Dynamical scale symmetry breaking and scale phase transition

at Scalars 2017

by Jisuke Kubo,
Kanazawa University

with M.B.Soesanto Qidir
and M. Yamada

$$\mathcal{L}_\phi = \frac{1}{2}\left(\partial_\mu\phi\;\partial^\mu\phi - m^2\phi^2\right) - \frac{\lambda}{4!}\phi^4$$

A slight extension to embed:

U(Nc) symmetry+ (classi.) Scale Invariance.

$$\mathcal{L}_S \;=\; \partial_\mu S_a^\dagger \, \partial^\mu S_a - \lambda_S \, (S_a^\dagger S_a)(S_b^\dagger S_b) \qquad (a, b = 1, \ldots, N_c)$$

★ **TheScale invariance is broken by anomaly.**

Callan, '70; Symanzik, '70

★ **The massless theory is free of IR divergences off-shell.**

Loewenstein+Zimmermann, '76

★ **The scale invariance can protect the Higgs mass from the fine tuning problem.**

Bardeen, '95

CLAIM:

(J.K and M.Yamada,PRD '16)

$\mathcal{L}$s is an effective Lagrangian for the strongly interacting non-abelian GT

$$\mathcal{L}_{\rm H} \;=\; -\frac{1}{2}\,{\rm tr}F_{\mu\nu}F^{\mu\nu} + (D_\mu S)^\dagger_a (D^\mu S)_a - \hat{\lambda}_S\;(S^\dagger_a S_a)(S^\dagger_b S_b)$$

We can use $\mathcal{L}_S$ to effectively describe dynamical scale symmetry breaking caused by the SU(Nc) invariant condensation of the scalar bi-linear in the confinement phase:

$$< S^\dagger_a S_a > \neq 0$$

A very brief review on Nambu-Jona-Lasinio (NJL) model

Massless QCD

$$\mathcal{L}_{QCD} = -\frac{1}{2}\mathrm{tr}F^2 + i\bar{\psi}_i\gamma^\mu D_\mu\psi_i$$

At low energy:

$$\langle\bar{\psi}_i\psi_j\rangle = \langle\sum_{c=1}^{N_c}\bar{\psi}_i^c\psi_j^c\rangle \propto \delta_{ij}$$

The effective theory for chiral symmetry breaking is:

The NJL model

$$\mathcal{L}_{\mathrm{NJL}} = i\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i + 2G\,\Phi^\dagger\Phi + \ldots = i\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i + G\left[(\bar{\psi}\lambda^a\psi)^2 - (\bar{\psi}\gamma_5\lambda^a\psi)^2\right] + \ldots$$

(4-fermi)

$$\Phi_{ij} = \bar{\psi}_i(1-\gamma_5)\psi_j = \frac{1}{2}\sum_{a=0}^{N_f^2-1}\lambda^a_{ji}\,\bar{\psi}\lambda^a(1-\gamma_5)\psi$$

The relevant global symmetry

$(N_f = 3, N_c = 3)$

★ **At the classical level**

$$SU(3)_L \times SU(3)_R \times U(1)_V \times \begin{cases} U(1)_A \\ Z_6 \end{cases}$$

QCD

NJL (4+6 fermi)

★ **At the quantum level**

$U(1)_A \to Z_6$ **Chiral anomaly in QCD**

$SU(3)_L \times SU(3)_R \to SU(3)_V$ **Dynamical chiral symmetry breaking**

★ **Finally** $SU(3)_V \times U(1)_V \times Z_6$

How to describe $SU(3)_L \times SU(3)_R \to SU(3)_V$

in the NJL theory?

Auxiliary field method

$$\mathcal{L}_{\rm NJL} = i\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4G}(\sigma^a \sigma^a + \pi^a \pi^a) - \bar{\psi}\lambda^a \sigma^a \psi - i\bar{\psi}\gamma_5 \pi^a \lambda^a \psi$$

Eqs of motion:

$$\sigma^a = -2G\ \bar{\psi}\lambda^a \psi\ ,\ \pi^a = -i2G\ \bar{\psi}\gamma_5 \lambda^a \psi$$

Auxiliary fields (mean fields)

Assume SU(Nc) xSU(Nf) invariant condensation:

$$\langle \bar{\psi}_i \psi_j \rangle = -\frac{1}{4G}\, \delta_{ij} \langle \hat{\sigma} \rangle \quad (\hat{\sigma} = \sqrt{3/2}\sigma^0)$$

Integrate out ψ to get the effective potential:

$$V_{\rm eff}(\hat{\sigma}) = N_f \left(\frac{\hat{\sigma}^2}{8G} - N_c\, I(\hat{\sigma}) \right) - V_{\rm eff}(0)$$

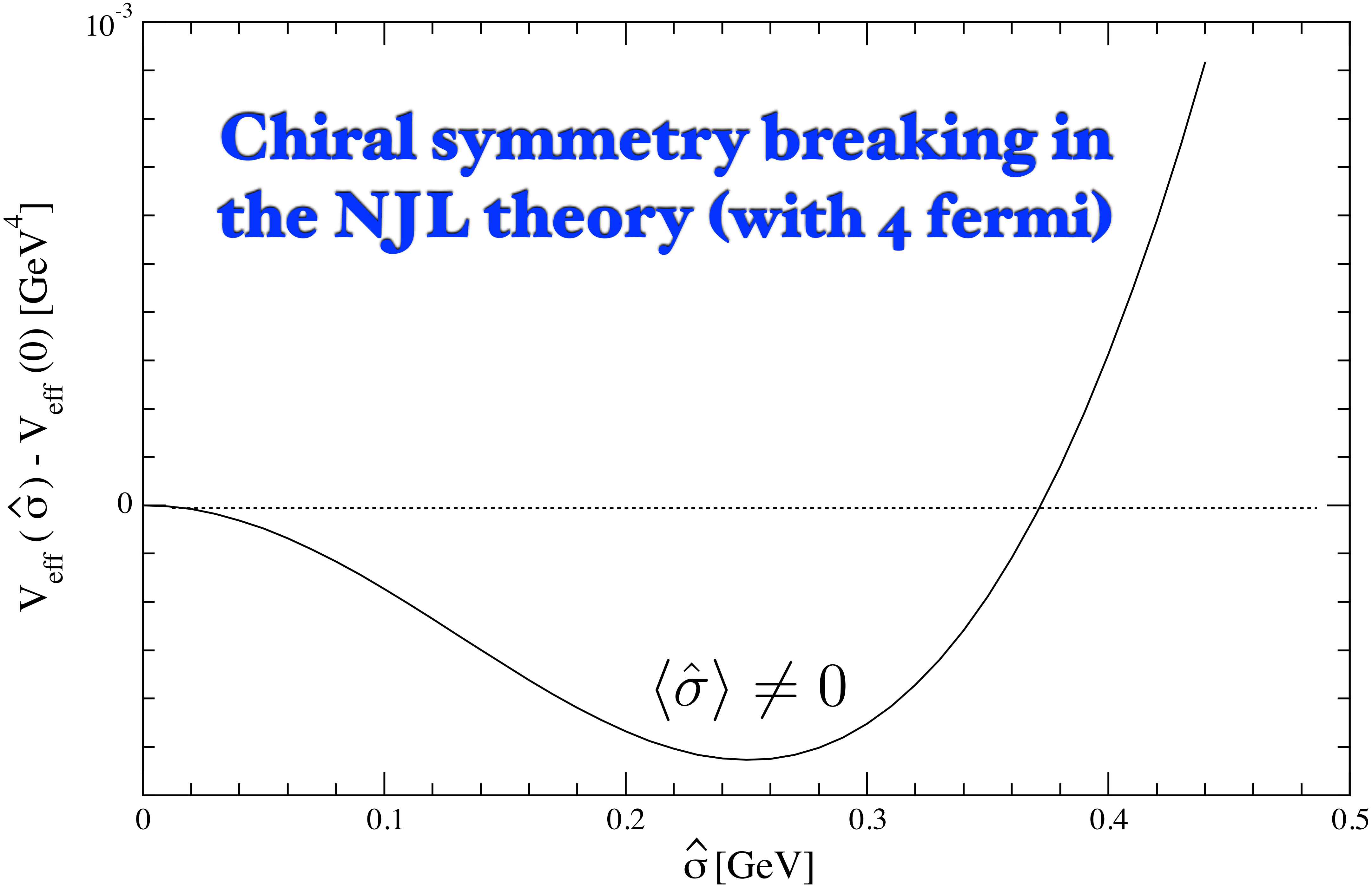
$$I(\hat{\sigma}) = \frac{\Lambda^4}{16\pi^2} \left(\ln\left(1 + \frac{\hat{\sigma}^2}{\Lambda^2}\right) - \frac{\hat{\sigma}^4}{\Lambda^4} \ln\left(1 + \frac{\Lambda^2}{\hat{\sigma}^2}\right) + \frac{\hat{\sigma}^2}{\Lambda^2} \right)$$

