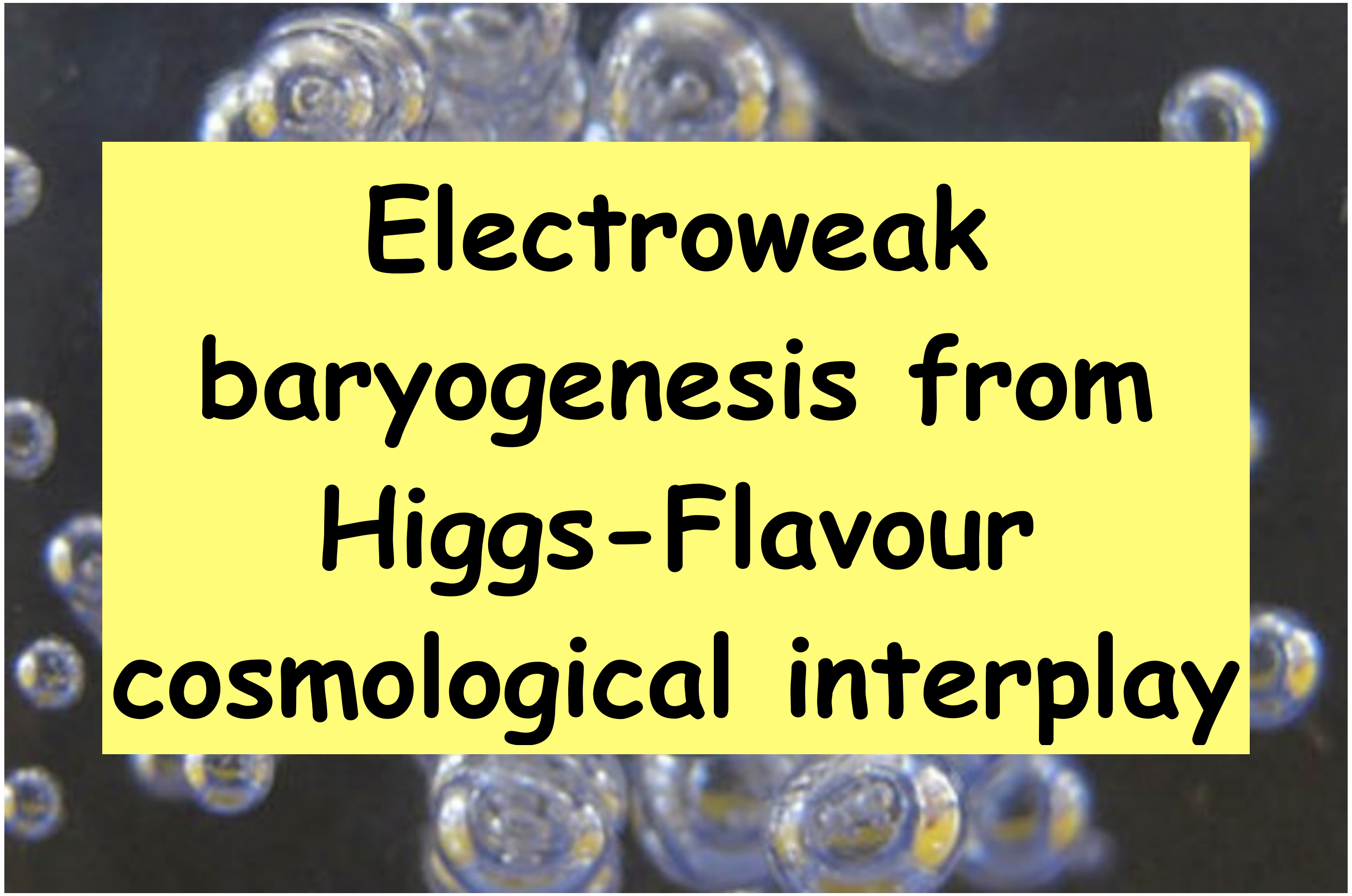
Electroweak baryogenesis from Higgs-Flavour cosmological interplay



Géraldine SERVANT

DESY/U.Hamburg

“20th Planck conference”
Warsaw,
May 26 2017

Aim: Exploring the cosmological interplay between Flavour dynamics and Electroweak baryogenesis

A rich programme

- Effect of varying Yukawas on EW phase transition

  Baldes, Konstandin, Servant, 1604.04526

- Implementation in Froggatt-Nielsen

  Baldes, Konstandin, Servant, 1608.03254

- Natural realisation of Yukawa variation in Randall-Sundrum

  Von Harling, Servant, 1612.02447

- Calculation of baryon asymmetry in models of variable Yukawas

  Bruggisser, Konstandin, Servant, to appear

see parallel talks by
Bruggisser and Von Harling

Matter Anti-matter asymmetry of the universe

$$\boxed{\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \equiv \eta_{10} \times 10^{-10}}$$

$$5.7 \leq \eta_{10} \leq 6.7 \ (95\%\mathrm{CL})$$

η remains unexplained within the Standard Model

double failure:

- lack of out-of-equilibrium condition

- so far, no baryogenesis mechanism that works with only SM CP violation (CKM phase)

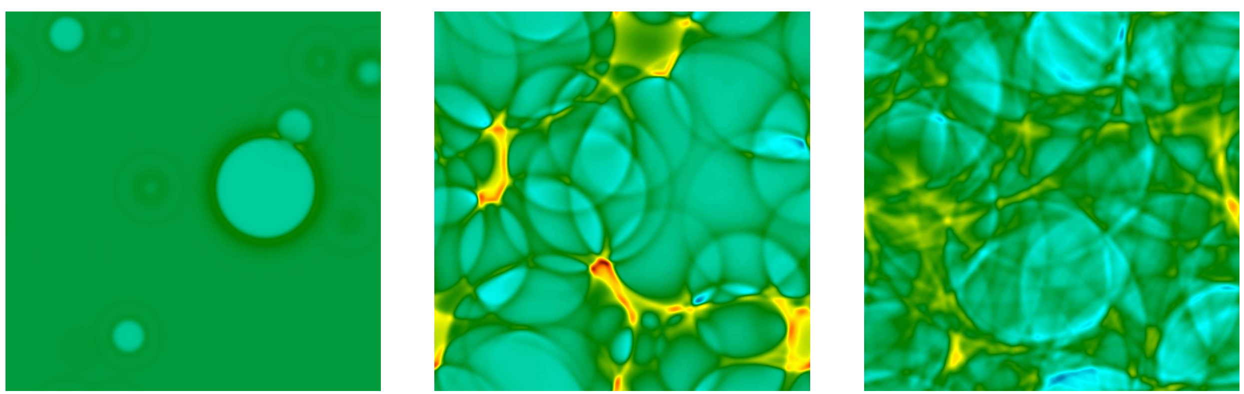
proven for standard EW baryogenesis

Gavela, P. Hernandez, Orloff, Pene '94
Konstandin, Prokopec, Schmidt '04

attempts in cold EW baryogenesis

Tranberg, A. Hernandez, Konstandin, Schmidt '09
Brauner, Taanila,Tranberg,Vuorinen '12

Baryogenesis at a first-order EW phase transition



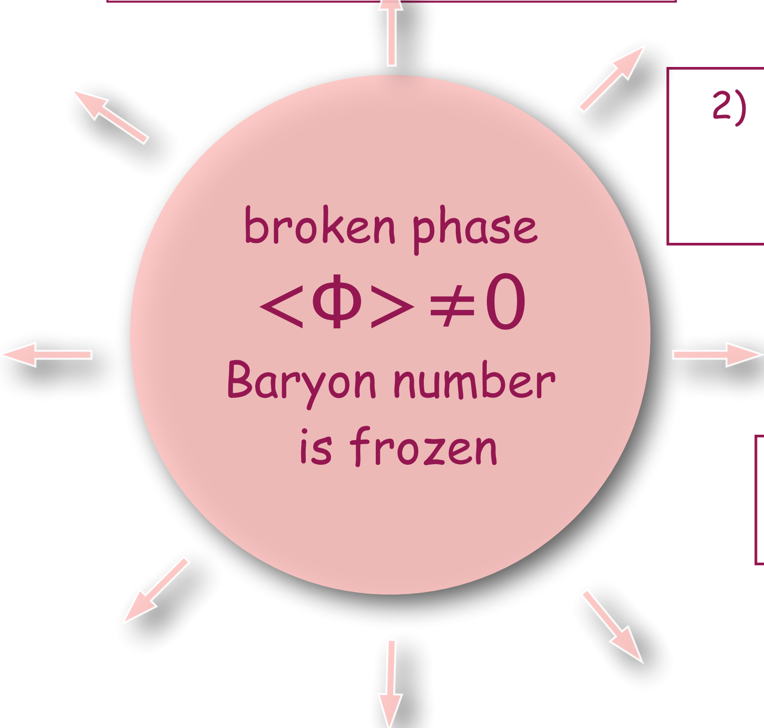
[image credit:1304.2433]

Baryon asymmetry and the EW scale

Kuzmin, Rubakov, Shaposhnikov'85

Cohen, Kaplan, Nelson'91

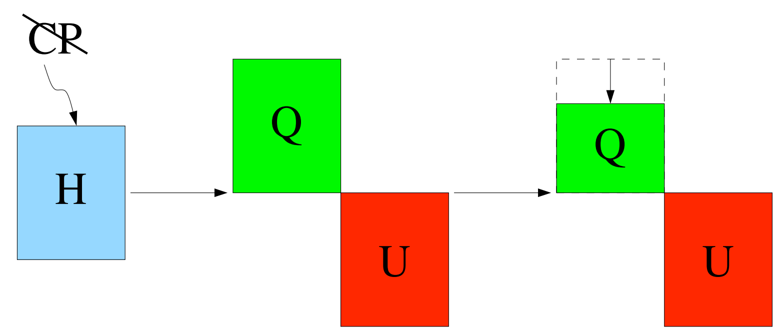
1) nucleation and expansion of bubbles of broken phase



2) CP violation at phase interface responsible for mechanism of charge separation

Chirality Flux in front of the wall

3) In symmetric phase, $\langle\Phi\rangle = 0$, very active sphalerons convert chiral asymmetry into baryon asymmetry

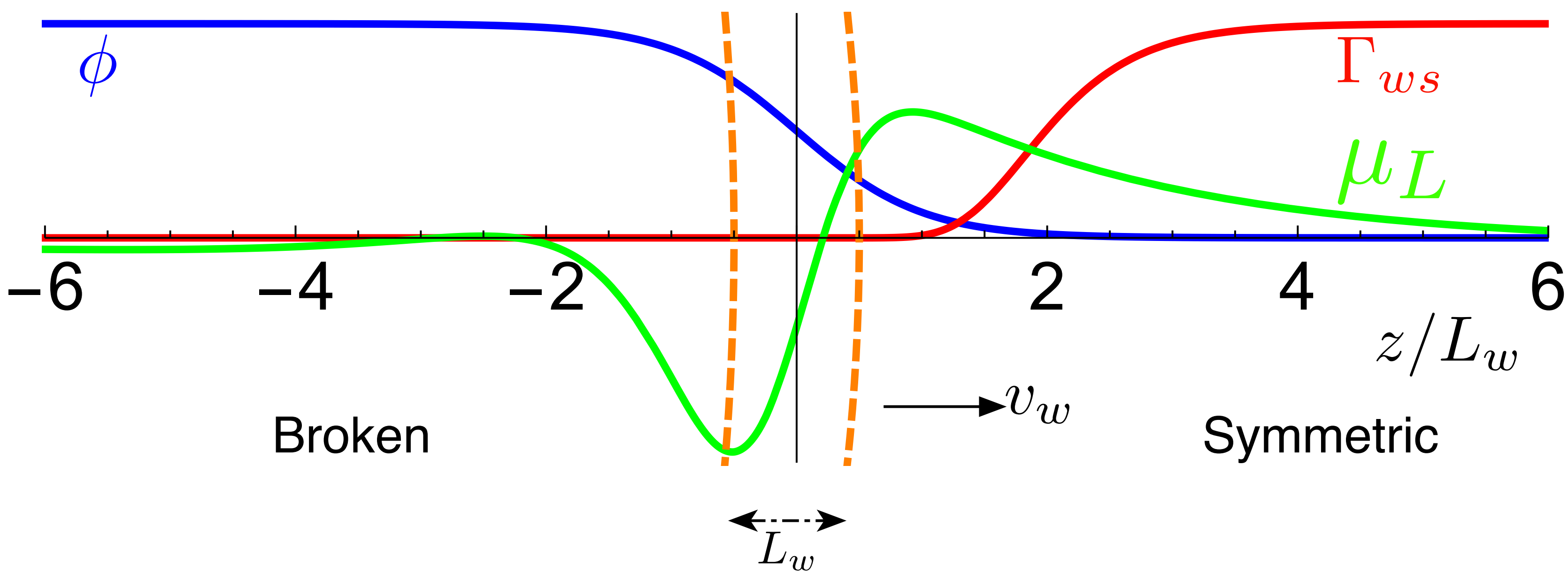


Electroweak baryogenesis mechanism relies on a first-order phase transition satisfying $\dfrac{\langle\Phi(T_n)\rangle}{T_n} \gtrsim 1$

