

On the Cosmological Quantum Frame Problem in Multifield Inflation

APOSTOLOS PILAFTSIS

*School of Physics and Astronomy, University of Manchester,
Manchester M13 9PL, United Kingdom*

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Based on

- S. Karamitsos, D. Burns and A.P., Nucl. Phys. B907 (2016) 785
- S. Karamitsos and A.P., arXiv:1706.07011

Motivation:

Frame Problems in the History of Science (from WIKIPEDIA)

- **Geocentric versus Heliocentric System**

Geocentric: Anaximander (6c BC), . . . , Plato (4c BC),
Aristotle (3c BC), Ptolemy (2c AD), . . .

Heliocentric: Aristarchus (3c BC), Seleucus (2c BC), . . . ,
Copernicus (15c AD), Kepler (16c AD), Galileo (16c AD), . . .

- **Absolute versus Relative/Local Inertial Frame in Gravitation**

Absolute: Newton (17c AD), . . .

Relative: Einstein (20c AD), . . .

- **Einstein versus Jordan Frame in Inflationary Cosmology** [this talk]

– Einstein versus Jordan Frame

Action in Einstein Frame: $S^{\text{EF}}[g_{\mu\nu}, \varphi] = \int_x \left[-\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) \right]$

Action in Jordan Frame: $S^{\text{JF}}[\tilde{g}_{\mu\nu}, \tilde{\varphi}] = \int_x \left[-\frac{1}{2}f(\tilde{\varphi})\tilde{R} + \frac{1}{2}(\partial_\mu \tilde{\varphi})^2 - \tilde{V}(\tilde{\varphi}) \right]$

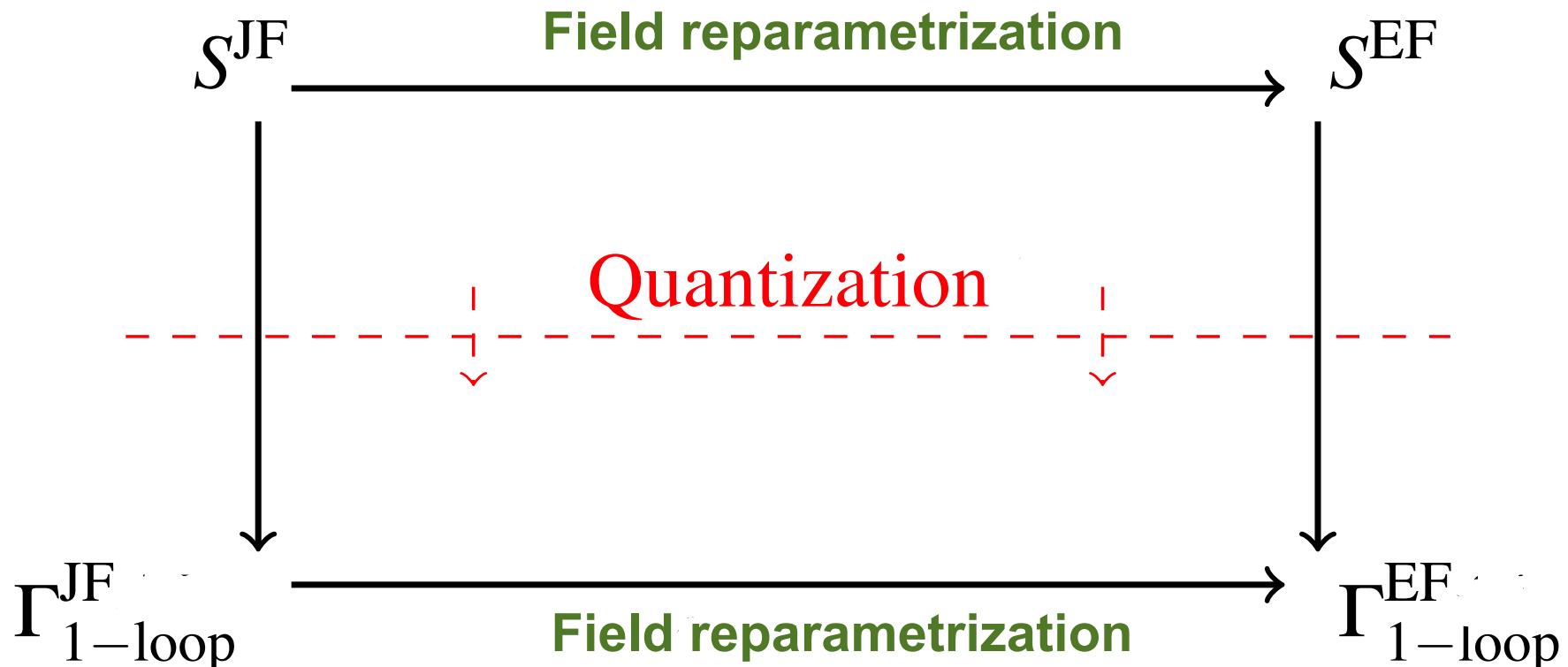
Frame equivalence $\implies S^{\text{JF}}[\tilde{g}_{\mu\nu}, \tilde{\varphi}] = S^{\text{EF}}[g_{\mu\nu}, \varphi]$ [R. H. Dicke '62]

– Einstein versus Jordan Frame

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Frame equivalence $\implies S^{\text{JF}}[\tilde{g}_{\mu\nu}, \tilde{\varphi}] = S^{\text{EF}}[g_{\mu\nu}, \varphi]$ [R. H. Dicke '62]



$\Gamma_{1-\text{loop}}^{\text{JF}}[\tilde{g}_{\mu\nu}, \tilde{\varphi}] \neq \Gamma_{1-\text{loop}}^{\text{EF}}[g_{\mu\nu}, \varphi]$: Effective action is **frame dependent**.