(with 4 fermi)

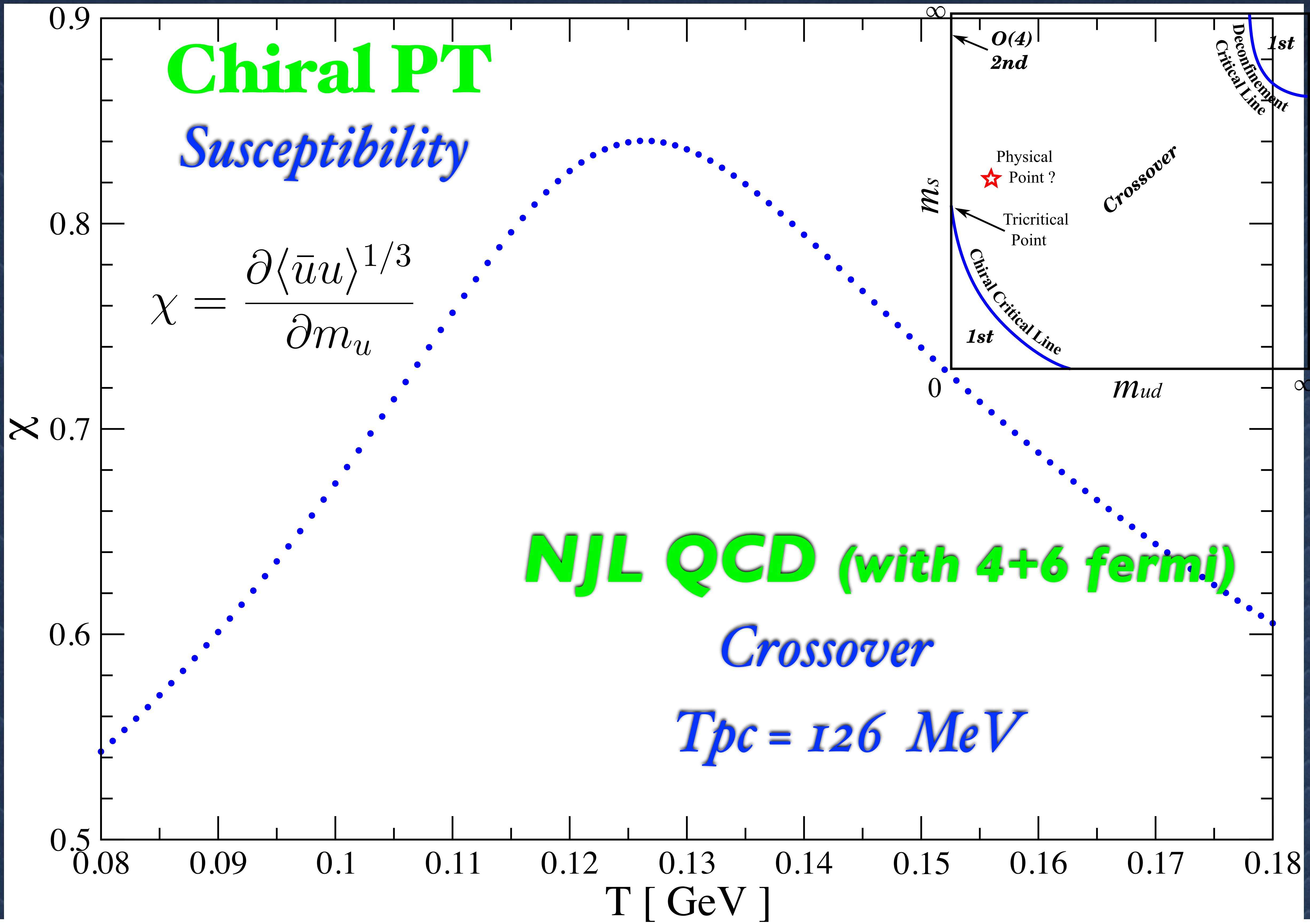
Λ is the physical cutoff.

$$\frac{\partial V_{\rm eff}}{\partial \hat{\sigma}} = 0 \text{ is a gap equation.}$$

$$\Lambda = 1\ \mathrm{GeV} \quad G = (0.5\ \mathrm{GeV})^{-2}$$



$$\langle\bar{\psi}_i\psi_j\rangle = -\frac{1}{4G}\,\delta_{ij}\langle\hat{\sigma}\rangle$$



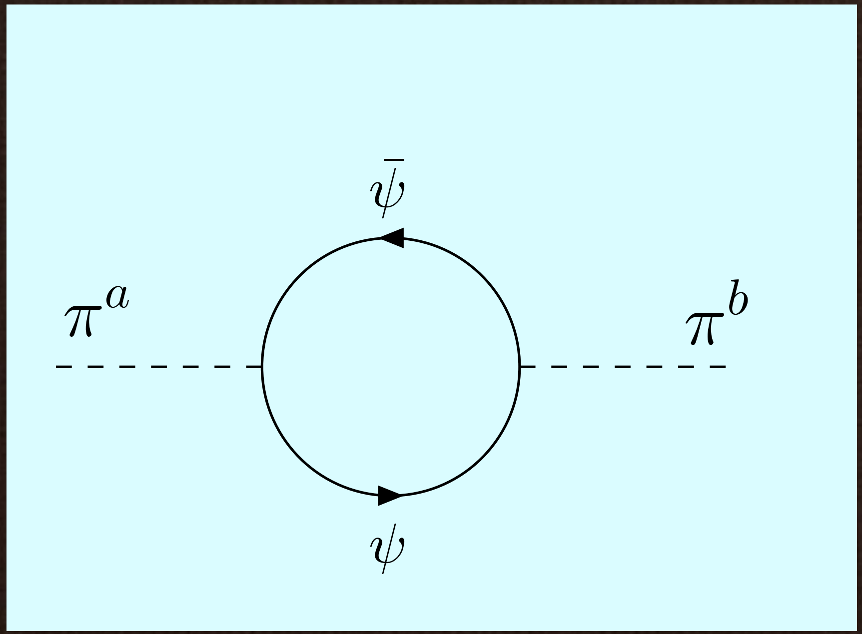
$T_{pc} = 154 \pm 9$ MeV (HotQCD, 2012)

$$\bar{\psi}_i(1-\gamma_5)\psi_j = -\frac{1}{4G}\ \left[\delta_{ij}\hat{\sigma} + \lambda^a(\sigma'^a + i\pi^a)\right]$$

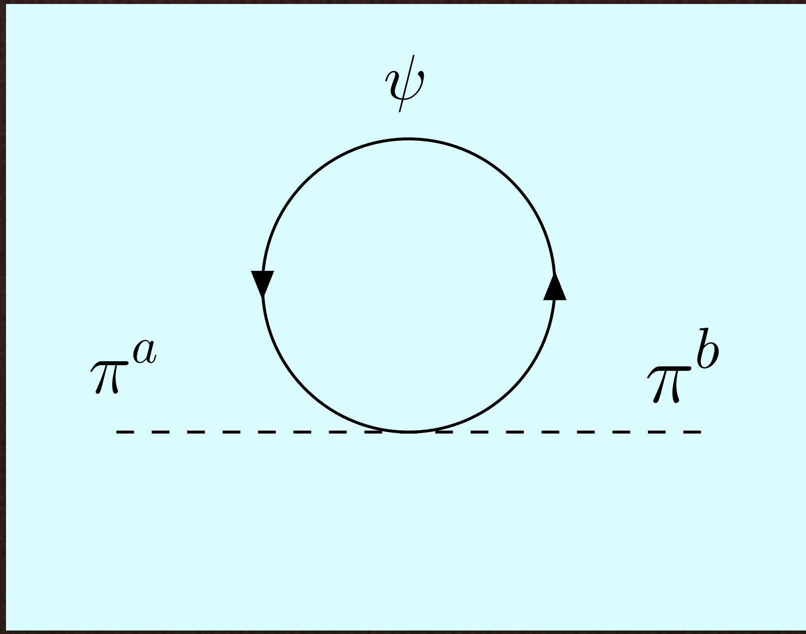
with $\langle\sigma'^a\rangle = \langle\pi^a\rangle = 0$

Excitations

The sigma and pion masses can be computed from



and



The NJL can reproduce the basic quantities of the mesons with O(80-90) % accuracy.

