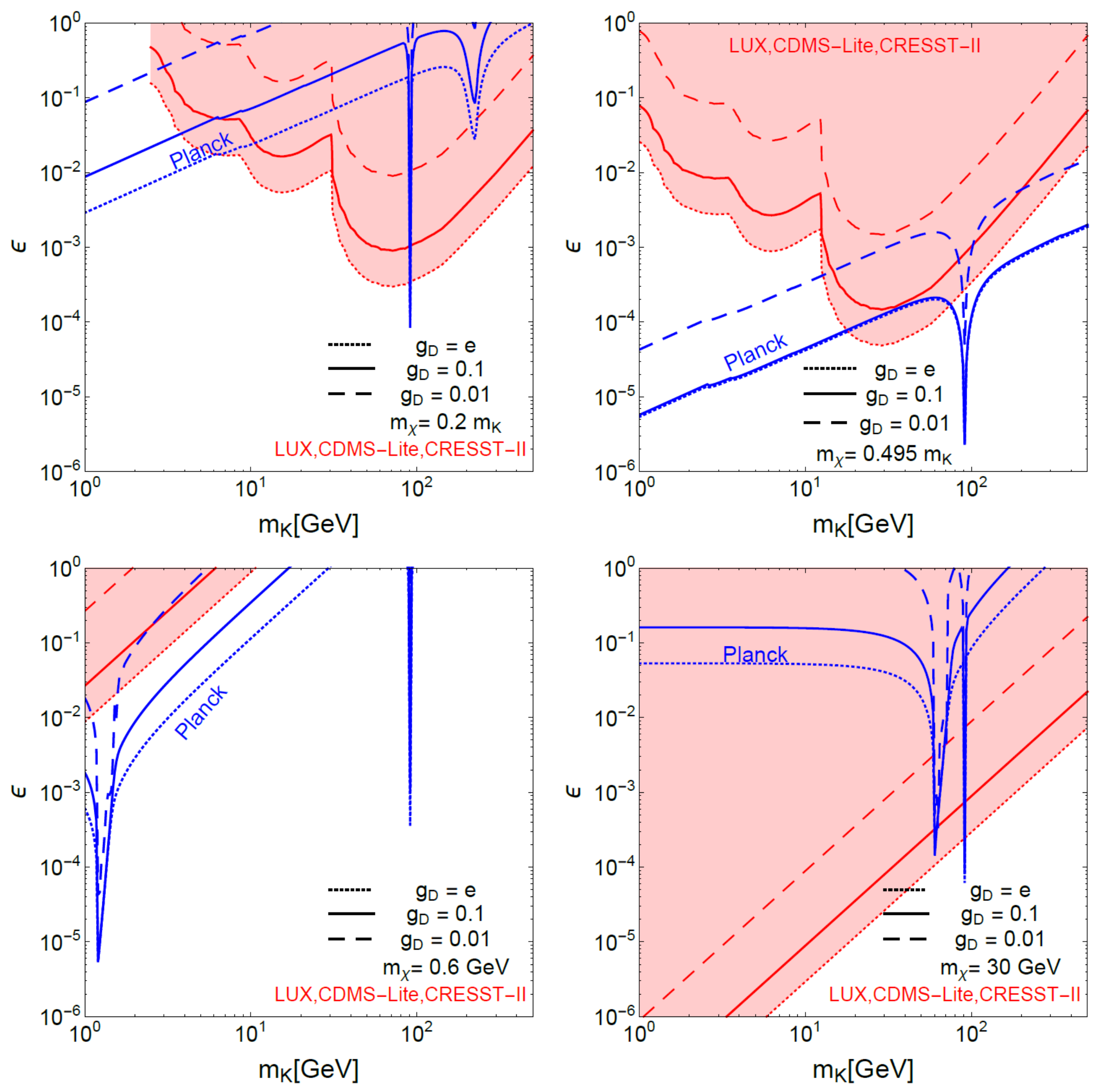
Dark matter direct detection

- Relic abundance (blue line) shows resonances at dark photon and *Z* masses
- DM is underabundant above blue line, overabundant below blue line