The Electroweak Baryogenesis Miracle:

$$\eta_B = \frac{n_B(-\infty)}{s} = \frac{135\ N_c}{4\pi^2 v_w g_* T} \int_{-\infty}^{+\infty} \mathrm{d}z\ \Gamma_{ws}\ \mu_L\ \mathrm{Exp}\left[-\frac{3}{2} A \frac{1}{v_w} \int_{-\infty}^{z} \mathrm{d}z_0 \Gamma_{ws}\right]$$

$$\Gamma_{ws} = 10^{-6}\ T\ e^{-\frac{E_{sph}}{T}\frac{\phi(T)}{v}}$$



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$$\Gamma_{ws} = 10^{-6}\ T\ e^{-\frac{E_{sph}}{T}\frac{\phi(T)}{v}}$$

$$\eta_B \sim \frac{\Gamma_{ws}\ \mu_L\ L_w}{g_*\ T}$$

$$\mu_L \sim M'' M \sim \frac{\delta_{CP}}{L_w^2\ T} \qquad\qquad L_w \sim \frac{1}{T}$$

$$\eta_B \sim \frac{\Gamma_{ws}\ \delta_{CP}}{g_*\ L_w\ T^2} \sim \frac{10^{-6}\ \delta_{CP}}{g_*} \sim 10^{-8} \delta_{CP}$$

All parameters fixed by electroweak physics. If new CP violating source of order 1 then we get just the right baryon asymmetry.

Objective # I

Strong 1st-order EW phase transition

An easy way: dilaton-like potential

naturally leads to supercooling

not a polynomial

$$V = V(\sigma) + \frac{\lambda}{4}(\phi^2 - c\sigma^2)^2 \qquad c = \frac{v^2}{\langle\sigma\rangle^2}$$

Higgs vev controlled by dilaton vev (e.g. Randall-Sundrum scenario)

$$\boxed{V(\sigma) = \sigma^4 \times f(\sigma^\epsilon)}$$

Creminelli, Nicolis, Rattazzi'01
Randall, Servant'06
Nardini,Quiros,Wulzer'07
Konstandin,Nardini,Quiros'10
Hassanain, March-Russell, Schwellinger'07
Servant'1
Konstandin, Servant I and II, '1
Bunk, Hubisz, Jain'1

a scale invariant function modulated by a slow evolution through the σ^ϵ term for $|\varepsilon|<<1$

similar to Coleman-Weinberg mechanism where a slow Renormalization Group evolution of potential parameters can generate widely separated scales

Nucleation temperature can be parametrically much smaller than the weak scale

Application:

Baryogenesis from strong CP violation and the QCD axion

A coupling of the type ~ $\frac{a(t)}{f_a} F\tilde{F}$ ← EW field strength

will induce from the motion of the axion field a chemical potential for baryon number given by

$$\frac{\partial_t a(t)}{f_a}$$

This is non-zero only once the axion starts to oscillate after it gets a potential around the QCD phase transition.

Time variation of axion field can be CP violating source for baryogenesis if EW phase transition is supercooled

Servant, 1407.0030

→ **Cold Baryogenesis**

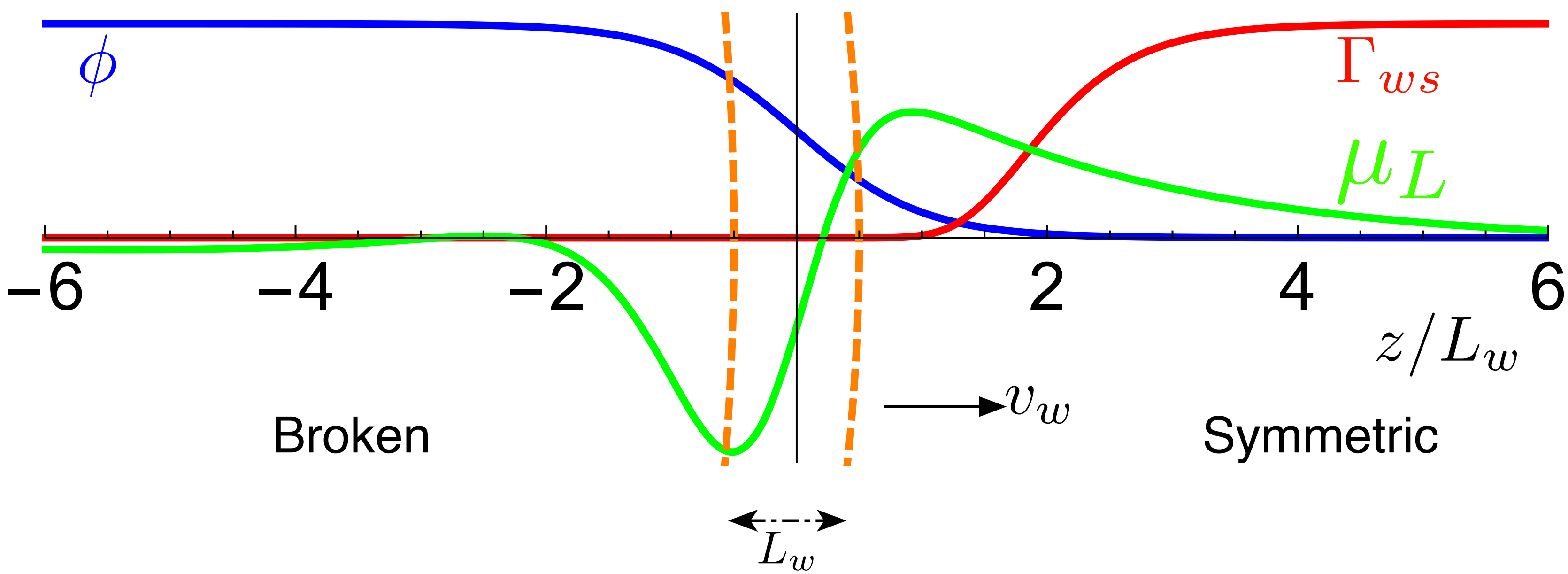
requires a coupling between the Higgs and an additional light scalar: testable @ LHC & compatible with usual QCD axion Dark matter predictions

Objective # II

New large sources of CP violation

$$\eta_B = \frac{n_B(-\infty)}{s} = \frac{135\ N_c}{4\pi^2 v_w g_* T} \int_{-\infty}^{+\infty} \mathrm{d}z\ \Gamma_{ws}\ \mu_L\ \mathrm{Exp}\left[-\frac{3}{2} A \frac{1}{v_w} \int_{-\infty}^{z} \mathrm{d}z_0 \Gamma_{ws}\right]$$

$$\Gamma_{ws} = 10^{-6}\ T\ e^{-\frac{E_{sph}}{T}\frac{\phi(T)}{v}}$$



Cline, Joyce, Kainulainen '00
Konstandin, Prokopec, Schmidt '04
Huber Fromme '06

Kinetic equations

$$\left(k_z \partial_z - \frac{1}{2}\left(\left[V^\dagger \left(m^\dagger m\right)' V\right]\right)_{ii} \partial_{k_z}\right) f_{L,i} \approx \mathbf{C} + \mathcal{S}$$

$$\left(k_z \partial_z - \frac{1}{2}\left(\left[V^\dagger \left(m^\dagger m\right)' V\right]\right)_{ii} \partial_{k_z}\right) f_{R,i} \approx \mathbf{C} - \mathcal{S}$$

collisions **source**

$$\eta_B = \sum_i \int_{-\infty}^{+\infty} \mathrm{d}y\; K_i(y)\; \bar{\mathcal{S}}_i(y)$$

diffusion effects & sphalerons

CP-violating source

Usual CP-violating sources in EW baryogenesis:

-Charginos/neutralinos (MSSM) **Cline et al, Carena et al...**

-Varying phase in effective Top quark Yukawa

SM+singlet, **Fromme-Huber**

Composite Higgs, **Espinosa, Gripaios, Konstandin, Riva, '11**

2-Higgs doublet model **Konstandin et al, Cline et al**

- two recent alternatives: strong CP

QCD axion (**Servant '14**)

and CP in DM sector (**e.g. Cline'17**)

the CKM matrix as the CP-violating source

In the SM: $\eta_B \lesssim 10^{-2}\Delta_{CP}$

Farrar, Shaposhnikov ‘93

$$\Delta_{CP} \sim \left(M_W^6 T_c^6\right)^{-1} \prod_{\substack{i>j\\u,c,t}} \left(m_i^2 - m_j^2\right) \prod_{\substack{i>j\\d,s,b}} \left(m_i^2 - m_j^2\right) J_{CP}$$

Gavela, *et al.* ’93
Huet, Sather ’94

Jarlskog constant

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) = (3.0 \pm 0.3) \times 10^{-5},$$

Based solely on reflection coefficients

If large masses during EW phase transition
->no longer suppression of CKM CP violation

Berkooz, Nir, Volansky ’04

New idea: Varying SM Yukawas as CP violating source

$$S \sim \mathrm{Im}\left[V_{CKM}^{\dagger} {m^{\dagger}}'' m V_{CKM}\right]$$

$$m = y(z) \cdot \frac{\phi(z)}{\sqrt{2}}$$

For constant y:

$$S \sim \underbrace{\mathrm{Im}\left[V_{CKM}^{\dagger} y^{\dagger} y V_{CKM}\right]}_{=0} \phi'' \phi$$

1-Flavour case

$$m = |m|e^{i\theta}$$

$$S \propto \mathrm{Im}\left[V^{\dagger} m^{\dagger\prime\prime} m V\right] = \left(|m|^2 \theta'\right)'$$

requires variation of phase

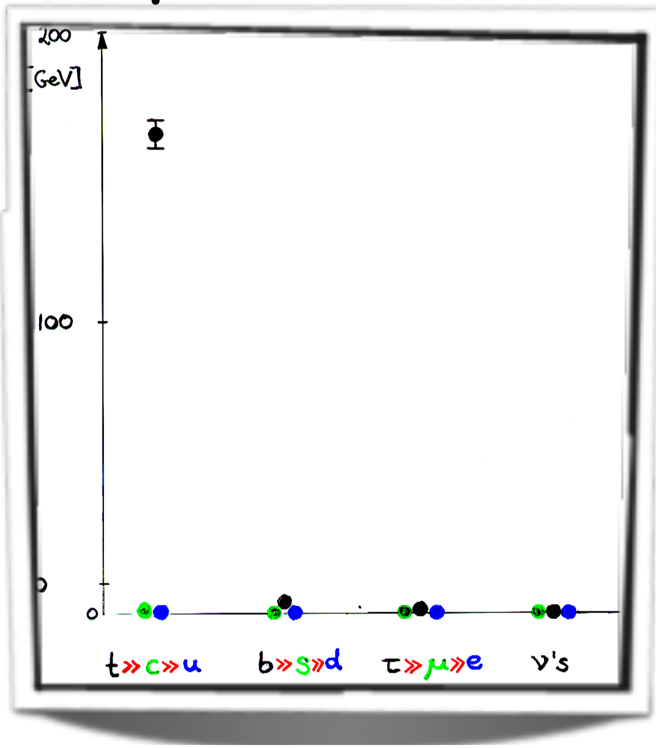
More than 1 flavour: no need for variation of phase

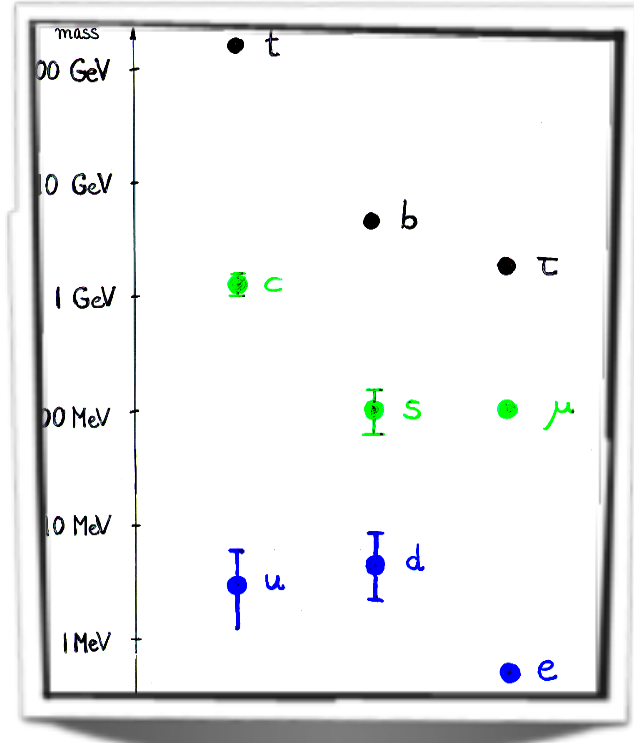
Flavour-EW symmetry breaking cosmological interplay

Baldes, Konstandin, Servant, 1604.04526
Baldes, Konstandin, Servant, 1608.03254
Von Harling, Servant, 1612.02447
Bruggisser, Konstandin, Servant, to appear

Origin of the fermion mass hierarchy?

the mass spectrum of the fermions is intriguing





fermion Yukawas $\quad y_{ij}\overline{f}^{i}_{L}\Phi^{(c)}f^{j}_{R}$

$$\langle\Phi\rangle = v/\sqrt{2}$$

fermion masses $\quad m_f = y_f v/\sqrt{2}$

There are three main mechanisms to describe fermion masses

$$m_f = y_f v/\sqrt{2}$$

1) Spontaneously broken abelian flavour symmetries as originally proposed by Froggatt and Nielsen

may be related by holography

2) Localisation of the profiles of the fermionic zero modes in extra dimensions

3) Partial fermion compositeness in composite Higgs models

The scale at which the flavour structure emerges is not known.

