Vacuum stability in the N2HDM

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THE MODEL

Two SU(2) doublets of hypercharge Y = 1, Φ_1 and Φ_2, with an added REAL SINGLET Φ_S (no hypercharge).

- **Discrete symmetries imposed on the model to reduce number of parameters AND obtain interesting phenomenology (such as *dark matter*).**

- **Reproduces the LHC-observed Higgs boson phenomenology**

- **A phase of the model, with an unbroken discrete symmetry by the vacuum, includes dark matter candidates, which comply with all current experimental bounds.**

- **The vacuum of the model can also, for some regions of parameter space, break charge or CP conservation, and the analysis of those possibilities is the purpose of this work.**

THE SCALAR POTENTIAL: we choose to impose *three discrete symmetries* on the model; the two doublets and the real singlet transform as:

(a) A $\mathbb{Z}_2$ symmetry of the form $\Phi_1 \to \Phi_1 \ , \ \Phi_2 \to -\Phi_2 \ , \ \Phi_S \to \Phi_S$

(b) A $\mathbb{Z}_2$ symmetry of the form $\Phi_1 \to \Phi_1 \ , \ \Phi_2 \to \Phi_2 \ , \ \Phi_S \to -\Phi_S$

(c) A CP symmetry of the form $\Phi_1 \to \Phi_1^* \ , \ \Phi_2 \to \Phi_2^* \ , \ \Phi_S \to \Phi_S$

The scalar potential therefore becomes

$$
\begin{aligned}
V = {} & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + h.c.\right) \\
& + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] \\
& + \frac{1}{2}m_S^2\Phi_S^2 + \frac{1}{8}\lambda_6\Phi_S^4 + \frac{1}{2}\lambda_7|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_8|\Phi_2|^2\Phi_S^2 \,,
\end{aligned}
$$

SOFT BREAKING TERM OF (a)!

where all the parameters are *REAL*.

- **There are different version of the N2HDM – same field content – but with different symmetries imposed, or with different soft breaking terms , both of which leads to *different phenomenology*.**

- **For instance, an early N2HDM had a softly broken $Z_2 \times Z_2$ symmetry, just like the one we considered, but with *a complex soft breaking term*.**

$$
\begin{aligned}
V \;=\; & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{1}{2}m_S^2\Phi_S^2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + h.c.\right) + \\
& + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] \\
& + \frac{1}{4}\lambda_6\Phi_S^4 + \frac{1}{2}\lambda_7|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_8|\Phi_2|^2\Phi_S^2 \,,
\end{aligned}
$$

Therefore, same quartic couplings, but the complex soft breaking quadratic term induces explicit CP-violation.

B. Grzadkowski and P. Osland, Phys. Rev. **D82**, 125026 (2010), 0910.4068

Another N2HDM had a different discrete symmetry, the two doublet and the real singlet transforming as

$$\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2 \quad , \quad \Phi_S \to -\Phi_S$$

The scalar potential therefore becomes

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{1}{2}m_S^2\Phi_S^2 + \left(A\Phi_1^\dagger\Phi_2\Phi_S + h.c.\right) \\ + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] \\ + \frac{1}{4}\lambda_6\Phi_S^4 + \frac{1}{2}\lambda_7|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_8|\Phi_2|^2\Phi_S^2\,,$$

CUBIC TERM!

where, with the exception of *A*, all the parameters are *REAL*. This model has explicit CP violation AND dark matter candidates, but CP breaking is "confined" to the "dark sector". Its only eventually observable consequences would be anomalous ZZZ vertex contributions.

D. Azevedo, P.M. Ferreira, M.M. Muhleitner , S. Patel, R. Santos, JHEP 1811 (2018) 091

- Coming back to the N2HDM we're considering:

(a) A $\mathbb{Z}_2$ symmetry of the form $\Phi_1 \to \Phi_1 \;,\; \Phi_2 \to -\Phi_2 \;,\; \Phi_S \to \Phi_S$

(b) A $\mathbb{Z}_2$ symmetry of the form $\Phi_1 \to \Phi_1 \;,\; \Phi_2 \to \Phi_2 \;,\; \Phi_S \to -\Phi_S$

(c) A CP symmetry of the form $\Phi_1 \to \Phi_1^* \;,\; \Phi_2 \to \Phi_2^* \;,\; \Phi_S \to \Phi_S$

The discrete symmetry (a) imposed prevents the occurrence of FCNC in the model, it can be extended to the Yukawa sector so that only Φ_1 couples to all fermions (type I-like model),

$$-\mathcal{L}_Y = \lambda_t \bar{Q}_L \tilde{\Phi}_1 t_R + \lambda_b \bar{Q}_L \Phi_1 b_R + \lambda_\tau \bar{L}_L \Phi_1 \tau_R + \ldots$$

- For what follows, the structure of the Yukawa terms will have little impact on the results obtained.