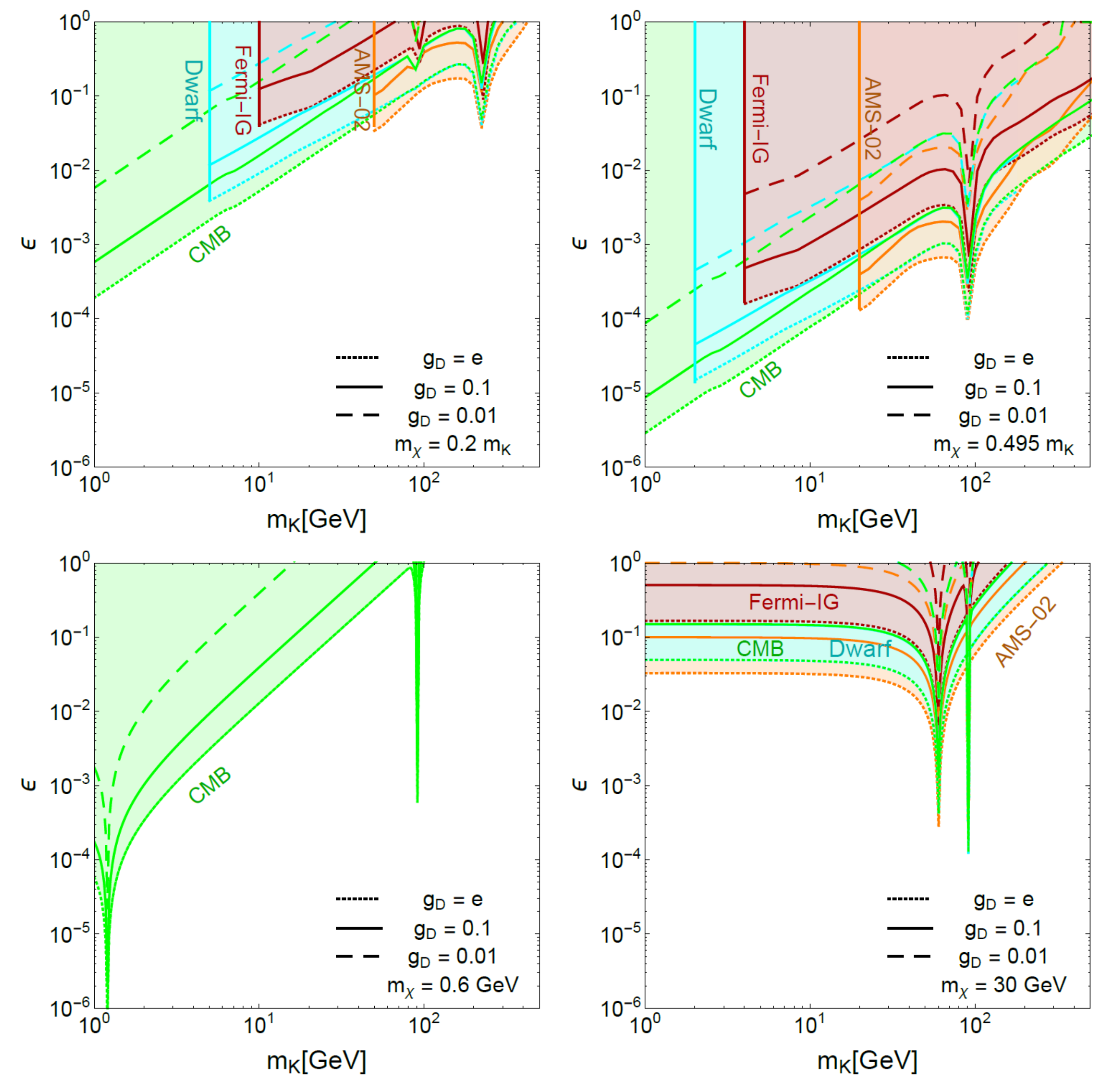
Dark matter direct detection

- Dark matter experiments fix the local relic abundance to 0.3 GeV/cm^3
  - On the other hand, the predicted dark matter relic abundance scales as ε^{-2}, while the scattering rate scales as ε^2
- Ratio of DD limits to relic abundance curve (for fixed m_K) gives the limit on local abundance



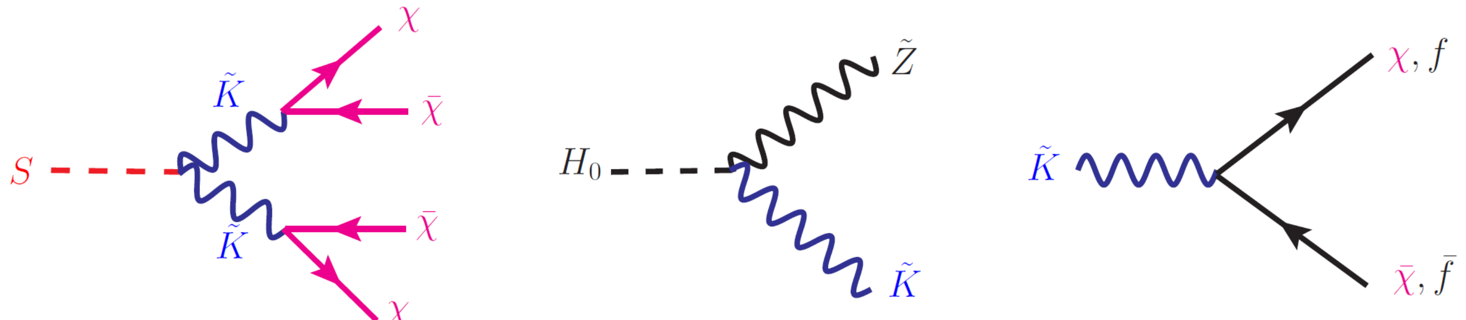
Dark matter indirect detection

- Present day annihilation constrained by observations of gamma ray spectra
- Early universe annihilation constrained by energy injection in CMB
- Strongest limits when DM mass is close to *Z* or dark photon resonance



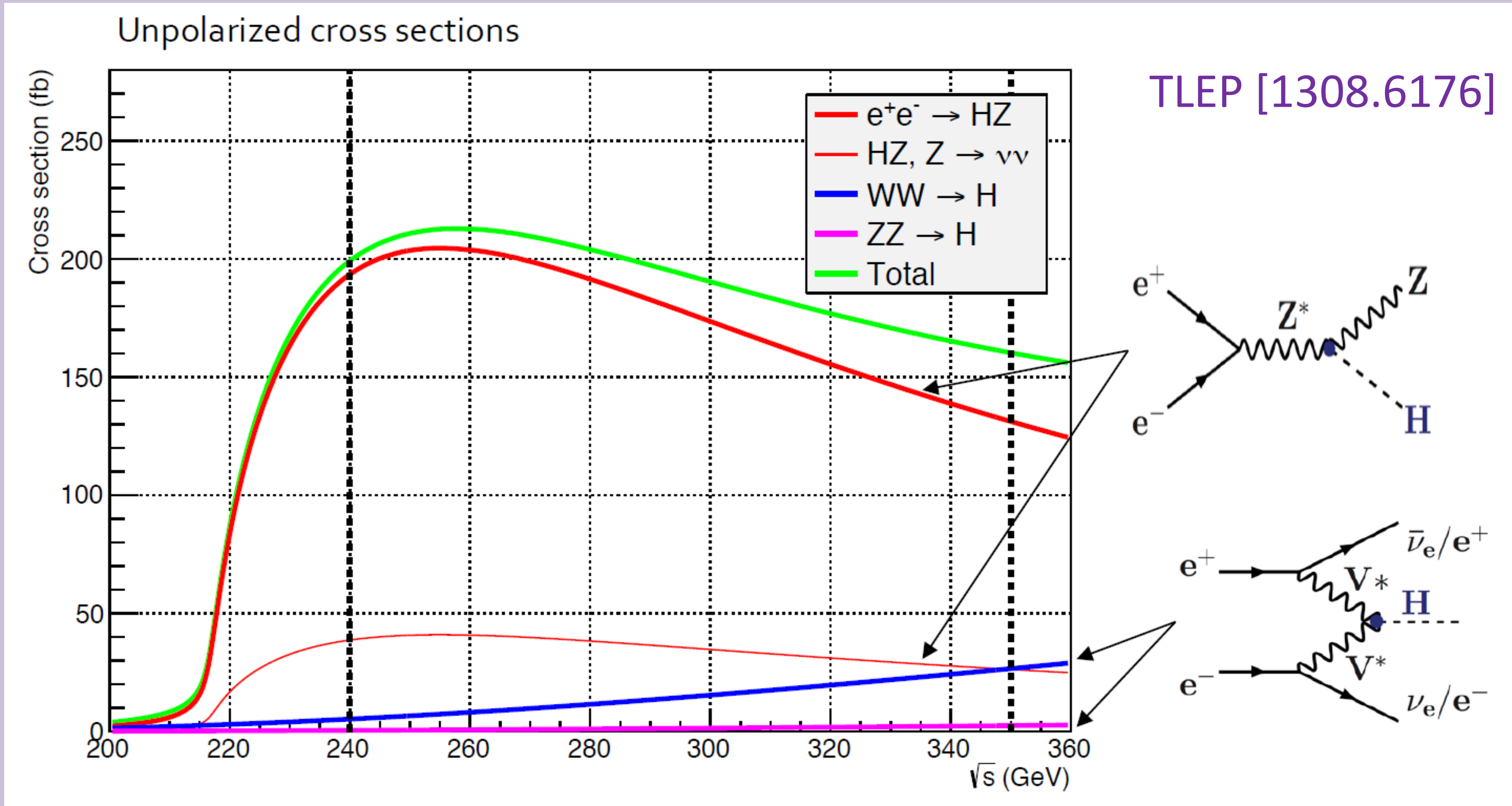
Collider phenomenology

- Modifications to *Z* couplings probed in precision electroweak observables
- Modifications to Higgs couplings tested by LHC and can be seen at a future Higgs factory
  - Also induce invisible and semi-visible exotic Higgs decays
- Will assume dark decays of *S* and *K* are on-shell
  - Ensured by kinematics and mild hierarchy for g_D and ε
    - (Can get displaced decays when dark matter is too heavy)