(Kunihiro+Hatsuda,'94)

Back to the scalar model with U(N_f) flavor symmetry.

$$\mathcal{L}_H = -\frac{1}{2}\mathrm{tr}F^2 + ([D_\mu S_i]^\dagger D^\mu S_i) - \hat{\lambda}_S(S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S(S_i^\dagger S_j)(S_j^\dagger S_i)$$

$$(i, j = 1, \ldots, N_f)$$

The color indices are suppressed.

The guiding principle:
The global symmetry.

★ **At the classical level:**

$U(N_f)$ flavor symmetry and scale invariance

★ **At the quantum level:**

$U(N_f)$ flavor symmetry and (anomalous) scale invariance, which is dynamically broken by $\langle S_i^\dagger S_j \rangle \neq 0$ with $\langle S_i \rangle = 0$.

$$\mathcal{L}_H = -\frac{1}{2}\mathrm{tr}F^2 + ([D_\mu S_i]^\dagger D^\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S (S_i^\dagger S_j)(S_j^\dagger S_i)$$

U(Nf)+classi. Scale Invariance
at low energy

UNIQUE !

$$\mathcal{L}_{\mathrm{eff}} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i)$$

It remains to show:
Scale invariance is dynamically broken.

1 Introduce the auxiliary field such that:

$$S_i^\dagger S_j = \delta_{ij} f + \delta_{ij} Z_\sigma^{1/2} \sigma + Z_\phi^{1/2} t^a_{ji} \phi^a \qquad (\langle \sigma \rangle = \langle \phi^a \rangle = 0)$$

Excitations

2 Integrate out the fluctuation of S to get:

$$V_{\rm eff}(f, \bar{S}) = M^2 (\bar{S}^\dagger \bar{S}) - N_f (N_f \lambda_S + \lambda'_S) f^2 + \frac{N_c N_f}{32\pi^2} M^4 \ln(M^2/\Lambda_H^2)$$

$$M^2 = 2(N_f \lambda_S + \lambda'_S) f \qquad \Lambda_H = \mu e^{3/4}$$

Earlier discussions in 70s and later in a different context:

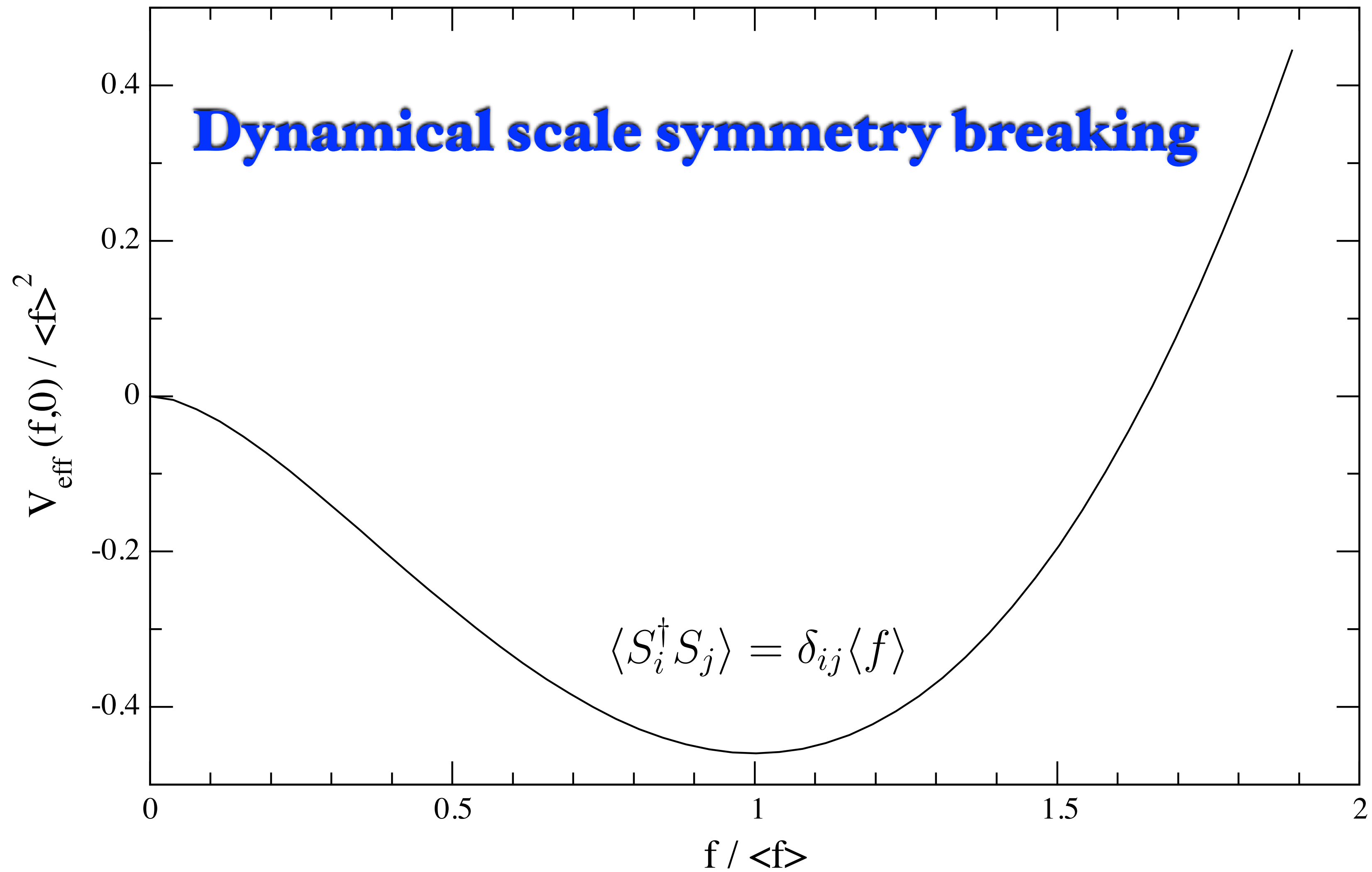
Coleman, Jackiw+Schnitzer,'74; Kobayashi+Kugo, '75; Bardeen+Moshe, '83;..........

The vacuum is unique:

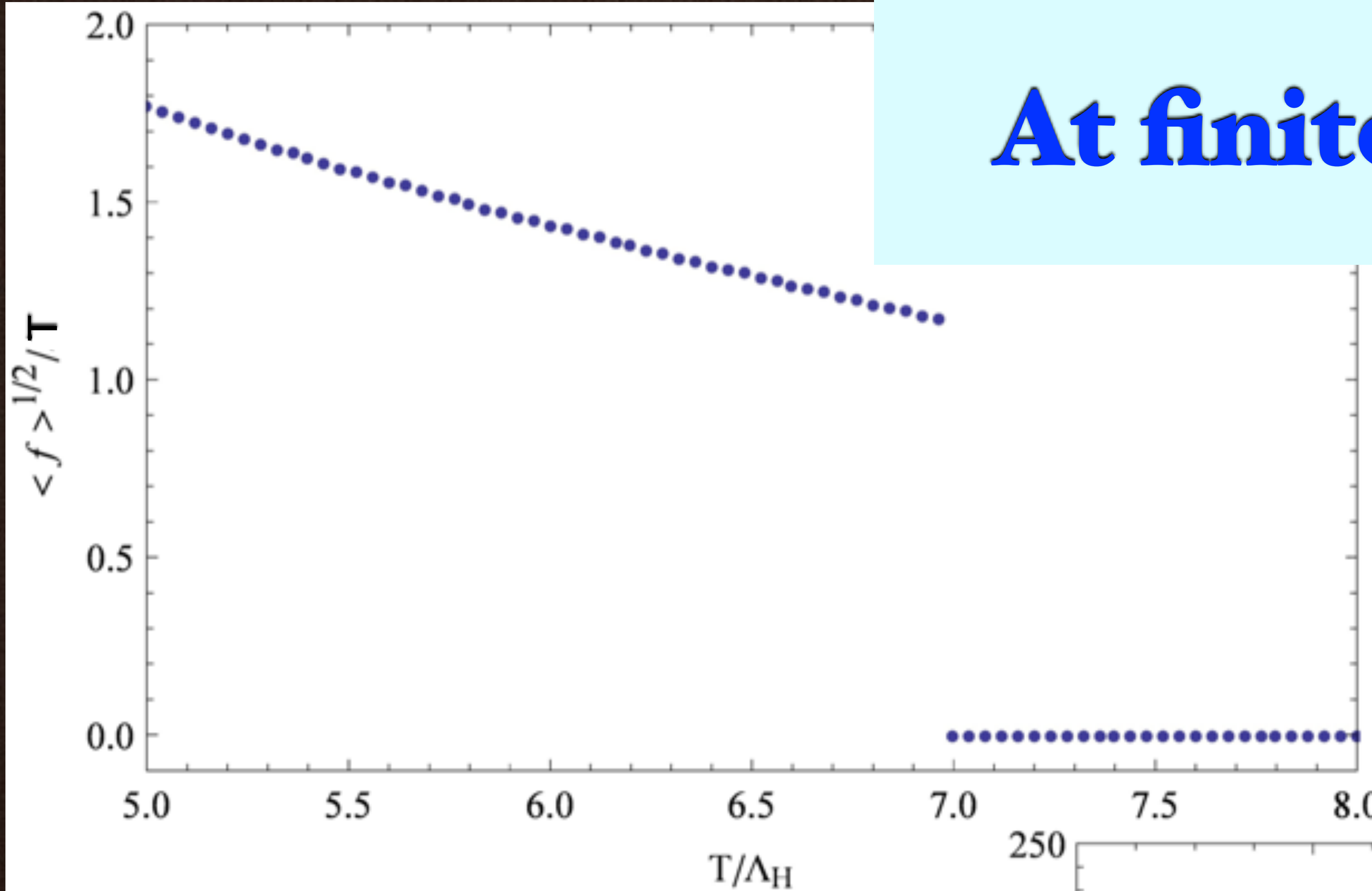
$$\langle f \rangle \neq 0, \langle S \rangle = 0$$