Usually assumed to be high but could be at the EW scale.

Origin of the fermion mass hierarchy?

Fermion Yukawas $$y_{ij}\overline{f}^{i}_{L}\Phi^{(c)}f^{j}_{R}$$

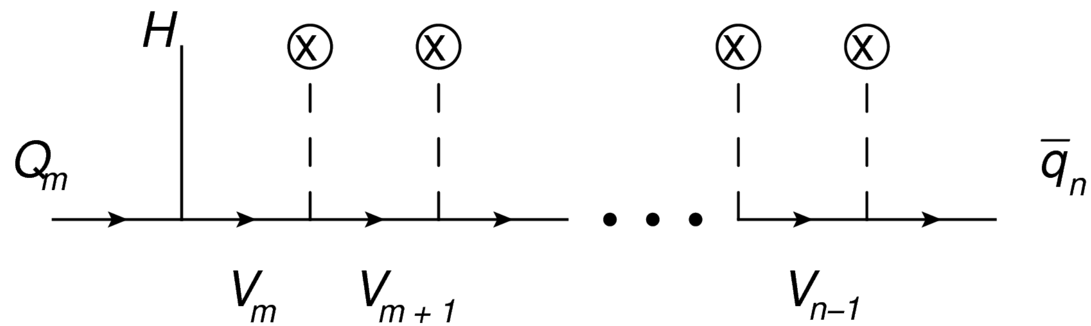
In Froggatt Nielsen constructions, the Yukawa couplings are controlled by the breaking parameter of a flavour symmetry. A scalar field "flavon" χ carrying a negative unit of the abelian charge develops a vacuum expectation value (VEV) and:

$$y_{ij} \sim (\langle\chi\rangle/M)^{-q_i+q_j+q_H}$$

flavor charges of the fermions

Froggatt-Nielsen

Fermion Yukawas $\qquad y_{ij}\overline{f}^{i}_{L}\Phi^{(c)}f^{j}_{R}$



$$y_{ij} \sim (\langle\chi\rangle/M)^{-q_i+q_j+q_H}$$

← flavor charges of the fermions

λ= $\langle\chi\rangle/M \ \sim \ 0.22$ ⇒

$$\begin{array}{lll} Y_t \sim 1, & Y_c \sim \lambda^3, & Y_u \sim \lambda^7, \\ Y_b \sim \lambda^2, & Y_s \sim \lambda^4, & Y_d \sim \lambda^6, \\ s_{12} \sim \lambda, & s_{23} \sim \lambda^2, & s_{13} \sim \lambda^3. \end{array}$$

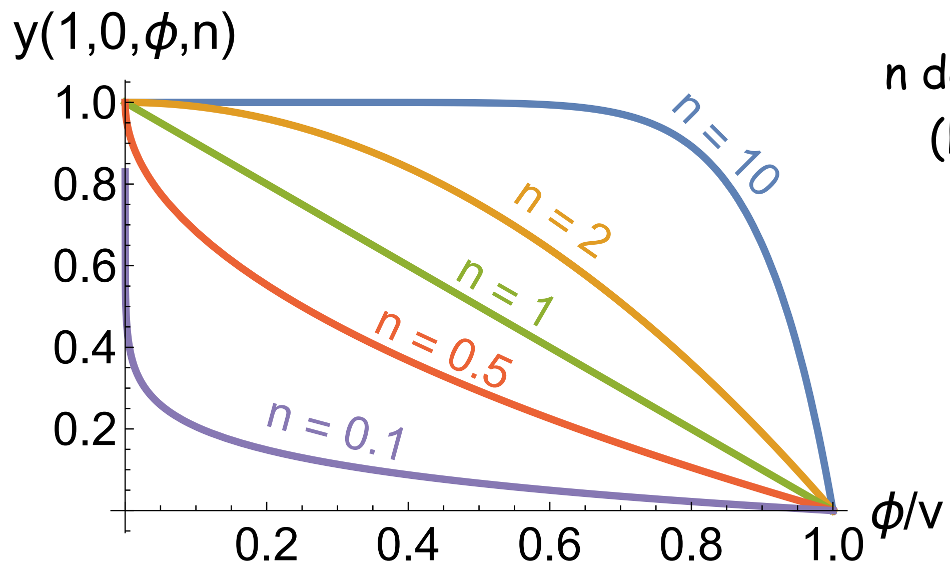
The scale M is usually assumed close to the GUT scale

Emerging Flavour during Electroweak symmetry breaking

Flavour structure could emerge during electroweak symmetry breaking if the "Flavon" field dynamics is linked to the Higgs field, e.g. via

$$\lambda_{\phi\chi}\phi^2\chi^2$$

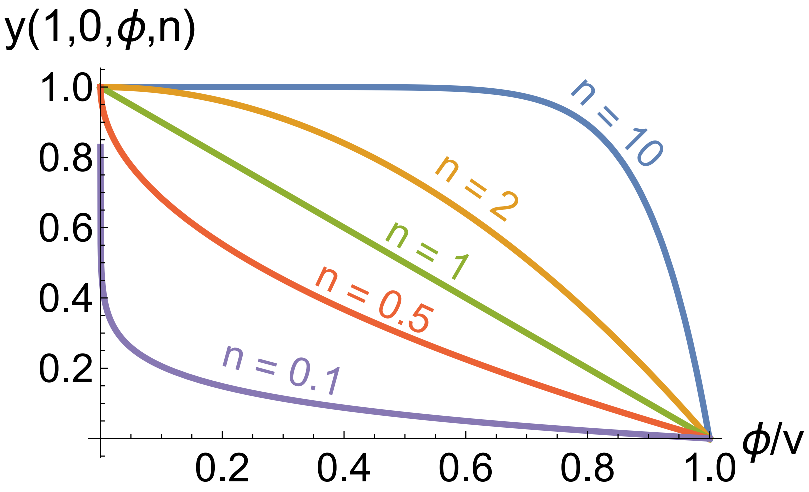
Induced Yukawa coupling variation across the bubble wall



n determined by trajectory in (Higgs,Flavon) field space

Not an EFT!

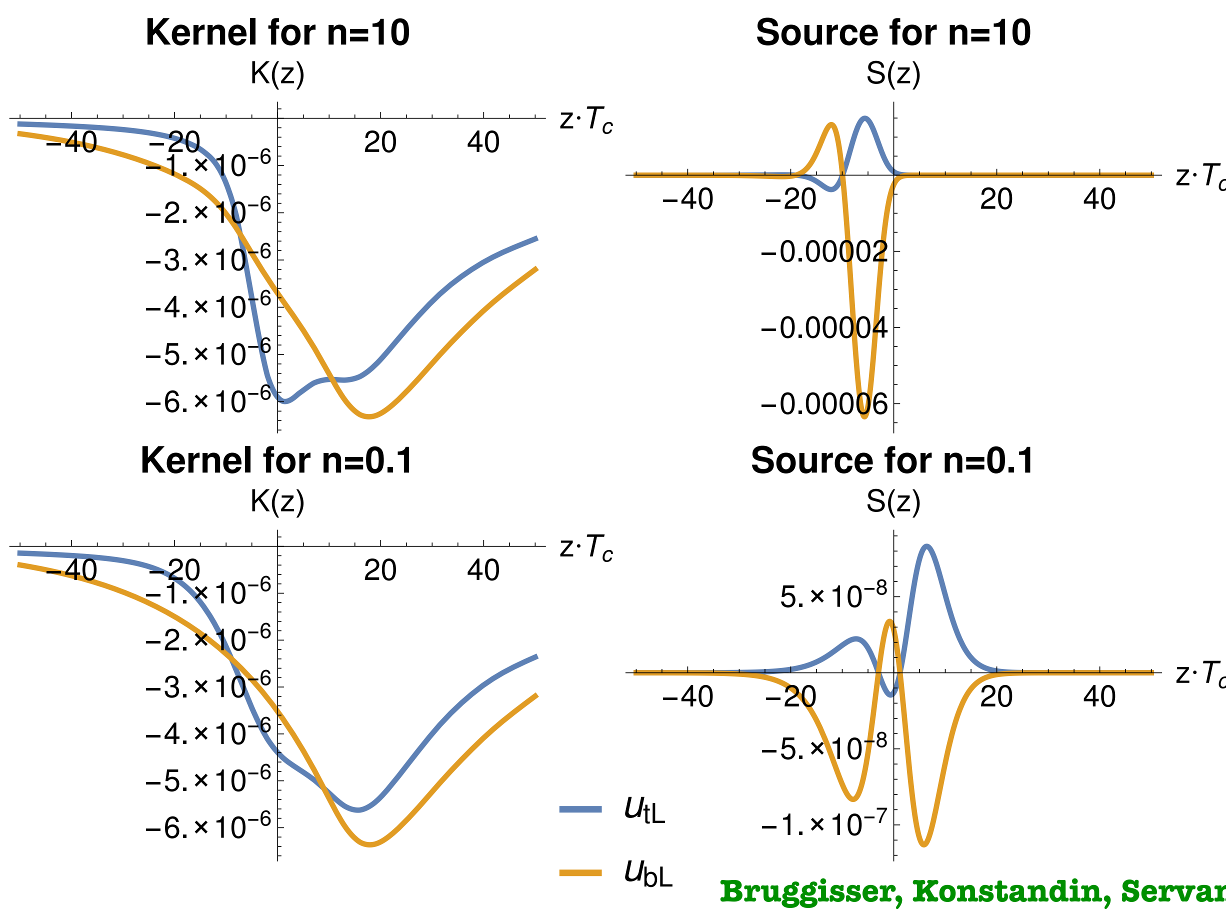
Yukawa does not depend on Φ today
Dependence induced only during EW phase transition.



