Effective Field Theories in R_ξ gauges

Mikołaj Misiak

University of Warsaw

in collaboration with M. Paraskevas, J. Rosiek, K. Suxho and B. Zglinicki

HARMONIA meeting, December 6-8th 2018, Warsaw

1. **Introduction**
2. **Operator basis reduction**
3. **Gauge fixing**
4. **Ghost sector and BRST**
5. **Summary**

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: **No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.**

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

First step: Simplify operators that contribute to bilinear terms in A_μ^a and $\varphi = \Phi - \langle \Phi \rangle \equiv \Phi - v$.

gauge fields real scalar fields $(|v| \ll \Lambda)$

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

First step: Simplify operators that contribute to bilinear terms in A_μ^a and $\varphi = \Phi - \langle \Phi \rangle \equiv \Phi - v$.

gauge fields — real scalar fields ($|v| \ll \Lambda$)

Notation: $F_{\mu\nu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c$, $D_\mu \Phi = (\partial_\mu + i A_\mu^a T^a)\Phi$, $(D_\rho F_{\mu\nu})^a = \partial_\rho F_{\mu\nu}^a - f^{abc} A_\rho^b F_{\mu\nu}^c$.

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

First step: Simplify operators that contribute to bilinear terms in A_μ^a (gauge fields) and $\varphi = \Phi - \langle \Phi \rangle \equiv \Phi - v$ (real scalar fields). $(|v| \ll \Lambda)$

Notation: $F_{\mu\nu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c$, $D_\mu \Phi = (\partial_\mu + i A_\mu^a T^a)\Phi$, $(D_\rho F_{\mu\nu})^a = \partial_\rho F_{\mu\nu}^a - f^{abc} A_\rho^b F_{\mu\nu}^c$.

Part of $\mathcal{L}^{(4)}$ that matters for the bilinear terms: $\mathcal{L}_{\Phi,A}^{(4)} = \frac{1}{2}(D_\mu \Phi)^T (D^\mu \Phi) - \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} - V(\Phi)$.

After perturbative decoupling of heavy particles with masses $\sim \Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

First step: Simplify operators that contribute to bilinear terms in A_μ^a (gauge fields) and $\varphi = \Phi - \langle \Phi \rangle \equiv \Phi - v$ (real scalar fields) $(|v| \ll \Lambda)$.

Notation: $F_{\mu\nu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c$, $\quad D_\mu \Phi = (\partial_\mu + i A_\mu^a T^a)\Phi$, $\quad (D_\rho F_{\mu\nu})^a = \partial_\rho F_{\mu\nu}^a - f^{abc} A_\rho^b F_{\mu\nu}^c$.

Part of $\mathcal{L}^{(4)}$ that matters for the bilinear terms: $\mathcal{L}_{\Phi,A}^{(4)} = \frac{1}{2}(D_\mu \Phi)^T (D^\mu \Phi) - \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} - V(\Phi)$.

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $\quad (D^\mu F_{\mu\nu})^a = \boxed{HL}$ $\quad$ ("Higher-dimensional or Lower-derivative terms").

After perturbative decoupling of heavy particles with masses $\sim\Lambda$, the EFT Lagrangian takes the form:

$$\mathcal{L} \;=\; \mathcal{L}^{(4)} \;+\; \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

R_ξ gauge fixing in SMEFT up to dimension-six operators:

A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143

A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. 120 (2018) 251801 (backround field method)

Advantages: No tree-level mixing of gauge bosons with scalars, simple relations among masses, etc.

Present goal: Extension to a generic class of EFTs with operators of arbitrary dimension.

First step: Simplify operators that contribute to bilinear terms in A_μ^a (gauge fields) and $\varphi = \Phi - \langle\Phi\rangle \equiv \Phi - v$ (real scalar fields) $(|v| \ll \Lambda)$.

Notation: $F_{\mu\nu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c$, $\quad D_\mu \Phi = (\partial_\mu + i A_\mu^a T^a)\Phi$, $\quad (D_\rho F_{\mu\nu})^a = \partial_\rho F_{\mu\nu}^a - f^{abc} A_\rho^b F_{\mu\nu}^c$.