$$\langle V_{\text{eff}} \rangle = -\frac{N_c N_f}{16\pi^2}(N_f \lambda_S + \lambda'_S)^2 \langle f \rangle^2 < 0$$

$\lambda_S = 0.13,\ \lambda'_S = 2.2,\ N_c = 6,\ N_f = 2$ $\quad m_\phi/\sqrt{\langle f \rangle} = 2.96,\ m_\sigma/\sqrt{\langle f \rangle} = 4.08$



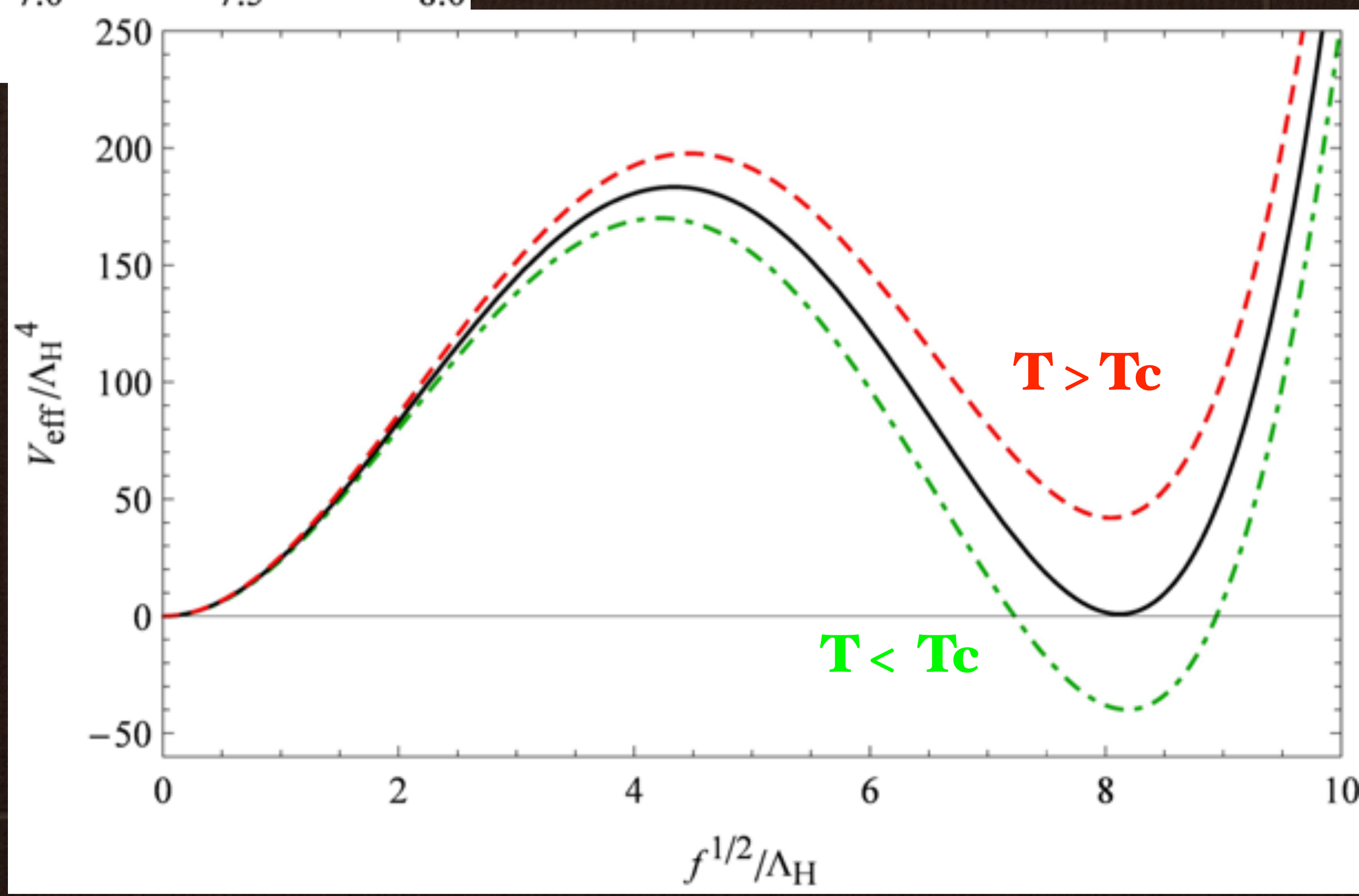
At finite temperature



J.K and M.Yamada, PTEP '15
See also Bardeen+Moshe, '83;

Scale Phase Transition is 1st order.

$$N_f = 1,\ N_c = 6,\ \lambda_S + \lambda'_S = 2.083$$



Application: Couple to the SM.

motivated by J.K+Lim+Lindner, PRL 113 (2014)091604

J.Kubo and M. Yamada,
PRD93 (2016) 075016;
PTEP (2015) 093B01.

$$
\begin{aligned}
\mathcal{L}_{\rm H} = & -\frac{1}{2}\,{\rm tr}F^2 + ([D^\mu S_i]^\dagger D_\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i)(S_j^\dagger S_j) \\
& - \hat{\lambda}'_S (S_i^\dagger S_j)(S_j^\dagger S_i) + \hat{\lambda}_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 + \mathcal{L}'_{\rm SM}
\end{aligned}
$$

No mass term

Higgs portal

$N_c = \#$of the hidden colors

$i, j = 1, \ldots, N_f$

The absolute minimum at

$$\langle S\rangle = 0$$

$$|\langle H\rangle|^2 = \frac{v_h^2}{2} = \frac{N_f\lambda_{HS}}{G}\Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right) = \frac{N_f\lambda_{HS}}{2\lambda_H}\langle f\rangle$$

$$G \equiv 4N_f\lambda_H\lambda_S - N_f\lambda_{HS}^2 + 4\lambda_H\lambda_S' > 0$$

with

$$\langle V_{\rm eff}\rangle = -\frac{N_c N_f}{16\pi^2}\frac{G^2}{16\lambda_H^2}\langle f\rangle^2 < 0$$

* Higgs mass

$$m_h^2 = \frac{N_f\lambda_{HS}}{2\lambda_H}\left(\frac{16\lambda_H^2(N_f\lambda_S + \lambda_S')}{G} + \frac{N_c N_f\lambda_{HS}^2}{8\pi^2}\right)\langle f\rangle$$

