

A CLOCKWORK WIMP

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PLAN

- the clockwork mechanism illustrated
- construction of a clockwork wimp
- Majorana neutrino masses
- a bit of deconstruction
- conclusions

Based on [arXiv:1612.06411](https://arxiv.org/abs/1612.06411), in collaboration with Thomas Hambye and Daniele Teresi

ON SCALES AND MASSES

new physics (local) $\longrightarrow$ effective operators $\longrightarrow$ new scale

Weinberg operator $\longrightarrow$ L breaking scale

Proton decay $\longrightarrow$ GUT scale

Axion $\longrightarrow$ PQ symmetry breaking scale

Quantum gravity $\longrightarrow$ Planck scale

...

but

MASS $\neq$ SCALE



$$G_F^{-1/2} \sim \frac{M_W}{g}$$

Fermi

$$\Lambda_\nu \sim \frac{M_R}{y^2}$$

Weinberg

$$M_P \sim \frac{M_s}{g_s}$$

Planck

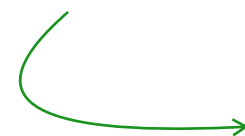
THE CLOCKWORK MECHANISM

Kaplan & Ratazzi (2015); Choi & Im (2015) ; Giudice & McCullough (2016)

$$\Lambda \sim \frac{M}{y^x} \quad \text{large scale physics : large mass or tiny coupling?}$$

We are often reluctant to introduce small parameters
(even if natural in the sense of 't Hooft)

The **clockwork** is a possible mechanism
to generate **small numbers**
out of a theory with $O(1)$ parameters
or large effective scales
out of
dynamics at **much lower energies**



reading accessible energies...

THE CLOCKWORK MECHANISM

a framework for model building....



THE CLOCKWORK MECHANISM

- **hierarchy problem** Guidice & McCullough (2016)
- **low scale invisible axion** Guidice & McCullough (2016); Farina *et al* (2016)
- **inflation** Kehagias & Riotto (2016)
- **neutrino physics** Hambye, Teresi & MT (2016); Carena *et al* (2017); Ibarra *et al* (2017)
- **sugra** Antoniadis, Delgado, Markou & Pokorski (2016)
- **dark matter** Hambye, Teresi & MT (2016) (this talk)
- ...



THE CLOCKWORK MECHANISM ILLUSTRATED

$$\begin{array}{ccccccc} \Phi_0 & \Phi_1 & \dots & \Phi_k \sim e^{i\phi_k/f} & \dots & & \Phi_N \\ \times & \times & \times & \times & \times & \times & \times \end{array}$$

The Scalar Clockwork

THE CLOCKWORK MECHANISM ILLUSTRATED

$$\begin{array}{ccccccc} \Phi_0 & \Phi_1 & \dots & \Phi_k \sim e^{i\phi_k/f} & \dots & & \Phi_N \\ \times & \times & \times & \times & \times & \times & \times \end{array}$$

$$\mathcal{L} = \frac{f^2}{2} \sum_{k=0}^N |\partial \Phi_k|^2 - \frac{m^2 f^2}{2} \sum_{k=0}^{N-1} (\Phi_k^\dagger \Phi_{k+1}^q + h.c.)$$



$$- \frac{m^2}{2} \sum_0^{N-1} (\phi_k - q\phi_{k+1})^2 + \dots$$


q > 1

$$G = U_0(1) \otimes U_1(1) \otimes \dots \otimes U_N(1) \xrightarrow{m^2 \neq 0} U(1)$$



one massless + N massive modes

THE CLOCKWORK MECHANISM ILLUSTRATED

$$\begin{array}{ccccccc} \Phi_0 & \Phi_1 & \dots & \Phi_k \sim e^{i\phi_k/f} & \dots & \Phi_N \\ \times & \times & \times & \times & \times & \times & \times \end{array}$$

$$-\frac{m^2}{2} \sum_0^{N-1} (\phi_k - q\phi_{k+1})^2 + \dots \longrightarrow M^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \dots & 0 \\ -q & 1+q^2 & -q & \dots & 0 \\ 0 & -q & 1+q^2 & -q & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -q & 1+q^2 & -q \\ & & & \dots & -q & q^2 \end{pmatrix}$$

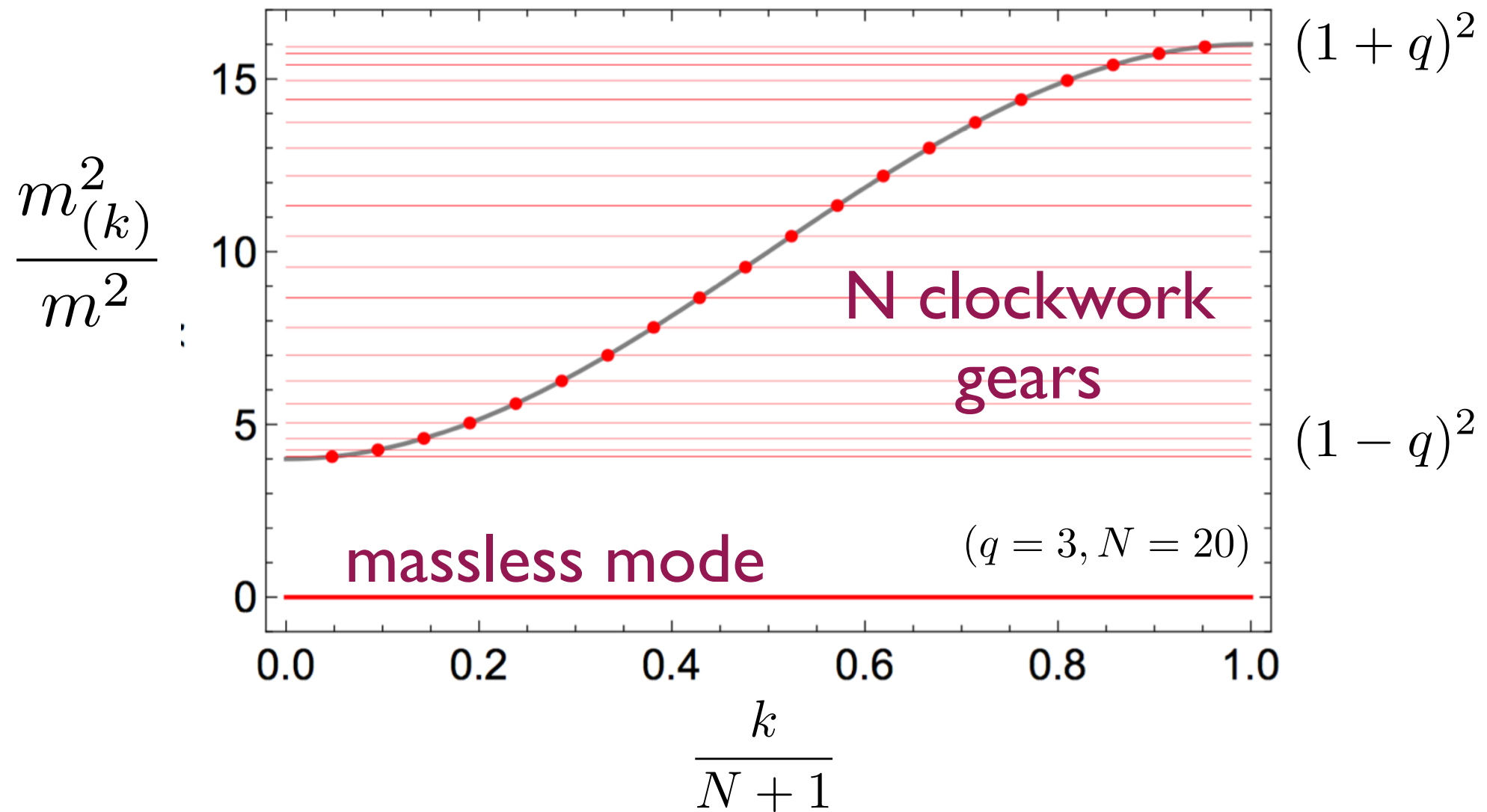
$$M^2 \varphi^{(0)} = 0 \longrightarrow \varphi^{(0)} \sim \phi_0 + \phi_1/q + \dots + \phi_N/q^N$$