- The second doublet and the singlet have "dark charges" under symmetries (a) and (b) and therefore there may be phases of the model which will originate dark matter – the lightest scalar from the dark sector will be stable in those phases.

SPONTANEOUS SYMMETRY BREAKING

Let us consider possible vacua which do not break neither charge nor CP conservation – **NORMAL VACUA**. There are two possibilities, depending on the singlet having a vev or not.

- No singlet vev: $$\langle\Phi_1\rangle_{\mathcal{N}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle_{\mathcal{N}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle\Phi_S\rangle_{\mathcal{N}} = 0$$

This vacuum preserves the $\mathbb{Z}_2$ symmetry **(b)** defined above – it contains possible dark matter candidates (**DARK PHASE**).

- Singlet with vev: $$\langle\Phi_1\rangle_{\mathcal{N}s} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v'_1 \end{pmatrix}, \quad \langle\Phi_2\rangle_{\mathcal{N}s} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v'_2 \end{pmatrix}, \quad \langle\Phi_S\rangle_{\mathcal{N}s} = v'_S$$

This vacuum breaks both the **(a)** and **(b)** $\mathbb{Z}_2$ symmetries defined above.

Notice that the doublet vevs can be quite different for each vacuum, and it is by no means guaranteed that they both correctly break electroweak symmetry.

CHARGE BREAKING VACUA

Unlike the SM (but like the 2HDM) it is not guaranteed that the global minimum of the N2HDM preserves $U(1)_{em}$. Depending on the values of the potential's parameters, there can be CHARGE BREAKING (CB) VACUA. As before, we can have two possibilites:

- No singlet vev:

$$\langle\Phi_1\rangle_{CB} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle\Phi_2\rangle_{CB} = \frac{1}{\sqrt{2}}\begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle\Phi_S\rangle_{CB} = 0$$

- Singlet with vev:

$$\langle\Phi_1\rangle_{CB_s} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ c'_1 \end{pmatrix}, \quad \langle\Phi_2\rangle_{CB_s} = \frac{1}{\sqrt{2}}\begin{pmatrix} c'_2 \\ c'_3 \end{pmatrix}, \quad \langle\Phi_S\rangle_{CB_s} = c'_4$$

The vevs c_2 and c'_2 break charge conservation. It is easy to show that these are the most general CB vevs one can have.

M. Mühlleitner, M. O. P. Sampaio, R. Santos, and J. Wittbrodt, JHEP **03**, 094 (2017), 1612.01309.

SPONTANEOUS CP BREAKING VACUA

This model explicitly preserves CP, so spontaneous CP violation is a possibility, like in the 2HDM. Depending on the values of the potential's parameters, there can be CP BREAKING (CP) VACUA. As before, we can have two possibilites:

- No singlet vev:

$$\langle \Phi_1 \rangle_{\mathcal{CP}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_{\mathcal{CP}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_2 + i\bar{v}_3 \end{pmatrix}, \quad \langle \Phi_S \rangle_{\mathcal{CP}} = 0$$

- Singlet with vev:

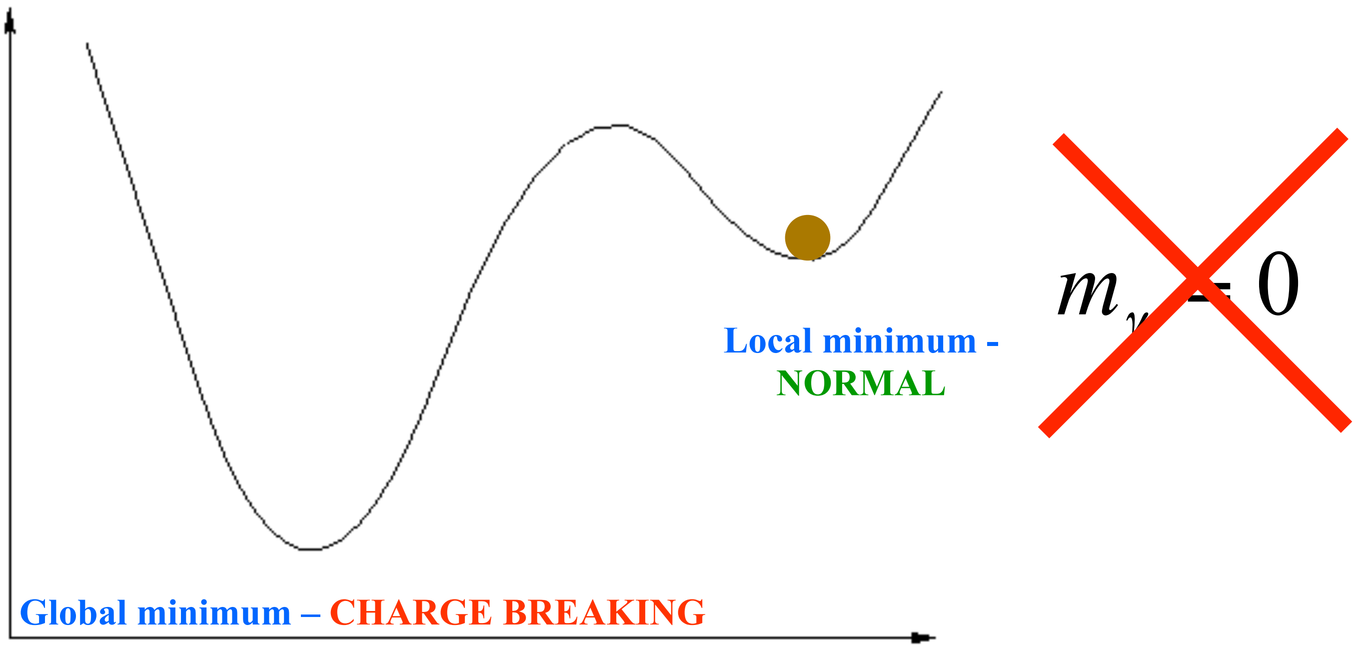
$$\langle \Phi_1 \rangle_{\mathcal{CP}_s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}'_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_{\mathcal{CP}_s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}'_2 + i\bar{v}'_3 \end{pmatrix}, \quad \langle \Phi_S \rangle_{\mathcal{CP}} = \bar{v}'_4$$

The complex vevs break CP. It can be shown that these are the most general CP vevs one can have.

M. Mühlleitner, M. O. P. Sampaio, R. Santos, and J. Wittbrodt, JHEP **03**, 094 (2017), 1612.01309.

Would there be any problem if the potential had two of these minima simultaneously?

Answer: there might be, if the CB minimum, for instance, were "deeper" than the normal one (metastable).



$m_\gamma \neq 0$!

It can be shown ANALYTICALLY, for the 2HDM that this cannot occur, and normal minima are stable against charge or CP breaking... However, the coexistence of Normal, CB and CP breaking minima is possible in other models with extended scalar sectors – and *if the tunneling time is small enough, they can be "dangerous".*

COEXISTENCE OF NORMAL AND CB VACUA

Assume now that the minimization conditions of the N2HDM admit two simultaneous stationary points: a NORMAL vacuum N (the *dark phase* of the mode, where the singlet has no vev) and a CHARGE BREAKING one, CB. Using a *bilinear formalism*, it is possible to find an analytic expression relating the value of the sclar potential at each of the stationary points. To wit,

$$V_{\mathcal{CB}} - V_{\mathcal{N}} = \frac{m_{H^\pm}^2}{4v^2} \left[(v_2 c_1 - v_1 c_3)^2 + v_1^2 c_2^2 \right]$$

In this expression, $m_{H^\pm}^2$ is the value of the charged Higgs mass *calculated at the N vacuum*. Since the quantity in square brackets is certainly positive, we conclude that, if is N a minimum then certainly one will have $m_{H^\pm}^2 > 0$ and therefore one necessarily obtains $V_{\mathcal{CB}} - V_{\mathcal{N}} > 0$. Therefore, it can be concluded that

If the potential has a minimum of type $\mathcal{N}$, any $\mathcal{CB}$ stationary point, if it exists, lies above $\mathcal{N}$.

COEXISTENCE OF NORMAL AND CBs VACUA

We obtain a similar situation if the potential admits two stationary points, one again a NORMAL vacuum $\mathcal{N}$ and a CHARGE BREAKING one including also a vev for the singlet, $\mathcal{CB}s$. We again obtain an analytical expression relating the depth of the potential at both stationary points, namely

$$V_{\mathcal{CB}s} - V_{\mathcal{N}} = \frac{1}{4}\left\{\frac{m^2_{H^\pm}}{v^2}\left[(v_2 c'_1 - v_1 c'_3)^2 + v_1^2 {c'_2}^2\right] + m^2_D {c'_4}^2\right\}$$

Again, $m^2_{H^\pm}$ is the value of the charged Higgs mass *calculated at the $\mathcal{N}$ vacuum*, and

$$m^2_D = m^2_S + \frac{1}{2}(\lambda_7 v_1^2 + \lambda_8 v_2^2)$$

is the squared mass of the singlet field (the dark matter particle mass, therefore). Again, if $\mathcal{N}$ is a minimum, all of its squared scalar masses need to be positive and hence, $V_{\mathcal{CB}s} - V_{\mathcal{N}} > 0$. Therefore,

If the potential has a minimum of type $\mathcal{N}$, any $\mathcal{CB}s$ stationary point, if it exists, lies above $\mathcal{N}$.