– Partial list of references

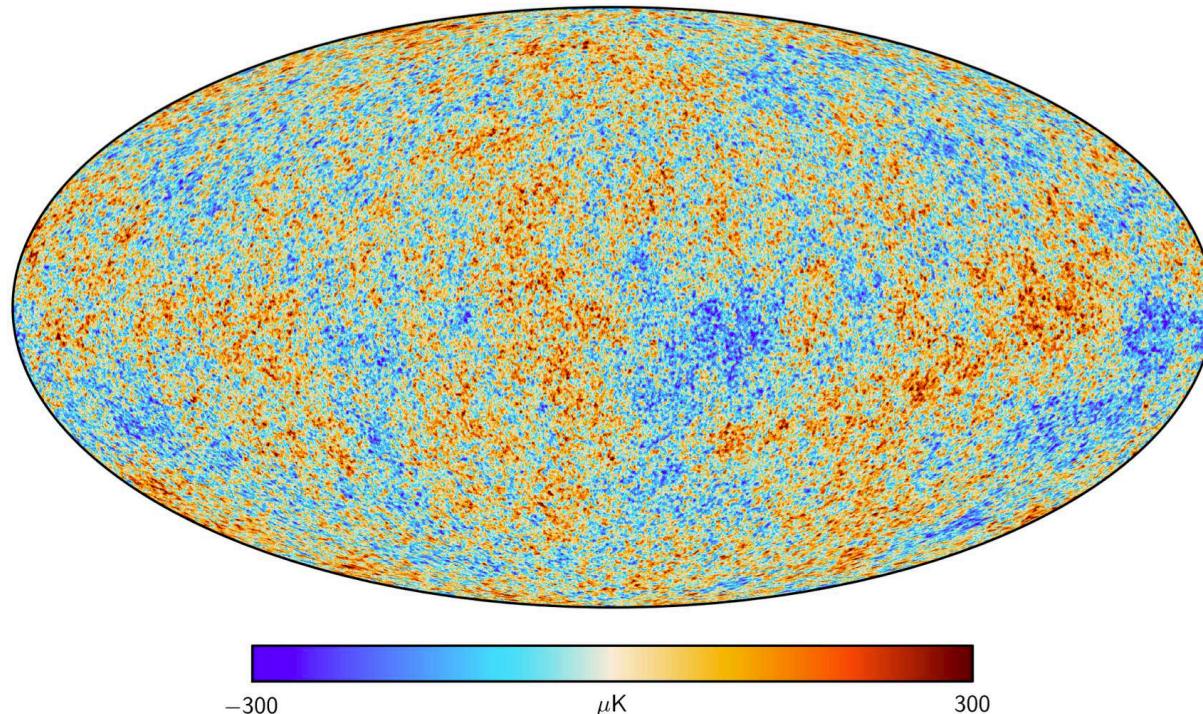
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Outline:

- A Profound View of Our Cosmos
- Frame Covariance in Curved Field Space
- Frame Covariant Quantum Perturbations
- Frame Invariant Cosmological Observables
- Two-Field Models
- Beyond the Tree Approximation
- Conclusions and Future Directions

- A Profound View of Our Cosmos

Standard Big-Bang Cosmology cannot naturally explain the *flatness* of our Universe and the *largeness* of its *causal horizon*.



$$\frac{\delta T}{T} \lesssim 10^{-4}$$

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Inflation provides a natural solution to these problems of our Cosmos, as well as a quantitative framework for explaining the CMB anisotropies.

Interesting class of viable inflationary models: Scalar-Curvature Multifield Theories, including Higgs Inflation.

- Frame Covariance in Curved Field Space

- Scalar-Curvature Multifield Theories in Jordan Frame

$$S = \int d^4x \sqrt{-g} \left[-\frac{f(\varphi)}{2}R + \frac{1}{2}k_{AB}(\varphi) g^{\mu\nu} (\nabla_\mu \varphi^A)(\nabla_\nu \varphi^B) - V(\varphi) \right],$$

$S = S[g_{\mu\nu}, \varphi, f(\varphi), k(\varphi), V(\varphi)]$: *classical action*,
 $f(\varphi)$, $k_{AB}(\varphi)$, $V(\varphi)$: *model functions*.

Frame transformations consist of

(i) *Conformal transformations*

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \varphi^A \rightarrow \tilde{\varphi}^A = \Omega^{-1} \varphi^A$$

(ii) *Field reparametrizations*

$$\varphi^A \rightarrow \varphi^{\tilde{A}} = \varphi^{\tilde{A}}(\varphi), \quad \frac{d\tilde{\varphi}^{\tilde{A}}}{d\varphi^B} = \Omega^{-1} K_B^{\tilde{A}}(\varphi)$$

Under a frame transformation: $\varphi^A \mapsto \tilde{\varphi}^{\tilde{A}} = \tilde{\varphi}^{\tilde{A}}(\varphi)$, the model functions transform as

$$\begin{aligned}\tilde{f}(\tilde{\varphi}) &= \Omega^{-2} f(\varphi), \\ \tilde{k}_{\tilde{A}\tilde{B}}(\tilde{\varphi}) &= \left[k_{AB} - 6f(\ln \Omega)_{,A}(\ln \Omega)_{,B} + 3f_{,A}(\ln \Omega)_{,B} + 3(\ln \Omega)_{,A}f_{,B} \right] K_{\tilde{A}}^A K_{\tilde{B}}^B, \\ \tilde{V}(\tilde{\varphi}) &= \Omega^{-4} V(\varphi).\end{aligned}$$

Functional form of the classical action S remains invariant:

$$S[\tilde{g}_{\mu\nu}, \tilde{\varphi}, \tilde{f}(\tilde{\varphi}), \tilde{k}(\tilde{\varphi}), \tilde{V}(\tilde{\varphi})] = S[g_{\mu\nu}, \varphi, f(\varphi), k(\varphi), V(\varphi)]$$

Models related by a frame transformation define an *equivalence class*

\implies Our starting point for *Frame Covariance*.

