Naturalness and the Weak Gravity Conjecture

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The Challenge Confronting Naturalness

A new scalar has been discovered at the LHC.

Where are the “naturalons”? 

Selected diphoton sample

<table>
<thead>
<tr>
<th>√s (TeV)</th>
<th>L (fb⁻¹)</th>
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<tbody>
<tr>
<td>7</td>
<td>4.8</td>
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<tr>
<td>8</td>
<td>5.9</td>
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CMS Preliminary

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<thead>
<tr>
<th>√s (TeV)</th>
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<tr>
<td>7</td>
<td>5.1</td>
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<tr>
<td>8</td>
<td>5.3</td>
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S/B Weighted Data

S+B Fit

Bkg Fit Component

±1 σ

±2 σ

ATLAS Preliminary
The principle of **naturalness** is the requirement that operators not protected by a symmetry are unstable to quantum corrections induced at the cutoff of the effective field theory.

This means that the numerical coefficients are $\mathcal{O}(1)$.

- No delicate cancellation allowed without additional symmetry (e.g., SUSY).

Naturalness has been a motivator of new physics for decades.

Currently being revisited:
- regulator tricks
- modified naturalness, ultraviolet conformal symmetry
- meso-tuning
The Weak Gravity Conjecture
The Weak Gravity Conjecture (WGC) is an ultraviolet consistency condition for quantum gravity.

**WGC statement:** For any Abelian gauge theory coupled to quantum gravity, there exists a state in the spectrum with charge $q$ and mass $m$ such that

\[ \frac{q}{m} > \frac{1}{m_{Pl}}. \]

In other words, “gravity is the weakest force.”

**Low-Cutoff Conjecture:** There is a low cutoff scale

\[ \Lambda \sim q m_{Pl}, \]

at which 4D QFT breaks down completely.
A black hole decay thought experiment:

- Theory with spectrum \( \{q_i, m_i\} \).
- Black hole decaying to one species \( i \) via Hawking or Schwinger process.
- Charge conservation: \( Q / q_i \) particles produced.
- Energy conservation: \( M > m_i Q / q_i \).
- Decay requires \( z_i > Z \).

\[
\begin{align*}
Z &= Q m_{\text{Pl}} / M \\
\therefore \quad z_i &= q_i m_{\text{Pl}} / m_i \\
\vdots
\end{align*}
\]
For extremal black holes ($\mathcal{Z} = 1$) to decay, there must be a particle in the spectrum with $\mathcal{z}_i > 1$. $\implies$ WGC.

Stable black holes $\implies$ very large number of stable states in the theory.

- Thermodynamic pathologies
- Virtual black hole loops in Feynman diagrams
- Tension with holography
Arkani-Hamed et al. supported the WGC with a host of examples in field theory and string theory.

- $SU(2) \rightarrow U(1)$ gauge theory: W-bosons and monopoles
- States in $SO(32)$ heterotic string theory
- Problems with DGP gravity and embedding extranatural inflation in string theory

If no WGC, then the $q \rightarrow 0$ limit of a gauge theory yields an exact global symmetry.

- Conflict with no-hair theorems: black holes labeled only by overall mass, spin, and charge.
The Limits of Naturalness
Scalar QED: Naturalness vs. Weak Gravity

- Simple example: scalar $U(1)$-charged particle with a hierarchy problem:
  \[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4, \]
  where $D_\mu = \partial_\mu + iqA_\mu$.

- WGC: $q(\mu) > m(\mu)/m_{\text{Pl}}$ — running with renormalization scale:
  - $q(\mu)$ logarithmically
  - $m(\mu)$ quadratically
  - Presages tension with naturalness. Evaluate $\mu$ at physical mass.

- Loop corrections to mass: $m^2 \rightarrow m^2_{\text{phys}} = m^2 + \delta m^2$, where
  \[ \delta m^2 = \frac{\Lambda^2}{16\pi^2} (aq^2 + b\lambda). \]

- Naturalness: $a$ and $b$ are incalculable, $O(1)$ coefficients.
Scalar QED: Naturalness vs. Weak Gravity

- Technically natural region of parameter space: $q^2 \ll \lambda$.
- Then naturalness $\implies \delta m^2 \gg \text{WGC bound}$.
- Conflict between ultraviolet consistency and low-energy effective field theory!