Part of $\mathcal{L}^{(4)}$ that matters for the bilinear terms: $\mathcal{L}^{(4)}_{\Phi,A} = \frac{1}{2}(D_\mu\Phi)^T(D^\mu\Phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - V(\Phi)$.

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $\quad (D^\mu F_{\mu\nu})^a = \boxed{HL}$ $\quad$ ("Higher-dimensional or Lower-derivative terms").

Claim: Out of all $\Phi^n F^m D^k$, only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction (actually, field redefinitions - see J. C. Criado, M. Perez-Victoria, arXiv:1811.09413).

Examples: Before the reduction, e.g., $(D_\mu\Phi)^T(D^\mu\Phi)F^a_{\nu\rho}F^{a\,\nu\rho}$ does not matter but, e.g., $(\Phi^T\Phi)(\Phi^T D^\mu D^\nu D_\mu D_\nu \Phi)$ may matter.

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $\quad (D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $\quad (D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

1. $D_{\mu_1} \ldots D_{\mu_k} \Phi$ with internal contractions. $D_\mu D_\nu = D_\nu D_\mu + \boxed{HL}$.

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $(D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

1. $D_{\mu_1} \dots D_{\mu_k} \Phi$ with internal contractions. $D_\mu D_\nu = D_\nu D_\mu + \boxed{HL}$.

2. $D_{\mu_1} \dots D_{\mu_k} \Phi$ without internal contractions must be contracted with $(\dots) D^{\mu_{\sigma(1)}} \dots D^{\mu_{\sigma(k)}} \Phi$ or $(\dots) D^{\mu_{\sigma(1)}} \dots D^{\mu_{\sigma(k-2)}} F^{\mu_{\sigma(k-1)} \mu_{\sigma(k-2)}}$.

At this point, no operators with second or higher derivatives of Φ need to be considered any longer.

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $(D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

1. $D_{\mu_1} \dots D_{\mu_k} \Phi$ with internal contractions. $D_\mu D_\nu = D_\nu D_\mu + \boxed{HL}$.

2. $D_{\mu_1} \dots D_{\mu_k} \Phi$ without internal contractions must be contracted with $(\dots) D^{\mu_{\sigma(1)}} \dots D^{\mu_{\sigma(k)}} \Phi$ or $(\dots) D^{\mu_{\sigma(1)}} \dots D^{\mu_{\sigma(k-2)}} F^{\mu_{\sigma(k-1)} \mu_{\sigma(k-2)}}$.

   At this point, no operators with second or higher derivatives of Φ need to be considered any longer.

3. $D_\mu \Phi$ contracted with $(\dots) D_\alpha F^{\alpha\mu}$ or $(\dots) D_\alpha \tilde{F}^{\alpha\mu}$

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $(D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

1. $D_{\mu_1} \ldots D_{\mu_k} \Phi$ with internal contractions. $D_\mu D_\nu = D_\nu D_\mu + \boxed{HL}$.

2. $D_{\mu_1} \ldots D_{\mu_k} \Phi$ without internal contractions must be contracted with $(\ldots) D^{\mu_{\sigma(1)}} \ldots D^{\mu_{\sigma(k)}} \Phi$ or $(\ldots) D^{\mu_{\sigma(1)}} \ldots D^{\mu_{\sigma(k-2)}} F^{\mu_{\sigma(k-1)} \mu_{\sigma(k-2)}}$.

   At this point, no operators with second or higher derivatives of Φ need to be considered any longer.

3. $D_\mu \Phi$ contracted with $(\ldots) D_\alpha F^{\alpha\mu}$ or $(\ldots) D_\alpha \tilde{F}^{\alpha\mu}$

4. $P^{ab}(\Phi)\ [(\ldots)(D_\mu F_{\nu\rho})]^a [(\ldots)(D^\mu F^{\nu\rho})]^b$ or $P^{ab}(\Phi)\ [(\ldots)(D_\mu F_{\nu\rho})]^a [(\ldots)(D^\nu F^{\mu\rho})]^b$.