Introduce *Field Space Metric*:

$$G_{AB} \equiv \frac{k_{AB}}{f} + \frac{3}{2} \frac{f_{,A}f_{,B}}{f^2}.$$

Frame Tensors $X_{B_1 \dots B_q}^{A_1 \dots A_p}$ in Field Space:

$$\tilde{X}_{\tilde{B}_1 \dots \tilde{B}_q}^{\tilde{A}_1 \dots \tilde{A}_p} = \Omega^{-(w_X + p - q)} (K_{A_1}^{\tilde{A}_1} \dots K_{A_p}^{\tilde{A}_p}) X_{B_1 \dots B_q}^{A_1 \dots A_p} (K_{\tilde{B}_1}^{B_1} \dots K_{\tilde{B}_q}^{B_q}).$$

X	conformal weight (w_X)	scaling dimension (d_X)
dx^μ	0	0
$d\varphi^A$	0	1
$d\varphi_A$	0	-1
$g_{\mu\nu}$	-2	-2
$g^{\mu\nu}$	2	2
N_L, a	-1	-1
$\mathcal{H} \equiv (\mathcal{D}_t a)/a$	1	1
f	2	2
G_{AB}	0	-2
G^{AB}	0	2
$U \equiv V/f^2$	0	0
$X_{B_1 B_2 \dots B_q}^{A_1 A_2 \dots A_p}$	w_X	$w_X + p - q$
$\nabla_A X_{B_1 B_2 \dots B_q}^{A_1 A_2 \dots A_p}$	w_X	$w_X - 1 + p - q$
$\mathcal{D}_\lambda X_{B_1 B_2 \dots B_q}^{A_1 A_2 \dots A_p}$	$w_X - d_{\delta\lambda}$	$w_X - d_{\delta\lambda} + p - q$

Fully Frame Covariant Differentiation:

$$\begin{aligned} \nabla_C X_{B_1 \dots B_q}^{A_1 \dots A_p} &\equiv X_{B_1 \dots B_q, C}^{A_1 \dots A_p} - \frac{w_X}{2} \frac{f_{,C}}{f} X_{B_1 \dots B_q}^{A_1 \dots A_p} \\ &\quad + \Gamma_{CD}^{A_1} X_{B_1 \dots B_q}^{D \dots A_p} + \dots + \Gamma_{CD}^{A_p} X_{B_1 \dots B_q}^{A_1 \dots D} \\ &\quad - \Gamma_{B_1 C}^D X_{D \dots B_q}^{A_1 \dots A_p} - \dots - \Gamma_{B_q C}^D X_{B_1 \dots D}^{A_1 \dots A_p}. \end{aligned}$$

Frame-Invariant Number of e-folds:

$$N(t) = - \int_{t_{\text{end}}}^t dt' \mathcal{H}(t') .$$

t_{end} : time of end of inflation.

Frame Covariant Cosmological Equations:

- Scalar-field equations: $\mathcal{D}_t \mathcal{D}_t \varphi^A + 3\mathcal{H}(\mathcal{D}_t \varphi^A) + f G^{AB} U_{,B} = 0$.
- Friedmann equation: $\mathcal{H}^2 = \frac{1}{3} \left[\frac{1}{2} G_{AB} (\mathcal{D}_t \varphi^A)(\mathcal{D}_t \varphi^B) + f U \right]$.

- **Frame Covariant Quantum Perturbations**

- Metric perturbations:

$$g_{\mu\nu} dx^\mu dx^\nu = (1 + 2\Phi) N_L^2 dt^2 - a^2 [(1 - 2\Phi)\delta_{ij} + h_{ij}] dx^i dx^j$$

- Frame Covariant *Mukhanov–Sasaki* variables:

$$Q^A = \delta\varphi^A + \frac{\mathcal{D}_t\varphi^A}{\mathcal{H}} \Phi$$

- Line element in the *Field Space*: $d\sigma^2 = G_{AB} d\varphi^A d\varphi^B$

- Vielbeins along Curvature (||) and Isocurvature (⊥) direction(s):

$$e_\sigma^A = \frac{\mathcal{D}_t\varphi^A}{\mathcal{D}_t\sigma}, \quad e_s^A = - \frac{s_B^A U^{,B}}{\sqrt{s^{AB} U_{,A} U_{,B}}},$$

with $s_B^A \equiv \delta_B^A - e_\sigma^A e_B^\sigma$.

- Comoving adiabatic (\mathcal{R}) and entropic ($\mathcal{S}^{(i)}$) perturbations:

$$\mathcal{R} \equiv \frac{\mathcal{H}}{\mathcal{D}_t \sigma} Q^\sigma, \quad \mathcal{S}^{(i)} \equiv \frac{\mathcal{H}}{\mathcal{D}_t \sigma} Q^{s_i}.$$

- Scalar power spectrum $P_{\mathcal{R}}$: $2\pi^2 p^{-3} P_{\mathcal{R}} \delta(\mathbf{p} + \mathbf{q}) \equiv \langle \mathcal{R}_{\mathbf{p}} | \mathcal{R}_{\mathbf{q}} \rangle$
- Frame-invariant scalar and tensor power spectra:

$$P_{\mathcal{R}} = \frac{1}{8\pi^2} \frac{\mathcal{H}^2}{f(\varphi) \bar{\epsilon}_H}, \quad P_T = \frac{2}{\pi^2} \frac{\mathcal{H}^2}{f(\varphi)},$$

with $\bar{\epsilon}_H \equiv -(\mathcal{D}_N \mathcal{H})/\mathcal{H}$.