$$q > 1 \rightarrow q^N \gg 1$$

massless mode
localized towards site k=0

THE CLOCKWORK MECHANISM ILLUSTRATED



THE CLOCKWORK MECHANISM ILLUSTRATED

$$\varphi^{(0)} \sim \underbrace{\phi_0}_{\times} \quad \underbrace{\phi_1}_{\times} \quad \underbrace{\dots}_{\times} \quad \times \quad \times \quad \times \quad \times \quad \underbrace{\phi_N}_{\times} \text{ --- SM}$$

massless mode
localized towards
site $k=0$

$$\mathcal{L} \supset \frac{\phi_N}{F} G \tilde{G}$$

$$\longrightarrow \frac{\varphi^{(0)}}{q^N F} G \tilde{G}$$



tiny
coupling

e.g.

$$N = 15, \quad q = 3 \quad \longrightarrow \quad q^N F \sim 10^{10} \left(\frac{F}{\text{TeV}} \right) \text{ GeV}$$

large effective scale

A CLOCKWORK WIMP ?

WIMP abundance from thermal freeze-out



$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2}$$

mass in hundreds
of GeV-TeV range

WIMP stability protected by a symmetry

SUSY, SO(10), exact gauge symmetry,
accidental symmetry,...

→ e.g. a Majorana SU(2) 5-plet (aka Minimal Dark Matter)

possible decay through
dim-6 op.

$$\tau \sim \frac{\Lambda^4}{M^5} \longrightarrow \Lambda \gtrsim 10^{16} \text{ GeV}$$

a large, difficult
to test scale

CAN WE STABILIZE A WIMP THROUGH THE CLOCKWORK MECHANISM ?



A CLOCKWORK WIMP - CONSTRUCTION

chiral chain	R_0	$L_1 R_1$	$\times$	$\dots$	$\times$	$\times$	$L_N R_N$
	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

complex scalars

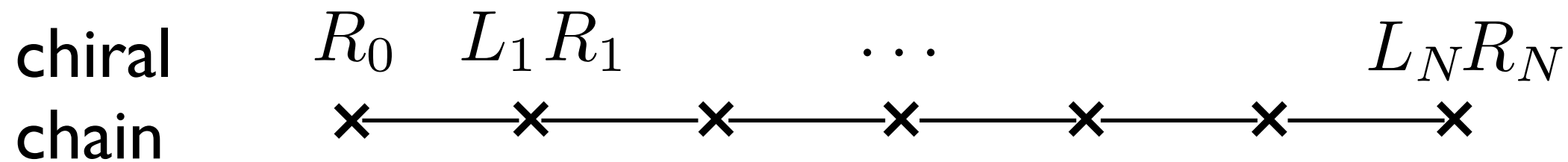
$$\begin{cases} S_i \sim (1, -1) & \text{under } U(1)_{L_{i+1}} \times U(1)_{R_i} \\ C_i \sim (1, -1) & \text{under } U(1)_{L_i} \times U(1)_{R_i} \end{cases}$$

basic ingredients:

a chiral chain (Weyl spinors) and
complex scalars (spurions or dynamical)

construction goes through 4 steps

A CLOCKWORK WIMP - CONSTRUCTION



complex scalars

$$\begin{cases} S_i \sim (1, -1) & \text{under } U(1)_{L_{i+1}} \times U(1)_{R_i} \\ C_i \sim (1, -1) & \text{under } U(1)_{L_i} \times U(1)_{R_i} \end{cases}$$

step 1: break the chiral symmetries

$$\begin{cases} m \rightarrow y_S \langle S_i \rangle \\ mq \rightarrow y_C \langle C_i \rangle \end{cases}$$

rem: these fields are spurions
in the original framework
they are dynamical in our case

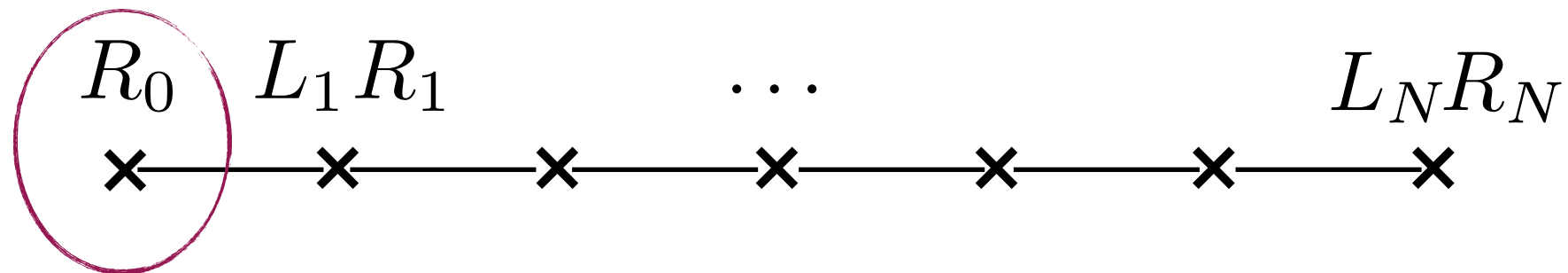
$$U(1)_{R_0} \times U(1)_{L_1} \times U(1)_{R_1} \times \dots \times U(1)_{L_N} \times U(1)_{R_N} \longrightarrow U(1)_R$$

one massless mode



$$\mathcal{L} \supset -m \sum_{i=1}^N \left(\bar{L}_i R_{i-1} - q \bar{L}_i R_i \right) + h.c.$$

A CLOCKWORK WIMP - CONSTRUCTION



$$\mathcal{L} \supset -m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i) + h.c.$$

$$N \approx R_0 + \frac{1}{q} R_1 + \frac{1}{q^2} R_2 + \dots + \frac{1}{q^N} R_N$$

