Back to Weinberg's 3HDM: boundedness from below

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3HDM with $U(1) \times U(1)$

The Higgs potential of the $U(1) \times U(1)$-symmetric 3HDM:

$$V = V_2 + V_N + V_{CB}\,,$$

where $V_2 = m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{33}^2(\phi_3^\dagger\phi_3)$ and

$$\begin{aligned} V_N &= \frac{\lambda_{11}}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_{22}}{2}(\phi_2^\dagger\phi_2)^2 + \frac{\lambda_{33}}{2}(\phi_3^\dagger\phi_3)^2 \\ &\quad + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)\,, \\ V_{CB} &= \lambda'_{12} z_{12} + \lambda'_{13} z_{13} + \lambda'_{23} z_{23}\,. \end{aligned}$$

Here, $z_{ij} = (\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) - (\phi_i^\dagger\phi_j)(\phi_j^\dagger\phi_i)$.
"Neutral" directions: $\phi_1 \propto \phi_2 \propto \phi_3 \to z_{ij} = 0$.

Other directions in the Higgs space are called "charge-breaking" directions.

3HDM with $\mathbb{Z}_2 \times \mathbb{Z}_2$

Weinberg's 3HDM [PRL37 (1976) 657] with $\approx$ 1000 citations:

$$V = V_2 + V_N + V_{CB} + V_{\mathbb{Z}_2\times\mathbb{Z}_2}\,,$$

where

$$V_{\mathbb{Z}_2\times\mathbb{Z}_2} = \bar\lambda_{12}(\phi_1^\dagger\phi_2)^2 + \bar\lambda_{13}(\phi_1^\dagger\phi_3)^2 + \bar\lambda_{23}(\phi_2^\dagger\phi_3)^2 + H.c.$$

with $\bar\lambda_{ij}$ in general complex (explicit CPV + natural flavor conservation).

- When exploring Weinberg's model, it is usually assumed that the potential is bounded from below (BFB);
- Necessary and sufficient BFB conditions are still not found!
- If we want to explore all pheno situations offered by Weinberg's model, we need to establish the exact BFB conditions.

3HDM with $\mathbb{Z}_2 \times \mathbb{Z}_2$

A recent twist:

- Recent years saw a revived interest in 3HDMs, in particular when combining active and inert doublets ($\to$ DM candidates), including cases with "dark democracy": [0904.2173, 1012.4680, 1302.3713, 1504.06432, 1608.01673].
- Although $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not always exact in these models, they are equivalent to Weinberg's model in what concerns the BFB conditions.
- In 2009, [Grzadkowski, Ogreid, Osland, 0904.2173] proposed a set of the exact BFB conditions.
- Very recently, in [Faro, Ivanov, 1907.01963], it was shown that these conditions are sufficient but not necessary.
- Sufficient conditions are easy to establish, but they miss viable, potentially intriguing parameter space regions offer by the model.

Towards the BFB conditions in Weinberg's model

We tried to derive the exact BFB conditions in Weinberg's model, but have not succeeded so far. Here is what we do have:

- Exact BFB conditions in $U(1) \times U(1)$ 3HDM [Faro, Ivanov, 1907.01963], which can be a viable model (if unbroken by vevs or with soft breaking). The method is to apply a couple of cute tricks to render the problem treatable by the copositivity conditions.
- A couple of ideas how to attack the $\mathbb{Z}_2 \times \mathbb{Z}_2$-symmetric case, not yet finished!
- The suspicion that it may be impossible to present these conditions as familiar inequalities on coefs!
- The hope that this is not the end of the story: a different way to parametrize the model may help!

Conclusions

- Our main goal, when building bSM models, is to look for new interesting pheno consequences. But all these examples must be technically self-consistent.
- A basic task needed for a systematic exploration of Weinberg's model (and similar 3HDMs) has not yet been solved in a satisfactory way. Previous attempts were too simplistic and the difficulty of the problem was not widely recognized.
- Finding the exact BFB conditions for Weinberg's model remains an intriguing and challenging problem. It may teach us something useful in general, applicable to other multi-Higgs models.