

Curved Spacetime Effective Field Theory (cEFT) – Formalism And Some Applications



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Outline

- 1 Effective Field Theory – general overview
 - Why or when to use a EFT?
- 2 Curved spacetime Effective Field Theory – cEFT
 - Method of construction
 - Example of application
- 3 Conclusions

EFT in particle physics

Effective Field Theory as a way to look for either the new physics or better understanding of the old one.

Effective Field Theory is useful when:

- we do not have all the data
- we do not now where to look for the new data

EFT in particle physics

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Way of construction of EFT

- bottom-up
- top-down

The Standard Model of particle physics contains operators of the dimension up to four.

New physics may be encoded by operators with dimension higher than four.

cEFT step by step

$$\begin{aligned}
 S_{UV}(\phi, \Phi) & \xrightarrow{\quad} S_{UV} = \int \sqrt{-g} d^4x \left\{ \right. \\
 & \quad \left. -\frac{1}{2} d_\mu H^\dagger d^\mu H - \frac{1}{2} m_H^2 |H|^2 - \frac{\lambda_H}{4!} |H|^4 - \xi_H R |H|^2 + \right. \\
 & \quad \left. -\frac{1}{2} d_\mu X d^\mu X - \frac{1}{2} m_X^2 X^2 - \xi_X R X^2 - \frac{1}{2} \lambda_{HX} X^2 |H|^2 \right\} \\
 S_{UV}^{heavy}(\Phi) & \xrightarrow{\quad} \left(-\frac{1}{2} d_\mu X d^\mu X - \frac{1}{2} m_X^2 X^2 - \xi_X R X^2 \right) \\
 S_{UV}^{light}(\phi) & \xrightarrow{\quad} \left(-\frac{\lambda_H}{4!} |H|^4 - \xi_H R |H|^2 - \frac{1}{2} \lambda_{HX} X^2 |H|^2 \right)
 \end{aligned}$$

cEFT step by step

$$\begin{aligned}
 S_{cEFT}(\phi) & \xrightarrow{\quad} S_{cEFT} = \int \sqrt{-g} d^4x \left\{ \right. \\
 & \quad \left. -\frac{1}{2} d_\mu H^\dagger d^\mu H - \frac{1}{2} m_H^2 |H|^2 - \frac{\lambda_H}{4!} |H|^4 - \xi_H R |H|^2 + \right. \\
 & \quad \left. -\frac{1}{2} c_{dHdH} d_\mu |H|^2 d^\mu |H|^2 - c_{GHH} G^{\mu\nu} d_\mu |H|^2 d_\nu |H|^2 + \right. \\
 & \quad \left. - c_H |H|^2 - c_{HH} |H|^4 - c_6 |H|^6 \right\} \\
 & \xrightarrow{\quad} S^{light, heavy}(\phi, \Phi)_{|\Phi=\Phi_d(\phi)} + \frac{i\hbar}{2} c_s \ln \text{sdet}(\mu^{-2} D_{ij}^2)_{|\Phi=\Phi_d(\phi)}
 \end{aligned}$$

$S^{light}(\phi)$ points to the red box terms in the integrand.
 S_{cEFT} points to the blue box S_{cEFT} label.
 $S^{light, heavy}(\phi, \Phi)_{|\Phi=\Phi_d(\phi)} + \frac{i\hbar}{2} c_s \ln \text{sdet}(\mu^{-2} D_{ij}^2)_{|\Phi=\Phi_d(\phi)}$ points to the green box terms in the integrand.

- Contribution to cEFT from heavy-heavy loops at one-loop order:

$$\Gamma_{\Phi\Phi}^{(1)} = \frac{i\hbar}{2} c_s \ln \text{sdet} (\mu^{-2} D^2),$$

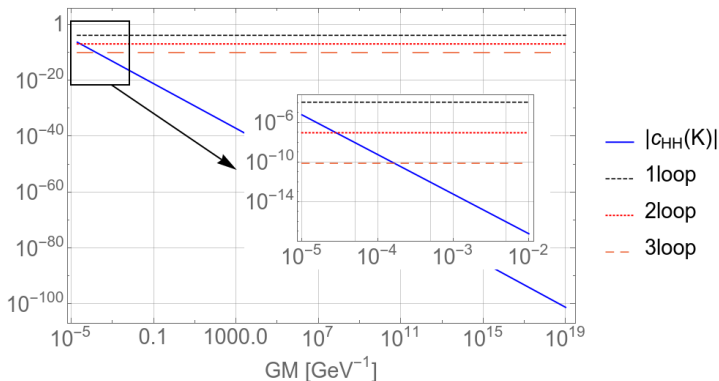
$$D^2 = \square + 2h^\mu(\phi, \Phi_{cl}(\phi))d_\mu + \Pi(\phi, \Phi_{cl}(\phi)) - m_\Phi^2.$$

- Local heat kernel representation for this contributions (containing operators of the dimension up to six)