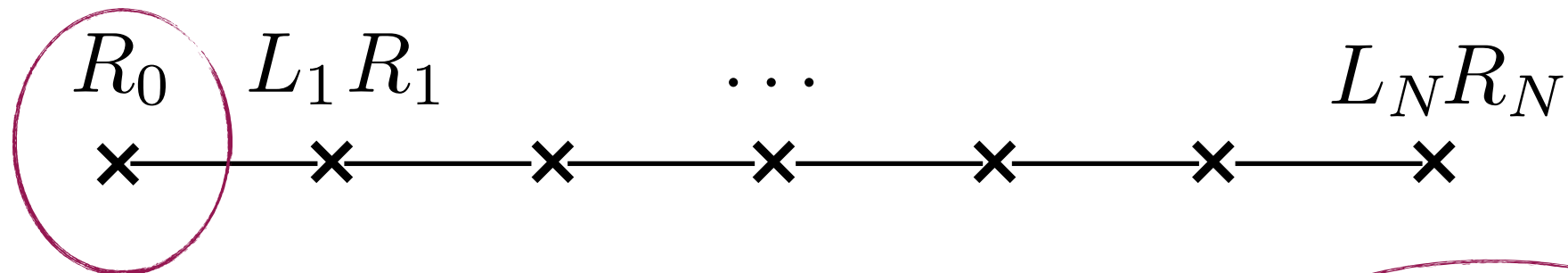
**massless
chiral mode**

$$\psi^{(n)} \approx \frac{1}{\sqrt{N}} \sum_k [\mathcal{O}(1) L_k + \mathcal{O}(1) R_k]$$

**N Dirac gears
mass $\sim q m$**

i.e. pretty much like the scalar clockwork

A CLOCKWORK WIMP - CONSTRUCTION



$$\mathcal{L} \supset -m \sum_{i=1}^N \left(\bar{L}_i R_{i-1} - q \bar{L}_i R_i \right) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0$$

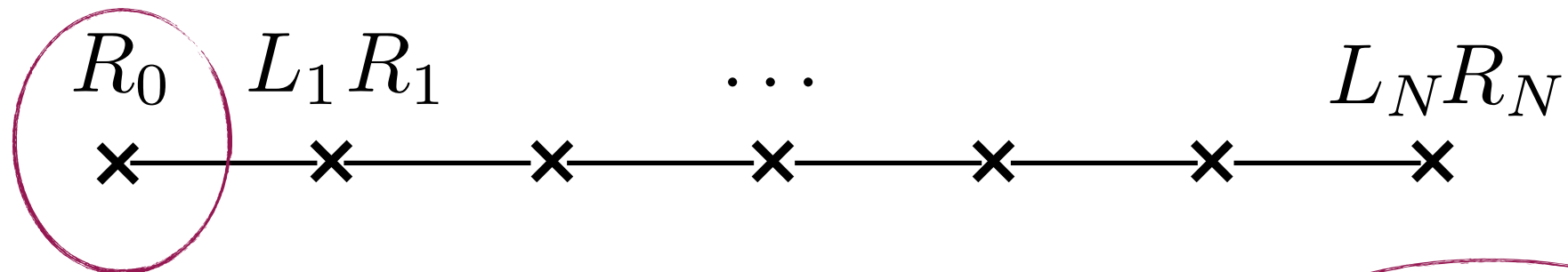
The last term, $-\frac{1}{2} m_N \bar{R}_0^c R_0$, is highlighted with a purple oval.

step 2: break the residual chiral symmetry

i.e.