$$\eta_B = \sum_i \int_{-\infty}^{+\infty} \mathrm{d}y \; K_i(y) \; \bar{\mathcal{S}}_i(y)$$

kernel does not depend much on Yukawa profile

source term gets shifted towards broken phase for large n



$\eta_B \approx 5.5 \cdot 10^{-10}$

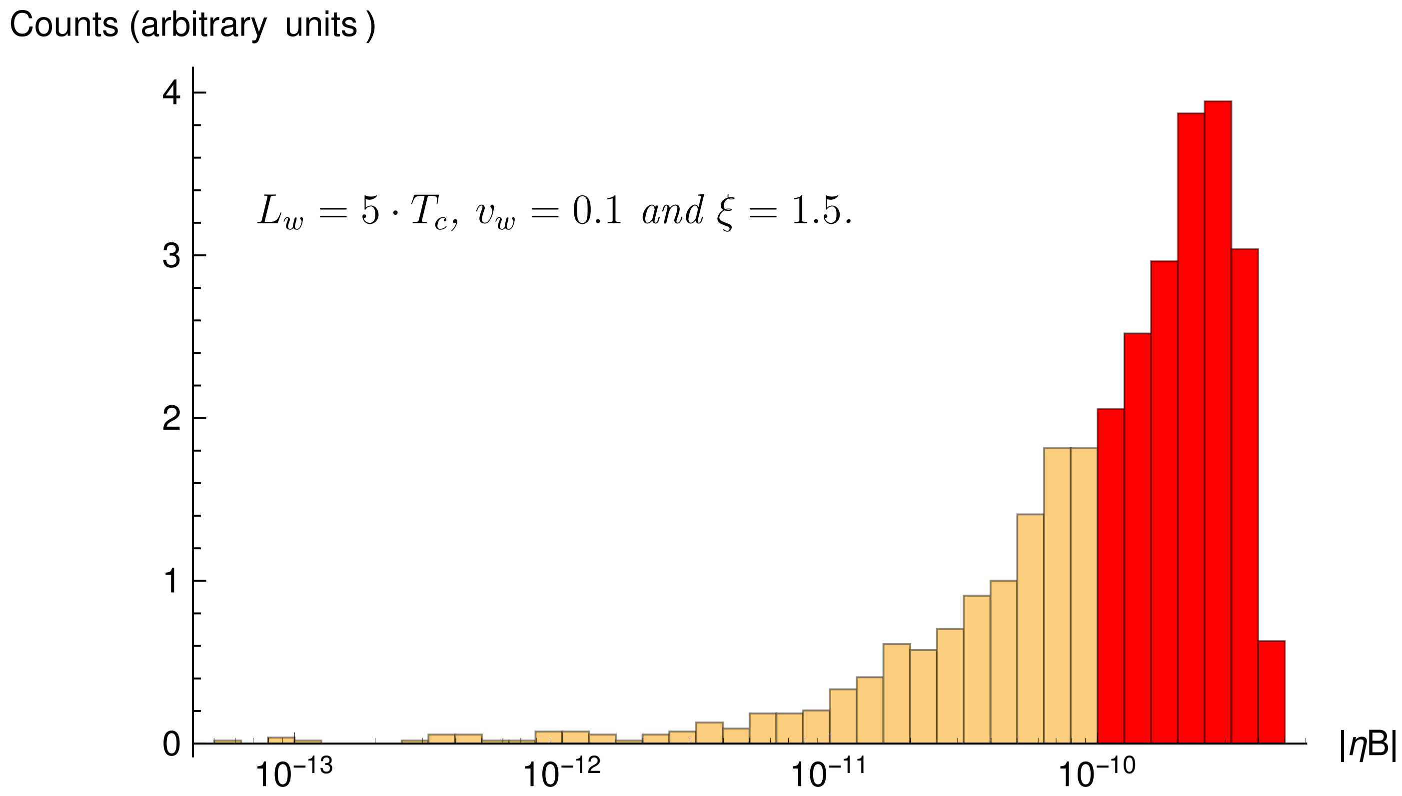
$\eta_B \approx 2.1 \cdot 10^{-12}$

Bruggisser, Konstandin, Servant, to appear

Baryon asymmetry for random distribution of n_i

(between 0 and 10)

$$Y(n_1,n_2,n_3,n_4) = \begin{pmatrix} e^{i3.17} y(1,0,\phi,n_1) & e^{i4.92} y(1,0,\phi,n_2) \\ e^{i5.29} y(1,0,\phi,n_3) & e^{i2.04} y(1,0,\phi,n_4) \end{pmatrix}$$



Bruggisser, Konstandin, Servant, to appear

A first-order Electroweak Phase Transition in the Standard Model from Varying Yukawas

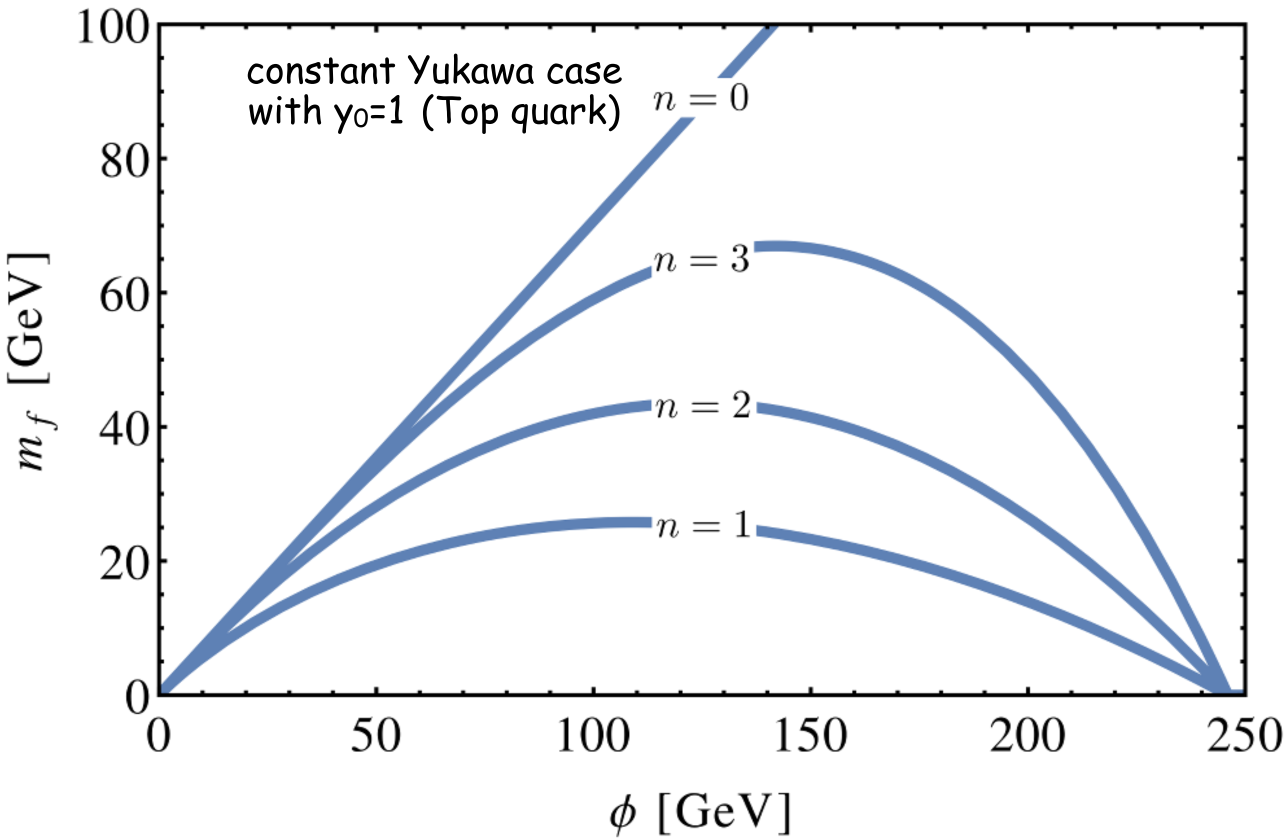
Baldes, Konstandin, Servant, 1604.04526

The nature of the EW phase transition is strongly modified when the Standard Model Yukawas vary at the same time as the Higgs is acquiring its vacuum expectation value.

Mass of fermionic species for varying Yukawas

$$m_f = \frac{y(\phi)\phi}{\sqrt{2}}$$

$$\boxed{y(\phi) = \begin{cases} y_1 \left(1 - \left[\frac{\phi}{v}\right]^n\right) + y_0 & \text{for} \quad \phi \leq v, \\ y_0 & \text{for} \quad \phi \geq v. \end{cases}}$$



y_0: Yukawa value today

y_1: Yukawa value before the EW phase transition ~ O(1)

High Temperature Effective Higgs Potential

At one-loop:

$$V_{\rm eff} = V_{\rm tree}(\phi) + V_1^0(\phi) + V_1^T(\phi,T) + V_{\rm Daisy}(\phi,T).$$

tree level piece	1-loop T=0 piece	1-loop T≠0 piece	Daisy resummation piece

2) Barrier from the $T \neq 0$ one-loop potential:

$$V_1^T(\phi, T) \;=\; \sum_i \frac{g_i(-1)^F T^4}{2\pi^2} \times \int_0^\infty y^2 \mathrm{Log}\left(1-(-1)^F e^{-\sqrt{y^2+m_i^2(\phi)/T^2}}\right) \mathrm{d}y.$$

$$V_f^T(\phi, T) = -\frac{gT^4}{2\pi^2} J_f\left(\frac{m_f(\phi)^2}{T^2}\right)$$

High-T expansion:

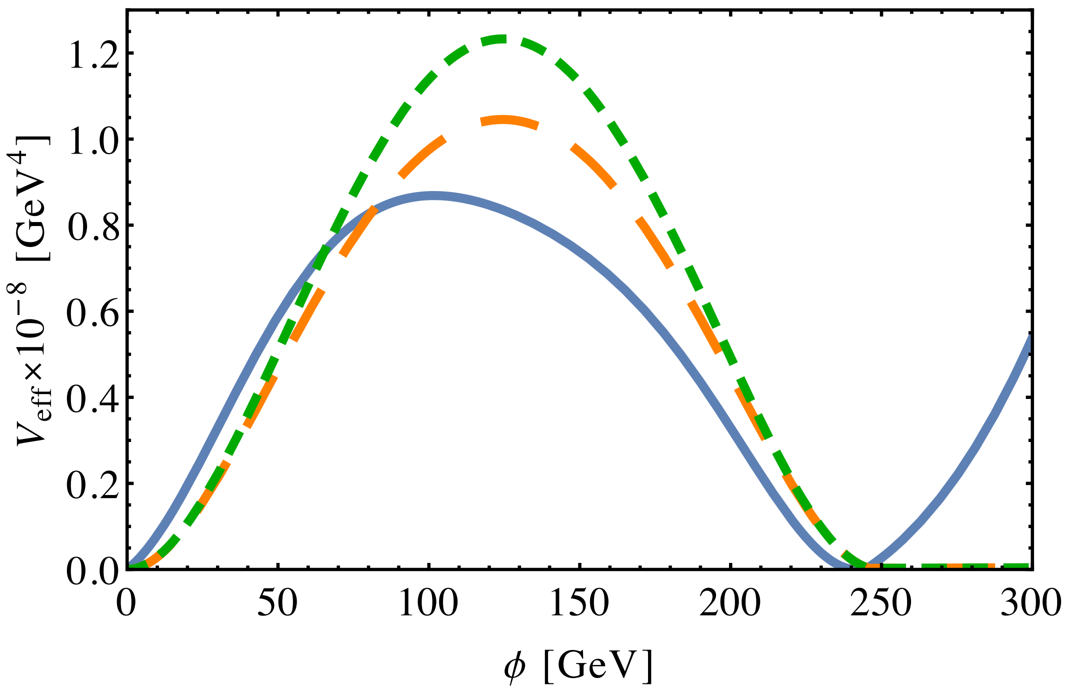
$$J_f(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{x^4}{32}\mathrm{Log}\left[\frac{x^2}{13.9}\right]$$

$$\delta V \equiv V_f^T(\phi, T) - V_f^T(0, T) \approx \frac{gT^2\phi^2[y(\phi)]^2}{96}$$

Fermionic fields create a barrier!