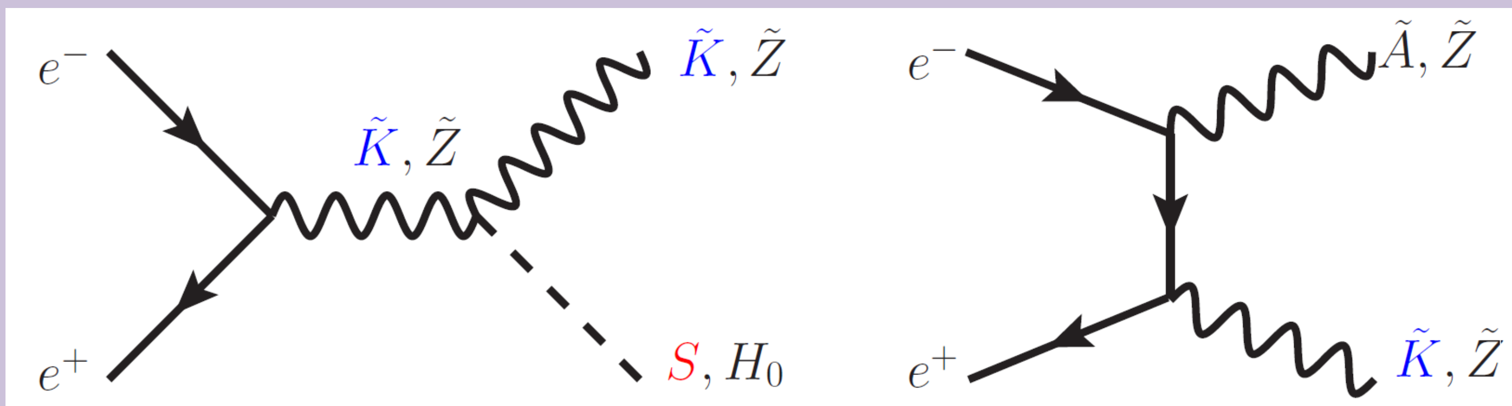
Going beyond κ-framework, Higgs EFT

- Higgsstrahlung production at e^+e^- machine
  - In Double Dark Portal model, both portal couplings give leading order deviations in Higgsstrahlung



Going beyond κ-framework, Higgs EFT

- New light states cause deviations in Higgs physics and can be directly produced



- Exploit radiative return process for hidden photon production
  - Recoil mass technique adapted to monophoton events and other SM candles as recoil taggers

New capabilities at e^+e^- machines

- Radiative return – use ISR photon to make 2-2 production on-shell
    - At LHC, "radiative return" is better known as "mono-jet"
- Recoil mass method – use four-momentum conservation in 2-2 process
    - In case of invisible decay and radiative return, equivalent to searching for a monophoton peak
        - Design driver for e^+e^- electromagnatic calorimeter

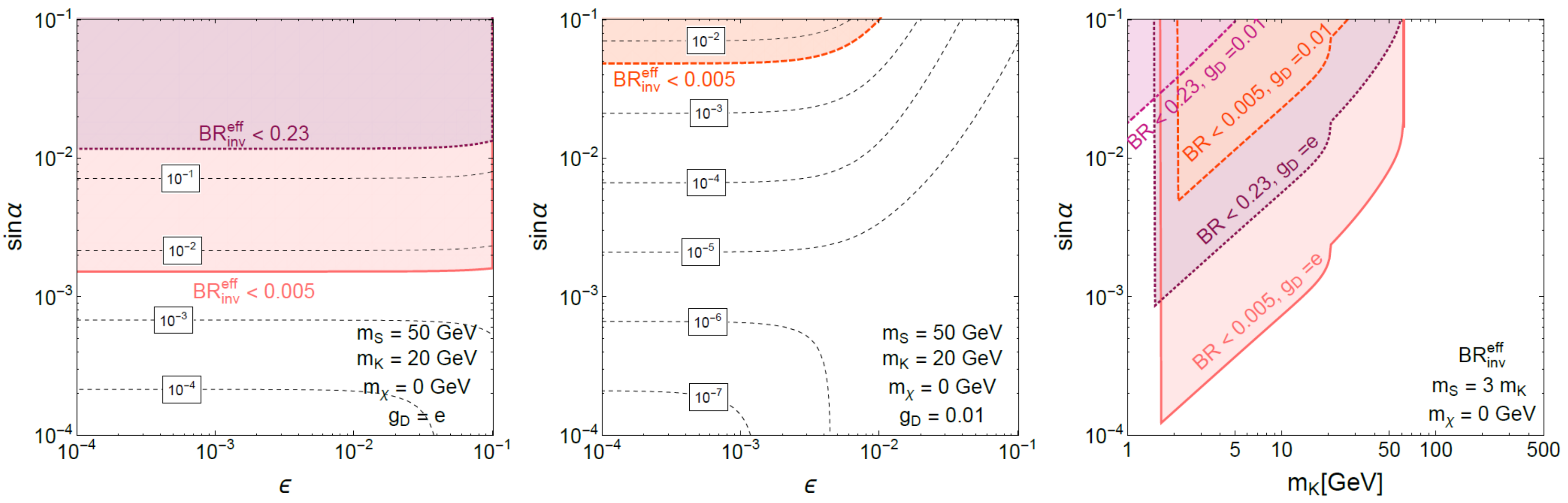
$$E_{\text{vis}} = \frac{\sqrt{s}}{2} + \frac{m_{\text{vis}}^2 - m_X^2}{2\sqrt{s}}$$

$$m_{\text{recoil}} = m_X = \sqrt{s + m_{\text{vis}}^2 - 2E_{\text{vis}}\sqrt{s}}$$