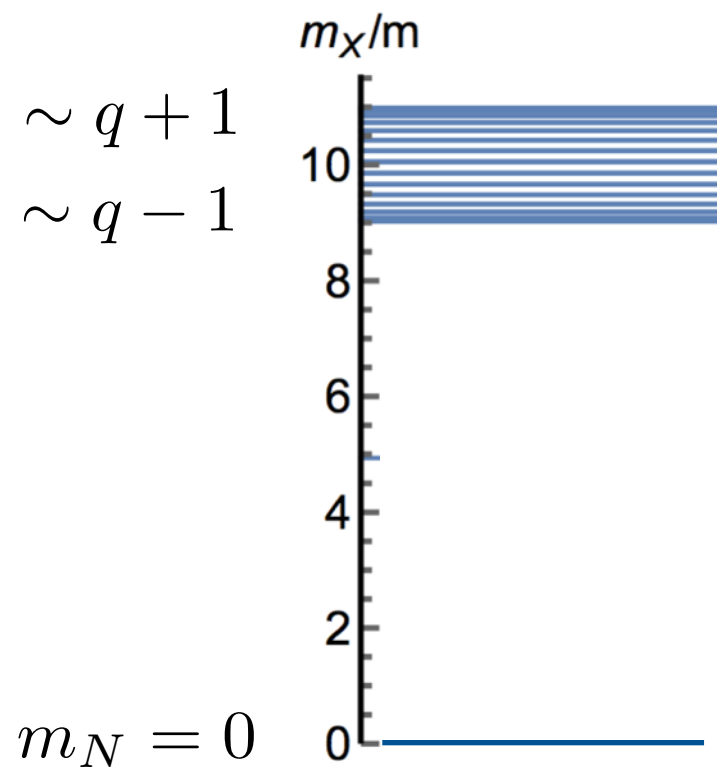
give a mass to the N state

A CLOCKWORK WIMP - CONSTRUCTION

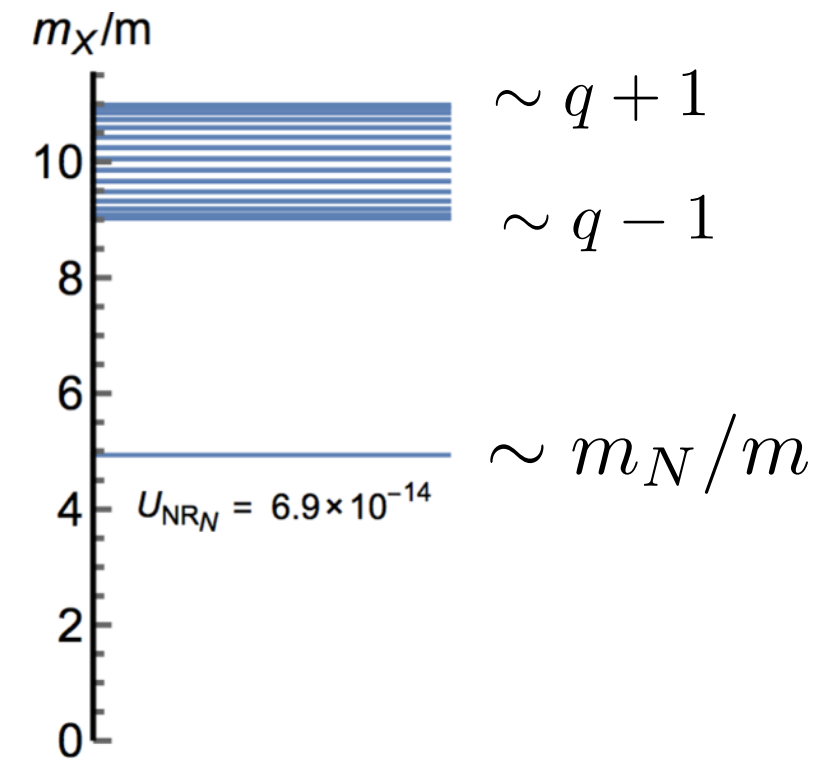


$$\mathcal{L} \supset -m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0$$

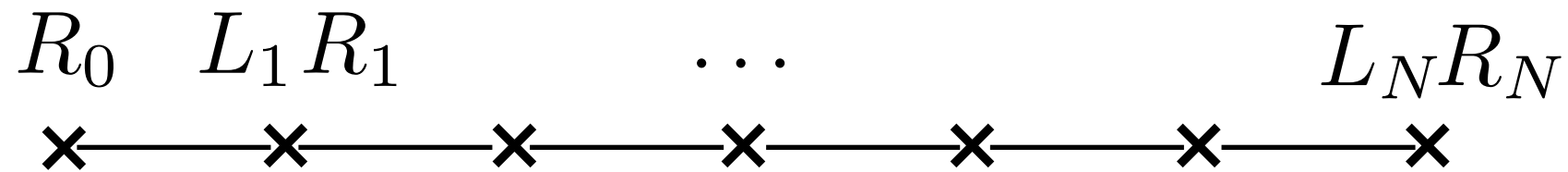
$(q = 10, N = 15, m_N = 0)$



$(q = 10, N = 15, m_N = 5m)$



A CLOCKWORK WIMP - CONSTRUCTION



$$\mathcal{L} \supset -m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0$$

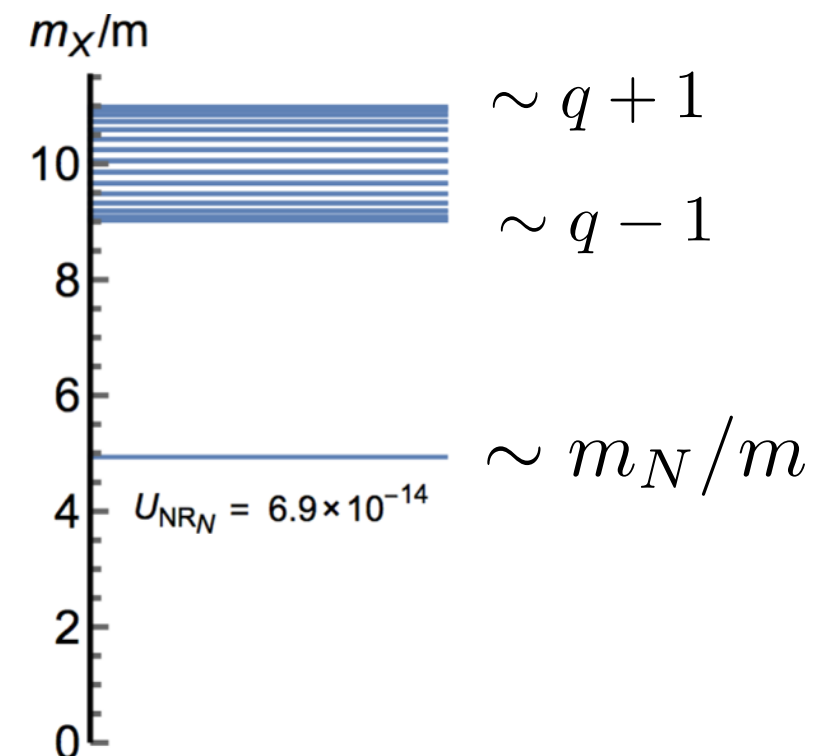
clockwork mechanism
unaffected provided

$$m_N \lesssim qm$$

{ one light & localized mode
N gears with $O(1)$ couplings

(pseudo-Dirac if $q \gg m_N/m$)

($q = 10, N = 15, m_N = 5m$)



Hambye, Teresi & MT (2016)

A CLOCKWORK WIMP - CONSTRUCTION

$$\begin{array}{ccccccc}
 R_0 & L_1 R_1 & & \dots & & L_N R_N & \text{--- SM} \\
 \times & \times & \times & \times & \times & \times & \times
 \end{array}$$

$$\mathcal{L} \supset -m \sum_{i=1}^N \left(\bar{L}_i R_{i-1} - q \bar{L}_i R_i \right) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0 \\
 - y (\bar{L}_{SM} \tilde{H} R_N + h.c.)$$

step 3: couple the chain to the SM

A CLOCKWORK WIMP - CONSTRUCTION

$$\begin{array}{ccccccc}
 R_0 & L_1 R_1 & & \dots & & L_N R_N & \text{--- SM} \\
 \times & \times & \times & \times & \times & \times & \times
 \end{array}$$

$$\mathcal{L} \supset -m \sum_{i=1}^N \left(\bar{L}_i R_{i-1} - q \bar{L}_i R_i \right) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0 \\
 - y (\bar{L}_{SM} \tilde{H} R_N + h.c.)$$

step 3: couple the chain to the SM

$$N \approx R_0 + \frac{1}{q} R_1 + \frac{1}{q^2} R_2 + \dots + \frac{1}{q^N} R_N \rightarrow \mathcal{L} \supset -\frac{y}{q^N} \bar{L}_{SM} \tilde{H} N + h.c.$$



tiny
coupling

A CLOCKWORK WIMP - CONSTRUCTION

$$\begin{array}{ccccccc}
 R_0 & L_1 R_1 & & \dots & & L_N R_N & \text{--- SM} \\
 \times & \times & \times & \times & \times & \times & \times
 \end{array}$$

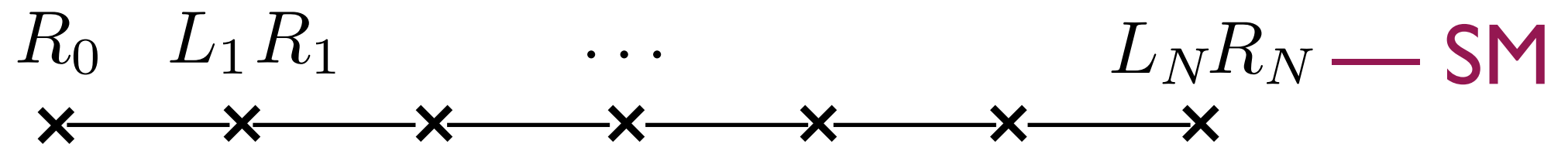
$$\mathcal{L} \supset -m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0 \\
 - y (\bar{L}_{SM} \tilde{H} R_N + h.c.)$$

$$\mathcal{L} \supset \frac{-y}{q^N} \bar{L}_{SM} \tilde{H} N + h.c. \longrightarrow \Gamma(N \rightarrow \nu h, \nu Z, lW) \sim \frac{m_N}{8\pi} \frac{y^2}{q^{2N}}$$

The N mode is **unstable** but lifetime $\gtrsim 10^{26} \text{sec}$ (gamma's, etc.)

$$\text{if } q^{2N} \gtrsim 10^{52} \left(\frac{m_N}{100 \text{GeV}} \right) y^2 \quad \text{e.g. } (q = 10, N = 26, y = 1)$$

A CLOCKWORK WIMP - CONSTRUCTION



$$\mathcal{L} \supset -m \sum_{i=1}^N \left(\bar{L}_i R_{i-1} - q \bar{L}_i R_i \right) + h.c. - \frac{1}{2} m_N \bar{R}_0^c R_0$$

$$- y (\bar{L}_{SM} \tilde{H} R_N + h.c.)$$

last (but not the least) step:

abundance from thermal freeze-out?

$$NN \rightarrow hh, hZ, l\bar{l}, \dots \quad \text{but rate} \propto \frac{y^4}{q^{4N}}$$

i.e. quite suppressed...