This leads to a cubic term in ϕ, e.g. for $y(\phi) = y_1(1 - \phi/v)$:

$$\delta V \approx \frac{g y_1^2 \phi^2 T^2}{96} \left(1 - 2\frac{\phi}{v} + \frac{\phi^2}{v^2}\right)$$



__ full potential

_ _ _ thermal contribution only

- - -thermal contribution only with high-T expansion

3) Effects from the Daisy correction:

come from resumming Matsubara zero-modes for the bosonic degrees of freedom

$$V_{\mathrm{Daisy}}(\phi,T)=\sum_i \frac{\overline{g}_i T}{12\pi}\left\{m_i^3(\phi)-\left[m_i^2(\phi)+\Pi_i(T)\right]^{3/2}\right\}$$

sum is over bosons

thermal mass

Consider the contribution from the Higgs:

$$V^{\phi}_{\mathrm{Daisy}}(\phi,T)=\frac{T}{12\pi}\left\{m_\phi^3(\phi)-\left[m_\phi^2(\phi)+\Pi_\phi(\phi,T)\right]^{3/2}\right\}$$

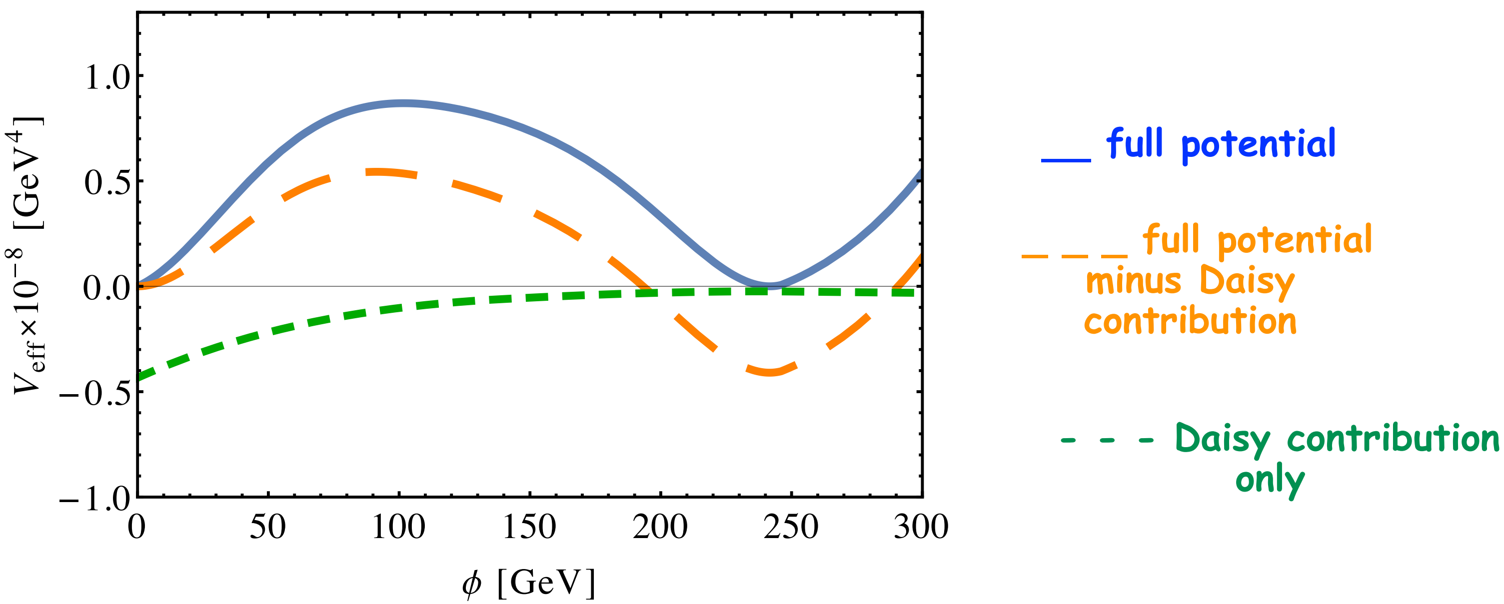
$$\Pi_\phi(\phi,T)=\left(\frac{3}{16}g_2^2+\frac{1}{16}g_Y^2+\frac{\lambda}{2}+\frac{y_t^2}{4}+\frac{gy(\phi)^2}{48}\right)T^2$$

The novelty is the dependence of the thermal mass on Φ, which comes from the Φ-dependent Yukawa couplings

3) Effects from the Daisy correction:

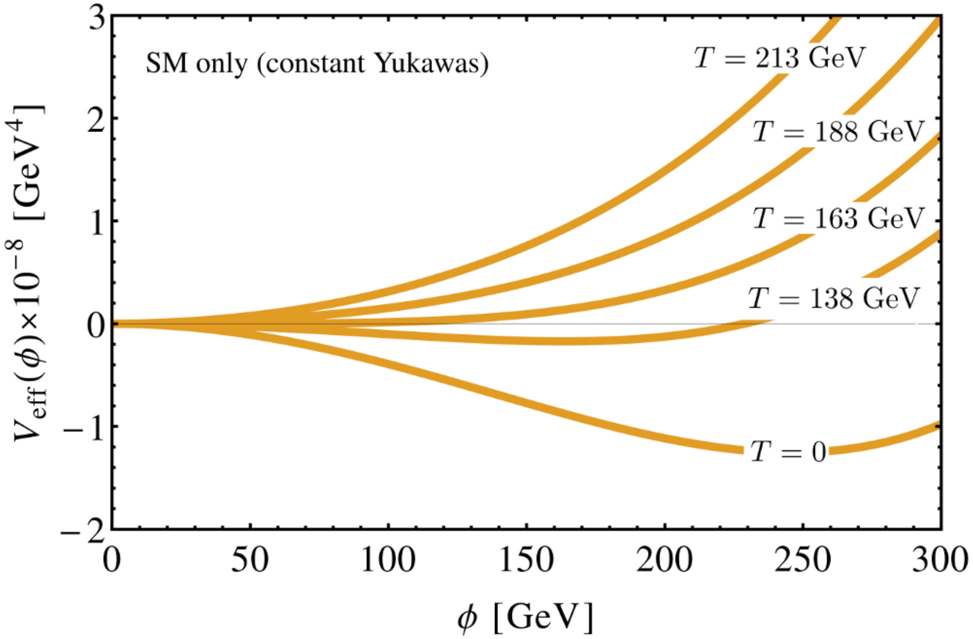
The effect is to lower the effective potential at Φ =0, with respect to the broken phase minimum.

By lowering the potential at Φ =0, the phase transition is delayed and strengthened.

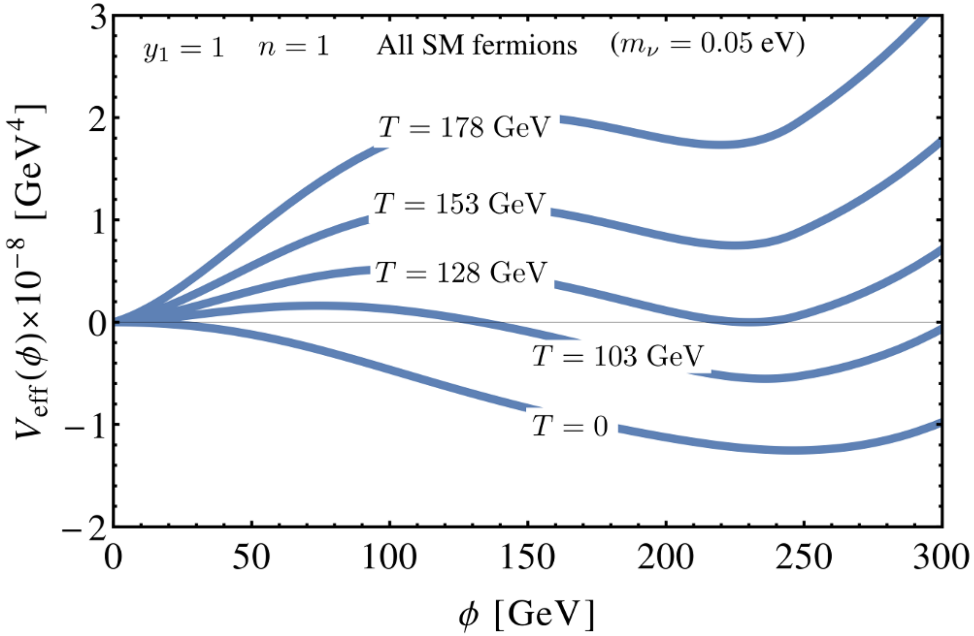


Full one-loop effective Higgs potential with Daisy Resummation

Standard Case (Constant Yukawas)



With varying Yukawas



Summary

Variation of the Yukawas of SM fermions from O(1) to their present value during the EW phase transition leads to a strong first-order EW phase transition

This offers new routes for generating the baryon asymmetry at the electroweak scale, strongly tied to flavour models.

Naturally varying Yukawas: The Froggatt-Nielsen case

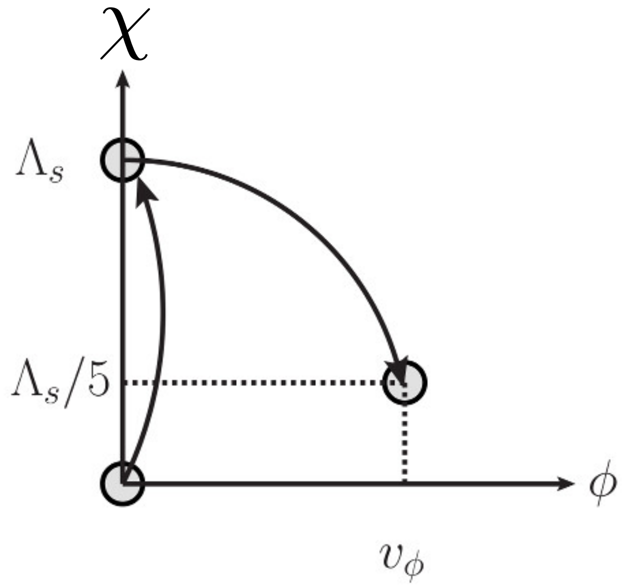
In simplest implementation: Tension between requirement of very light flavon (for sufficient Yukawa variation) and Flavour constraints (meson oscillations)

Baldes, Konstandin, Servant, 1608.03254

But: We did not take into account dynamics of Froggatt-Nielsen fermions. Follow-up study in progress.

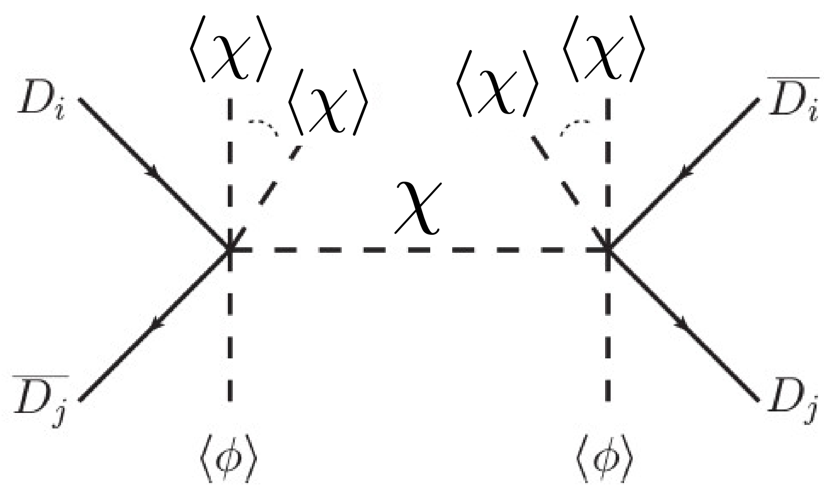
Baldes, Servant, Suresh, in progress.

The tension in low-scale Froggatt-Nielsen models



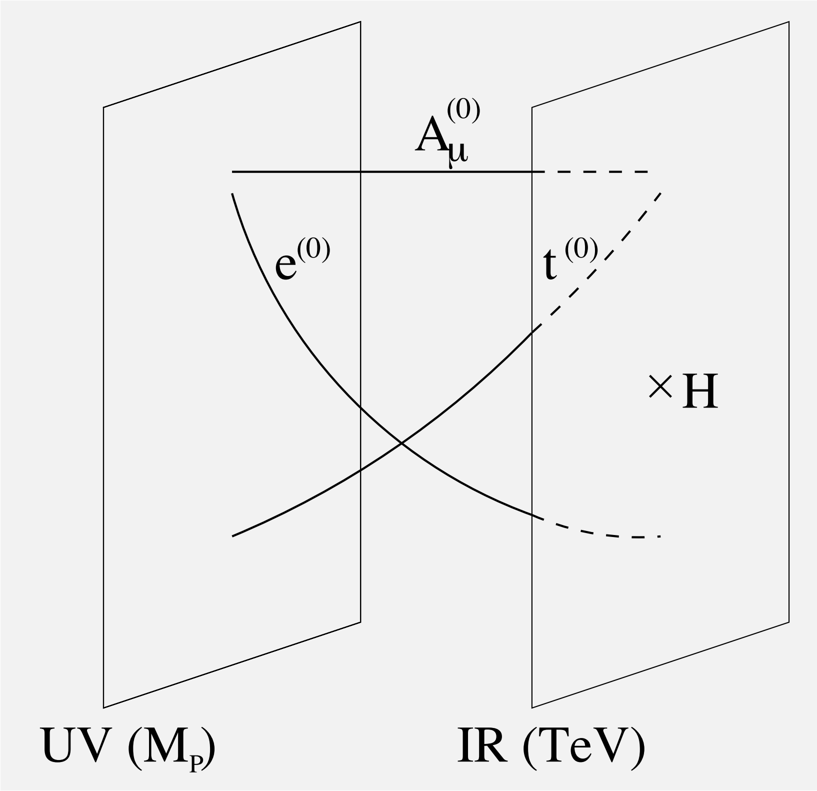
Such dynamics requires a light χ

But a light χ induces large B-meson oscillations



Naturally varying Yukawas: The Randall-Sundrum case

Von Harling, Servant '16



AdS_5 metric

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$S \supset \int d^4x \left(\eta^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - e^{-2ky_{\rm IR}} M_P^2 |H|^2 + \lambda |H|^4\right)$$

radion $\sigma \equiv e^{-ky_{\rm IR}}$

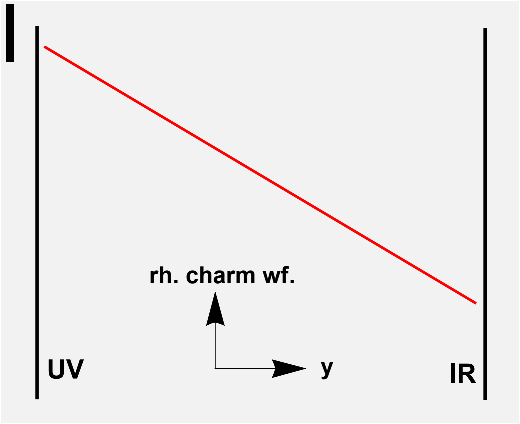
In minimal Randall-Sundrum models, Yukawas decrease across the bubble wall