Exotic invisible decay of Higgs

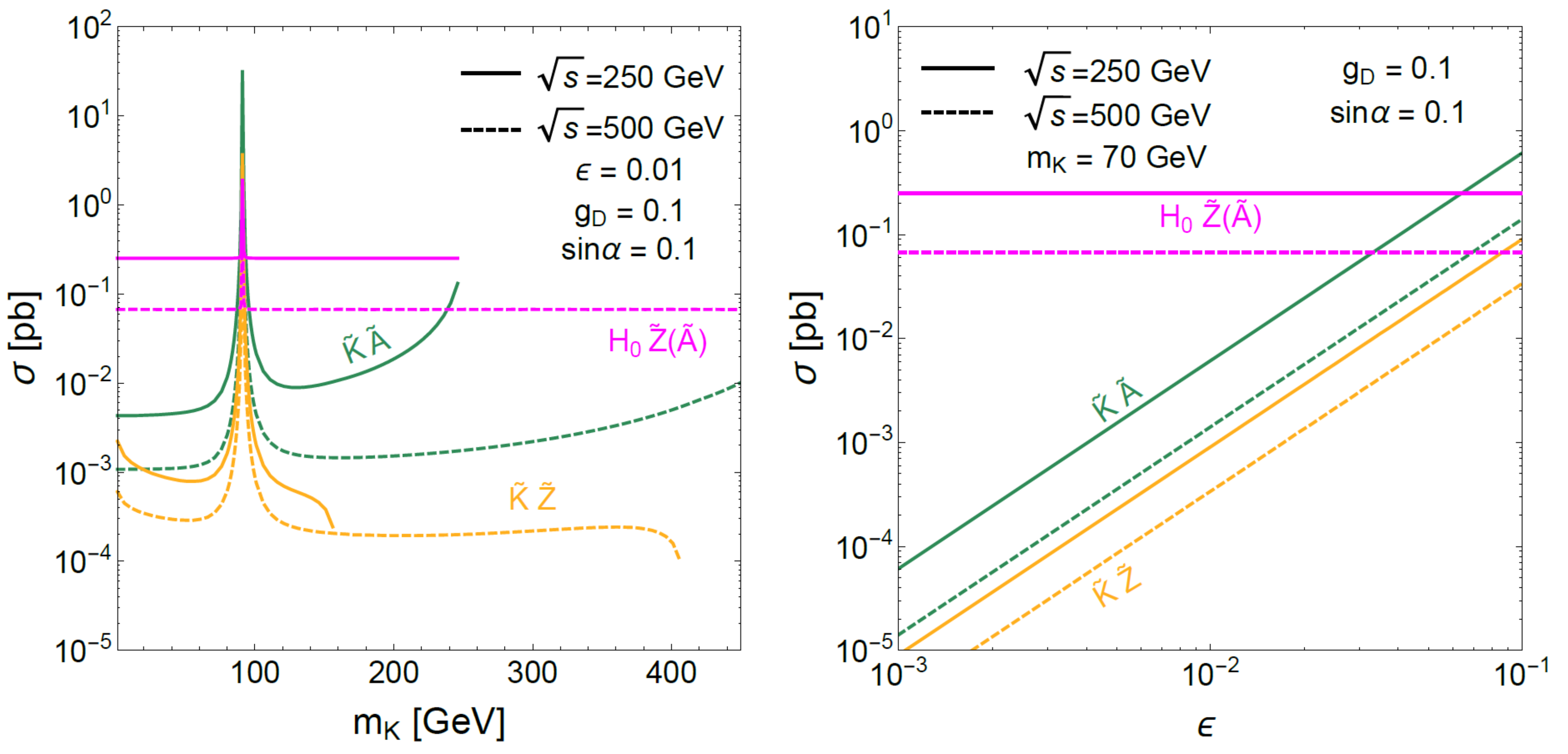
- Familiar case: Higgs recoiling against Z for invisible Higgs decays
  - Invisible decay combines sensitivity to sin α and ε, overall rate driven by g_D

$$\Gamma(H_0 \to \text{inv}) \approx \Gamma(H_0 \to SS) + \Gamma(H_0 \to \tilde{K}\tilde{K}) + 0.2 \times \Gamma(H_0 \to \tilde{K}\tilde{Z})$$



Direct production of new light states

- Possible new physics within kinematic reach
  - Signatures too difficult at LHC, exploit e^+e^- capabilities



Prospects for dark photon

- Many possible visible and invisible final states

$e^+e^- \to \tilde{Z}H_0$ Study $\tilde{Z} \to \ell\ell$ and semi-visible $H_0 \to (\ell\ell)_Z \chi\chi$

$e^+e^- \to \tilde{Z}\tilde{K}$ Study $\tilde{Z} \to \ell\ell$ and $\tilde{K} \to \bar{\chi}\chi$ or $\ell\ell$

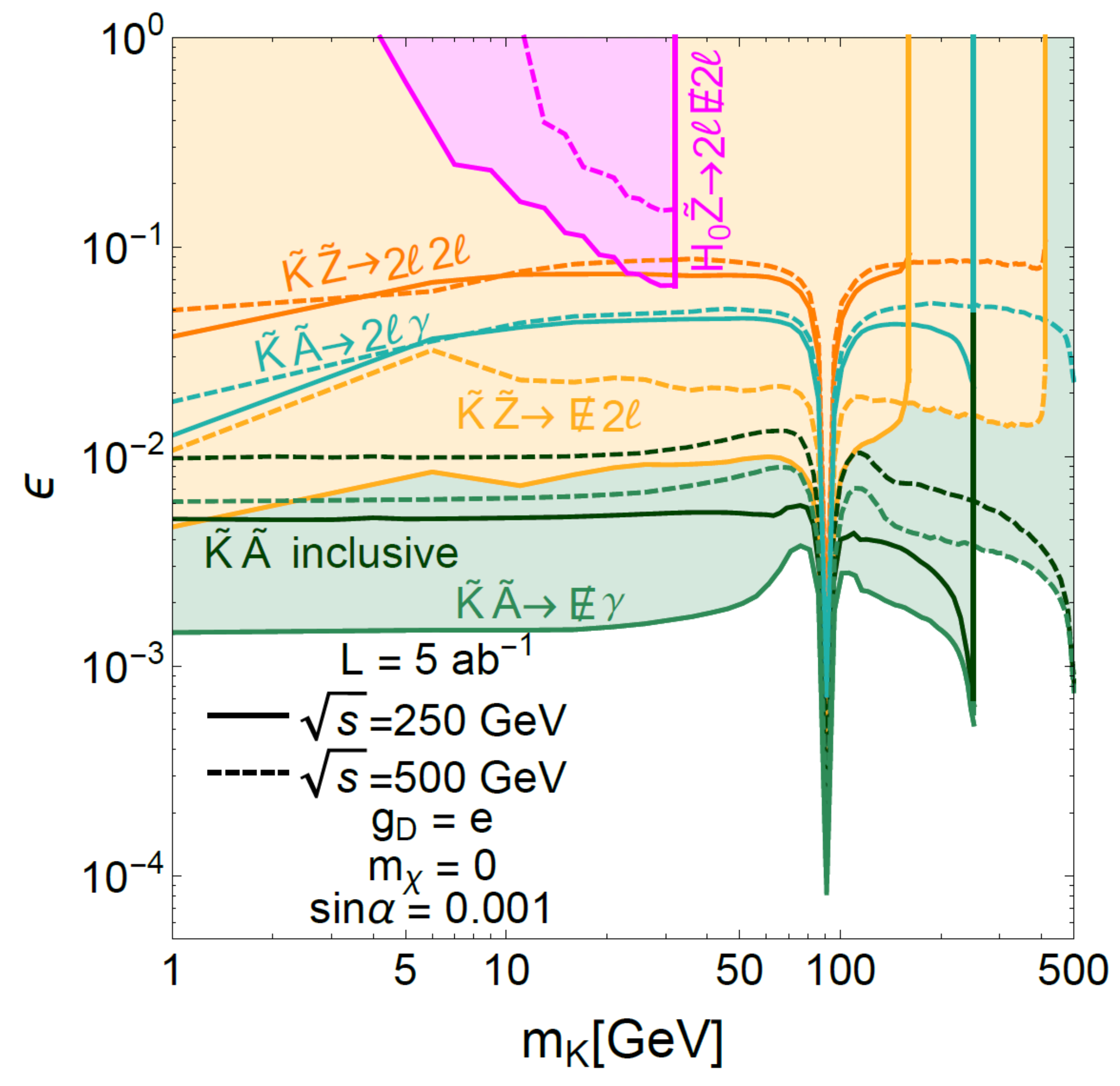
$e^+e^- \to \gamma\tilde{K}$ Study $\tilde{K}$ inclusive decays, and exclusive $\tilde{K} \to \bar{\chi}\chi$ or $\ell\ell$