- Power spectrum modified by Entropy Transfer effects:

$$P_{\mathcal{R}}(t) \approx P_{\mathcal{R}}(t_*) (1 + T_{\mathcal{R}\mathcal{S}}^2),$$

t_* : time at horizon exit

t : time at observation.

- Frame Invariant Cosmological Observables

- Cosmological observables:

$$n_{\mathcal{R}} - 1 \equiv \left. \frac{d \ln P_{\mathcal{R}}}{d \ln k} \right|_{k=a\mathcal{H}}, \quad n_T \equiv \left. \frac{d \ln P_T}{d \ln k} \right|_{k=a\mathcal{H}}, \quad r \equiv \frac{P_T}{P_{\mathcal{R}}},$$

$$\alpha_{\mathcal{R}} \equiv \left. \frac{dn_{\mathcal{R}}}{d \ln k} \right|_{k=a\mathcal{H}}, \quad \alpha_T \equiv \left. \frac{dn_T}{d \ln k} \right|_{k=a\mathcal{H}}, \quad f_{NL} \equiv \frac{5}{6} \frac{N^{,A} N^{,B} \nabla_A \nabla_B N}{(N_{,A} N^{,A})^2}.$$

- Frame-invariant potential slow-roll parameters:

$$\bar{\epsilon}_{U,1} \equiv \frac{1}{2} (\ln U)_{,A} G^{AB} (\ln U)_{,B},$$

⋮

$$\bar{\epsilon}_{U,n} \equiv - (\ln \bar{\epsilon}_{U,n-1})_{,A} G^{AB} (\ln U)_{,B},$$

with $\mathcal{D}_t \mathcal{D}_t \varphi^A \ll \mathcal{H}(\mathcal{D}_t \varphi^A)$, and $\bar{\epsilon}_U \equiv \bar{\epsilon}_{U,1}$, $\bar{\eta}_U \equiv \bar{\epsilon}_{U,2}$ and $\bar{\xi}_U \equiv \bar{\epsilon}_{U,3}$.

- Cosmological Observables in the slow-roll approximation:

$$n_{\mathcal{R}} = 1 - 2\bar{\epsilon}_U + \bar{\eta}_U - \mathcal{D}_N \ln(1 + T_{\mathcal{R}\mathcal{S}}^2) , \quad r = 16\bar{\epsilon}_U (1 + T_{\mathcal{R}\mathcal{S}}^2)^{-1} ,$$

$$\alpha_{\mathcal{R}} = -2\bar{\epsilon}_U \bar{\eta}_U - \bar{\eta}_U \bar{\xi}_U + \mathcal{D}_N \mathcal{D}_N \ln(1 + T_{\mathcal{R}\mathcal{S}}^2) , \quad \alpha_T = -2\bar{\epsilon}_U \bar{\eta}_U .$$

- Isocurvature perturbation(s) \mathcal{S} evolve outside the horizon as follows:

$$\mathcal{D}_N \mathcal{R} = -2\omega \mathcal{S} , \quad \mathcal{D}_N \mathcal{S} = -B \mathcal{S} .$$

- Entropy Transfer depends on the norm ω of the *acceleration vector*:

$$\omega^A \equiv \mathcal{D}_N \left(\frac{\mathcal{D}_t \varphi^A}{\mathcal{D}_t \sigma} \right) = \frac{1}{\mathcal{H}} \frac{e_s^A U, s}{\mathcal{D}_t \sigma} .$$