   Bianchi identity: $D_{[\mu} F_{\nu\rho]} = 0$, EOM: $D_{[\mu} \tilde{F}_{\nu\rho]} = \boxed{HL}$

Equations of Motion (EOM): $D^\mu D_\mu \Phi = \boxed{HL}$, $(D^\mu F_{\mu\nu})^a = \boxed{HL}$.

Claim: Only Φ^n, $\Phi^n D^2$ and $\Phi^n F^2$ matter for the bilinear terms after the EOM reduction.

In the following, F may stand for $\tilde{F}$, too.

Step-by-step EOM reduction (starting from the lowest dimension, and highest number of derivatives at a given dimension):

1. $D_{\mu_1} \ldots D_{\mu_k} \Phi$ with internal contractions. $D_\mu D_\nu = D_\nu D_\mu + \boxed{HL}$.

2. $D_{\mu_1} \ldots D_{\mu_k} \Phi$ without internal contractions
   must be contracted with $(\ldots) D^{\mu_{\sigma(1)}} \ldots D^{\mu_{\sigma(k)}} \Phi$ or $(\ldots) D^{\mu_{\sigma(1)}} \ldots D^{\mu_{\sigma(k-2)}} F^{\mu_{\sigma(k-1)} \mu_{\sigma(k-2)}}$.

   At this point, no operators with second or higher derivatives of Φ need to be considered any longer.

3. $D_\mu \Phi$ contracted with $(\ldots) D_\alpha F^{\alpha\mu}$ or $(\ldots) D_\alpha \tilde{F}^{\alpha\mu}$

4. $P^{ab}(\Phi)\ [(\ldots)(D_\mu F_{\nu\rho})]^a [(\ldots)(D^\mu F^{\nu\rho})]^b$ or $P^{ab}(\Phi)\ [(\ldots)(D_\mu F_{\nu\rho})]^a [(\ldots)(D^\nu F^{\mu\rho})]^b$.

   Bianchi identity: $D_{[\mu} F_{\nu\rho]} = 0$, EOM: $D_{[\mu} \tilde{F}_{\nu\rho]} = \boxed{HL}$

5. Skip $\Phi^n F \tilde{F}$ (total derivative after $\Phi \to v$).

$\square$

The sum of all the $\Phi^n D^2$ and $\Phi^n F^2$ terms can be written as:

$$\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu\Phi)_i \; h_{ij}[\Phi] \; (D^\mu\Phi)_j - \frac{1}{4}F^a_{\mu\nu} \; g^{ab}[\Phi] \; F^{b\,\mu\nu}$$

(position-dependent metric in the field space).

[arXiv:1803.08001]

The sum of all the $\Phi^n D^2$ and $\Phi^n F^2$ terms can be written as:

$$\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu \Phi)_i \; h_{ij}[\Phi] \; (D^\mu \Phi)_j - \frac{1}{4} F^a_{\mu\nu} \; g^{ab}[\Phi] \; F^{b\,\mu\nu}$$

(position-dependent metric in the field space).
[arXiv:1803.08001]

The bilinear terms are selected by setting

$$h_{ij}[\Phi] \to h_{ij}[v] \equiv h_{ij} \qquad \text{and} \qquad g^{ab}[\Phi] \to g^{ab}[v] \equiv g^{ab}.$$

Then $\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu \Phi)^T \, h \, (D^\mu \Phi) - \frac{1}{4} A^T_{\mu\nu} \, g \, A^{\mu\nu} + \text{(interactions)}$ with $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$.

The sum of all the $\Phi^n D^2$ and $\Phi^n F^2$ terms can be written as:

$$\mathcal{L}_{h,g} = \tfrac{1}{2}(D_\mu \Phi)_i \; h_{ij}[\Phi] \; (D^\mu \Phi)_j - \tfrac{1}{4} F^a_{\mu\nu} \; g^{ab}[\Phi] \; F^{b\,\mu\nu}$$

(position-dependent metric in the field space).
[arXiv:1803.08001]

The bilinear terms are selected by setting

$$h_{ij}[\Phi] \to h_{ij}[v] \equiv h_{ij} \qquad \text{and} \qquad g^{ab}[\Phi] \to g^{ab}[v] \equiv g^{ab}.$$

Then $\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu \Phi)^T h \, (D^\mu \Phi) - \frac{1}{4} A^T_{\mu\nu} \, g \, A^{\mu\nu} + (\text{interactions})$ with $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$.