$e^+e^- \to \tilde{Z}S$ Study $\tilde{Z} \to \ell\ell$ and $S \to 4\chi$

  - Event simulation using MG5+Pythia+Delphes
    - Use parametrized preliminary CEPC detector card
  - Backgrounds, cuts in backup
- Rates for visible states are lower by $(\varepsilon/g_D)^2$, best sensitivity from requiring missing energy threshold
  - LEP direct constraints ($\varepsilon < 0.03$) not competitive

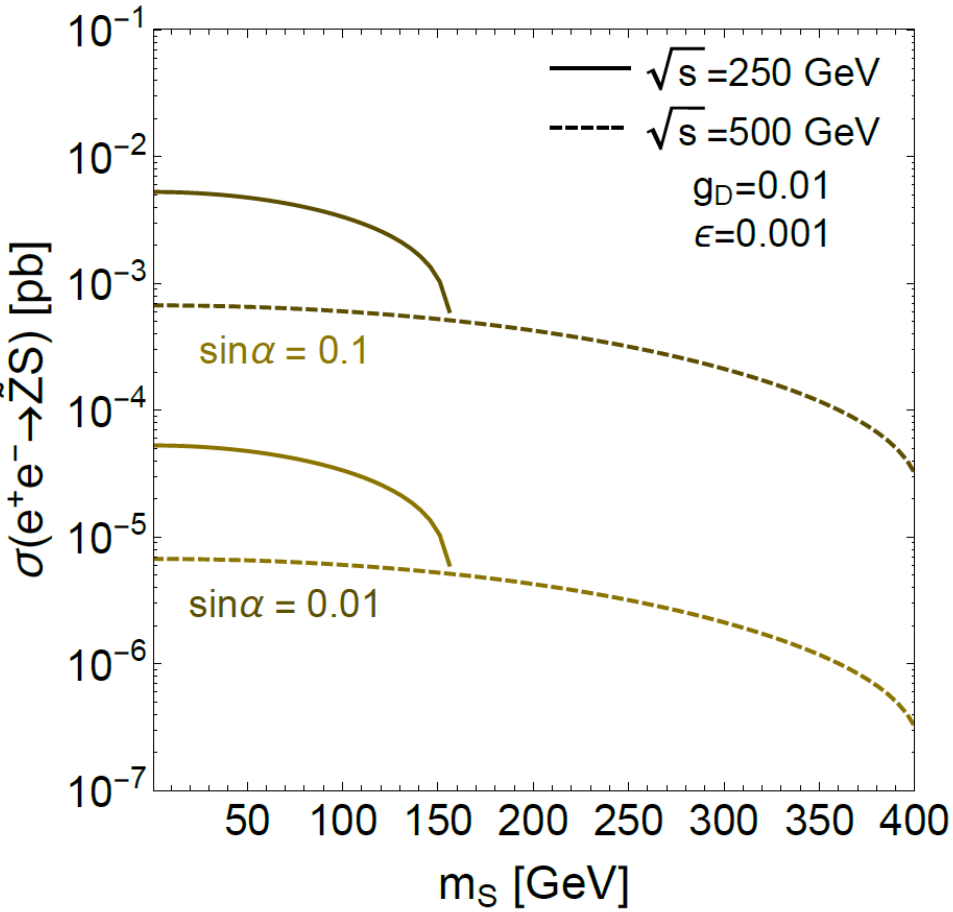
Hook, Izaguirre, Wacker [1006.0973]