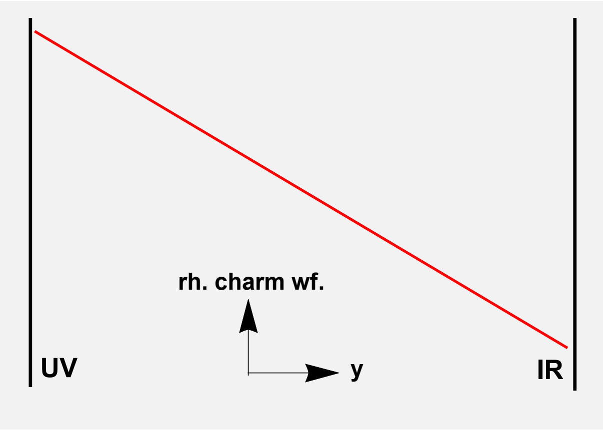
CONSTANT bulk fermion mass term:

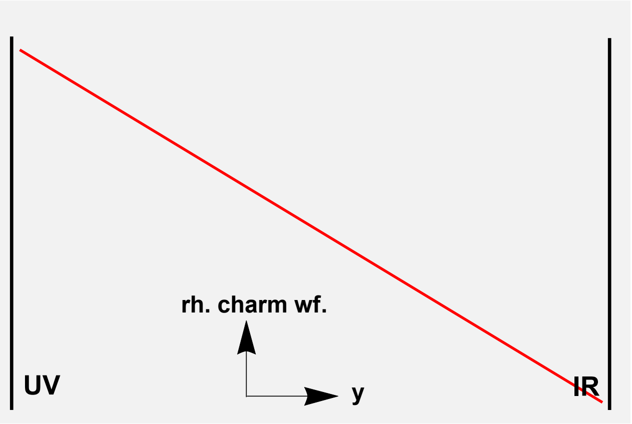
$$S \supset -\int d^5x \sqrt{g}\, c\, k\, \overline{\psi}\psi$$

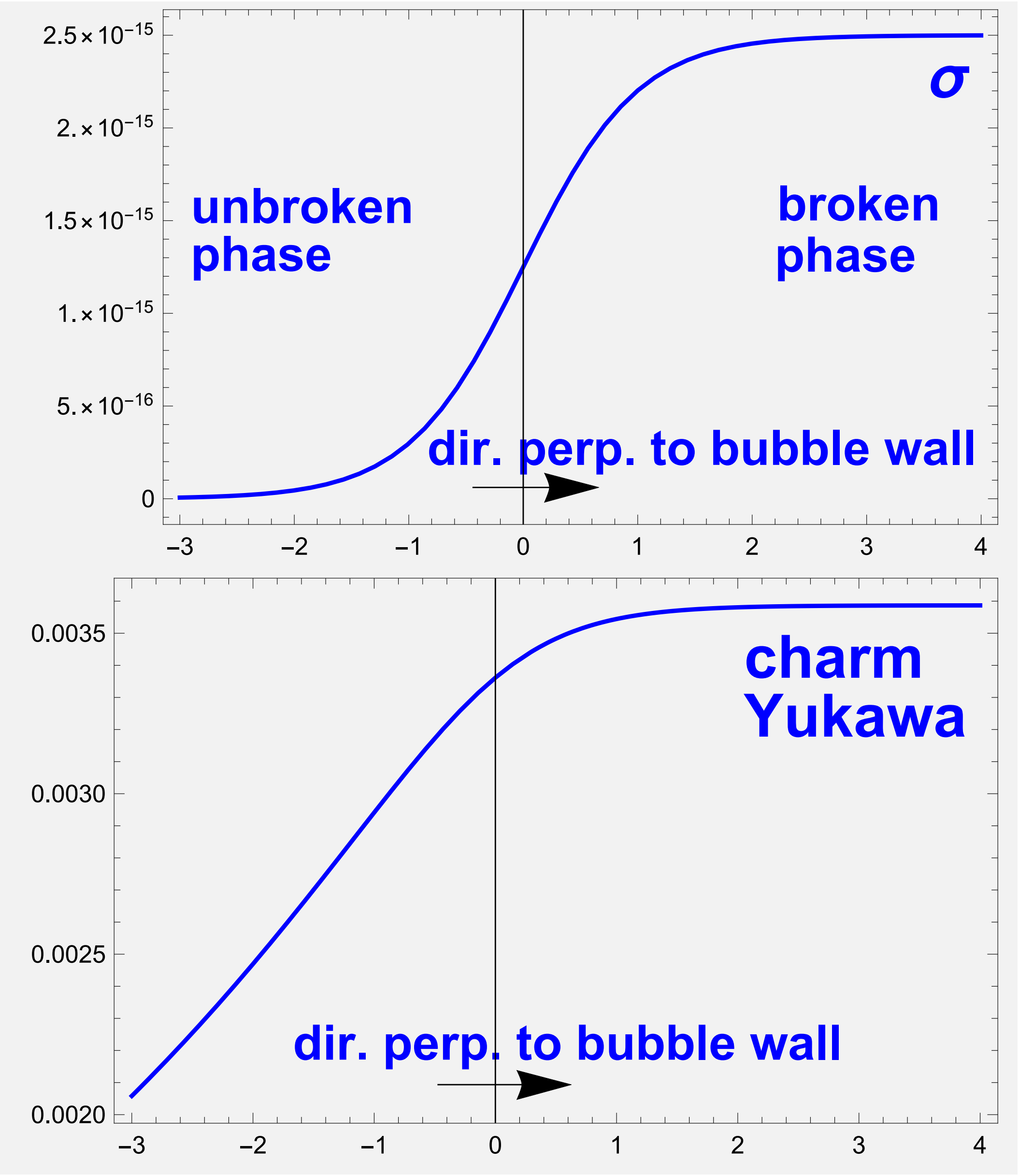
resulting 4D effective Yukawas:

$$y(\sigma) = \lambda \sqrt{\frac{1-2c_L}{1-\sigma^{1-2c_L}}} \sqrt{\frac{1-2c_R}{1-\sigma^{1-2c_R}}}$$









$\Rightarrow$ Yukawas decrease along bubble wall

$\Rightarrow$ not enough CP-violation from $S_{\not{C}\not{P}} \propto \mathrm{Im}\left[V^\dagger M^{\dagger\prime\prime} M\, V\right]$

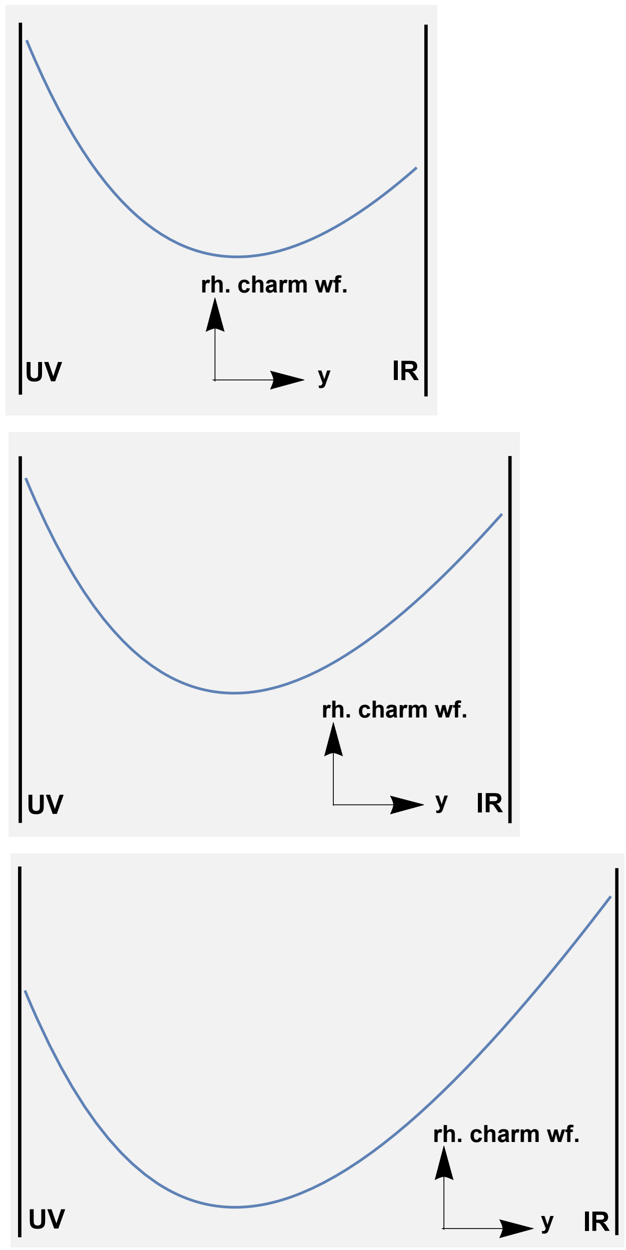
Now, assume following natural possibility: bulk fermion mass term comes from Yukawa coupling with Goldberger-Wise scalar:

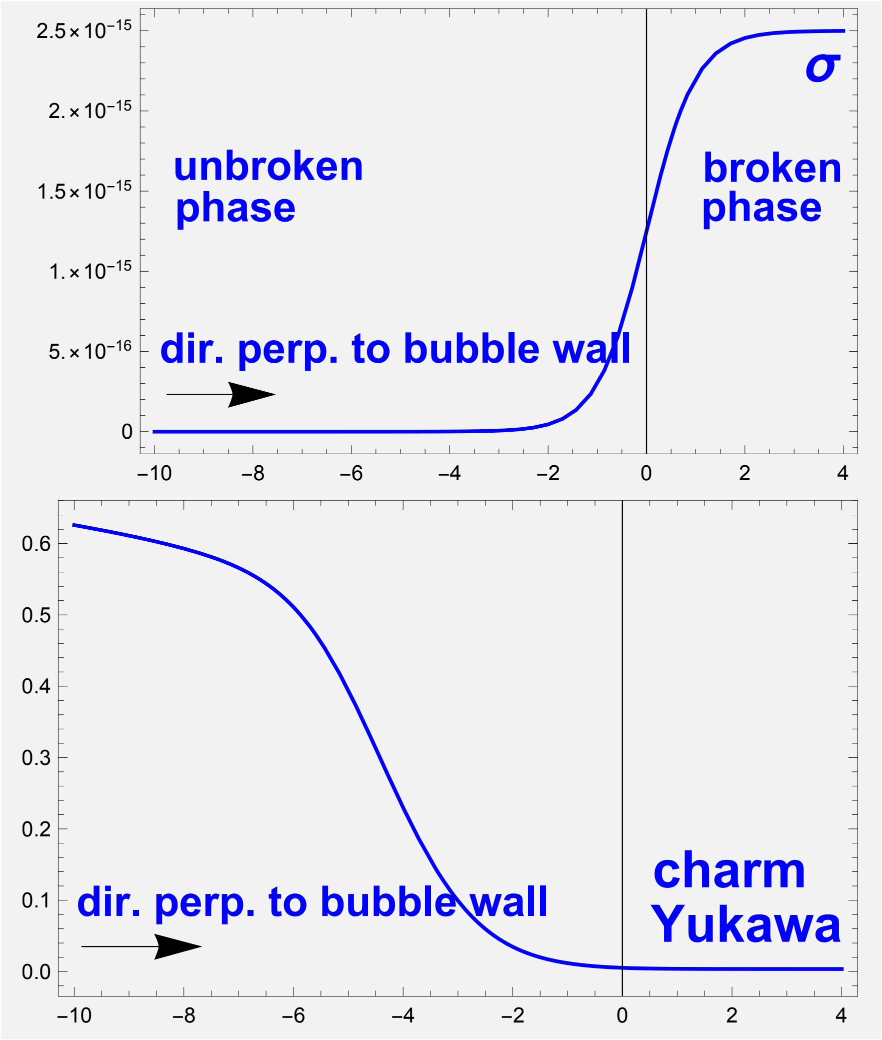
$$S \supset -\int d^5x\sqrt{g}\,\rho\langle\phi\rangle\,\overline{\psi}\psi$$

