Detecting symmetries in 3HDM in any basis

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based on:

I. P. Ivanov, C. Nishi, J. P. Silva, A. Trautner, PRD99 (2019) 015039
I. P. Ivanov, C. Nishi, A. Trautner, EPJ C79 (2019) 315
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Is there life beyond the SM Higgs?

The minimal Higgs sector of the SM is overstretched. As a result:

- does not explain fermion masses and mixing, neutrino masses, *CP*-violation;
- has boring flavor properties: no tree-level FCNCs;
- does not help explain DM or baryon asymmetry.

These issues can be successfully addressed in models with extended scalar sectors.

A conservative but rich class of models: *N*-Higgs-doublet models (NHDMs).

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3HDM

What's new in 3HDM compared to 2HDM:

- richer pheno (both scalar and fermion sectors);
- combining nice features of 2HDM, e.g. NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009];
- new options for *CP* violation, e.g. geometrical CPV [Branco, Gerard, Grimus, 1984],
- *CP* symmetry of order 4 (CP4) [Ivanov, Silva, 2015]:
    - mass degeneracy, *CP* eigenstates beyond *CP*-even/odd [Ivanov, Silva, 2015; Haber et al, 2018];
    - DM stabilized by CP4: [Koepke, 2018; Ivanov, Laletin, 2018];
    - quark/neutrino patterns from CP4: [Ferreira et al, 2017; Ivanov, 2018];
    - solution to strong *CP* problem: [Cherchiglia, Nishi, 2019].
- symmetries, lots of symmetries in the 3HDM scalar sector!

Symmetries in 3HDM

Particular examples of 3HDMs with symmetries begin in 1970's;
full classification only recently.

- abelian groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2\times\mathbb{Z}_2, \quad U(1), \quad U(1)\times\mathbb{Z}_2, \quad U(1)\times U(1)\,.$$

- discrete non-abelian groups: [Ivanov, Vdovin, 1210.6553]

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- symmetry breaking patterns $G \to G_v$: [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested, but
  - if G is large → severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small → too many free parameters, no predictive power.
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]:

  If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.

  But for large G, this is algebraically impossible.

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Proximity to a symmetric 3HDM

For large G:

- imposing an exact G $\to$ some observables $= 0$;
- a 3HDM in the vicinity, ϵ, of an exact G $\to$ observables depend as ϵ^α.
- a 3HDM can be close to several distinct symmetric situations $\to$ competing symmetries.

Challenge

When scanning the 3HDM parameter space,

one must detect (proximity to) a G-symmetric situations.

Basis-invariant methods

Large freedom of basis changes: $\phi_a \mapsto U_{ab}\phi_b$, $U \in U(N)$.

Physics does not change upon basis changes!

A symmetry can be evident in one basis and hidden in another $\rightarrow$ challenge!

The goal

Detecting structural properties of NHDMs irrespective of the basis choice!

General recipe [Botella, Silva, 1995]:

- write down all couplings as tensors under basis changes,
- take their product and contract all indices $\rightarrow$ basis invariants J_k,
- find algebraically independent J_k,
- link them to the phenomenon you study.

Explicit *CP* conservation in 2HDM scalar sector

The most general 2HDM potential:

$$V = Y_{ab}(\phi_a^\dagger\phi_b) + Z_{ab,cd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d)\,,$$

or, in the explicit form,

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \mathrm{h.c.}\right] \end{aligned}$$

It contains $4 + 10 = 14$ free parameters.

General 2HDM scalar sector

Checking explicit CP-conservation [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

- There exists of a basis with all coefs real → symmetry $\phi_a \to \phi_a^*$.
- Construct invariants with Y_{ab} and $Z_{ab,cd}$ and establish independent ones;
- Basis-invariant criterion: check the following four invariants

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$
$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$
$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

Basis invariants

Drawbacks:

- non-intuitive, relies on computer algebra; one needs to find the generating set of the ring of symmetry-related invariants;
  NB! [Trautner, 1812.02614] shows how to derive them in 2HDM.
- becomes even more complicated beyond 2HDM; conditions for *CP* symmetry in 3HDM via basis invariants still not established [Varzielas et al, 1603.06942];
- not all information can be easily retrieved! *CP*-odd basis invariants in 3HDM cannot tell the usual *CP* from CP4 (order-4 *CP* symmetry).

A more efficient solution to the basis-invariant challenge:
basis-invariant statements via basis-covariant objects.