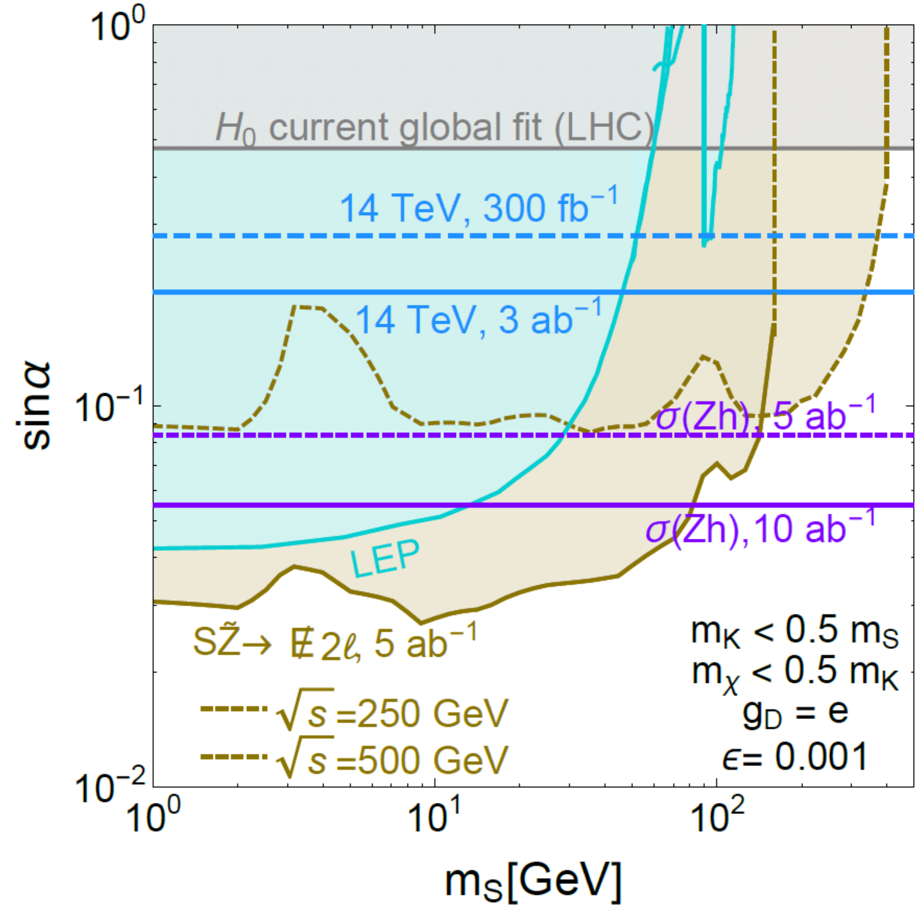
Dark photon sensitivity



Prospects for dark scalar

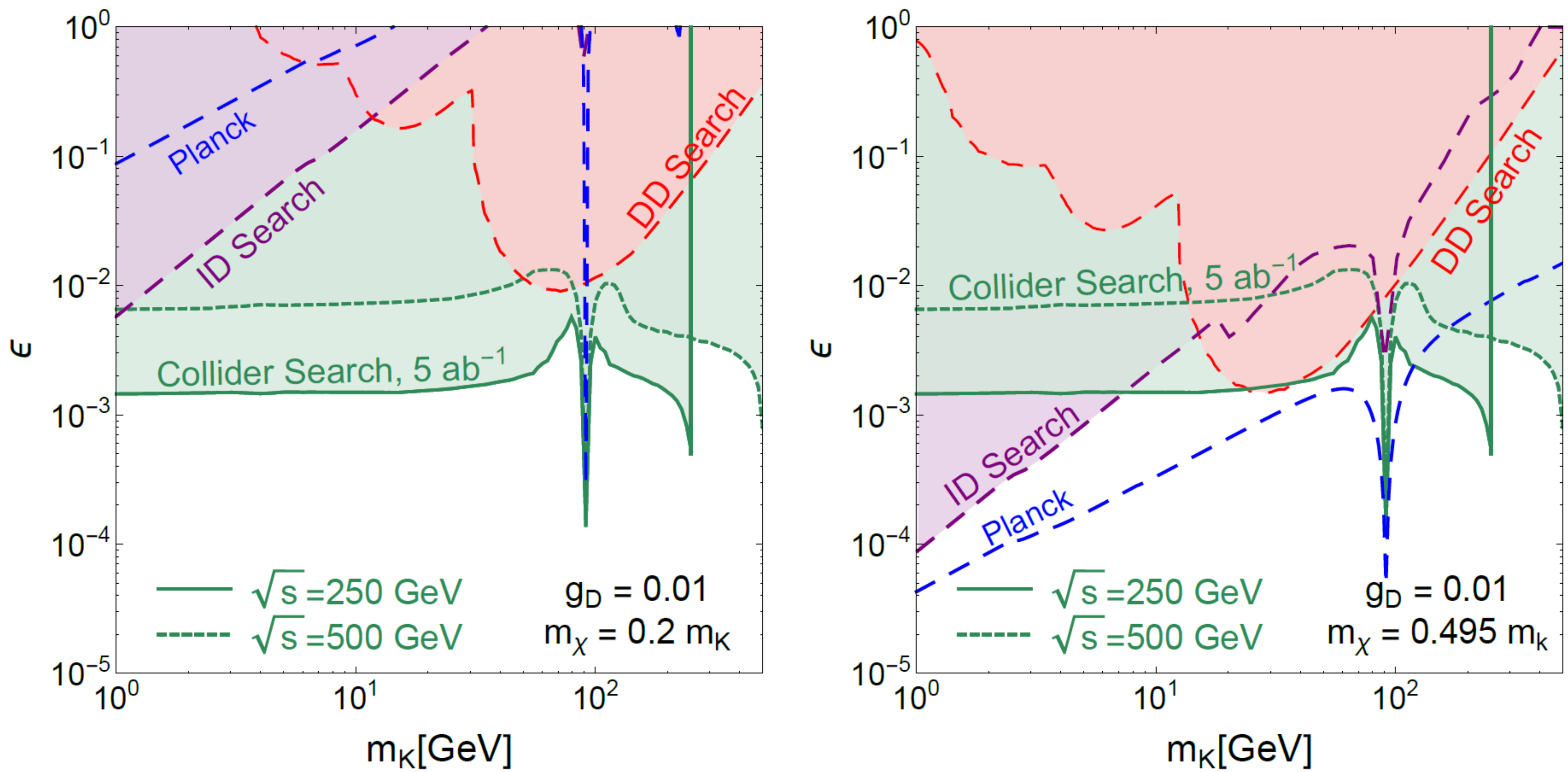
- Similarly, Higgs physics from invisible decays and precision Higgs measurements





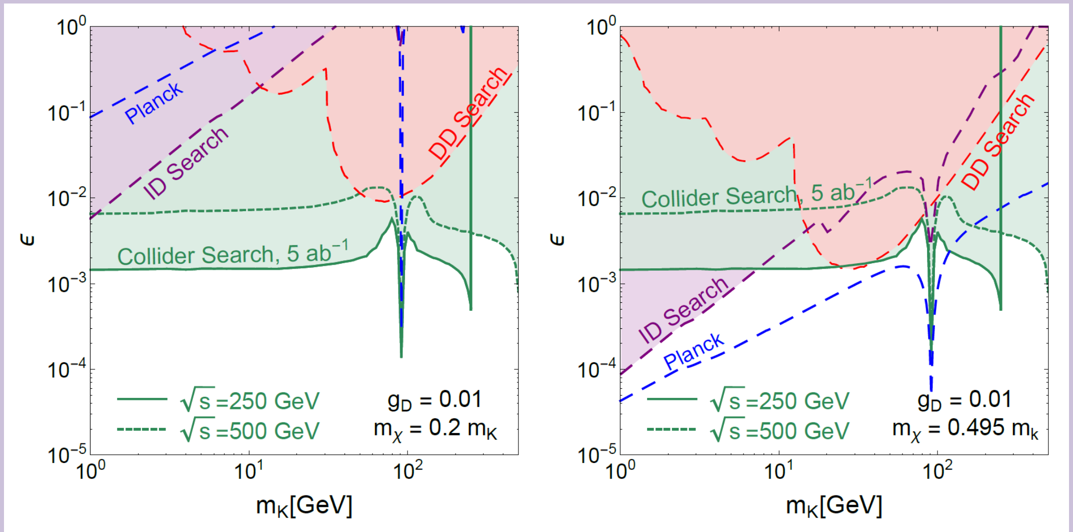
Combining complementary probes

- Dark matter discovery possible at e^+e^- machines



Conclusions

- Physics potential of e^+e^- machine goes well beyond precision Standard Model program
- Direct production of new, light, hidden particles possible
- Double Dark Portal model is a concrete framework for studying two marginal couplings in tandem