The “unwanted” tree-level mixing:

$$\mathcal{L}_{A\varphi} = -i \left(\partial^\mu A^a_\mu \right) \left[\varphi^T h T^a v \right]$$

(identification of the would-be Goldstone and physical scalars).

The sum of all the $\Phi^n D^2$ and $\Phi^n F^2$ terms can be written as:

$$\mathcal{L}_{h,g} = \tfrac{1}{2}(D_\mu \Phi)_i \; h_{ij}[\Phi] \; (D^\mu \Phi)_j - \tfrac{1}{4} F^a_{\mu\nu} \; g^{ab}[\Phi] \; F^{b\,\mu\nu}$$

(position-dependent metric in the field space).
[arXiv:1803.08001]

The bilinear terms are selected by setting

$h_{ij}[\Phi] \to h_{ij}[v] \equiv h_{ij}$ and $g^{ab}[\Phi] \to g^{ab}[v] \equiv g^{ab}$.

Then $\mathcal{L}_{h,g} = \tfrac{1}{2}(D_\mu \Phi)^T \, h \, (D^\mu \Phi) - \tfrac{1}{4} A^T_{\mu\nu} \, g \, A^{\mu\nu} + (\text{interactions})$ with $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$.

The "unwanted" tree-level mixing:

$$\mathcal{L}_{A\varphi} = -i \left(\partial^\mu A^a_\mu \right) \left[\varphi^T h T^a v \right]$$

(identification of the would-be Goldstone and physical scalars).

Now, we can specify the R_ξ gauge-fixing term as:

$\mathcal{L}_{GF} = -\frac{1}{2\xi} \mathcal{G}^a g^{ab} \mathcal{G}^b$ with $\mathcal{G}^a = \partial^\mu A^a_\mu - i\xi (g^{-1})^{ab} \left[\varphi^T h T^b v \right]$.

The sum of all the $\Phi^n D^2$ and $\Phi^n F^2$ terms can be written as:

$$\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu\Phi)_i \; h_{ij}[\Phi] \; (D^\mu\Phi)_j - \frac{1}{4}F^a_{\mu\nu} \; g^{ab}[\Phi] \; F^{b\,\mu\nu}$$

(position-dependent metric in the field space).
[arXiv:1803.08001]

The bilinear terms are selected by setting

$$h_{ij}[\Phi] \to h_{ij}[v] \equiv h_{ij} \qquad \text{and} \qquad g^{ab}[\Phi] \to g^{ab}[v] \equiv g^{ab}.$$

Then $\mathcal{L}_{h,g} = \frac{1}{2}(D_\mu\Phi)^T h \,(D^\mu\Phi) - \frac{1}{4}A^T_{\mu\nu}\, g\, A^{\mu\nu} + (\text{interactions})$ with $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$.

The “unwanted” tree-level mixing:

$$\mathcal{L}_{A\varphi} = -i\left(\partial^\mu A^a_\mu\right)\left[\varphi^T h T^a v\right]$$

(identification of the would-be Goldstone and physical scalars).

Now, we can specify the R_ξ gauge-fixing term as:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\mathcal{G}^a g^{ab}\mathcal{G}^b \qquad \text{with} \qquad \mathcal{G}^a = \partial^\mu A^a_\mu - i\xi (g^{-1})^{ab}\left[\varphi^T h T^b v\right].$$

The kinetic terms are rendered canonical via: $\tilde{\varphi}_i = (h^{\frac{1}{2}})_{ij}\varphi_j$, $\tilde{A}^a_\mu = (g^{\frac{1}{2}})^{ab}A^b_\mu$,

which brings the bilinear terms to the familiar form:

$$\mathcal{L}_{\text{kin,mass}} = -\frac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} + \frac{1}{2}\tilde{A}^T_\mu(M^T M)\tilde{A}^\mu + \frac{1}{2}(\partial_\mu\tilde{\varphi})^T(\partial^\mu\tilde{\varphi}) - \frac{1}{2\xi}(\partial^\mu\tilde{A}_\mu)^T(\partial^\nu\tilde{A}_\nu) - \frac{\xi}{2}\tilde{\varphi}^T(M M^T)\tilde{\varphi},$$

with $M_j{}^b \equiv [h^{\frac{1}{2}}(iT^a)v]_j\,(g^{-\frac{1}{2}})^{ab}$ (real matrix).