A CLOCKWORK WIMP - AT LAST

$$\begin{array}{ccccccc}
 R_0 & L_1 R_1 & & \dots & & L_N R_N & \text{--- SM} \\
 \times & \times & \times & \times & \times & \times & \times
 \end{array}$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kinetic}}$$

$$- \sum_{i=1}^N (y_S S_i \bar{L}_i R_{i-1} - y_C C_i \bar{L}_i R_i + h.c.) \quad \text{☞} \quad \chi SB$$

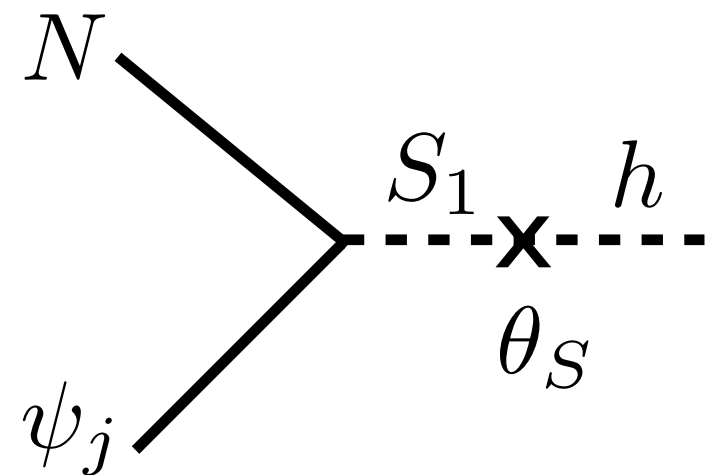
$$- (y \bar{L}_{SM} \tilde{H} R_N + h.c.) - \frac{1}{2} (m_N \bar{R}_0^c R_0 + h.c.) .$$

$$- \sum_{i=1}^N (\lambda_S S_i^\dagger S_i H^\dagger H + \lambda_C C_i^\dagger C_i H^\dagger H) \quad \text{☞} \quad \text{Higgs portal}$$

A CLOCKWORK WIMP - AT LAST

$$\mathcal{L} \supset -\frac{\xi_j^S}{\sqrt{2}} S_1 \bar{\psi}_j P_R N + h.c.$$

$$-\frac{\xi_j}{\sqrt{2}} h \bar{\psi}_j P_R N + h.c.$$



with $\xi_j^S \approx \sqrt{\frac{2}{N+1}} \sin\left(\frac{j\pi}{N+1}\right) y_S$ and $\xi_j \approx \theta_S \xi_j^S$

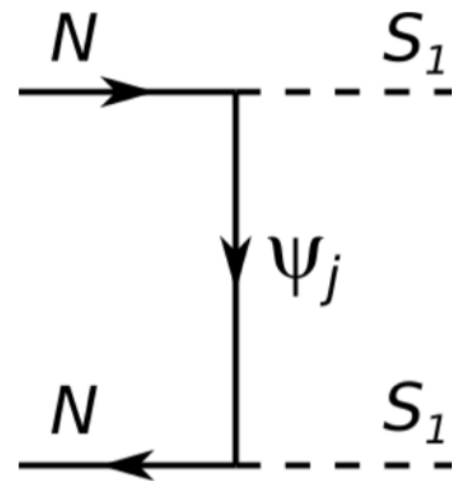
$$NN \leftrightarrow S_1 S_1, S_1 h, hh, \dots \rightarrow \propto \xi_S^2 (\xi^2) = \sum_{j=1}^N |\xi_j^S|^2 \approx y_S^2 (\theta_S)^2$$



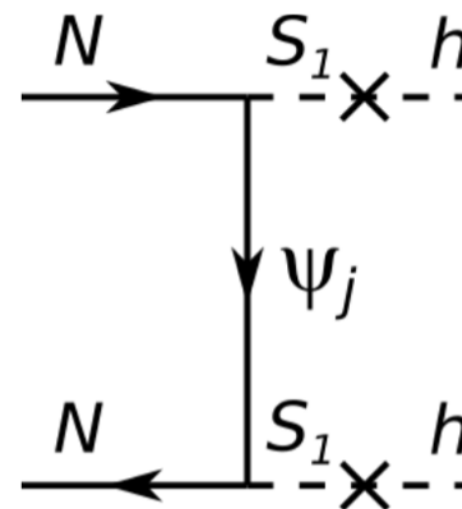
O(1) couplings, the N could be a WIMP!

A CLOCKWORK WIMP - TO RECAP

Abundance



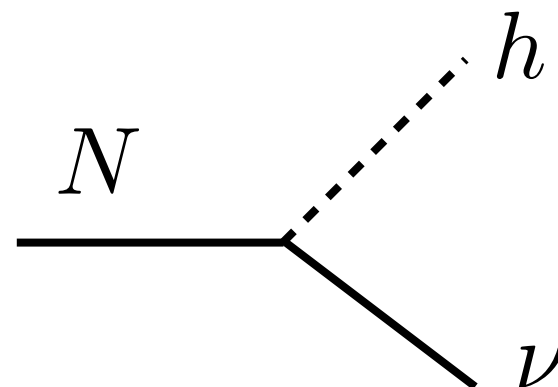
(scenario A)



(scenario B)