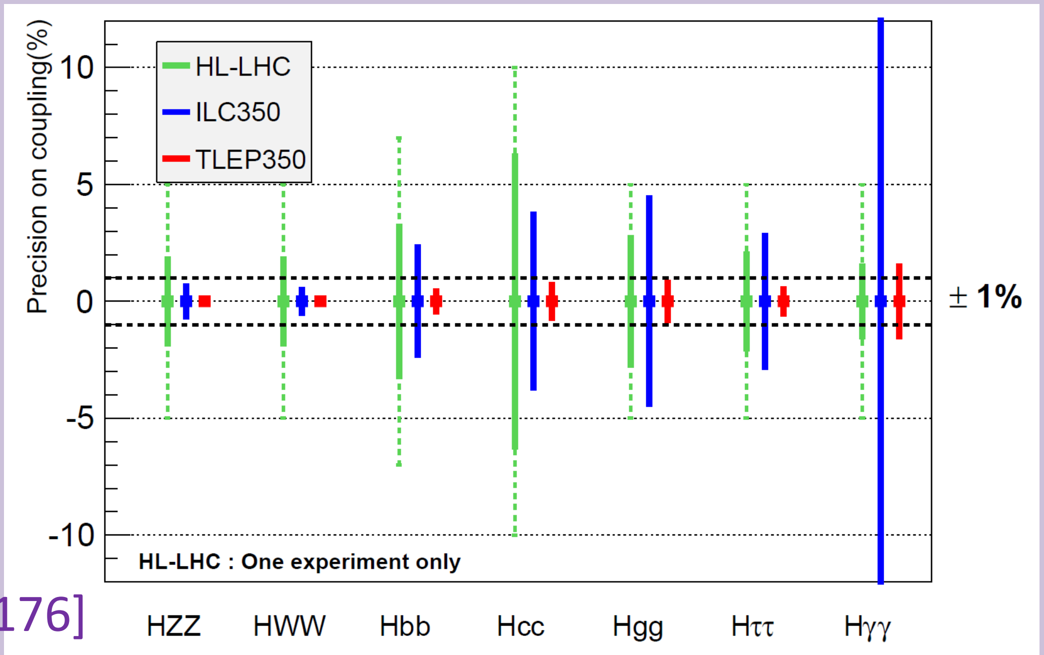


Collider study cuts

Parameter	Signal process		Background (pb)		Signal region
ϵ	$\tilde{Z}\tilde{K}$	$\tilde{Z} \to \bar{\ell}\ell$, $\tilde{K} \to \bar{\chi}\chi$	$\bar{\ell}\ell\bar{\nu}\nu$	0.929 (250 GeV)	$N_\ell \geq 2$, $\lvert m_{\ell\ell} - m_Z \rvert < 10$ GeV,
				0.545 (500 GeV)	and $\lvert m_{\text{recoil}} - m_{\tilde{K}} \rvert < 2.5$ GeV
		$\tilde{Z} \to \bar{\ell}\ell$, $\tilde{K} \to \bar{\ell}\ell$	$\bar{\ell}\ell\bar{\ell}\ell$	0.055 (250 GeV)	$N_\ell \geq 4$, $\lvert m_{\ell\ell} - m_Z \rvert < 10$ GeV,
				0.023 (500 GeV)	and $\lvert m_{\ell\ell} - m_{\tilde{K}} \rvert < 2.5$ GeV
	$\tilde{A}\tilde{K}$	$\tilde{K}$ inclusive decay	$\gamma\bar{f}f$	23.14 (250 GeV)	$N_\gamma \geq 1$, and
				8.88 (250 GeV)	$\lvert E_\gamma - (\frac{\sqrt{s}}{2} - \frac{m_{\tilde{K}}^2}{2\sqrt{s}}) \rvert < 2.5$ GeV
		$\tilde{K} \to \bar{\ell}\ell$	$\gamma\bar{\ell}\ell$	12.67 (250 GeV)	$N_\gamma \geq 1$, $N_\ell \geq 2$, $\lvert E_\gamma - (\frac{\sqrt{s}}{2} - \frac{m_{\tilde{K}}^2}{2\sqrt{s}}) \rvert < 2.5$ GeV,
				4.38 (500 GeV)	and $\lvert m_{\ell\ell} - m_{\tilde{K}} \rvert < 5$ GeV
		$\tilde{K} \to \bar{\chi}\chi$	$\gamma\bar{\nu}\nu$	3.45 (250 GeV)	$N_\gamma \geq 1$, $\lvert E_\gamma - (\frac{\sqrt{s}}{2} - \frac{m_{\tilde{K}}^2}{2\sqrt{s}}) \rvert < 2.5$ GeV,
				2.92 (500 GeV)	and $\not{E} > 50$ GeV
	$\tilde{Z}H_0$	$H_0 \to \tilde{K}\tilde{Z}$ with $\tilde{K} \to \bar{\chi}\chi$, $\tilde{Z} \to \bar{\ell}\ell$	$\bar{\ell}\bar{\ell}\ell\ell\bar{\nu}\nu$	1.8×10^{-5} (250 GeV)	$N_\ell \geq 4$, $\lvert m_{\ell\ell} - m_Z \rvert < 10$ GeV,
				3.5×10^{-4} (500 GeV)	and $\lvert m_{\text{recoil}} - m_{\tilde{K}} \rvert < 2.5$ GeV
$\sin\alpha$	$\tilde{Z}S$	$\tilde{Z} \to \bar{\ell}\ell$, $S \to \tilde{K}\tilde{K} \to 4\chi$	$\bar{\ell}\ell\bar{\nu}\nu$	0.87 (250 GeV)	$N_\ell \geq 2$, $\lvert m_{\ell\ell} - m_Z \rvert < 10$ GeV,
				0.87 (250 GeV)	and $\lvert m_{\text{recoil}} - m_S \rvert < 2.5$ GeV

Future e^+e^- collider(s)

- In particular, precision Higgs machines are strongly motivated, including
  - International Linear Collider — ILC [0709.1893]
  - TLEP / Future Circular Collider-ee — TLEP [1308.6176]
  - Circular electron-positron collider — CEPC-SPPC Pre-CDR, IHEP-TH-2015-01



From TLEP [1308.6176]

Double Dark Portal model

- Steps for solving the neutral vector Lagrangian (pedagogical)
  - Diagonalize gauge boson mass matrix
    - Usual $t_W = g' / g$ rotation corresponds to

$$\mathcal{L} \supset \frac{-1}{4} \begin{pmatrix} Z^{\mu\nu}_{\rm SM} & A^{\mu\nu}_{\rm SM} & K^{\mu\nu} \end{pmatrix} \begin{pmatrix} 1 & 0 & \epsilon t_W \\ 0 & 1 & -\epsilon \\ \epsilon t_W & -\epsilon & 1 \end{pmatrix} \begin{pmatrix} Z_{\mu\nu,\ \rm SM} \\ A_{\mu\nu,\ \rm SM} \\ K_{\mu\nu} \end{pmatrix}$$

$$+ \frac{1}{2} \begin{pmatrix} Z^{\mu}_{\rm SM} & A^{\mu}_{\rm SM} & K^{\mu} \end{pmatrix} \begin{pmatrix} m^2_{Z,\ \rm SM} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m^2_K \end{pmatrix} \begin{pmatrix} Z_{\mu,\ \rm SM} \\ A_{\mu,\ \rm SM} \\ K_{\mu} \end{pmatrix}$$