$\Rightarrow$ Position-dependent mass term!

resulting 4D effective Yukawas:

$$y(\sigma) = \lambda k\,\mathcal{N}^{(0)}_{\tilde{c}_L}\mathcal{N}^{(0)}_{\tilde{c}_R}\,\sigma^{-1}\,e^{\frac{(\tilde{c}_L+\tilde{c}_R)\,\sigma^{\epsilon}}{\epsilon}}$$

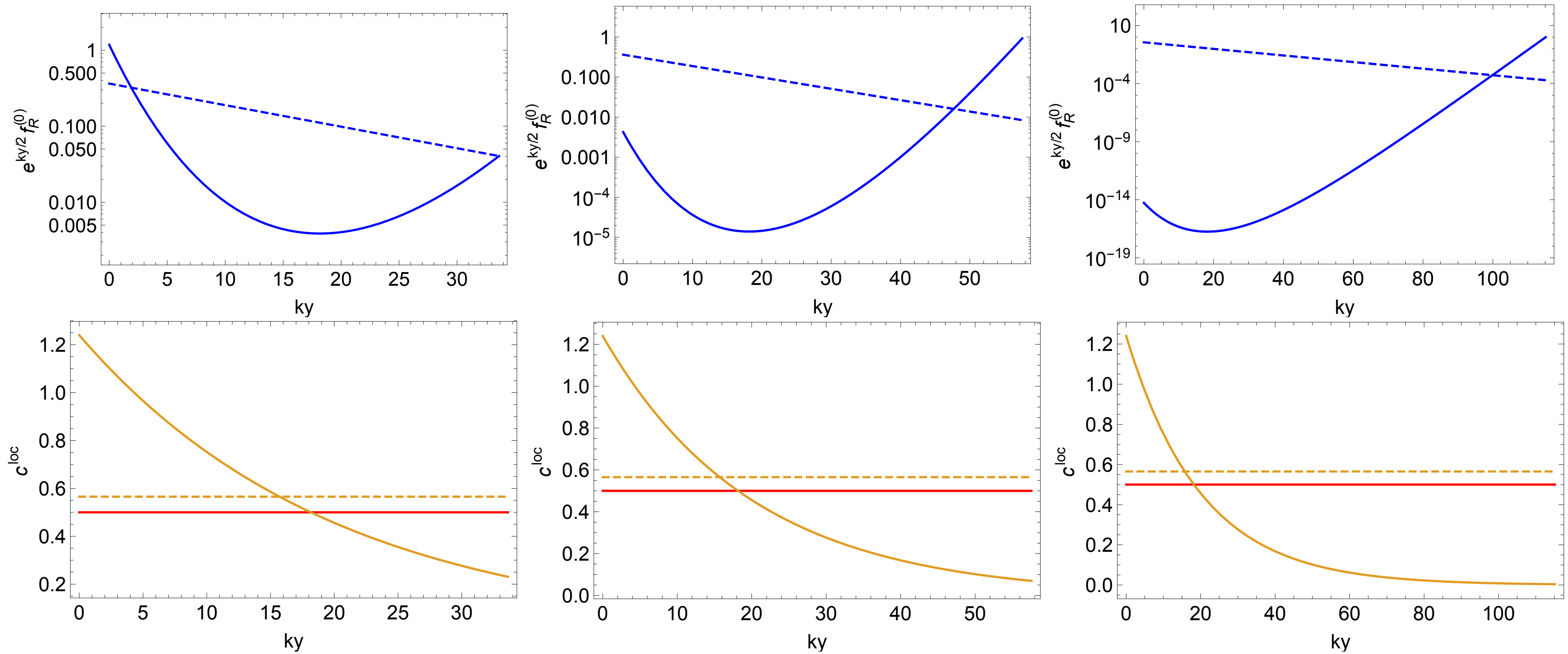




$\Rightarrow$ Yukawas increase along bubble wall

$\Rightarrow$ more CP-violation from $S_{\not{C}\not{P}} \propto \mathrm{Im}\left[V^\dagger M^{\dagger\prime\prime} M\, V\right]$

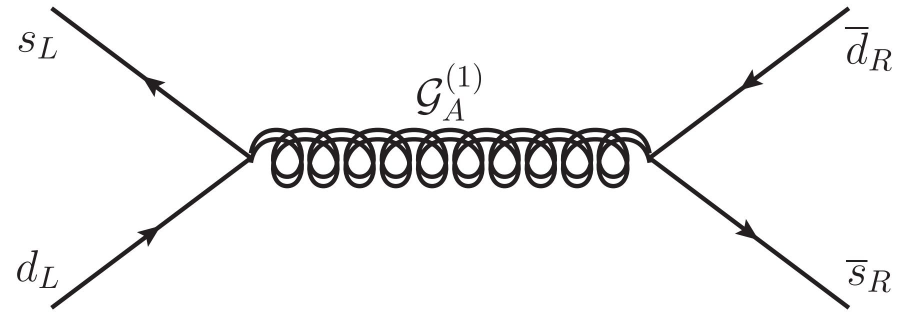
Wave function when going back in time



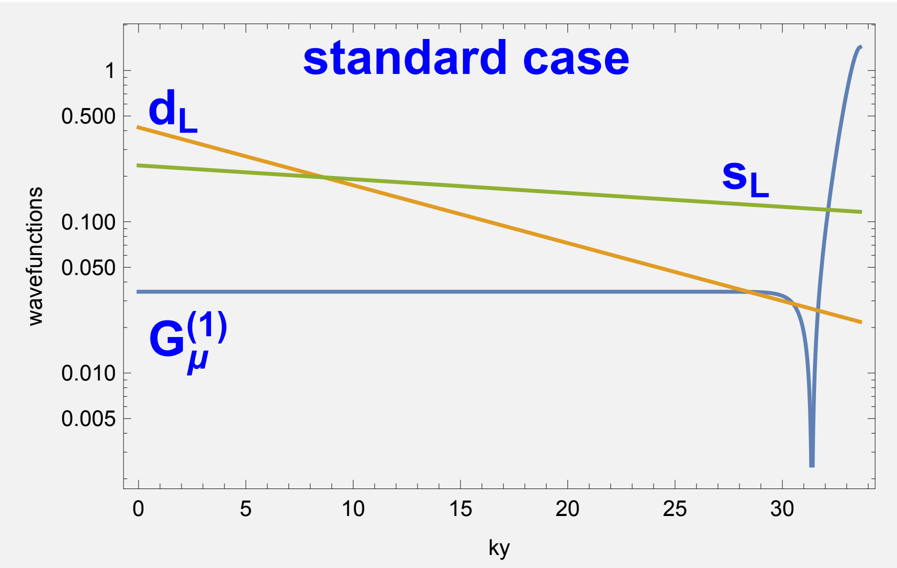
Bonus:

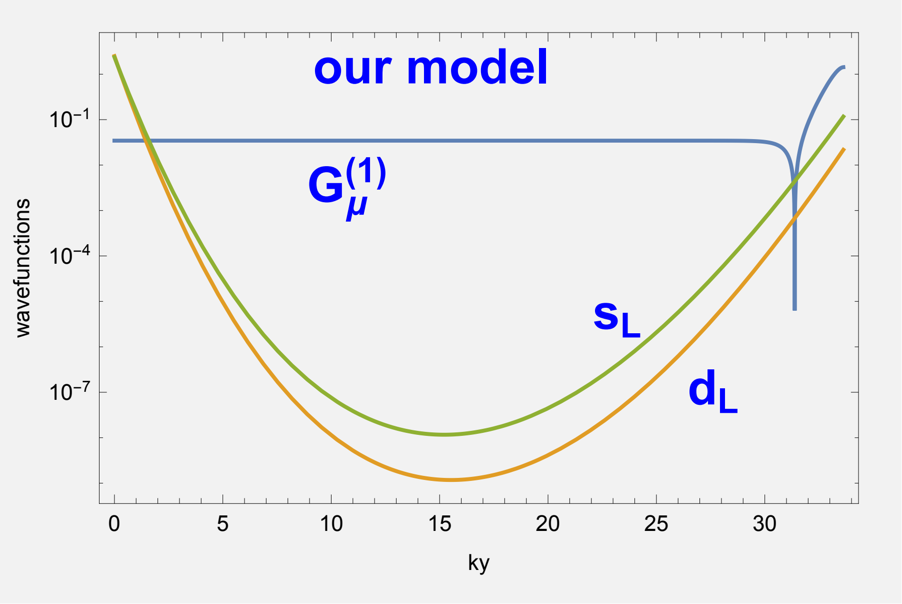
Modified wave functions give suppression of CP-violating processes which are very constraining in the standard case

CP violation in K-Kbar mixing



Suppression of overlap integral





Constraint for standard case of constant bulk mass terms:

$$m_{\mathcal{G}}^{(1)} \gtrsim \frac{3}{\lambda_*}(22 \pm 6)\ \text{TeV} \quad \Rightarrow \quad e^{-ky_{\rm IR}}k \gtrsim \frac{3}{\lambda_*}(9 \pm 3)\ \text{TeV}$$

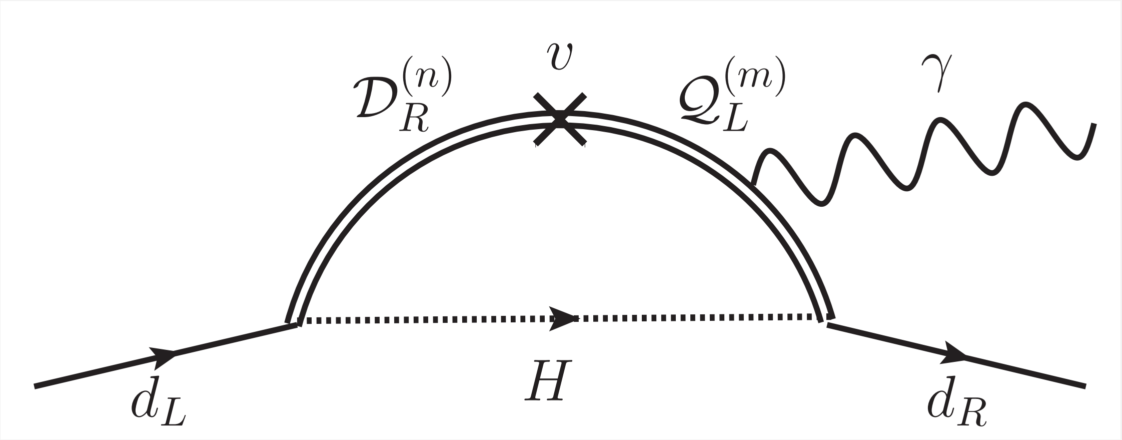
- In our scenario instead:

$$m_{\mathcal{G}}^{(1)} \gtrsim \frac{3}{\lambda_*}(7 \pm 2)\ \text{TeV} \quad \Rightarrow \quad e^{-ky_{\rm IR}}k \gtrsim \frac{3}{\lambda_*}(3 \pm 1)\ \text{TeV}$$

$\Rightarrow$ Significant improvement!