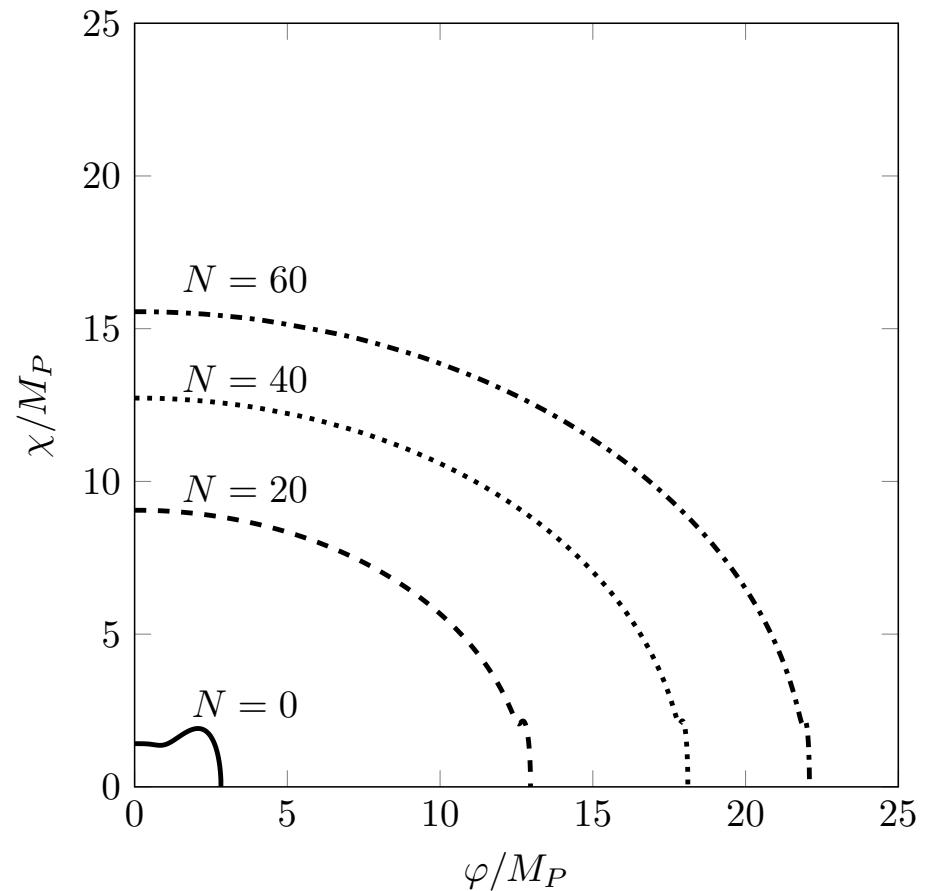
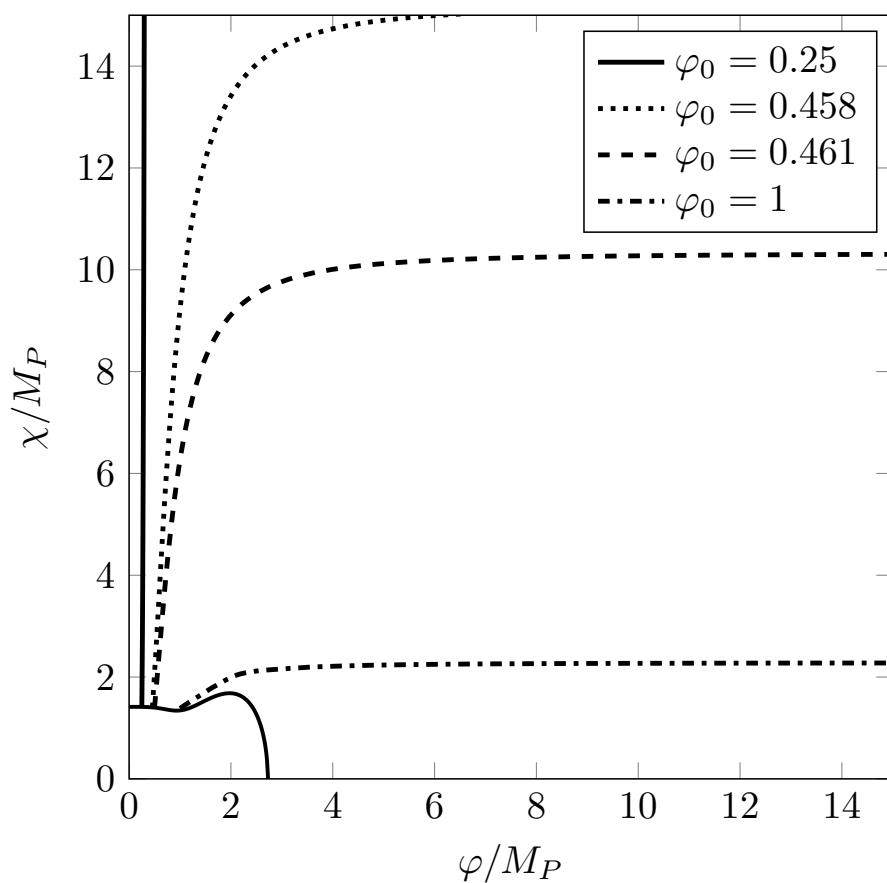
- Entropy Generation and Transfer up to the end of inflation:

$$T_{\mathcal{R}\mathcal{S}} \sim \frac{2\omega}{B} \left(e^{-B N} - 1 \right) ; \quad B \sim \bar{\eta}_{ss} = \frac{\omega^A \omega^B}{\omega^2} \frac{\nabla_A \nabla_B U}{U} + \frac{2\bar{\epsilon}_U}{3} R_{s\sigma s\sigma} .$$

- Two-Field Models

- Minimal Model:

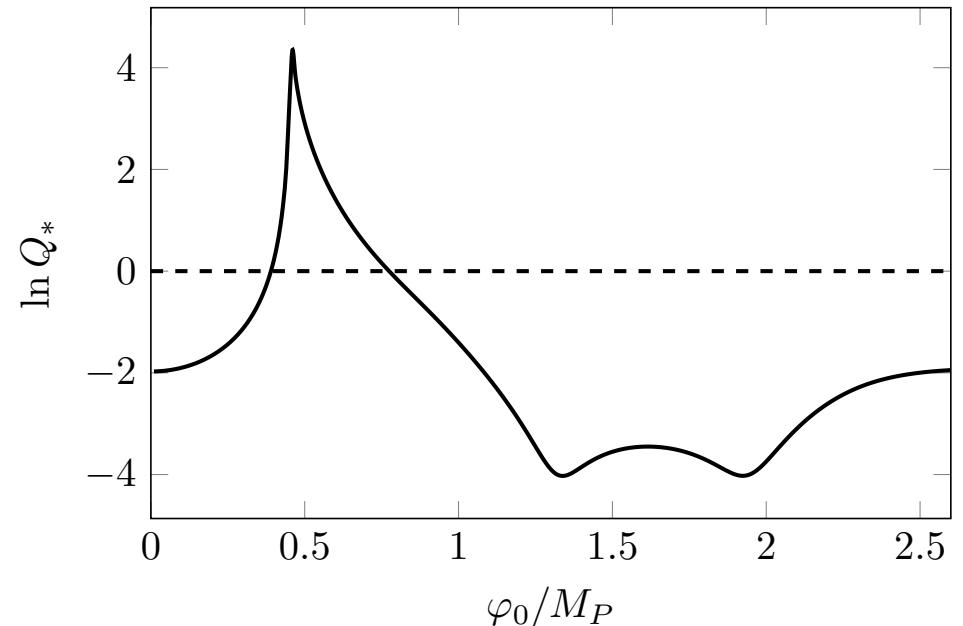
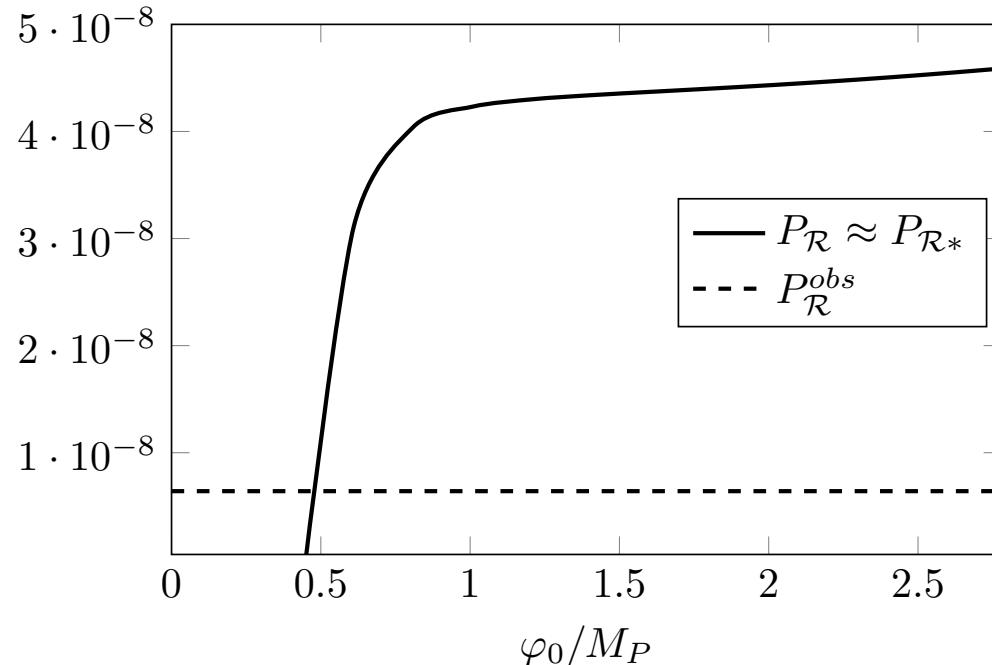
$$\mathcal{L} = \frac{M_P^2 R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda\varphi^4}{4} - \frac{m^2\chi^2}{2}$$



– Predictions in the Minimal Model

[S. Karamitsos, A.P., arXiv:1706.07011]

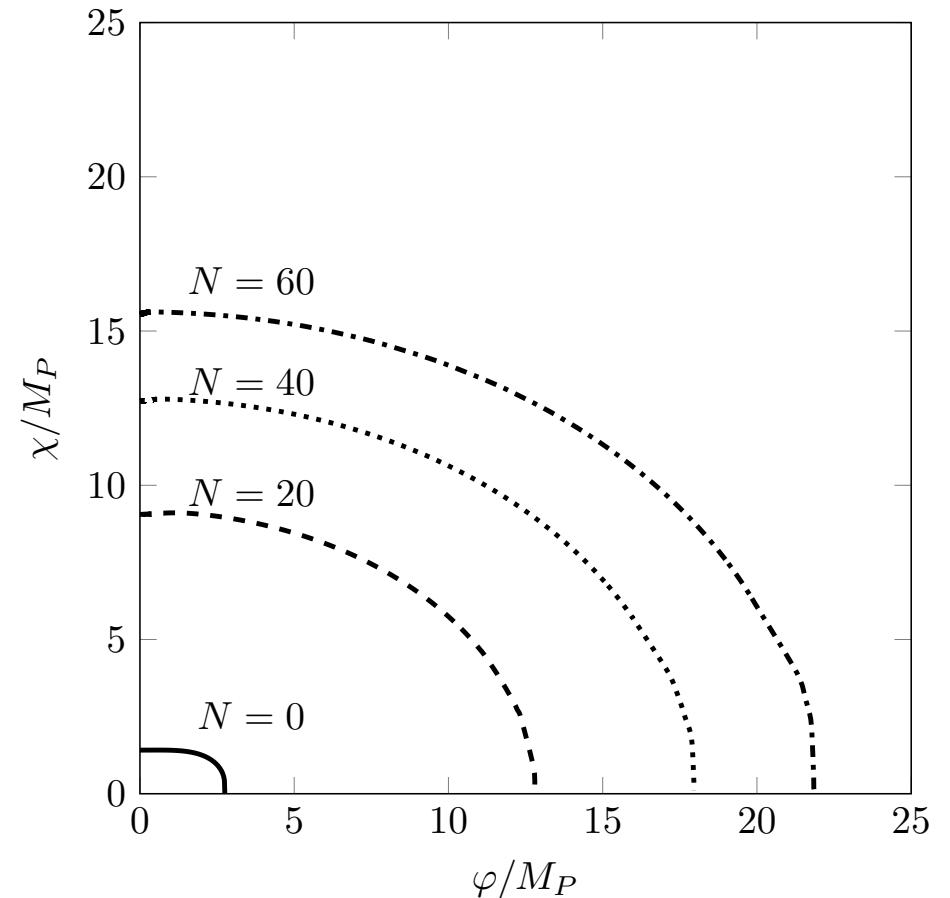
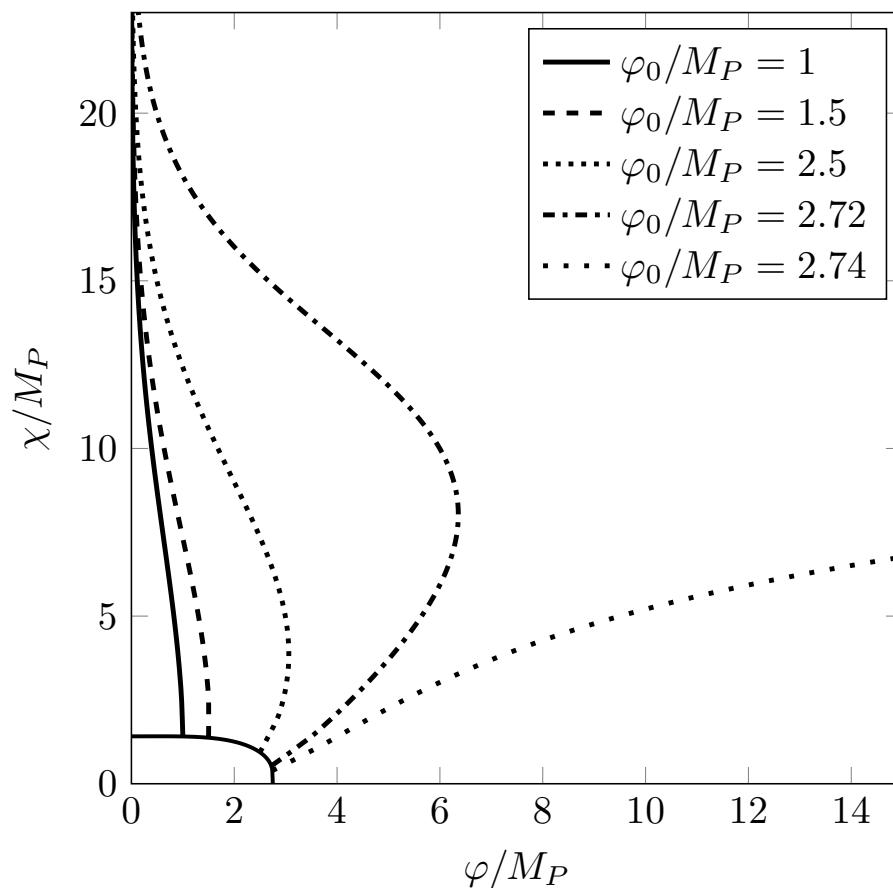
$m/M_P = 10^{-6}$ and $\lambda = 10^{-12}$:



	$\varphi_0/M_P = 0.498$	PLANCK 2015
r	0.501	≤ 0.12 (95% CL)
$n_{\mathcal{R}}$	0.906	0.968 ± 0.006 (68% CL)
$\alpha_{\mathcal{R}}$	-0.00288	-0.003 ± 0.008 (68% CL)
α_T	-0.0019	-0.000167 ± 0.000167 (68% CL)
f_{NL}	-0.0000509	0.8 ± 5.0 (68% CL)

– Non-Minimal Model

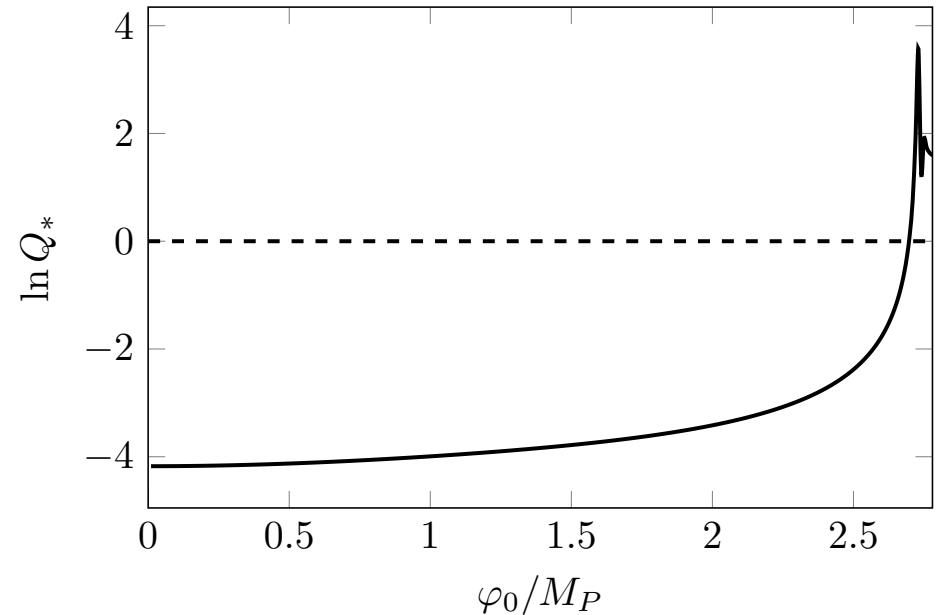
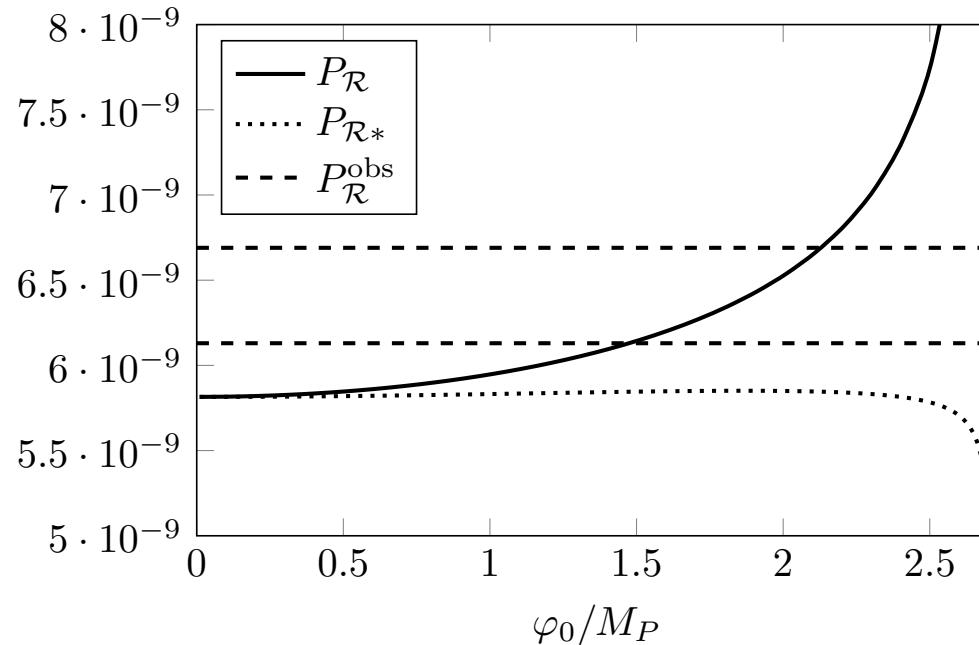
$$\mathcal{L} = \frac{(M_P^2 + \xi\varphi^2)R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda(\varphi^2 - v^2)^2}{4} - \frac{m^2\chi^2}{2}$$