Origin of the Higgs mass

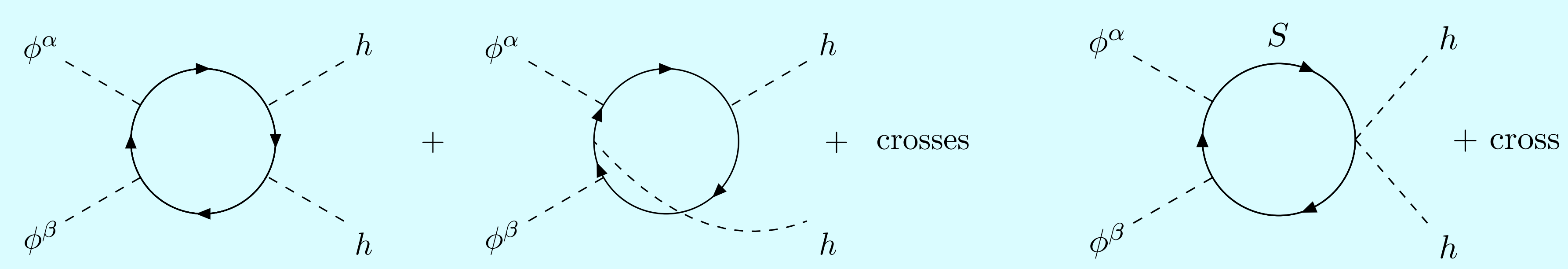
Dark Matter

$$S_i^\dagger S_j = \delta_{ij} f + \delta_{ij} Z_\sigma^{1/2} \sigma + Z_\phi^{1/2} t_{ji}^a \phi^a$$

Excitations

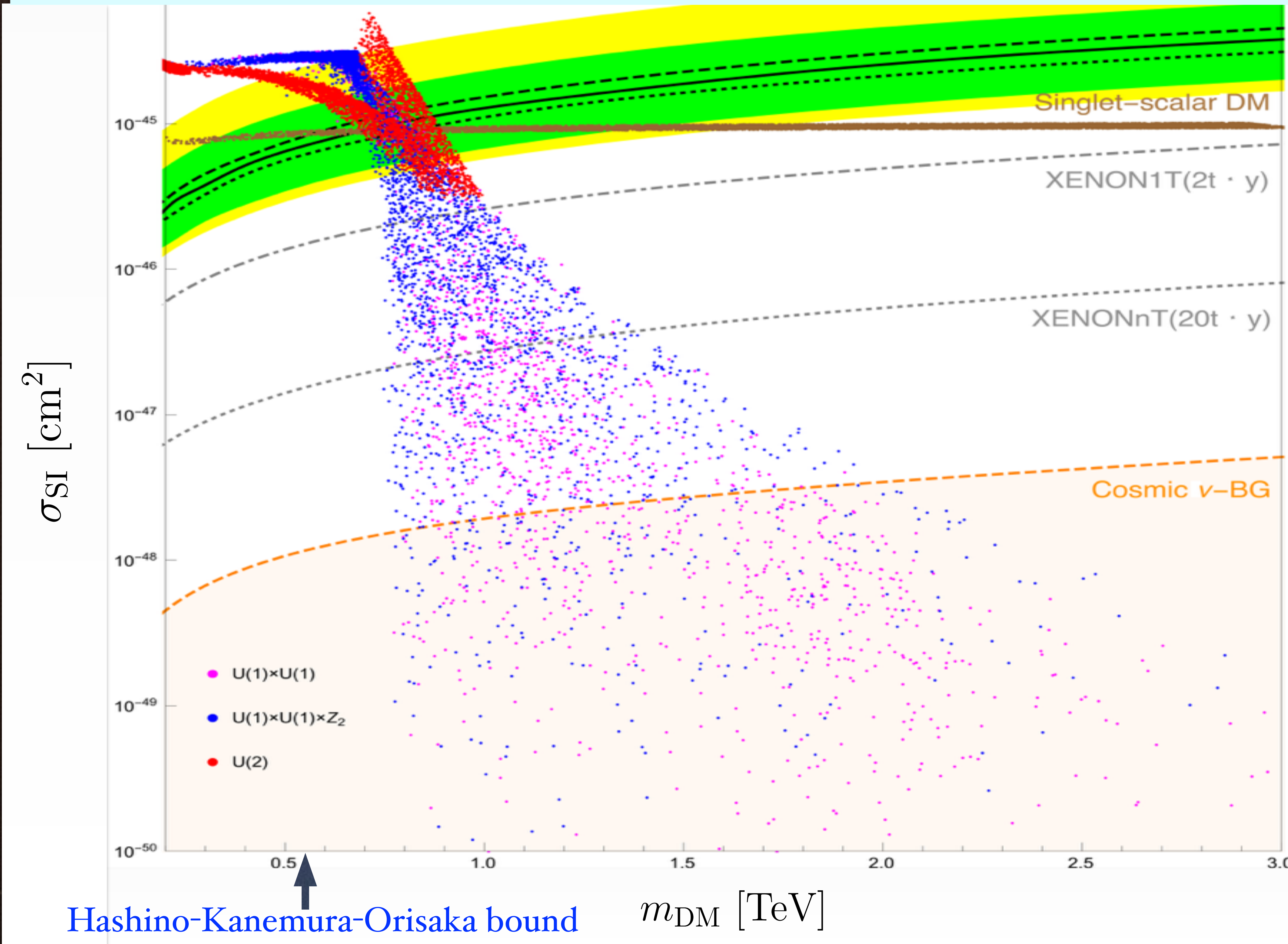
Since U(Nf) is unbroken,

ϕ^a **is stable and can be a DM candidate.**



Independent parameters : λ_S , λ'_S , λ_{SH} , λ_H , Λ_H

Input : $v_h = 246$ GeV , $m_h = 126$ GeV , $\Omega_{\rm DM}h^2 = 0.120 \pm 0.005$



Hashino-Kanemura-Orisaka bound

Phase Transitions (PT)

Two order parameters:

$< h >$ $\quad\quad$ $< \sigma >$

EWPT $\quad\quad$ **Scale PT**

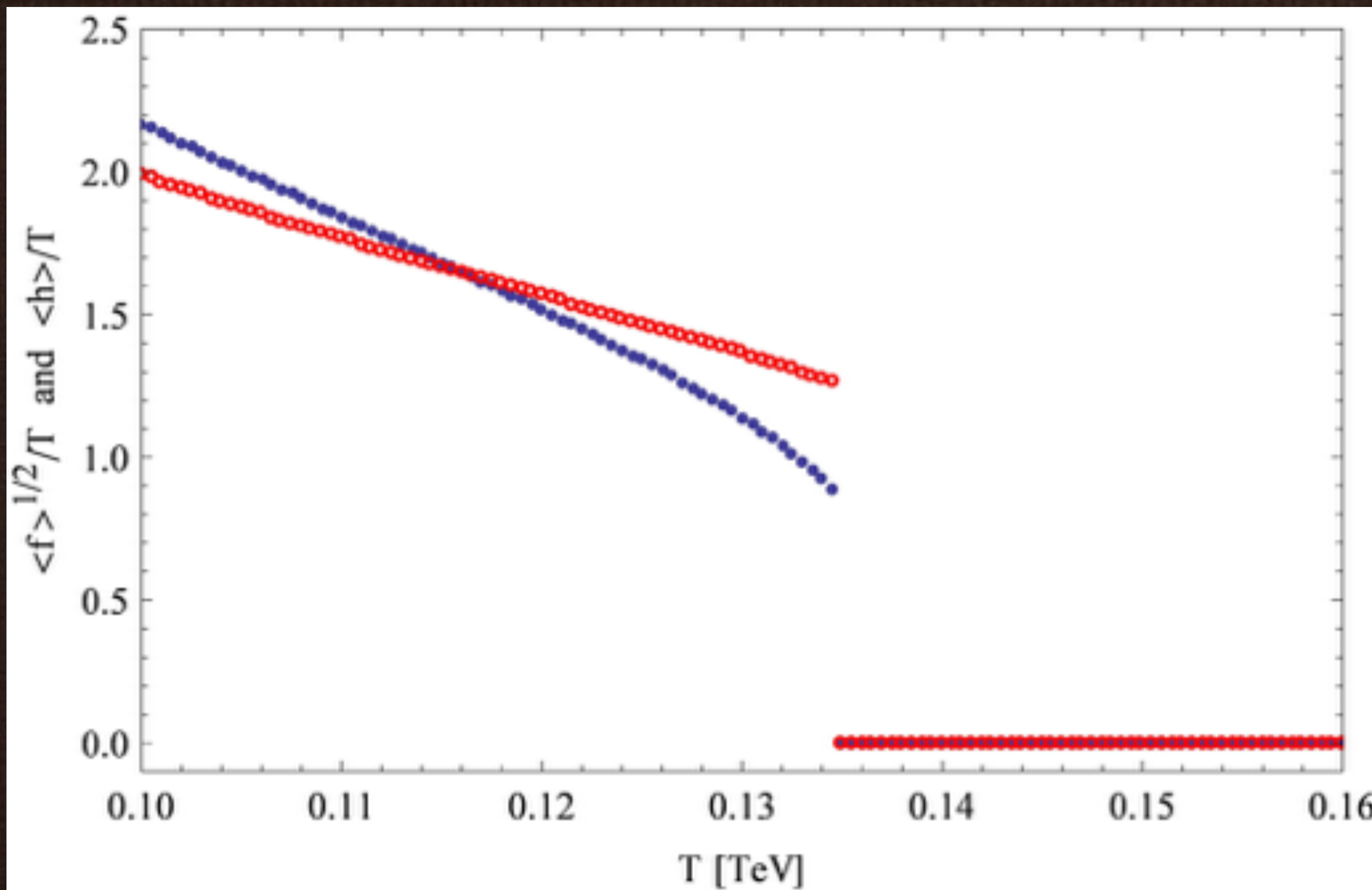
EW Baryogenesis

Kuzmin+Rubakov+Shaposhnikov,`85;
Klinkhamer+Manton,`84; ...