annihilation
through the
chiral gears

Stability/decay

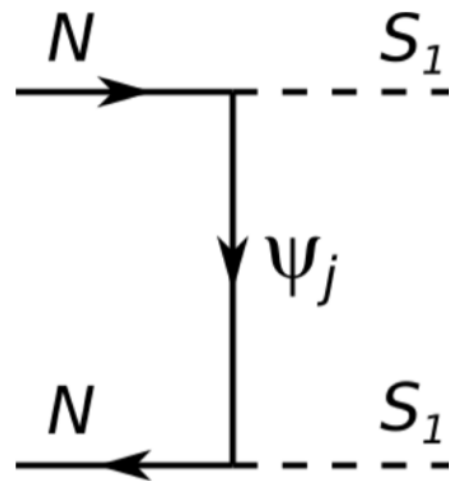


$$\frac{1}{q^N}$$

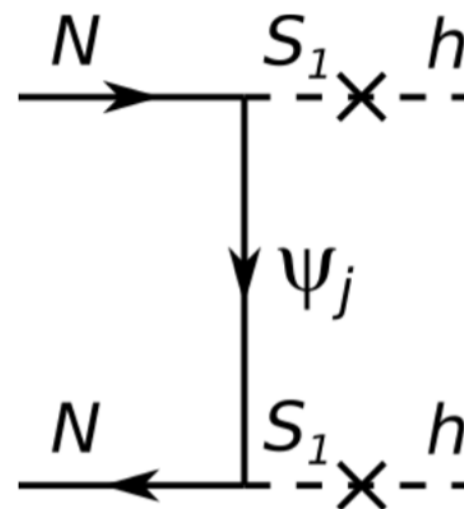
suppressed by
the chiral chain

A CLOCKWORK WIMP - TO RECAP

Abundance



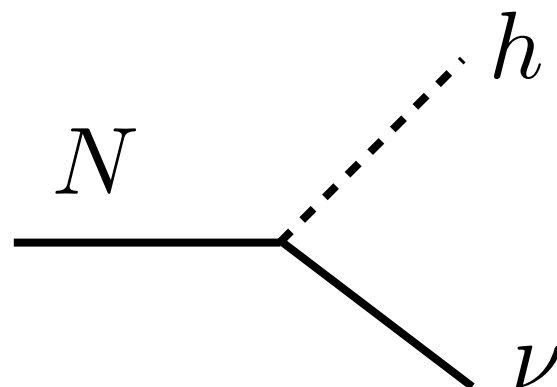
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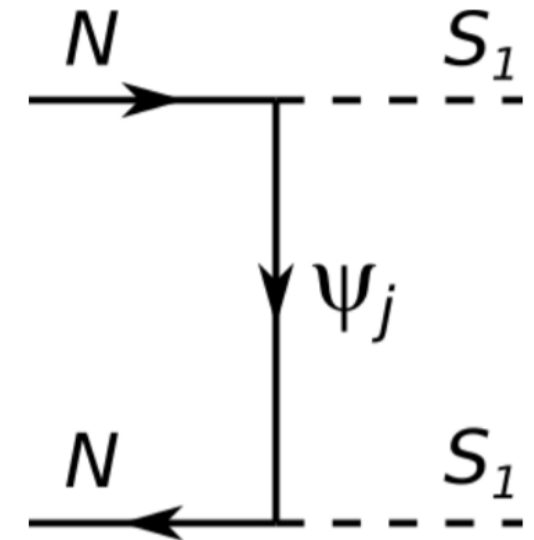
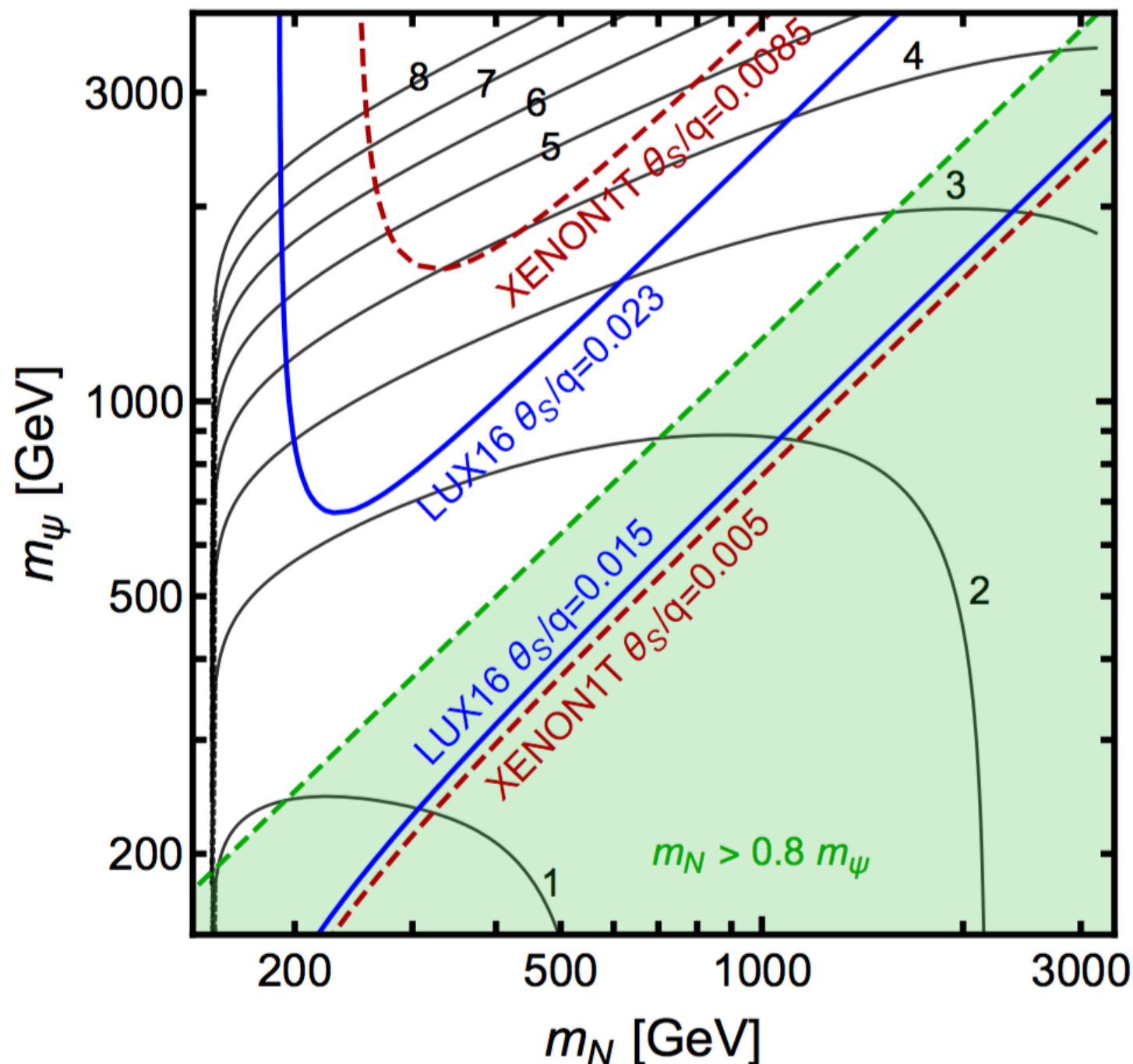
Diagonal coupling to Higgs
(taking into account the pseudo-Dirac nature
of the clockwork gears)

$$\mathcal{L} \supset \frac{m_N}{\sqrt{2}m} \frac{\xi}{q^2} \bar{N}^c N h + h.c.$$