COEXISTENCE OF $\mathcal{N}s$ AND CHARGE BREAKING VACUA

The situation changes if we consider a NORMAL vacuum $\mathcal{N}s$ - for which he singlet field does have a vev – and one of the two CHARGE BREAKING vacua, CB and CBs. We again obtain analytical expressions relating the depth of the potential at both stationary points. For coexisting $\mathcal{N}s$ and CBs, we obtain

$$V_{\mathcal{CB}s} - V_{\mathcal{N}s} = \left(\frac{m^2_{H\pm}}{4v^2}\right)_{\mathcal{N}s} \left[(v'_2 c'_1 - v'_1 c'_3)^2 + {v'_1}^2 {c'_2}^2\right]$$

The charged Higgs mass in this expression is now *calculated at the $\mathcal{N}s$ vacuum*, but the conclusions are the same – if $\mathcal{N}s$ is a minimum, it is deeper than CBs.

For coexisting $\mathcal{N}s$ and CB stationary points, however, we find

$$V_{\mathcal{CB}} - V_{\mathcal{N}s} = \left(\frac{m^2_{H\pm}}{4v^2}\right)_{\mathcal{N}s} \left[(v'_2 c_1 - v'_1 c_3)^2 + {v'_1}^2 c_2^2\right] - \frac{1}{4} s^2 m^2_{S1}$$

(m^2_{S1} is one of the squared masses at the CB stationary point)

There is now no mandatory relationship between the depths of both stationary points – *either of then can be deeper, and therefore the stability of $\mathcal{N}s$ against charge breaking is not guaranteed.*

COEXISTENCE OF NORMAL AND CP VACUA

Similar conclusions are reached when one considers the possibility of Normal and CP breaking vacua coexisting in the potential.

$$V_{\mathcal{CP}} - V_{\mathcal{N}} = \frac{m_A^2}{4v^2}\left[(v_2\bar{v}_1 - v_1\bar{v}_2)^2 + v_1^2\bar{v}_3^2\right]$$

$$V_{\mathcal{CP}s} - V_{\mathcal{N}} = \frac{1}{4}\left\{\frac{m_A^2}{v^2}\left[(v_2\bar{v}'_1 - v_1\bar{v}'_2)^2 + v_1^2\bar{v}'^2_3\right] + m_D^2\bar{v}'^2_4\right\}$$

N is stable against CP breaking

If $\mathcal{N}$ is a minimum, it is deeper than any $\mathcal{CP}$ or $\mathcal{CP}s$ stationary points.

$$V_{\mathcal{CP}s} - V_{\mathcal{N}s} = \left(\frac{m_A^2}{4v^2}\right)_{\mathcal{N}s}\left[(v'_2\bar{v}'_1 - v'_1\bar{v}'_2)^2 + {v'_1}^2\bar{v}'^2_3\right]$$

$$V_{\mathcal{CP}} - V_{\mathcal{N}s} = \left(\frac{m_A^2}{4v^2}\right)_{\mathcal{N}s}\left[(v'_2\bar{v}_1 - v'_1\bar{v}_2)^2 + {v'_1}^2\bar{v}_3^2\right] - \frac{1}{4}m_{\bar{D}}^2\, s^2$$

Ns is stable against CPs vacua, but not against CP ones.

COEXISTENCE OF TWO NORMAL OF DIFFERENT TYPES

Another possibility to be considered is the coexistence of N and Ns minima. We obain

$$V_{\mathcal{N}s} - V_{\mathcal{N}} = \frac{1}{4}\left[\left(\frac{m_{H^\pm}^2}{4v^2}\right)_{\mathcal{N}} - \left(\frac{m_{H^\pm}^2}{4v^2}\right)_{\mathcal{N}s}\right](v_1 v_2' - v_2 v_1')^2 + \frac{1}{4} m_D^2\, s^2$$

Therefore, not one of the types of minima is guaranteed to be the deepest.

OVERALL VACUUM STABILITY

We can resume the picture of stability of the N2HDM in the following table:

Extrema	$\mathcal{N}$	$\mathcal{N}s$	$\mathcal{CB}$	$\mathcal{CB}s$	$\mathcal{CP}$	$\mathcal{CP}s$
$\mathcal{N}$	×	×	Stability	Stability	Stability	Stability
$\mathcal{N}s$	×	×	×	Stability	×	Stability

Here, "stability" means that if N (Ns) is a minimum, it is deeper than the corresponding other stataionary point, and its stability is guaranteed for that type of symmetry breaking.