Gravitational wave BG

Hogan,`83; Witten,`84; ....

See J.K+Yamada,'16
for the present model.

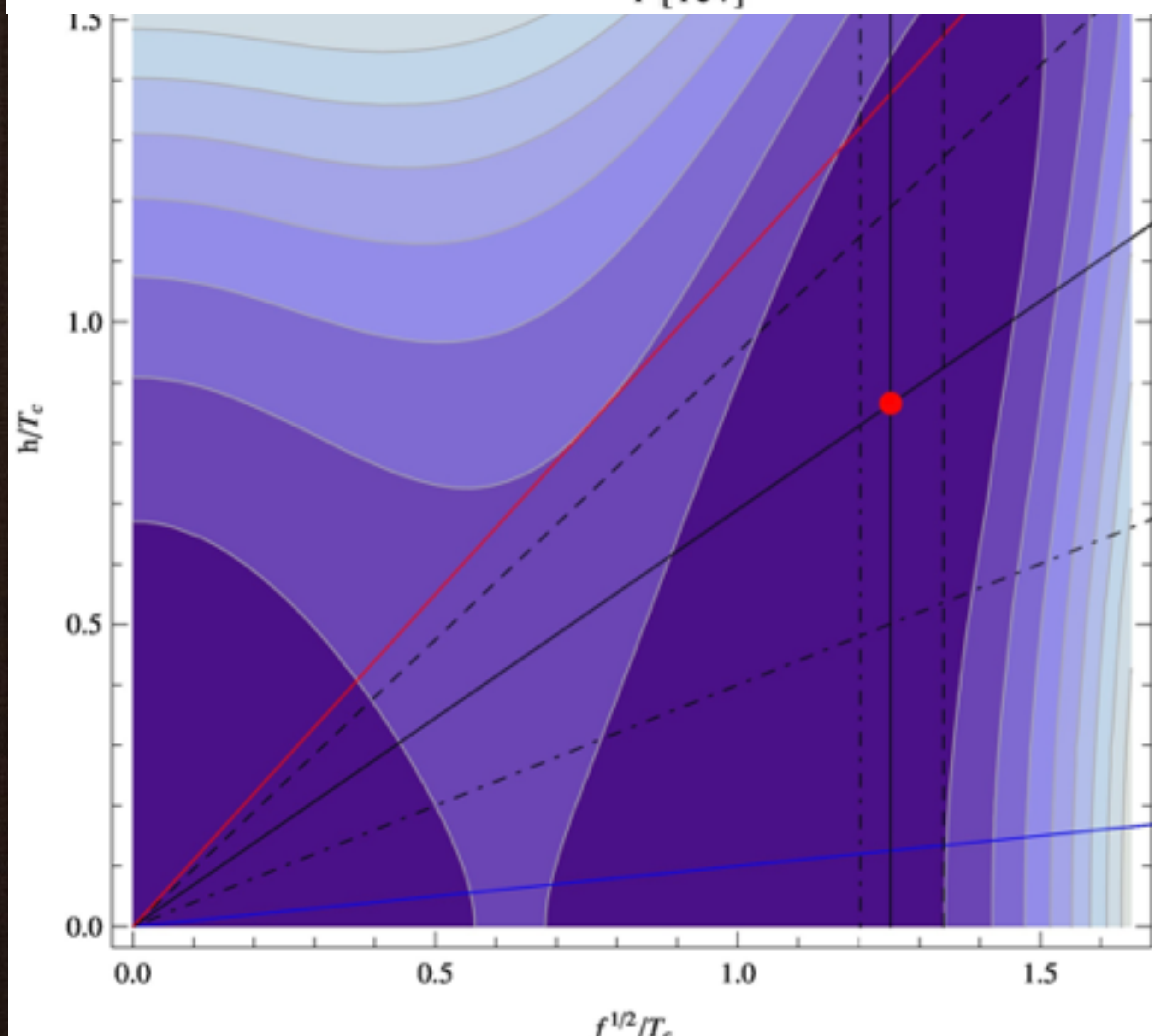


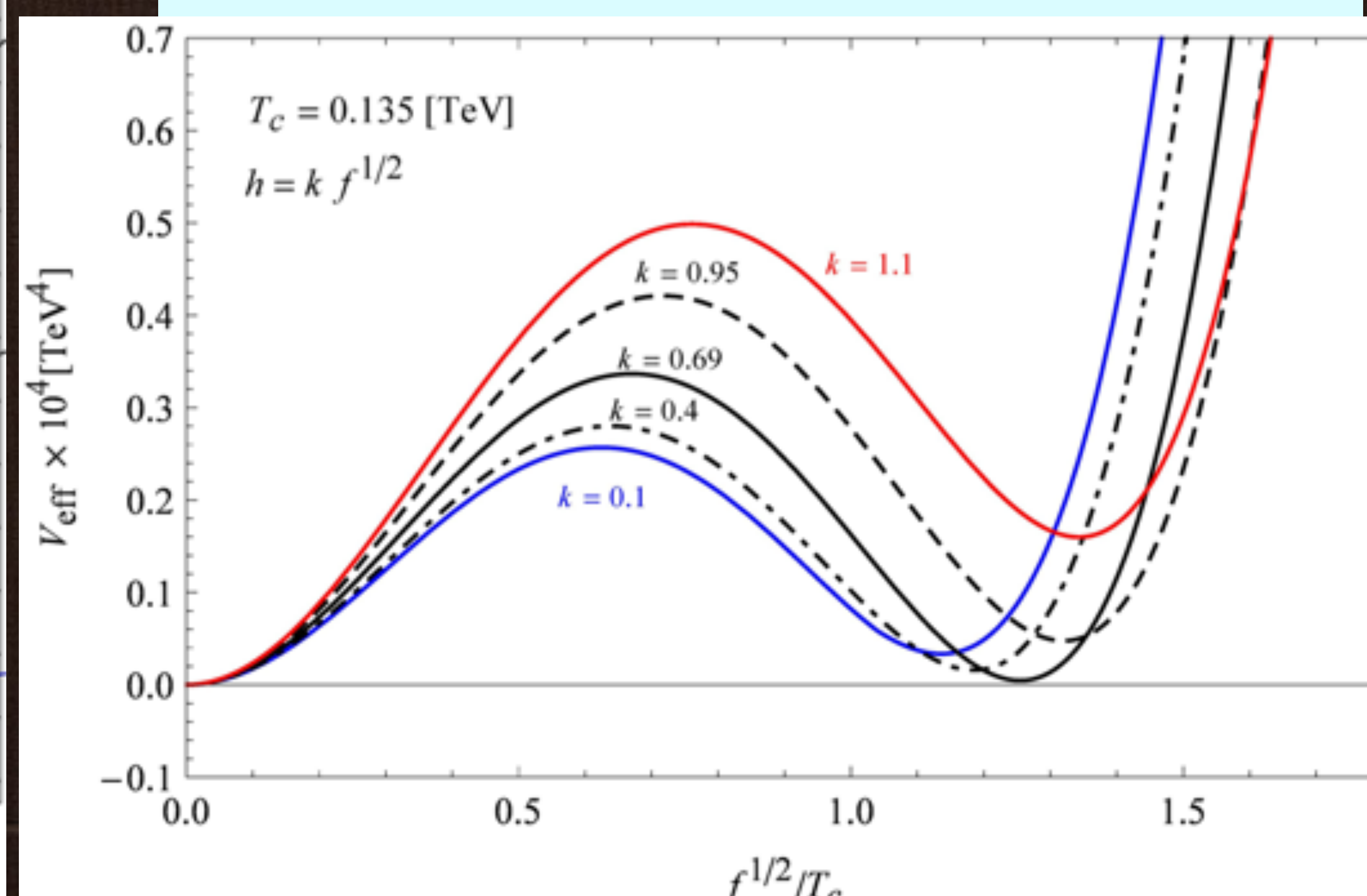
Strong 1st-order EW and Scale PT

No DM

$$N_f = 1,\ N_c = 6,\ \lambda_S + \lambda_S' = 2.083,$$

$$\lambda_{HS} = 0.296,\ \lambda_H = 0.208.$$





A benchmark point:

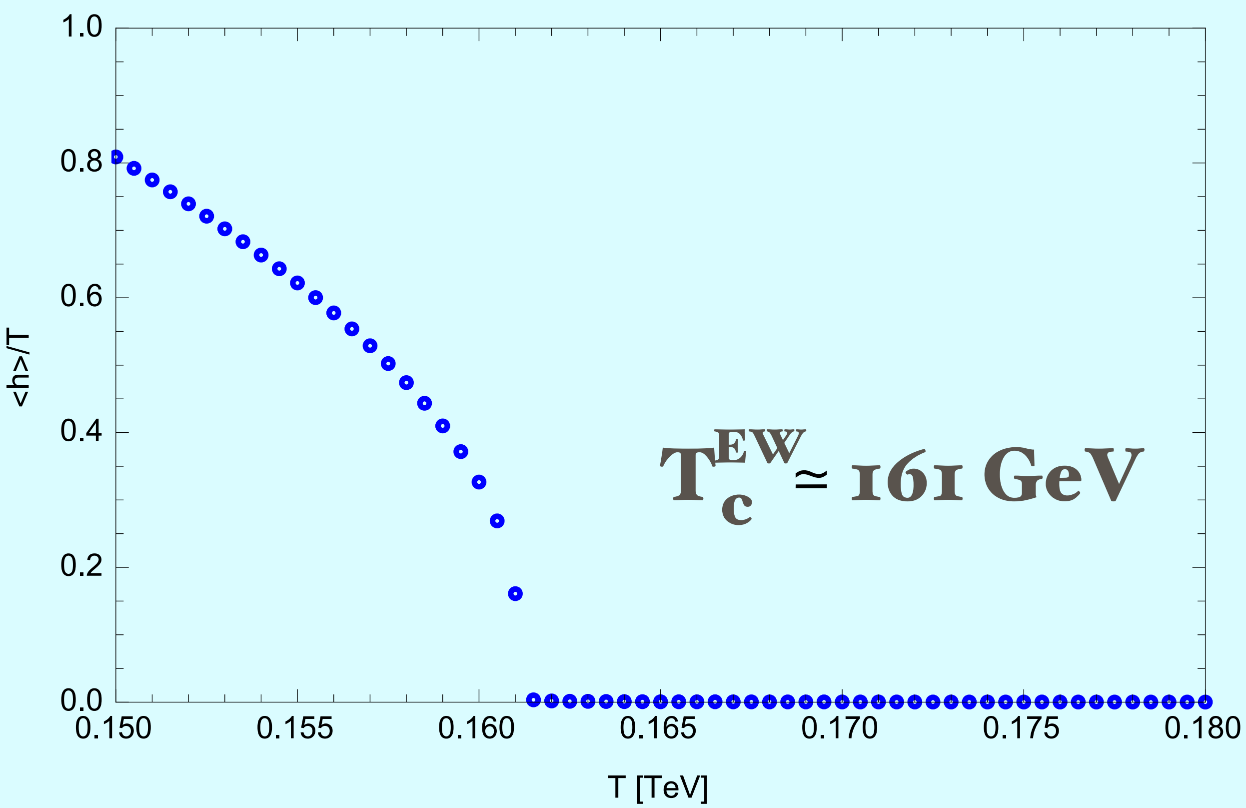
$$\lambda_S = 0.145, \lambda_S' = 2.045, \lambda_H = 0.15, \lambda_{HS} = 0.032$$

with $N_f = 2, N_c = 6$

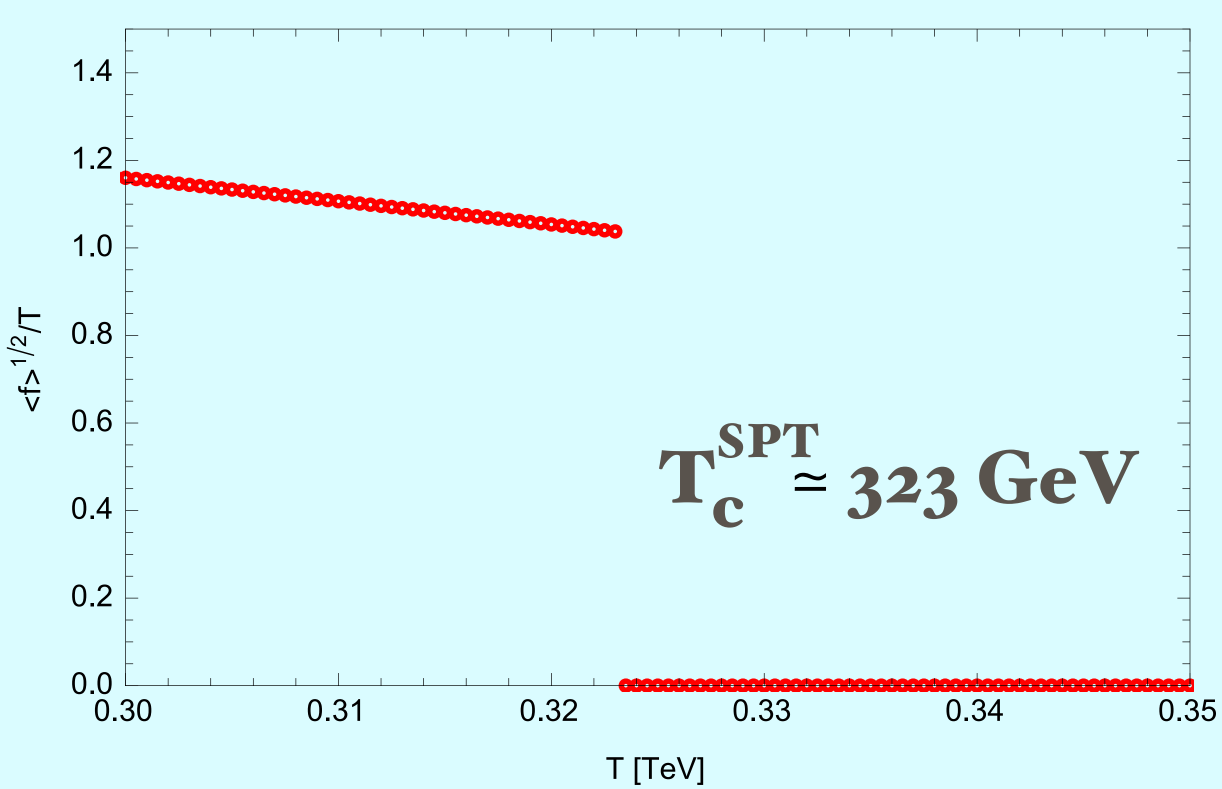
Output:

$$m_h = 0.126 \text{ TeV}, v_h = 0.246 \text{ TeV}, m_{\mathrm{DM}} = 0.856 \text{ TeV},$$
$$\Omega_{\mathrm{DM}} \hat{h}^2 = 0.122, \sigma_{SI} = 5.12 \times 10^{-46} \text{ cm}^2$$

EW PT



Scale PT



???