A CLOCKWORK WIMP — SCENARIO A

$$m_{S_1} = 150 \text{ GeV}$$



black solid: y_S required
for $\Omega_{dm} \approx 0.25$

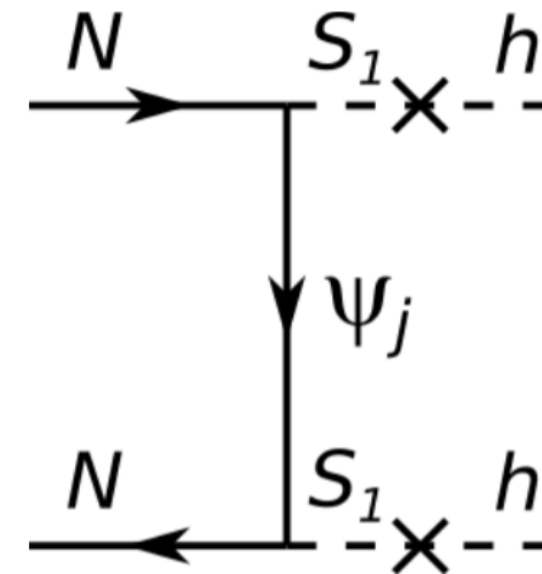
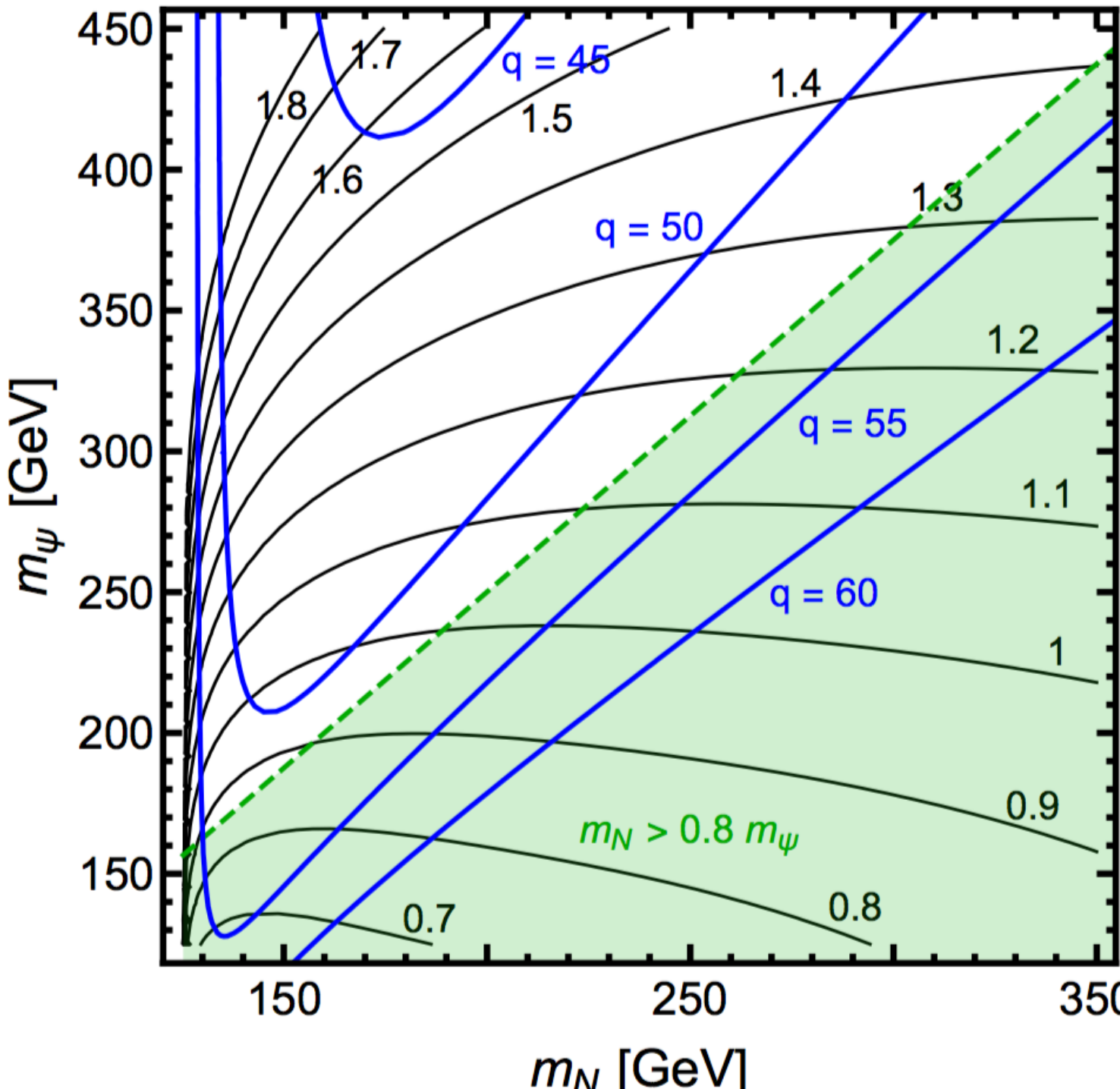
blue solid: LUX16
exclusions (below lines)

red dashed: Xenon1T
reach



Higgs mediated $\propto (\theta_S/q)^2$

A CLOCKWORK WIMP — SCENARIO B



$$\theta_S \lesssim \frac{0.4}{\sqrt{N}}$$

black solid: $y_S \theta_S$ required
for $\Omega_{dm} \approx 0.25$

blue solid: LUX16
exclusions (below lines)

A CLOCKWORK WIMP — BASIC PHENO

Indirect detection p-wave annihilation, but decay into
monochromatic neutrinos $N \rightarrow h\nu$

e.g. El Aisati, Gustafsson & Hambye (2015)

A CLOCKWORK WIMP — BASIC PHENO

Indirect detection p-wave annihilation, but decay into
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Collider searches ψ_j in the hundred GeV/TeV range, coupled via
 $y\bar{L}_{SM}HR_N$ with y sizable and $\psi_j \supset R_N$



observable pseudo-Dirac sterile RH neutrinos!

EWPT: $|\theta_{l\psi}|^2 \equiv y^2 v^2 / (2m_\psi^2) \lesssim 10^{-3}$

LFV: $BR(\mu \rightarrow e\gamma) \sim 10^{-3} |\theta_{e\psi}|^2 |\theta_{\mu\psi}|^2 \lesssim 10^{-13}$

L conserving searches: $m_\psi \lesssim 200 \text{ GeV}$ with 300 fb^{-1}

Das, Dev & Okada (2014)

L violating searches: $m_\psi \lesssim 300 \text{ GeV}$ with 300 fb^{-1}

$m_N \lesssim m_\psi$

Deppisch, Dev & Pilaftsis (2015)

assuming
single state
approximation

A CLOCKWORK WIMP — BASIC PHENO

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Deppisch, Dev & Pilaftsis (2015)

large mixing h-S in scenario B $\theta_S \lesssim 0.3 - 0.4$ (direct or EWPT)

e.g. Falkowski, Gross & Lebedev (2015)



assuming
single state
approximation

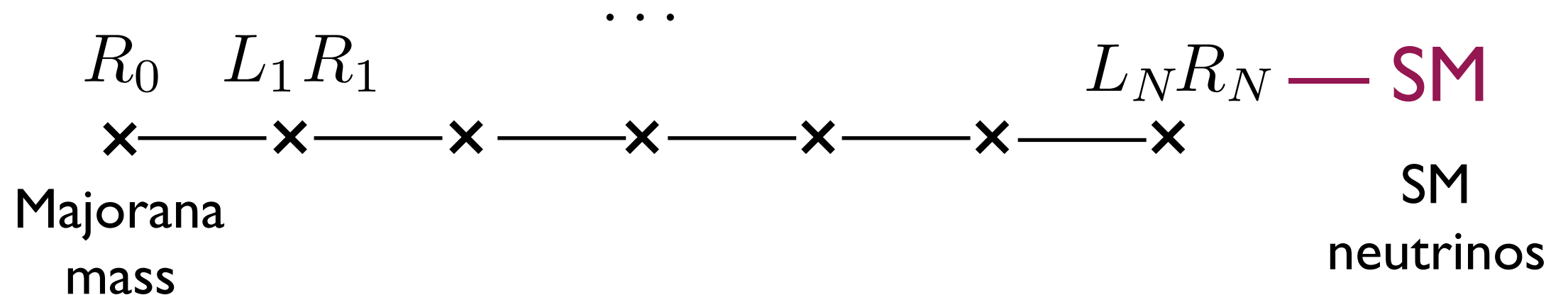
A CLOCKWORK WIMP IS POSSIBLE



CLOCKWORK MAJORANA NEUTRINO MASS

Hambye, Teresi & MT (2016)

another chain...



has to go through the whole chain

$$m_\nu \sim y^2 \frac{v^2}{q^N M}$$

smaller chain than for DM stability ($q = 10, N = 7, M = 1 \text{ TeV}$)

but one/SM neutrino mass!

(i.e. at least 2 chains for neutrino, one for the DM...)

See also Carena, Li, Machado, Machado & Wagner (2017); Ibarra, Kushwaha & Vempati (2017)

A LOT OF FIELDS, WITH VERY SPECIFIC COUPLINGS...