PHENOMENOLOGICAL ANALYSIS

- **From our analytical investigation, the minimum for which we expect to obtain more significant restrictions from vacuum stability is the *Ns* one, so all calculations and bounds presented pertain to that case.**

- **Necessary and Sufficient bounded from below conditions, unitarity bounds and electroweak precision variables S, T and U already known for this model, used results from ScannerS, see:**

M. Mühlleitner, M. O. P. Sampaio, R. Santos, and J. Wittbrodt, JHEP **03**, 094 (2017), 1612.01309.

- **Bounds from flavour physics (from 2HDM type I) imposed on the model, yielding large constraints on the m_{H+} - $\tan\beta$ plane.**

- **Agreement with collider Higgs data found by using HiggsSignals and HiggsBounds (very approximately requiring a 2σ agreement with all data).**

- **A sizeable parameter scan was obtained, by varying the model's parameters in large intervals.**

	m_{H_y}, m_{H_z}, m_A	$m_{H\pm}$	$\tan\beta$	m_{12}^2	v_S
min	30 GeV	150 GeV	0.8	0 GeV^2	1 GeV
max	1.5 TeV	1.5 TeV	20	5×10^5 GeV^2	3 TeV

Numerical analysis

In imposing vacuum stability constraints we distinguish the following cases:

- parameter points where the EW vacuum is the only vacuum,
- absolutely stable parameter points where secondary minima exist but are never deep,
- long-lived parameter points where secondary vacua are deep but never dangerous,
- short-lived parameter points that have dangerous secondary minima.

The value of the scalar potential at each of these stationary points is compared to the depth of the EW vacuum.

If there is no stationary point deeper than the EW vacuum we consider the EW vacuum at this parameter point as absolutely stable.

If stationary points deeper than the EW vacuum exist we calculate the tunnelling time to each of these deeper extrema.

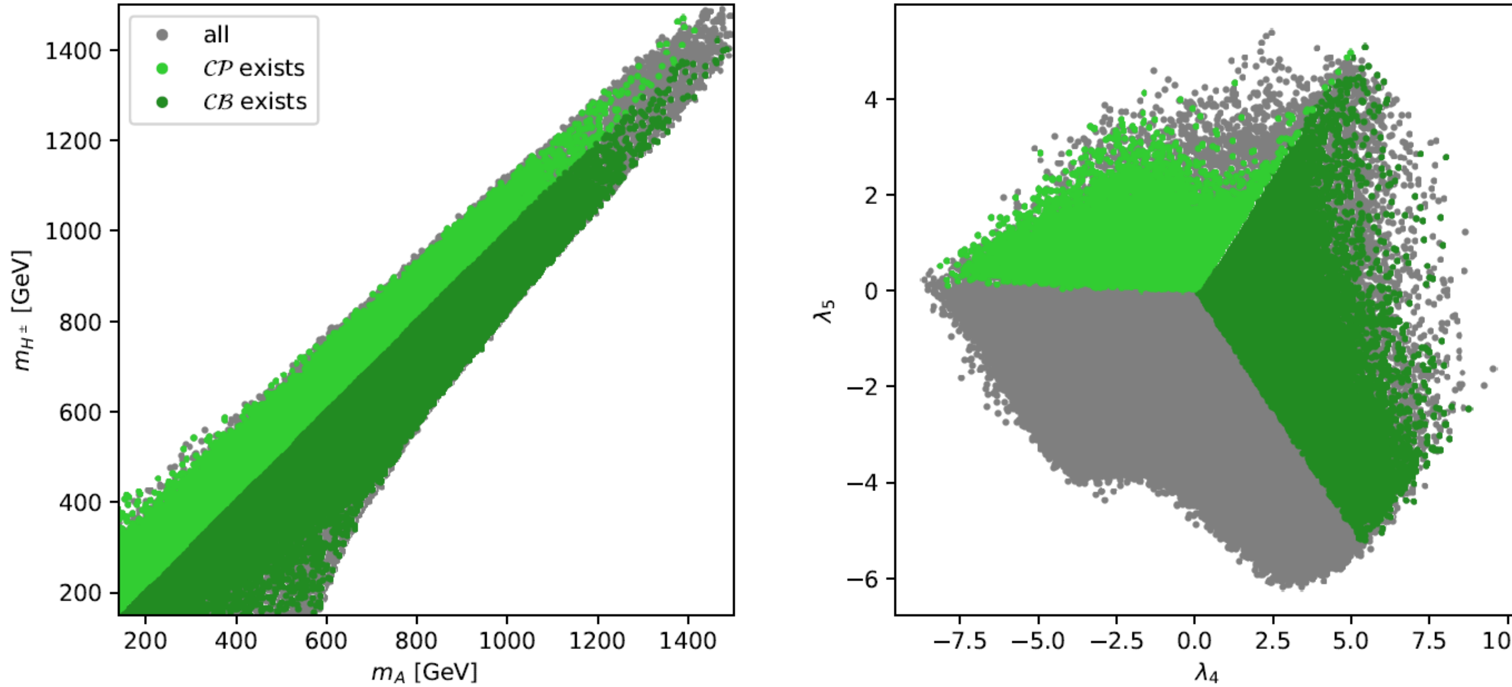
HOLLIK, WEIGLEIN, WITTBRODT, 1812.04644

$\mathcal{N}s'$ are minima of type $\mathcal{N}s$ with different values for the vevs – they originate from the fact the N2HDM can have multiple solutions for the same type of vacua.