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Bilinears in 3HDM

Geometric constructions in the adjoint space [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008]. V is built of 9 bilinears $\phi_a^\dagger \phi_b$.

$$r_0 = \frac{1}{\sqrt{3}} \phi_a^\dagger \phi_a \,, \quad r_i = \phi_a^\dagger (t^i)_{ab} \phi_b \,, \quad i = 1, \ldots, 8 \,,$$

where $t_i = \lambda_i/2$ are $SU(3)$ generators satisfying

$$[t_i, t_j] = i f_{ijk} t_k \,, \quad \{t_i, t_j\} = \frac{1}{3}\delta_{ij} \mathbf{1}_3 + d_{ijk} t_k \,.$$

The orbit space:

$$r_0 \geq 0 \,, \quad r_0^2 - r_i^2 \geq 0 \,, \quad \sqrt{3} d_{ijk} r_i r_j r_k + (r_0^2 - 3 r_i^2) r_0/2 = 0 \,.$$

Basis changes $\to$ $SO(8)$ rotations of r_i.

$SU(3) \subset SO(8) \Rightarrow$ not all $SO(8)$ rotations are basis changes!

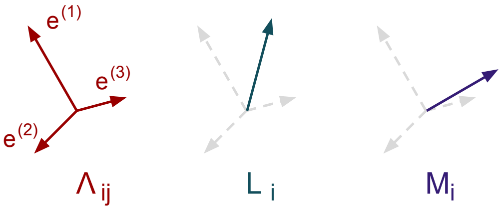
Adjoint space

The NHDM potential takes the simple form

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j \,,$$

with vectors $M, L \in \mathbb{R}^{N^2-1}$ and an $(N^2-1)\times(N^2-1)$ matrix Λ.

In 2HDM: 3×3 matrix Λ can be always diagonalized by basis change.



Orientation of M and L with respect to eigenvectors of Λ
$\Rightarrow$ immediate connection to symmetries.

Adjoint space

In 3HDM, we lack the full $SO(8)$ rotation group:

- directions in $\mathbb{R}^8$ are not equivalent!
- Λ is not in general diagonalizable by a basis change.

We need to make sense of the adjoint space.

The toolbox

Suppose vectors $a, b \in \mathbb{R}^8$. Define new products:

$$F_i^{(ab)} = f_{ijk} a_j b_k \,, \quad D_i^{(ab)} = \sqrt{3} d_{ijk} a_j b_k \,, \quad D_i^{(aa)} = \sqrt{3} d_{ijk} a_j a_k \,.$$

Applied to the eigenvectors of Λ, these products help detect basis-invariant structures in $\Lambda \Rightarrow$ symmetries in 3HDM.

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Detecting special subspaces

- **Check-(8)**. Consider $a \in \mathbb{R}^8$, $|a| = 1$. Compute vector $D^{(aa)}$.

  If $D^{(aa)} = -a$, then there is a basis in which a is along x_8.

  If an eigenvector of Λ passes Check-(8), then in this basis

$$\Lambda = \begin{pmatrix} \square_{7\times 7} & 0 \\ 0 & \Lambda_{88} \end{pmatrix}.$$

- **Check-(38)**. Consider $a, b \in \mathbb{R}^8$, $|a| = |b| = 1$.

  If $F^{(ab)} = 0$, then there is a basis in which $a, b \in (x_3, x_8)$.

  If two eigenvectors of Λ pass Check-(38), then in this basis

$$\Lambda = \begin{pmatrix} \square_{6\times 6} & 0 \\ 0 & \square_{2\times 2} \end{pmatrix}.$$

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$$\Lambda = \begin{pmatrix} \square_{6\times6} & 0 \\ 0 & \square_{2\times2} \end{pmatrix}.$$

Detecting further splitting of Λ

Check-(12)(45)(67)

Suppose Λ passes Check-(38). Then, in a certain basis, it has a generic 6×6 block within the subspace

$$V_6 = (x_1, x_2;\, x_4, x_5;\, x_6, x_7)\,.$$

Take 6 eigenvectors from this subspace. If they break into three pairs such that each pair of eigenvectors a', b' satisfies

$$D^{(a'b')} = 0 \quad \text{and} \quad D^{(a'a')} = D^{(b'b')} \in (x_3, x_8)\,,$$

then Λ splits into four 2×2 blocks within subspaces

$$(x_3, x_8)\,, \quad (x_1, x_2)\,, \quad (x_4, x_5)\,, \quad (x_6, x_7)\,.$$

Detecting special subspaces

- Such Checks give necessary and sufficient conditions for the corresponding features to occur.
- They can be performed in any basis and can ne automatized.
- One just needs to relate them to symmetries.