– Predictions in the Non-Minimal Model

[S. Karamitsos, A.P., arXiv:1706.07011]

$m/M_P = 5.6 \times 10^{-6}$, $\lambda = 10^{-12}$ and $\xi \approx 0.01$:



	$\varphi_0/M_P = 1.885^{+0.243}_{-0.411}$	PLANCK 2015
r	$0.12056^{+0.005}_{-0.005}$	≤ 0.12 (95% CL)
$n_{\mathcal{R}}$	$0.949^{+0.005}_{-0.003}$	0.968 ± 0.006 (68% CL)
$\alpha_{\mathcal{R}}$	$-0.0003^{+0.0001}_{-0.00008}$	-0.008 ± 0.008 (68% CL)
α_T	$-0.000276^{+0.000003}_{-0.000003}$	-0.000155 ± 0.00016 (68% CL)
f_{NL}	$0.033^{+0.00017}_{-0.0002}$	0.8 ± 5.0 (68% CL)

- **Beyond the Tree Approximation**

- Standard 1PI Effective action ($\Gamma[\varphi] = W[J(\varphi)] - J_a \varphi^a$):

$$\exp\left(\frac{i}{\hbar}\Gamma[\varphi]\right) = \int [D\phi] \mathcal{M}[\phi] \exp\left\{\frac{i}{\hbar}\left[S[\phi] + \Gamma_{,a}(\varphi^a - \phi^a)\right]\right\},$$

with $\Gamma_{,a} \equiv \delta\Gamma[\varphi]/\delta\varphi^a = -J_a$.

- $\Gamma[\varphi]$ is **not frame-invariant**, i.e.

$$\Gamma[\varphi] \neq \tilde{\Gamma}[\tilde{\varphi}],$$

because φ^a is **not** a **frame covariant** vector.

- Vilkovisky–De Witt formalism: $\varphi^a - \phi^a \rightarrow \Sigma^a(\varphi, \phi)$.

Under $\varphi \rightarrow \tilde{\varphi} = \tilde{\varphi}(\varphi)$, $\Sigma^a(\varphi, \phi)$ transforms as a vector.

Under $\phi \rightarrow \tilde{\phi} = \tilde{\phi}(\phi)$, $\Sigma^a(\varphi, \phi)$ is a (frame-invariant) scalar.

- Vilkovisky–De Witt 1PI effective action:

$$\exp\left(\frac{i}{\hbar}\Gamma[\varphi]\right) = \int [\mathcal{D}\phi] \mathcal{M}[\phi] \exp\left\{\frac{i}{\hbar}\left[S[\phi] + \nabla_a \Gamma[\varphi] \Sigma^a(\varphi, \phi)\right]\right\},$$

with $\mathcal{M}[\phi] \equiv \sqrt{\det \mathcal{G}_{ab}}$, and a configuration field space metric:

$$\mathcal{G}_{ab} \equiv G_{AB} \delta(x_A - x_B) = \left(\frac{k_{AB}}{f} + \frac{3}{2} \frac{f_{,A} f_{,B}}{f^2} \right) \delta(x_A - x_B).$$

- \hbar -Expansion of the Frame-Invariant Vilkovisky–De Witt effective action:

$$\Gamma[\varphi] = \sum_n \hbar^n \Gamma_n[\varphi] = \tilde{\Gamma}[\tilde{\varphi}],$$

with $\Gamma_0[\varphi] = S[\varphi]$, and

$$\Gamma_1[\varphi] = -\frac{i}{2} \text{tr} \ln \mathcal{G}_{ab} + \frac{i}{2} \text{tr} \ln (\nabla_a \nabla_b S[\varphi]) = \frac{i}{2} \text{tr} \ln (\nabla^a \nabla_b S[\varphi]).$$

• Conclusions

- Frame Covariant Approach to Multifield Inflation in Scalar-Curvature Theories.
- Use of Differential Geometry in Curved Multifield Space to covariantly compute Curvature and Isocurvature Perturbations.
- Entropic Effects significant in Non-Minimal Two-Field Models.
- Full Frame Covariance beyond the Tree Approximation by virtue of the Vilkovisky–De Witt Formalism. (*in progress*)

- Future Directions

- Extension of the Frame Covariant Approach to $F(R)$ and $F(\varphi, R)$ Theories.
- Inclusion of Matter and Quantum Gravity Effects.
- Higher-Loop Impact of Curved Field Space on Cosmological Observables.
- Consideration of Curved Field Space Effects in the Standard Model and Beyond.

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