	$\mathcal{N}s'$	$\mathcal{N}$	$\mathcal{CB}$	$\mathcal{CP}$
exists	0.05%	23.3%	4.49%	2.80%
deep	0.0015%	20.9%	4.11%	2.55%
dangerous	0%	6.89%	1.12%	0.678%

(from Rui Santos' 7th RISE Workshop presentation, Helsinki)

WHERE DO MINIMA COEXIST?



- The disjoint regions found in these plots can be explained analytically. For instance, one of the squared masses at the CB and CP stationary points is

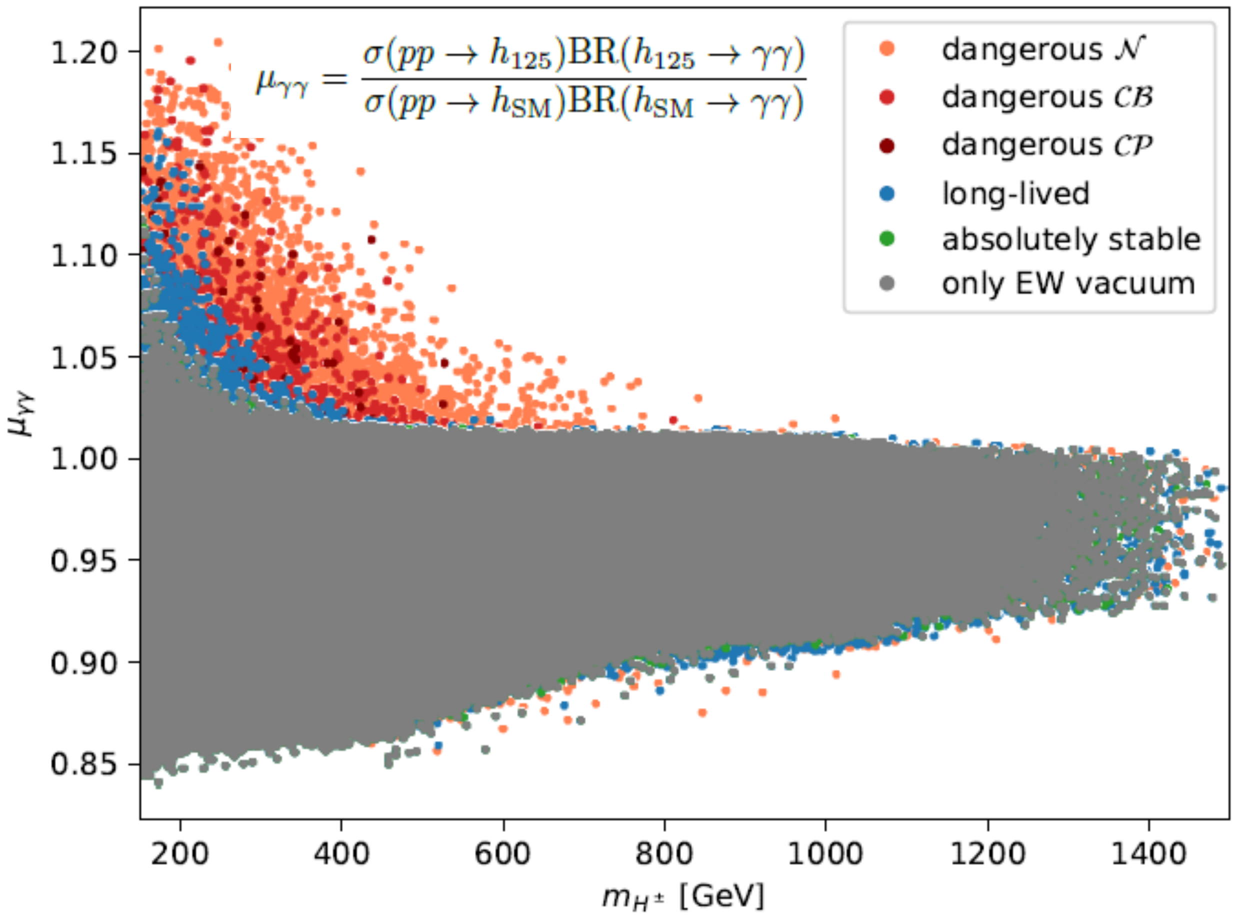
$$m^2_{\mathcal{CB}} = \frac{1}{2}\left(\lambda_4 - \lambda_5\right)\left(c_1^2 + c_2^2 + c_3^2\right) \quad , \quad m^2_{\mathcal{CP}} = -\frac{1}{2}\left(\lambda_4 - \lambda_5\right)\left(\bar{v}_1^2 + \bar{v}_2^2 + \bar{v}_3^2\right)$$