Neutron EDM

- Important constraint on IR scale $e^{-ky_{\mathrm{IR}}} M_P$ also from neutron EDM. Dominant contribution:



- Constraint for standard case of constant bulk mass terms:

$$m_\psi^{(1)} \gtrsim \frac{\lambda_*}{3}\, 26\ \mathrm{TeV} \quad \Rightarrow \quad e^{-ky_{\mathrm{IR}}} k \gtrsim \frac{\lambda_*}{3}\, 11\ \mathrm{TeV}$$

- Again expect that constraints eased in our scenario since first fermionic KKs are heavier than for constant bulk mass terms

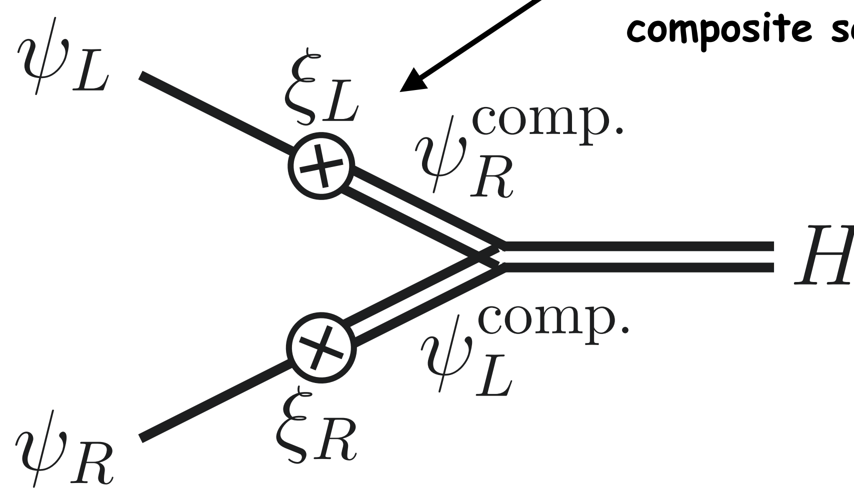
CFT Interpretation

$$\mathcal{L} \supset \mathcal{L}_{\rm CFT} + i\,\mathcal{Z}\,\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \frac{\omega}{\Lambda_{\rm UV}^{\Delta-5/2}} (\bar{\psi}_L \mathcal{O}_R + {\rm h.c.})\,,$$

mixing between elementary and composite sector

Effective Yukawa coupling

$$y(\Lambda_{\rm IR}) \propto \xi_L(\Lambda_{\rm IR}) \times \xi_R(\Lambda_{\rm IR})$$



mu: scale of spontaneous breaking of conformal invariance in the IR

$$\xi(\mu) \equiv \frac{\omega(\mu)}{\sqrt{\mathcal{Z}(\mu)}} \left(\frac{\mu}{\Lambda_{\rm UV}}\right)^{\Delta(\mu)-5/2}$$

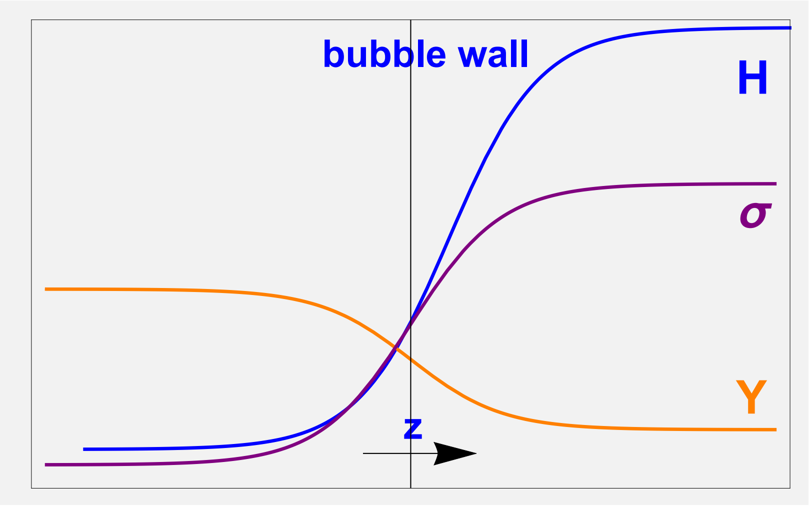
$$\Delta(\mu) = 2 + \tilde{c}\left(\frac{\mu}{\Lambda_{\rm UV}}\right)^{\epsilon}$$

Summary

Minimal modification of RS: Yukawa coupling between Goldberger-Wise scalar and bulk fermions

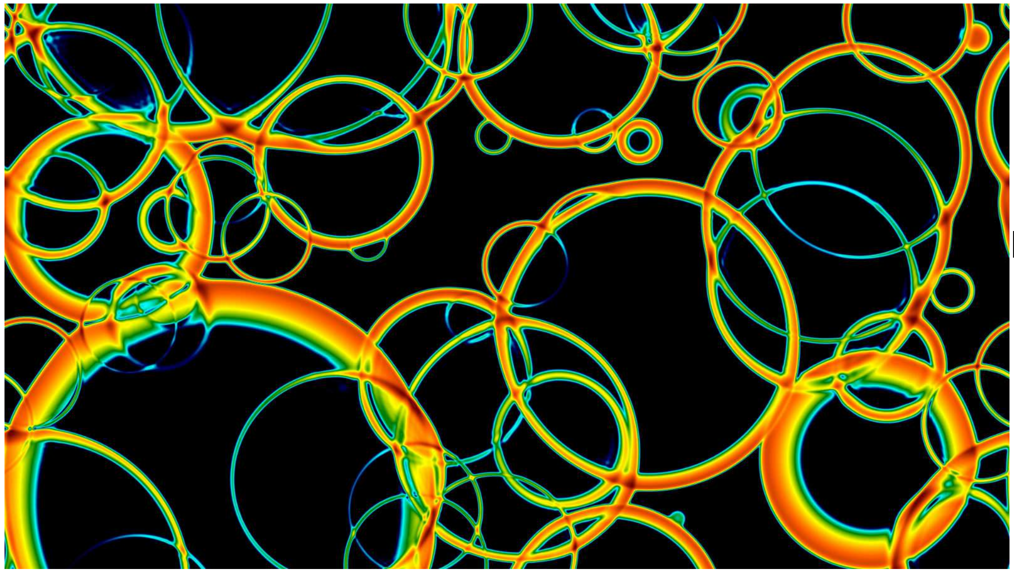
naturally large yukawas and enhanced CP violation in bubble walls during EW phase transition

eases constraints from CP violation n K Kbar mixing



Gravity wave signals from 1st order cosmological phase transitions

[eLISA Cosmology Working group, 1512.06239]



[Credit:David Weir]

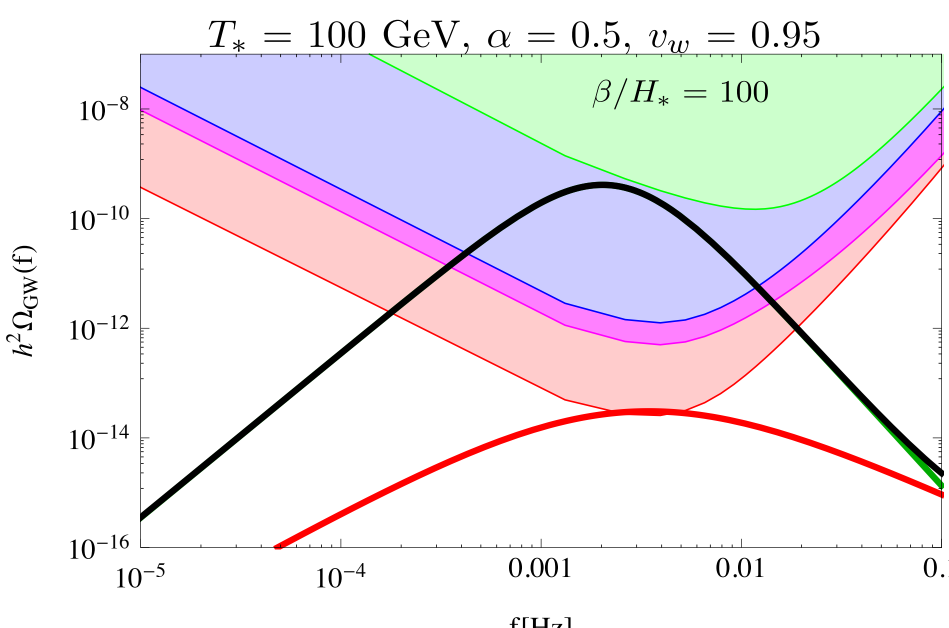
Fluid flows

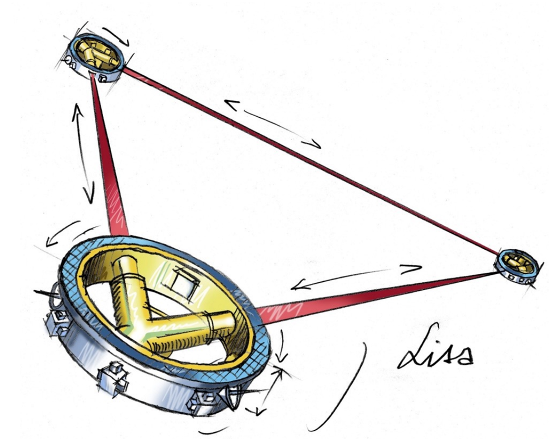
turbulence

Magnetic fields

→ Stochastic background of gravitational radiation

EW phase transition -> mHz -> LISA!





Conclusion

Time-dependent CP-violating sources can make EW baryogenesis compatible with Electric Dipole Moment constraints and can be well-motivated theoretically. We provided 2 examples:

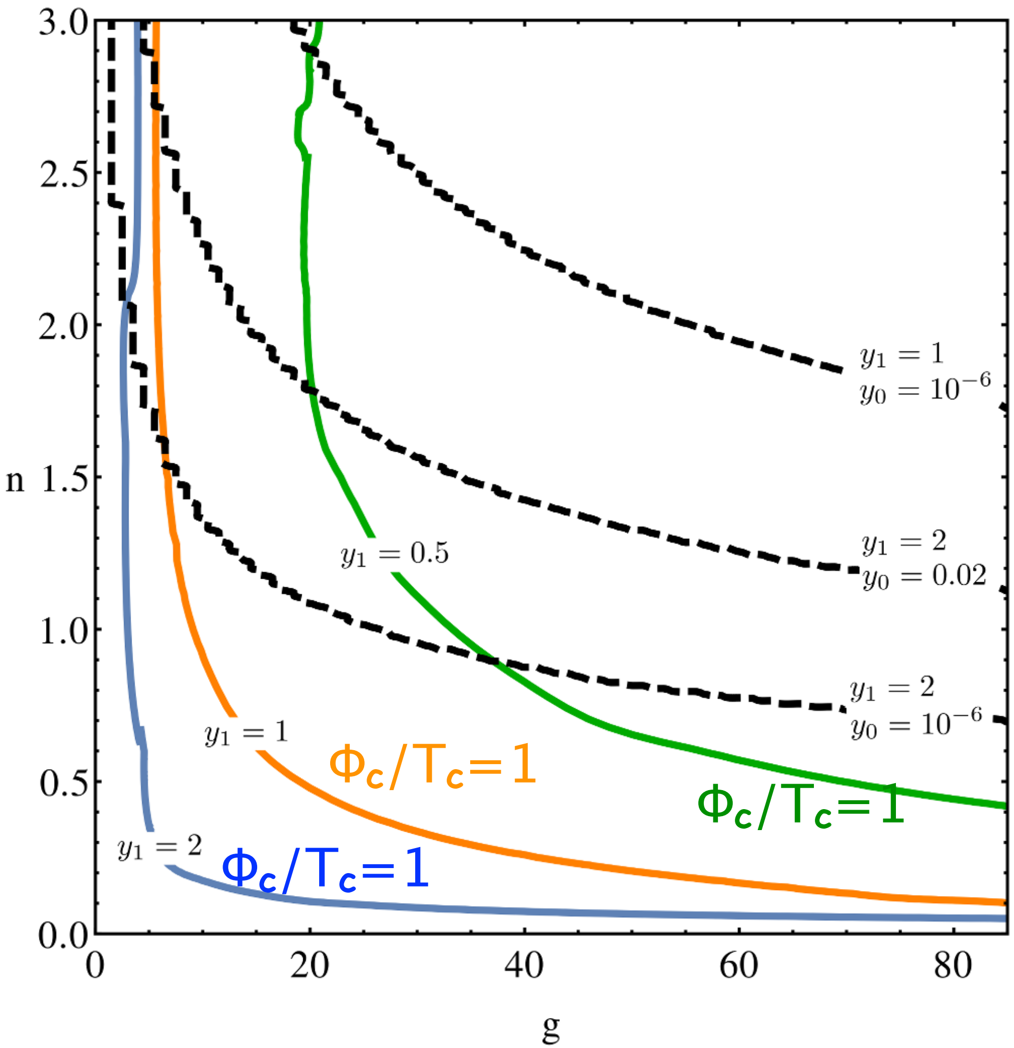
1) strong CP from QCD axion,

2) weak CP from dynamical Yukawas

2) —> Flavour cosmology! New window of opportunities. Dynamical interplay between flavour and electroweak symmetry breaking.

Annexes

Contours of $\Phi_c/T_c=1$ for different choices of y_1 and y_0, areas above these lines allow for EW baryogenesis.



Dashed lines: areas above these lines are disallowed (for the indicated choices of y_1 and y_0 due to the EW minimum not being the global one.

n characterizes how fast the Yukawa variation is taking place. Depending on the underlying model, the Higgs field variation will follow the flavon field variation at different speeds. Large n means the Yukawa coupling remains large for a greater range of phi away from zero. It strengthens the phase transition.

Baldes, Konstandin, Servant, 1604.04526