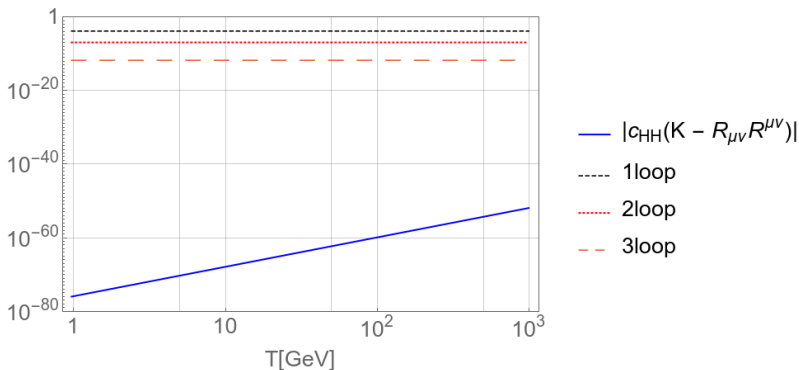
$$\Gamma_{\Phi\Phi}^{(1)} = c_s \int \sqrt{-g} d^4x \frac{\hbar}{64\pi^2} \text{Tr} \left\{ \frac{1}{3} \frac{a_3}{m_\Phi^2} + \frac{1}{12} \frac{a_4}{m_\Phi^4} \right\}.$$

- Example of gravity induced coefficient

$$c_{HH} = \frac{\hbar}{(4\pi)^2} \left[\frac{\lambda_{HX}^2}{4m_X^2} \left(2\xi_X - \frac{1}{6} \right) R - \frac{\lambda_{HX}^2}{8m_X^4} \left(2\xi_X - \frac{1}{6} \right)^2 R^2 + \right. \\ \left. - \frac{\lambda_{HX}^2}{720m_X^4} (\mathcal{K} - R_{\mu\nu} R^{\mu\nu}) + \frac{\lambda_{HX}^2}{m_X^4} \left(-\frac{1}{4}\xi_X + \frac{1}{40} \right) \square R - \frac{\lambda_{HX}^2}{90m_X^4} \nabla_\mu \nabla_\nu R^{\mu\nu} \right],$$



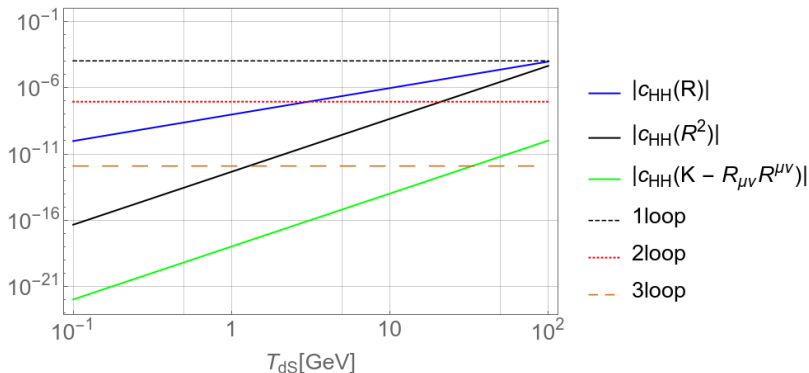
The contribution of the gravity induced part of the c_{HH} coefficient to the Higgs quartic coupling in the black hole background. $|c_{HH}(K)| = \left| -\frac{1}{(4\pi)^2} \frac{\lambda_{HX}^2}{720} \frac{\mathcal{K}}{m_X^4} \right|$, G is the Newton constant, M is the black hole mass and loops prefactors are given by the formula $n\text{loop} = \frac{\lambda_H^{n+1}}{(16\pi^2)^n}$. For the plot we chose $\lambda_{HX} = 0.25$, $\lambda_H = 0.13$ and $m_X = 10\text{TeV}$.



The contribution of the gravity induced part of the c_{HH} coefficient to the Higgs quartic coupling in the radiation dominated FLRW background.

$$|c_{HH}(K - R_{\mu\nu}R^{\mu\nu})| = \left| -\frac{1}{(4\pi)^2} \frac{\lambda_{HX}^2}{720} \frac{K - R_{\mu\nu}R^{\mu\nu}}{m_X^4} \right|, \quad T \text{ is the temperature and loops}$$

prefactors are given by the formula $n\text{loop} = \frac{\lambda_H^{n+1}}{(16\pi^2)^n}$. For the plot we chose $\lambda_{HX} = 0.25$, $\lambda_H = 0.13$ and $m_X = 10\text{TeV}$. For comparison $T_\odot \sim 10^{-13}\text{GeV}$, and $T_{EW} \sim 10^2\text{GeV}$.



The contribution of the gravity induced part of the c_{HH} coefficient to the Higgs quartic coupling in the de Sitter like FLRW background. Loops prefactors are given by the formula $n\text{loop} = \frac{\lambda_H^{n+1}}{(16\pi^2)^n}$ and T_{dS} is the temperature of the de Sitter spacetime. For the plot we chose $\lambda_{HX} = 0.25$, $\lambda_H = 0.13$, $m_X = 10\text{TeV}$ and $\xi_X = 10$. For comparison $T_\odot \sim 10^{-13}\text{GeV}$, and $T_{EW} \sim 10^2\text{GeV}$.

Conclusions

- The heat kernel may serve as a method for systematic construction of cEFT.
- Gravity may either introduce new operators or contribute to the ones that already exist in the flat spacetime.
- Gravity induced contributions to cEFT in some cases may be of the same order like two-loop effects, on the other hand for most purposes going beyond terms quadratic in spacetime curvatures is not necessary.

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Thank you for your attention.

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