A CLOCKWORK WIMP FROM DECONSTRUCTION



Lagrangian can be related to a discretized flat 5th dimension (Z)

$$1) \quad \mathcal{L}_5 \supset \bar{\psi}(i \overleftrightarrow{\not{D}} - M)\psi = i\bar{\psi}\gamma^\mu \partial_\mu \psi + \left[\frac{1}{2} (\bar{L} \partial_Z R - \partial_Z \bar{L} R) - M \bar{L} R + h.c. \right]$$

2) naive discretization $\Rightarrow$ fermion doubling

$\Rightarrow$ add a Wilson term $-\frac{a}{2} \partial_Z \bar{\psi} \partial_Z \psi$ with lattice spacing $a = \frac{\pi R}{N} \rightarrow 0$

3) Dirichlet condition $L(0) = 0 \Rightarrow$ surviving chiral mode R_0

4) Standard Model degrees of freedom at $Z = \pi R$ ($\sim$ braneworld)

$$\mathcal{L} \supset \sum_{i=0}^{N-1} \frac{1}{a} \bar{L}_{i+1} R_i - \sum_{i=1}^N \left(\underbrace{\frac{1}{a} + M}_{qm} \right) \bar{L}_i R_i$$

$\downarrow$
 m

well-defined
continuum limit

$$q^N = \left(1 + \frac{\pi R M}{N} \right)^N \rightarrow e^{\pi R M}$$

$$1/M \ll \pi R$$

CONCLUSIONS

We have build a **clockwork WIMP**

It is accidentally stable (not protected by a symmetry)

Its decay is mediated by the very same
degrees of freedom that determine its abundance

These many degrees of freedom lie in the 100 GeV-TeV range

They could be seen at the LHC as RHN

There are also (possibly) plenty of scalars

Notice that it's also a framework for low scale SM neutrino Majorana mass

If you care, the clockwork could be seen as an extra spatial dimension





A Clockwork Wimp

by [Lord_Gino_X](#) ©

A Clockwork Wimp (Book 1 of 2)

Chapter 1

I slowly slide the large window open and climbed inside without making any noise. But there were noises. Noises that were tearing my ear drums apart. Noises that were directly coming from my chest. I knew I have to control my breathing in order to slow down my heart. It was becoming a distraction. A distraction I cannot afford. Luckily, I need less than a minute to do what I am here to do.

ON SCALES AND MASSES

following Giudice & McCullough

new physics (local) $\longrightarrow$ effective operators $\longrightarrow$ new scale

$$\mathcal{L} \sim \frac{M^4}{g^2} \longrightarrow [\phi] = \frac{M}{g}, \quad [\psi] = \frac{M^{3/2}}{g}$$

a scale a mass

$$\mathcal{O} = \frac{1}{\Lambda^{d-4}} \phi^{n_B} \psi^{n_F} \longrightarrow \Lambda = \frac{M}{g^{\frac{n_B + n_F - 2}{d-4}}}$$


$$G_F^{-1/2} \sim \frac{M_W}{g}$$

Fermi

$$\Lambda_\nu \sim \frac{M_R}{y^2}$$

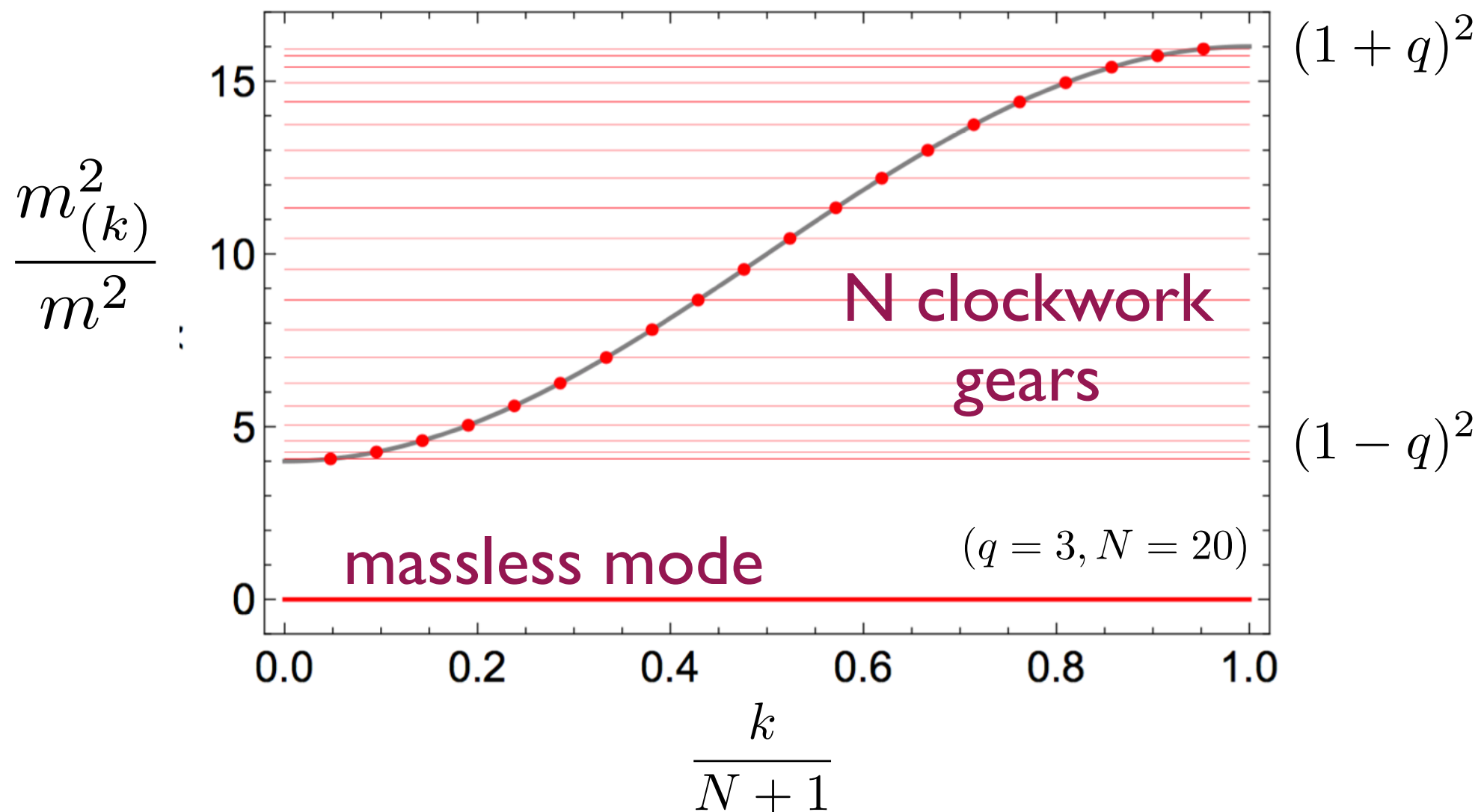
Weinberg

$$M_P \sim \frac{M_s}{g_s}$$

Planck

THE CLOCKWORK MECHANISM ILLUSTRATED

$$m_{(k)}^2 = m^2 \left(1 + q^2 - 2q \cos \left(\frac{k\pi}{N+1} \right) \right)$$



THE CLOCKWORK MECHANISM ILLUSTRATED

$$\varphi^{(k)} = \sum_{n=0}^N a_n^{(k)} \phi_n$$

clockwork gears
have unsuppressed
overlap at all sites