  - Require $|\varepsilon| < c_W$ for positive kinetic mixing determinant
  - Field strengths are Abelian kinetic terms, non-Abelian interactions inherited from transformations

Double Dark Portal model

- Steps for solving the neutral vector Lagrangian (pedagogical)
  - Remove kinetic mixing and canonically normalize

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\epsilon^2 t_W & 1 & \epsilon \\ -\epsilon t_W & 0 & 1 \end{pmatrix} \qquad U_2 = \begin{pmatrix} \sqrt{\dfrac{1-\epsilon^2}{1-\epsilon^2 c_W^{-2}}} & 0 & 0 \\ 0 & 1 & 0 \\ \dfrac{-\epsilon^3 t_W}{\sqrt{(1-\epsilon^2)(1-\epsilon^2 c_W^{-2})}} & 0 & \dfrac{1}{\sqrt{1-\epsilon^2}} \end{pmatrix}$$

$$\mathcal{L} \supset \frac{-1}{4} \begin{pmatrix} Z_{\rm SM}^{\mu\nu} & A_{\rm SM}^{\mu\nu} & K^{\mu\nu} \end{pmatrix} (U_1^T)^{-1} (U_2^T)^{-1} \mathbb{I}_3 U_2^{-1} U_1^{-1} \begin{pmatrix} Z_{\mu\nu,\ \rm SM} \\ A_{\mu\nu,\ \rm SM} \\ K_{\mu\nu} \end{pmatrix}$$

$$+ \frac{1}{2} \begin{pmatrix} Z_{\rm SM}^{\mu} & A_{\rm SM}^{\mu} & K^{\mu} \end{pmatrix} (U_1^T)^{-1} (U_2^T)^{-1} \begin{pmatrix} \dfrac{m_{Z,\ \rm SM}^2 (1-\epsilon^2)^2 + m_K^2 \epsilon^2 t_W^2}{(1-\epsilon^2)(1-\epsilon^2 c_W^{-2})} & 0 & \dfrac{-m_K^2 \epsilon t_W}{(1-\epsilon^2)\sqrt{1-\epsilon^2 c_W^{-2}}} \\ 0 & 0 & 0 \\ \dfrac{-m_K^2 \epsilon t_W}{(1-\epsilon^2)\sqrt{1-\epsilon^2 c_W^{-2}}} & 0 & \dfrac{m_K^2}{1-\epsilon^2} \end{pmatrix}$$

$$\times U_2^{-1} U_1^{-1} \begin{pmatrix} Z_{\mu,\ \rm SM} \\ A_{\mu,\ \rm SM} \\ K_\mu \end{pmatrix}$$

Double Dark Portal model

- Steps for solving the neutral vector Lagrangian (pedagogical)
  - Rediagonalize mass matrix via Jacobi rotation (exact)
  - To O(ε³), masses and fields are

$$m_{\tilde{K}}^2 = m_K^2 + \frac{m_K^2 c_W^{-2} \epsilon^2 (m_{Z,\,\mathrm{SM}}^2 c_W^2 - m_K^2)}{m_{Z,\,\mathrm{SM}}^2 - m_K^2}\,, \quad m_{\tilde{Z}}^2 = m_{Z,\,\mathrm{SM}}^2 + \frac{m_{Z,\,\mathrm{SM}}^4 t_W^2 \epsilon^2}{m_{Z,\,\mathrm{SM}}^2 - m_K^2}$$

$$\begin{pmatrix} \tilde{Z}_\mu \\ \tilde{A}_\mu \\ \tilde{K}_\mu \end{pmatrix} = \begin{pmatrix} Z_{\mu,\,\mathrm{SM}} - \frac{t_W m_K^2}{m_{Z,\,\mathrm{SM}}^2 - m_K^2} \epsilon K_\mu - \frac{m_{Z,\,\mathrm{SM}}^4 t_W^2}{2(m_{Z,\,\mathrm{SM}}^2 - m_K^2)^2} \epsilon^2 Z_{\mu,\,\mathrm{SM}} \\ A_{\mu,\,\mathrm{SM}} - \epsilon K_\mu \\ K_\mu + \frac{t_W m_{Z,\,\mathrm{SM}}^2}{m_{Z,\,\mathrm{SM}}^2 - m_K^2} \epsilon Z_{\mu,\,\mathrm{SM}} - \left(\frac{1}{2} + \frac{m_K^4 t_W^2}{2(m_{Z,\,\mathrm{SM}}^2 - m_K^2)^2} \right) \epsilon^2 K_\mu \end{pmatrix}$$

    - Singular behavior at $m_K = m_{Z,\,SM}$ is maximal mixing limit
  - Effects from field redefinitions seen in dark, SM currents

Exotic invisible decay of Higgs

$$\Gamma(H_0 \to SS) = g_D^2 \sin^2\alpha \frac{m_{H_0}}{32\pi}\sqrt{1-\frac{4m_S^2}{m_{H_0}^2}}\frac{(m_{H_0}^2+2m_S^2)^2}{m_{H_0}^2 m_K^2} \ ,$$

$$\Gamma(H_0 \to \tilde{K}\tilde{K}) = g_D^2 \sin^2\alpha \frac{m_{H_0}}{32\pi}\sqrt{1-\frac{4m_{\tilde{K}}^2}{m_{H_0}^2}}\frac{m_{H_0}^4 - 4m_{H_0}^2 m_{\tilde{K}}^2 + 12 m_{\tilde{K}}^4}{m_{H_0}^2 m_{\tilde{K}}^2}\frac{m_K^2}{m_{\tilde{K}}^2} \ ,$$

$$\Gamma(H_0 \to \tilde{K}\tilde{Z}) = \frac{\epsilon^2 t_W^2 \left(\frac{\cos\alpha}{v_H}+\frac{\sin\alpha}{v_D}\right)^2}{16\pi m_{H_0}^3 \left(m_K^2 - m_{Z,\,\mathrm{SM}}^2\right)^2}\frac{m_K^4 m_{Z,\,\mathrm{SM}}^4}{m_{\tilde{K}}^2 m_{\tilde{Z}}^2}\sqrt{m_{H_0}^4 + \left(m_{\tilde{K}}^2 - m_{\tilde{Z}}^2\right)^2 - 2m_{H_0}^2\left(m_{\tilde{K}}^2 + m_{\tilde{Z}}^2\right)}$$
$$\times\left((m_{H_0}^2 - m_{\tilde{K}}^2 - m_{\tilde{Z}}^2)^2 + 8 m_{\tilde{K}}^2 m_{\tilde{Z}}^2\right)$$