$$\mathcal{L}_{\text{kin,mass}} = -\tfrac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} + \tfrac{1}{2}\tilde{A}^T_\mu(M^T M)\tilde{A}^\mu + \tfrac{1}{2}(\partial_\mu\tilde{\varphi})^T(\partial^\mu\tilde{\varphi}) - \tfrac{1}{2\xi}(\partial^\mu\tilde{A}_\mu)^T(\partial^\nu\tilde{A}_\nu) - \tfrac{\xi}{2}\tilde{\varphi}^T(MM^T)\tilde{\varphi}.$$

Singular Value Decomposition: $M = U^T\Sigma V,$ $\quad\Sigma_{ij} = 0$ **when** $i \neq j,$ $\quad U, V$ – **orthogonal matrices.**

$$\Rightarrow \qquad MM^T = U^T(\Sigma\Sigma^T)U \qquad \text{and} \qquad M^T M = V^T(\Sigma^T\Sigma)V.$$

Mass eigenstates: $\phi_i = U_{ij}\tilde{\varphi}_j,$ $\qquad W^a_\mu = V^{ab}\tilde{A}^b_\mu.$

Diagonal mass matrices: $m^2_\phi = \Sigma\Sigma^T = \begin{bmatrix} D & \\ & 0 \end{bmatrix}_{m\times m}$ $\qquad m^2_W = \Sigma^T\Sigma = \begin{bmatrix} D & \\ & 0 \end{bmatrix}_{n\times n}$

The bilinear terms in the mass eigenbasis take the standard form:

$$\mathcal{L}_{\text{kin,mass}} = -\tfrac{1}{4}W^T_{\mu\nu}W^{\mu\nu} + \tfrac{1}{2}W^T_\mu m^2_W W^\mu + \tfrac{1}{2}(\partial_\mu\phi)^T(\partial^\mu\phi) - \tfrac{1}{2\xi}(\partial^\mu W_\mu)^T(\partial^\nu W_\nu) - \tfrac{\xi}{2}\phi^T m^2_\phi\phi.$$

Ghost sector and BRST

Infinitesimal gauge transformations in the initial basis:

$$\delta\varphi = -i\alpha^a T^a (\varphi + v), \qquad \delta A^a_\mu = \partial_\mu \alpha^a - f^{abc} A^b_\mu \alpha^c.$$

The corresponding BRST variations:

$$\delta_{\text{BRST}}\varphi = -i\epsilon N^a T^a (\varphi + v)\,, \qquad \delta_{\text{BRST}} A^a_\mu = \epsilon \left(\partial_\mu N^a - f^{abc} A^b_\mu N^c\right).$$

Ghost sector and BRST

Infinitesimal gauge transformations in the initial basis:

$$\delta\varphi = -i\alpha^a T^a\,(\varphi + v), \qquad \delta A_\mu^a = \partial_\mu \alpha^a - f^{abc} A_\mu^b \alpha^c.$$

The corresponding BRST variations:

$$\delta_{\rm BRST}\varphi = -i\epsilon N^a T^a\,(\varphi + v)\,, \qquad \delta_{\rm BRST} A_\mu^a = \epsilon\left(\partial_\mu N^a - f^{abc} A_\mu^b N^c\right).$$

Gauge-fixing functional: $\mathcal{G}^a = \partial^\mu A_\mu^a - i\xi (g^{-1})^{ac}\left[\varphi^T h T^c v\right].$