$m^2 \to m^2 + \delta m^2$

"natural" mass

forbidden by WGC

$m_{\text{phys}}$

$q m_{\text{Pl}}$
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$m^2 \rightarrow m^2 + \delta m^2$

un-“natural” mass

forbidden by WGC
Quantifying the Tension

Charge-to-mass ratio for $\phi$:

$$z = \frac{4\pi m_{P1}}{\Lambda} \frac{1}{\sqrt{a + b\lambda/q^2}} > 1$$

so

$$\Lambda < \begin{cases} \frac{4\pi m_{P1}}{\sqrt{a}}, & q^2 \gg \lambda \\ 4\pi m_{P1}\sqrt{\frac{q^2}{b\lambda}}, & q^2 \ll \lambda \end{cases}.$$ 

- $q^2 \gg \lambda \implies$ sub-Planckian cutoff (reasonable).
- $q^2 \ll \lambda \implies$ mandatory, quantifiable fine-tuning to satisfy the WGC.

Small coupling $q$ is technically natural: $q$ runs only logarithmically.
How can naturalness and the WGC be reconciled in a theory containing charged scalars?

- Forbid coupling hierarchies?
  - Perhaps $q^2 \ll \lambda$ is strictly forbidden (e.g. SUSY $D$-terms), though SUSY is not enough to ensure this.

- Better options:
  1. Higgs phase
  2. New physics below the Planck scale
Higgsing the Theory

- If the mass term $m^2|\phi|^2$ becomes tachyonic after loop corrections, the gauge field $A_\mu$ acquires a mass.

- The original WGC argument of Arkani-Hamed et al. was: “no stable extremal black holes” $\implies$ WGC.

- No-hair theorems: No stationary black hole solutions supporting classical hair from a massive photon.
  - A black hole charged under a massive $U(1)$ becomes “bald” on timescale of order $1/m_\gamma$.

- So the WGC is not justified for a Higgsed $U(1)$. 
New Physics

Simplest and most interesting option for reconciling the WGC and naturalness in scalar QED: **Introduce new physics!**

- If the EFT has cutoff at $\Lambda \sim qm_{\text{Pl}}$, then the WGC can still be satisfied.
- This is exactly the cutoff conjectured by Arkani-Hamed et al.
- Here, we see the first known effective-field-theoretic evidence for the weak interpretation of the low-cutoff conjecture.

![Trivial case diagram](image)
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More Forces, More Particles
More Forces, More Particles

To be more realistic, we’d like to generalize the WGC to a much broader class of theories than a single $U(1)$.

Consider:

- A product gauge group $\prod_{a=1}^{N} U(1)_a$ with $N$ Abelian factors.
- States $i$
  - masses $m_i$
  - charges $q_{ia} \equiv \vec{q}_i$, vectors of $SO(N)$
  - charge-to-mass vectors $\vec{z}_i = q_{ia} m_{Pl}/m$

What is the correct justification of the WGC?

- At least one species $i$ with $|\vec{z}_i| > 1$? Not sufficient!
- A $SO(N)$ basis of such species? Not sufficient!
- True generalized WGC is even stronger!
A black hole decay thought experiment:

- Theory with spectrum \( \{ q_i, m_i \} \rightarrow z_i \).
- Black hole decaying to \( n_i \) particles of species \( i \).
- Charge conservation: \( \vec{Q} = \sum_i n_i \vec{q}_i \).
- Energy conservation: \( M > \sum_i n_i m_i \).
- Decay requires \( \vec{Z} = \sum_i \sigma_i \vec{z}_i \), where \( \sigma_i = n_i m_i / M \) and \( \sum_i \sigma_i < 1 \).
We can understand the generalized WGC bound geometrically:

- Draw charge-to-mass vectors $\pm \vec{z}_i$ for all particles in the theory.
- Unit ball $|\vec{Z}| \leq 1$: all black hole states.
- The convex hull spanned by $\vec{z}_i$ must contain the unit ball.
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A Stronger WGC Bound

The generalized WGC bound is parametrically stronger than the bound for a single $U(1)$.