Planck

cl. scale invariant,
renormalizable, perturbative
gauge theory

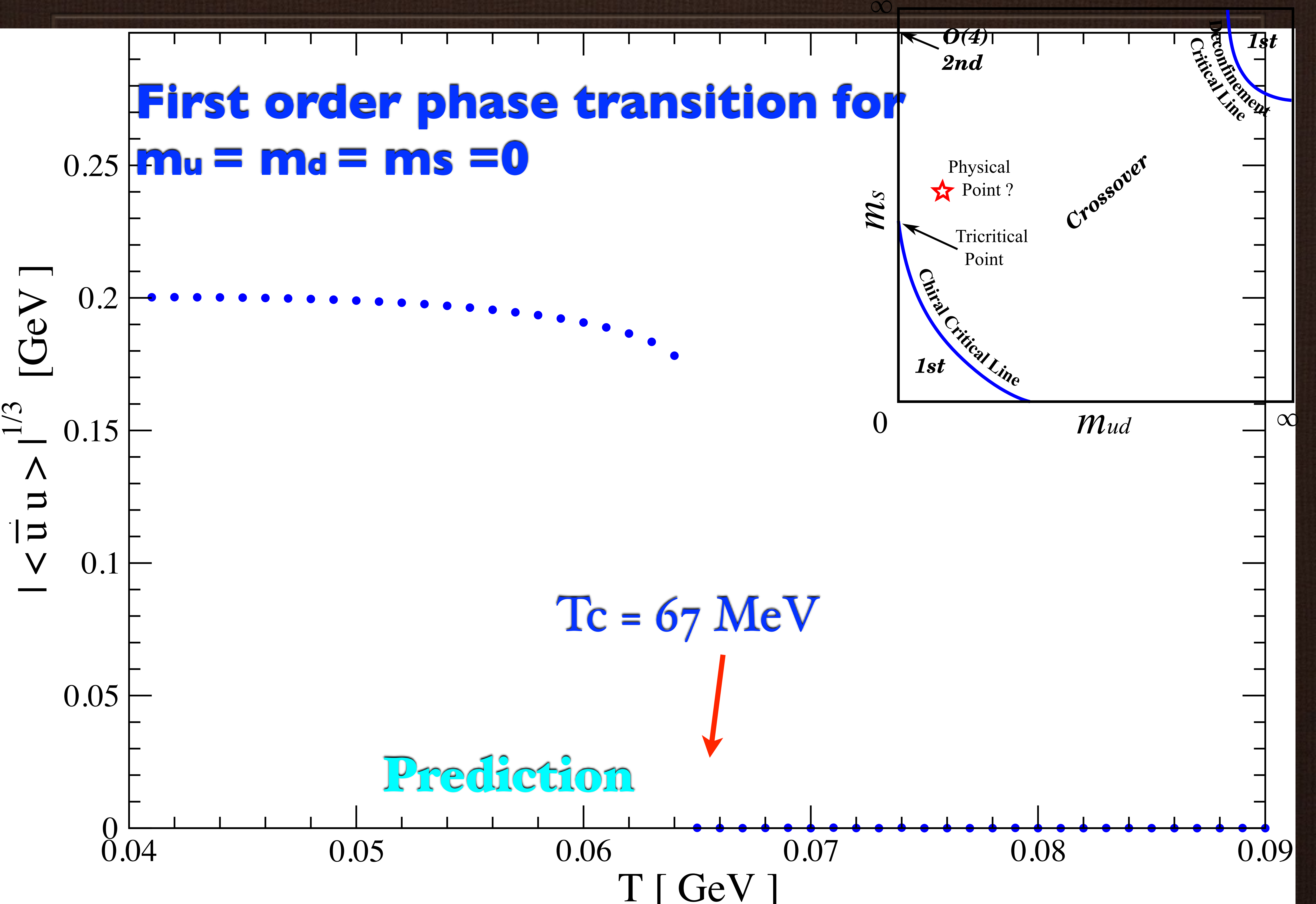
TeV — Dynamical scale SB -> DM +Higgs mass term

EWSB

GeV — DχSB in QCD -> Hadrons

25

podziękowaniaa



First order phase transition for
mu = md = ms =0
Tc = 67 MeV
Prediction
0.25
0.2
0.15
0.1
0.05
0
0.04
0.05
0.06
0.07
0.08
0.09
T [GeV]
| < ū u > |^1/3 [GeV]
O(4)
2nd
1st
Deconfinement
Critical Line
Physical
Point ?
Crossover
Tricritical
Point
Chiral Critical Line
1st
ms
mud
0
∞
∞

$$\Lambda = 0.93\ ,\ (2G)^{-1/2} = 0.361\ ,\ (-G_D)^{-1/5} = 0.406\ ,\ m_u = 0.006\ ,\ m_s = 0.163$$

in GeV

	Exp.	**NJL**
$m_{\pi^0}(m_{\pi^\pm})$	0.135(0.140)	0.136
f_π	0.092	0.093
$m_{K^0}(m_{K^\pm})$	0.498(0.494)	0.499
f_K	0.110	0.105
m_η	0.548	0.460
$m_{\eta'}$	0.958	0.960

in GeV

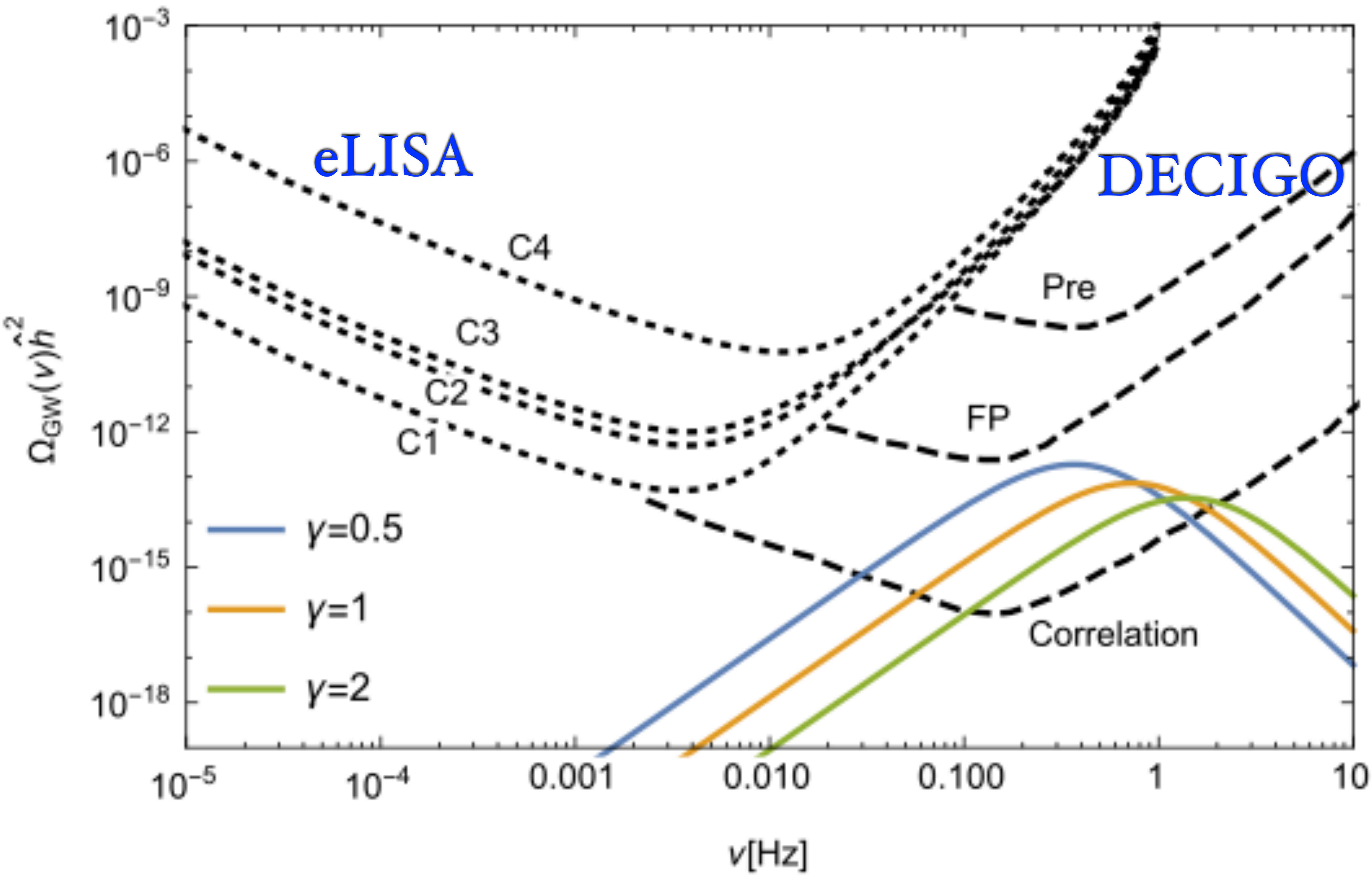
***Goldberger-Treiman relation:**

$$f_\pi G_{\pi qq} = 0.98 \times M$$

***Gell-Mann-Oakes-Renner relation:**

$$f_\pi^2 m_\pi^2 = -1.00 \times \frac{1}{2}(m_u + m_d) < \bar{u}u + \bar{d}d >$$

See also:Hatsuda+Kunihiro, `94



γ	T_t [TeV]	$S_3(T_t)/T_t$	α	$\tilde{\beta}$	$\tilde{\Omega}_{\rm sw}h^2$	$\tilde{\nu}_{\rm sw}$ [Hz]
0.5	0.300	149	0.070	3.7×10^3	1.9×10^{-13}	0.37
1.0	0.311	145	0.062	7.0×10^3	7.4×10^{-14}	0.73
2.0	0.316	146	0.059	13×10^3	3.4×10^{-14}	1.4

J.Kubo and M. Yamada,
arXiv:1610.02241.