Symmetries in 3HDM

The NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

can be invariant under global symmetries:

- family symmetries: $\phi_a \to U_{ab}\phi_b$, with $U \in U(N)$,
- GCP symmetries: $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, with $X \in U(N)$.

A symmetry group G and its breaking by vevs $G_v \subseteq G$ lead to a characteristic phenomenology (scalars, DM candidates, fermion masses, mixing, sources of CPV, etc).

In 3HDM, a novel form of CP-symmetry (CP4) [Ivanov, Silva, 1512.09276] which is physically distinct from the usual CP (CP2) [Haber, Ogreid, Osland, Rebelo, 1808.08629].

Explicit CP2 conservation

CP2: there exists a basis in which it takes the standard form: $\phi_a \to \phi_a^*$.

In the adjoint space, the standard CP is the following reflection:

- vectors from $V_+ = (x_3, x_8, x_1, x_4, x_6)$ stay unchanged,
- vectors from $V_- = (x_2, x_5, x_7)$ flip signs.

3HDM potential is explicitly CP2-invariant if there exists a basis in which:

- Λ has the block-diagonal form:

$$\Lambda = \begin{pmatrix} \square_{5\times 5} & 0 \\ 0 & \square_{3\times 3} \end{pmatrix}$$

  with generic blocks within V_+ and V_-.

- vectors $M, L \in V_+$,

Detecting explicit CP2 conservation

Detecting $\square_{3\times 3}$ in (x_2, x_5, x_7):

- There exist three mutually orthogonal eigenvectors a, b, c such that

$$2F^{(ab)} = c\,, \quad 2F^{(bc)} = a\,, \quad 2F^{(ca)} = b\,.$$

- vectors M, L are orthogonal to these a, b, c.

Derived first in [Nishi, hep-ph/0605153].

Explicit CP4 conservation

CP4 leads in a certain basis in the bilinear space to

$$x_8 \to x_8\,, \quad (x_1, x_2, x_3) \to -(x_1, x_2, x_3)$$
$$x_4 \to x_6\,, \quad x_6 \to -x_4\,, \quad x_5 \to -x_7\,, \quad x_7 \to x_5\,.$$

3HDM potential is explicitly CP4-invariant iff there exists a basis in which

- the matrix Λ is
$$\Lambda = \begin{pmatrix} \square_{3\times 3} & 0 & 0 \\ 0 & \square_{4\times 4} & 0 \\ 0 & 0 & \Lambda_{88} \end{pmatrix}$$
with a specific pattern in the 4×4 block,
- all possible vectors M, L, $(\Lambda^n)L$, $K_i \equiv d_{ijk}\Lambda_{jk}, \ldots$ are all parallel to x_8 (complete alignment).

Detecting explicit CP4 conservation

Basis invariant necessary and sufficient conditions for explicit CP4 conservation [Ivanov, Nishi, Silva, Trautner, 1810.13396]:

- Λ passes Check-(8): three exists an eigenvector $e^{(8)}$ such that $D^{(88)} = -e^{(8)}$;
- There exist three other eigenvectors a, b, c such that

$$F^{(a8)} = F^{(b8)} = F^{(c8)} = 0\,,$$

  which guarantees the 3×3 block within (x_1, x_2, x_3) subspace.

- M, L, $K_i = d_{ijk}\Lambda_{jk}$, and $K_i^{(2)} = d_{ijk}(\Lambda^2)_{jk}$ are aligned with $e^{(8)}$.

Weinberg's model

Weinberg's model ($\mathbb{Z}_2 \times \mathbb{Z}_2$):

- Λ passes Check-(38) and Check-(12)(45)(67);
- $M, L \in (x_3, x_8)$.

If, in addition, there are degenerate eigenvalues within V_6:

- if the degeneracy pattern is $1+1+2+2 \to U(1) \times \mathbb{Z}_2$;
- if the degeneracy pattern is $2+2+2 \to U(1) \times U(1)$.

We found basis-invariant conditions for all symmetry groups in 3HDM.

See the full list in [Ivanov, Varzielas, 1903.11110].

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Conclusions

Done:

- Efficient parameter space scans in multi-Higgs models must be able to detect symmetries in a basis invariant way.
- We found a way how to do it in the scalar sector of 3HDM: via subspace detection techniques applied to eigenvectors of Λ.

To do:

- Implement the algorithms in a working computer code.
- Go beyond 3HDM.
- Apply the idea to the fermion sector.