- Thus CB minima need $\lambda_4 - \lambda_5 > 0$, whereas CP ones require $\lambda_4 - \lambda_5 < 0$. Since

$$m^2_A - m^2_{H\pm} = \frac{1}{2}(\lambda_4 - \lambda_5)\, v^2$$

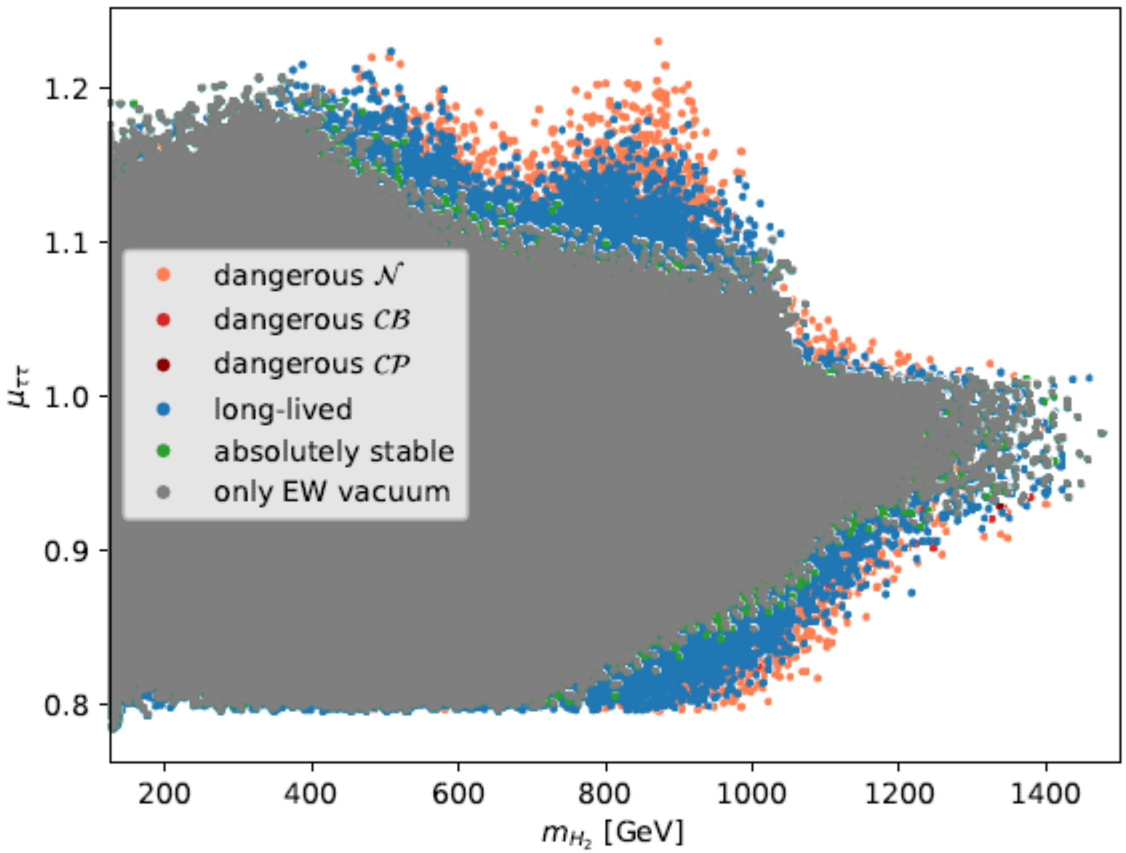
this justifies the diagonal separation between CB and CP minima in the first plot.

ARE THERE EXPERIMENTAL OBSERVABLES THAT COULD BE SENSITIVE TO DANGEROUS DEEPER MINIMA?



- **Grey, green and blue points acceptable.**
- **If for instance a charged higgs mass of ~ 400 GeV is found but with a diphoton rate of $\mu_{\gamma\gamma}$ ~ 1.1, then *the N2HDM would be excluded on stability grounds*!**

THE IMPORTANCE OF NOT BEING TOO STRICT IN YOUR VACUUM STABILITY CUTS...