- Example: $U(1)^2$, with states $\vec{z}_1$ and $\vec{z}_2$ requires
  \[ (\vec{z}_1^2 - 1) (\vec{z}_2^2 - 1) > (1 + |\vec{z}_1 \cdot \vec{z}_2|)^2. \]

- For $U(1)^N$ with $z_{ia} = \delta_{ia}z$, we require $z > \sqrt{N}$.

- Considering an $N$-charge theory of $N$ scalars, where
  \[ \delta m_i^2 = \frac{\Lambda^2}{16\pi^2} \left( a_i \vec{q}_i^2 + b_i \lambda_i \right), \]
  we require
  \[ \Lambda < \frac{4\pi m_{Pl}}{\mathcal{O}(\sqrt{N})} \times \begin{cases} \frac{1}{\sqrt{a_i}}, & \vec{q}_i^2 \gg \lambda_i \quad \text{[sub – Planckian cutoff]} \\ \sqrt{\frac{\vec{q}_i^2}{b_i \lambda_i}}, & \vec{q}_i^2 \ll \lambda_i \quad \text{[tension with naturalness]} \end{cases}. \]
The Hierarchy Problem
We’ve seen that naturalness can contradict the WGC.

Let’s now apply this tension to address the electroweak scale.

- In the SM, the Higgs mass gets loop corrections that make $v \sim 246$ GeV unnatural.
- Extra symmetry, SUSY, usually invoked to avoid this.
- We’ll try to just use the WGC and as little new low-energy physics as possible.

It’s tempting to charge the Higgs...

- We can’t do this, since that would give the photon a mass!
Example Model for the SM Hierarchy Problem

- Charge SM fermions under unbroken $U(1)_{B-L}$ Abelian gauge symmetry.
- To cancel anomalies, add right-handed neutrino $\nu_R$, with Dirac mass $m_\nu \sim y_\nu v \lesssim 0.1 \text{ eV}$:

  $m_\nu \bar{\nu}_L \nu_R + \text{h.c.}$

- Set $q$ very small:

  $q \sim m_\nu / m_{\text{Pl}} \sim 10^{-29}$ \quad (technically natural)

- Then the lightest neutrino has the largest $z$ and just marginally satisfies the WGC.
- At fixed Yukawa coupling $y_\nu$, a heavier, more natural, electroweak scale $v$ is forbidden!
Discussion

- This model is a proof of concept, but it does have a **prediction**: a new, weakly coupled massless gauge boson.
- Not ruled out by torsion balance and equivalence principle tests:
  \[ q \lesssim 10^{-24} \]
- May be probed in future!
- Makes the naturalness principle **directly experimentally testable**.
- Assuming naturalness in this theory requires Higgs phase or low cutoff \( \Lambda \lesssim \text{keV} \).
- Given predictions in the string landscape, such a fifth-force discovery would also falsify string theory.
- Can be adapted to other models to solve the SM hierarchy problem: charge weak-scale dark matter with \( q \sim 10^{-16} \) under a \( U(1) \) dark force.
Conclusions
Summary

- We showed that ultraviolet consistency can be at odds with naturalness.
- We showed that certain natural parameter regions of scalar QED are actually in the swampland.
- We extended the WGC to generic theories.
- We exhibited models in which the SM hierarchy is mandated by the WGC.
  - Lesson: Big hierarchies can be misleading without knowledge of the UV completion!
The WGC is still a **conjecture**.

All current evidence is either:

- Specific examples in string theory
- UV-dependent reasoning (black hole remnants, etc.)

We’d like to understand the WGC from a low-energy effective field theory perspective.

We’re developing WGC-like bounds on $q/m$ by observing violation of

- analyticity
- causality
- unitarity

in the effective photon-graviton theory if the WGC fails.