Its BRST variation: $\delta_{\rm BRST}\mathcal{G}^a = \epsilon M_F^{ab} N^b.$

Introducing the ghost term: $\mathcal{L}_{GF} + \mathcal{L}_{FP} \;=\; -\frac{1}{2\xi}\,\mathcal{G}^a g^{ab} \mathcal{G}^b \;+\; \bar{N}^a g^{ab} M_F^{bc} N^d.$

Explicitly: $\mathcal{L}_{FP} = g^{ab}\bar{N}^a \Box N^b + \xi \bar{N}^a [v^T T^a h T^b v] N^b + \bar{N}^a \overleftarrow{\partial}^\mu g^{ab} f^{bcd} A_\mu^c N^d + \xi \bar{N}^a [v^T T^a h T^b \varphi] N^b.$

Ghost sector and BRST

Infinitesimal gauge transformations in the initial basis:

$$\delta\varphi = -i\alpha^a T^a\,(\varphi + v), \qquad \delta A^a_\mu = \partial_\mu \alpha^a - f^{abc} A^b_\mu \alpha^c.$$

The corresponding BRST variations:

$$\delta_{\text{BRST}}\varphi = -i\epsilon N^a T^a\,(\varphi + v)\,, \qquad \delta_{\text{BRST}} A^a_\mu = \epsilon\left(\partial_\mu N^a - f^{abc} A^b_\mu N^c\right).$$

Gauge-fixing functional: $\mathcal{G}^a = \partial^\mu A^a_\mu - i\xi (g^{-1})^{ac}\left[\varphi^T h T^c v\right].$

Its BRST variation: $\delta_{\text{BRST}}\mathcal{G}^a = \epsilon M_F^{ab} N^b.$

Introducing the ghost term: $\mathcal{L}_{GF} + \mathcal{L}_{FP} \;=\; -\,\frac{1}{2\xi}\,\mathcal{G}^a g^{ab}\mathcal{G}^b \;+\; \bar{N}^a g^{ab} M_F^{bc} N^d.$

Explicitly: $\mathcal{L}_{FP} = g^{ab}\bar{N}^a \Box N^b + \xi \bar{N}^a[v^T T^a h T^b v] N^b + \bar{N}^a\,\overleftarrow{\partial}^\mu g^{ab} f^{bcd} A^c_\mu N^d + \xi \bar{N}^a [v^T T^a h T^b \varphi] N^b.$

BRST variations of the ghosts: $\delta_{\text{BRST}} N^a = \frac{\epsilon}{2} f^{abc} N^b N^c, \qquad \delta_{\text{BRST}} \bar{N}^a = \frac{\epsilon}{\xi}\mathcal{G}^a.$

$$\Rightarrow \qquad \delta_{\text{BRST}}\left(M_F^{ab} N^b\right) = 0 \qquad \Rightarrow \qquad \delta_{\text{BRST}}\left(\mathcal{L}_{GF} + \mathcal{L}_{FP}\right) = 0.$$

Ghost sector and BRST

Infinitesimal gauge transformations in the initial basis:

$$\delta\varphi = -i\alpha^a T^a\,(\varphi + v), \qquad \delta A^a_\mu = \partial_\mu \alpha^a - f^{abc} A^b_\mu \alpha^c.$$

The corresponding BRST variations:

$$\delta_{\text{BRST}}\varphi = -i\epsilon N^a T^a\,(\varphi + v)\,, \qquad \delta_{\text{BRST}} A^a_\mu = \epsilon\left(\partial_\mu N^a - f^{abc} A^b_\mu N^c\right).$$

Gauge-fixing functional: $\mathcal{G}^a = \partial^\mu A^a_\mu - i\xi (g^{-1})^{ac}\left[\varphi^T h T^c v\right].$

Its BRST variation: $\delta_{\text{BRST}}\mathcal{G}^a = \epsilon M_F^{ab} N^b.$

Introducing the ghost term: $\mathcal{L}_{GF} + \mathcal{L}_{FP} \;=\; -\frac{1}{2\xi}\,\mathcal{G}^a g^{ab} \mathcal{G}^b \;+\; \bar{N}^a g^{ab} M_F^{bc} N^d.$