- **Requiring ABSOLUTE stability would limit us to grey and green points.**

- **This would exclude all long-lived (blue) points, which are perfectly viable parameter regions of the model, yielding a sound description of phenomenology.**

CONCLUSIONS

- **The N2HDM is a 2HDM complemented with a real singlet and discrete symmetries and is a valid description of particle physics, with a possible explanation for dark matter.**

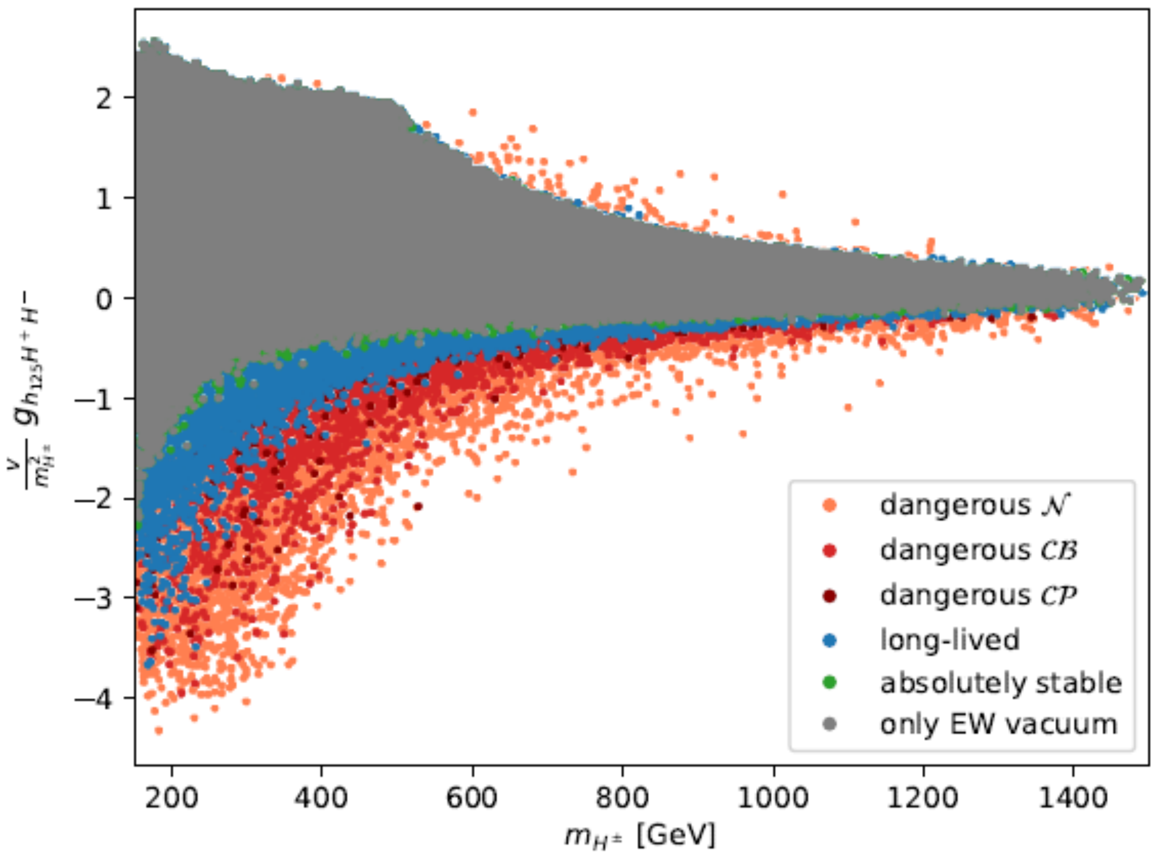
- **It is not guaranteed that the global minimum of the model preserves electric charge conservation, or indeed CP. Minima of different natures are found to possibly coexist, which opens the possibility of tunneling from "good" minima to dangerous ones.**

- **Analytical calculations have shown that a Normal minima with vevless singlet – the Dark Matter phase of the model – is stable against tunneling to a CB or CP breaking deeper minimum.**

- **But a Normal minimum where the singlet acquires a vev is no longer guaranteed to be absolutely stable, and tunneling to a deeper CB or CP breaking minimum is possible.**

- **A numerical analysis of the paramter space of the N2HDM has identified obsrvables for which one *could* exclude the N2HDM as a viable theory on stability issues. *But this is a tree-level result...***

ARE THERE EXPERIMENTALOBSERVABLES THAT COULD BE SENSITIVE TO DANGEROUS DEEPER MINIMA?



- **Diphoton rate different sensitivity to vacuum instability points motiviated by coupling between the 125 GeV scalar and the charged Higgs.**

- **Instability points concentrated at large negative values of the couplig, which enhances the diphoton rate through constrauctive interference with the W loop.**