Explicitly: $\mathcal{L}_{FP} = g^{ab}\bar{N}^a \Box N^b + \xi \bar{N}^a [v^T T^a h T^b v] N^b + \bar{N}^a \overleftarrow{\partial}^\mu g^{ab} f^{bcd} A^c_\mu N^d + \xi \bar{N}^a [v^T T^a h T^b \varphi] N^b.$

BRST variations of the ghosts: $\delta_{\text{BRST}} N^a = \frac{\epsilon}{2} f^{abc} N^b N^c, \qquad \delta_{\text{BRST}} \bar{N}^a = \frac{\epsilon}{\xi}\mathcal{G}^a.$

$$\Rightarrow \qquad \delta_{\text{BRST}}\left(M_F^{ab} N^b\right) = 0 \qquad \Rightarrow \qquad \delta_{\text{BRST}}\left(\mathcal{L}_{GF} + \mathcal{L}_{FP}\right) = 0.$$

Mass eigenstates: $\eta = V g^{\frac{1}{2}} N, \qquad \bar{\eta} = V g^{\frac{1}{2}} \bar{N}.$

The ghost bilinear terms in the mass eigenbasis take the standard form:

$$\mathcal{L}_{FP} \;=\; \bar{\eta}^T \Box \eta \;+\; \xi\,\bar{\eta}^T m_W^2 \eta \;+\; \text{(interactions)}.$$

Summary

- **We considered R_ξ gauge fixing in EFTs arising after decoupling of heavy ($M \sim \Lambda$) particles, assuming that scalar VEVs ($\langle \Phi \rangle \ll \Lambda$) give masses to some of the gauge bosons via the Higgs mechanism.**

Summary

- **We considered R_ξ gauge fixing in EFTs arising after decoupling of heavy ($M \sim \Lambda$) particles, assuming that scalar VEVs ($\langle \Phi \rangle \ll \Lambda$) give masses to some of the gauge bosons via the Higgs mechanism.**

- **The essential starting point was the operator basis reduction via EOM. It allowed us to express all the bosonic bilinears as stemming from operators with at most first (covariant) derivatives of Φ, and no derivatives of F.**

Summary

- **We considered R_ξ gauge fixing in EFTs arising after decoupling of heavy ($M \sim \Lambda$) particles, assuming that scalar VEVs ($\langle \Phi \rangle \ll \Lambda$) give masses to some of the gauge bosons via the Higgs mechanism.**

- **The essential starting point was the operator basis reduction via EOM. It allowed us to express all the bosonic bilinears as stemming from operators with at most first (covariant) derivatives of Φ, and no derivatives of F.**

- **Once this is done, all such bilinear terms can be resummed into the propagators.**

Summary

- We considered R_ξ gauge fixing in EFTs arising after decoupling of heavy ($M \sim \Lambda$) particles, assuming that scalar VEVs ($\langle\Phi\,\rangle \ll \Lambda$) give masses to some of the gauge bosons via the Higgs mechanism.

- The essential starting point was the operator basis reduction via EOM. It allowed us to express all the bosonic bilinears as stemming from operators with at most first (covariant) derivatives of Φ, and no derivatives of F.

- Once this is done, all such bilinear terms can be resummed into the propagators.

- Specifying the gauge-fixing and ghost terms, as well as the BRST variations proceeds along the same lines as in a renormalizable theory with non-canonical kinetic terms.

Summary

- **We considered R_ξ gauge fixing in EFTs arising after decoupling of heavy ($M \sim \Lambda$) particles, assuming that scalar VEVs ($\langle \Phi \rangle \ll \Lambda$) give masses to some of the gauge bosons via the Higgs mechanism.**

- **The essential starting point was the operator basis reduction via EOM. It allowed us to express all the bosonic bilinears as stemming from operators with at most first (covariant) derivatives of Φ, and no derivatives of F.**

- **Once this is done, all such bilinear terms can be resummed into the propagators.**

- **Specifying the gauge-fixing and ghost terms, as well as the BRST variations proceeds along the same lines as in a renormalizable theory with non-canonical kinetic terms.**

- **Standard relations between the masses of gauge bosons, would-be Goldstone bosons and ghosts remain valid. However, their interactions are affected by the presence of higher-dimensional operators.**