Finite family groups for fermionic and leptoquark mixing patterns

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Based on

[Re]constructing Finite Flavour Groups: Horizontal Symmetry Scans from the Bottom-Up
Jim Talbert (Oxford U., Theor. Phys.). Sep 25, 2014. 18 pp.
Published in **JHEP 1412 (2014) 058**
DOI: 10.1007/JHEP12(2014)058
e-Print: **arXiv:1409.7310 [hep-ph] | PDF**

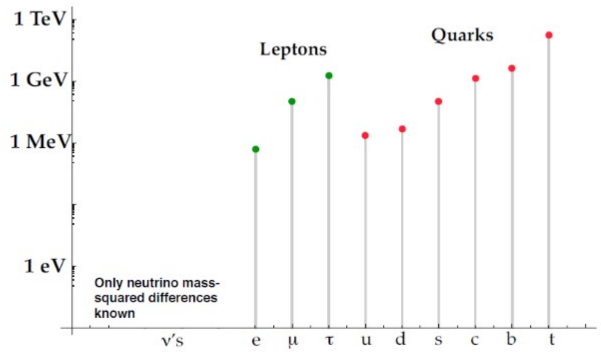
Clues for flavor from rare lepton and quark decays
Ivo de Medeiros Varzielas (Southampton U.), Gudrun Hiller (Tech. U., Dortmund (main)). Mar 3, 2015. 30 pp.
Published in **JHEP 1506 (2015) 072**
DO-TH-15-02, QFET-2015-04
DOI: 10.1007/JHEP06(2015)072
e-Print: **arXiv:1503.01084 [hep-ph] | PDF**

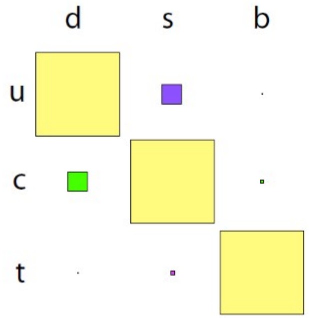
Bottom-Up Discrete Symmetries for Cabibbo Mixing
Ivo de Medeiros Varzielas (Southampton U.), Rasmus W. Rasmussen (DESY, Zeuthen), Jim Talbert (Oxford U., Theor. Phys. & Oxford U., Theor. Phys.). May 11, 2016. 21 pp.
Published in **Int.J.Mod.Phys. A32 (2017) no.06n07, 1750047**
DESY-16-125
DOI: 10.1142/S0217751X17500476
e-Print: **arXiv:1605.03581 [hep-ph] | PDF**

Simplified Models of Flavourful Leptoquarks
Ivo de Medeiros Varzielas (Lisbon U. & Lisbon, CFTP), Jim Talbert (DESY). Jan 29, 2019. 29 pp.
Published in **Eur.Phys.J. C79 (2019) no.6, 536**
DESY-18-210, DESY 18-210
DOI: 10.1140/epjc/s10052-019-7047-2
e-Print: **arXiv:1901.10484 [hep-ph] | PDF**

Finite Family Groups for Fermionic and Leptoquark Mixing Patterns
Jordan Bernigaud (Annecy, LAPTH), Ivo de Medeiros Varzielas (Lisbon, CFTP), Jim Talbert (DESY). Jun 26, 2019. 39 pp.
LAPTH-033/19, DESY-19-091
e-Print: **arXiv:1906.11270 [hep-ph] | PDF**

Flavour problem

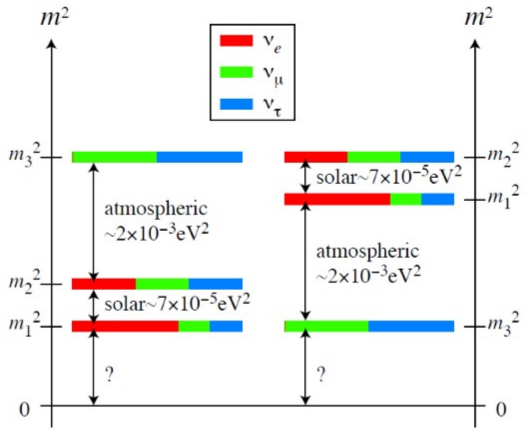


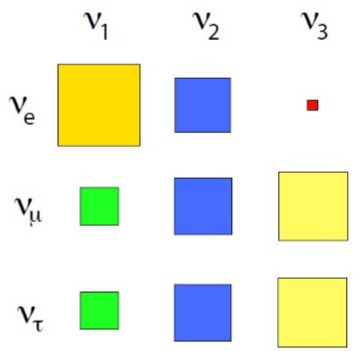


Quarks

CKM (PDG)

$$\begin{pmatrix} \binom{0.97456}{0.97436} & \binom{0.22496}{0.22408} & \binom{0.00377}{0.00353} \\ \binom{0.22482}{0.22394} & \binom{0.97369}{0.97348} & \binom{0.04290}{0.04138} \\ \binom{0.00920}{0.00873} & \binom{0.04207}{0.04059} & \binom{0.999137}{0.999073} \end{pmatrix}$$





Leptons

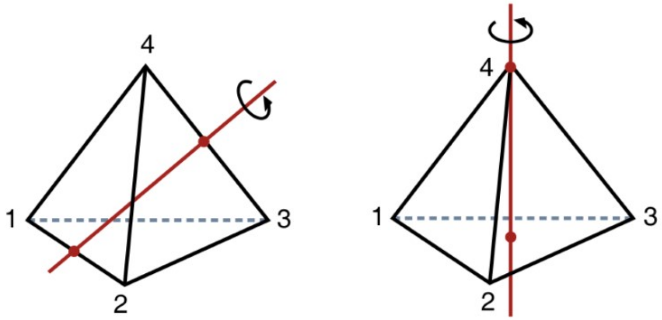
PMNS (Nufit)

$$\begin{pmatrix} \binom{0.844}{0.799} & \binom{0.582}{0.516} & \binom{0.156}{0.141} \\ \binom{0.494}{0.242} & \binom{0.678}{0.467} & \binom{0.774}{0.639} \\ \binom{0.521}{0.284} & \binom{0.695}{0.490} & \binom{0.754}{0.615} \end{pmatrix}$$

Non-Abelian Discrete Symmetries (NADS)

A_4 example

Group that leaves the tetrahedron invariant



$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \; , \quad \omega = e^{i\frac{2\pi}{3}} \quad \omega^3 = 1$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \; ; \quad T^3 = S^2 = \mathbb{1}$$

Field	L	e^c	μ^c	τ^c	ν^c	ϕ_T	ϕ_S
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$

Yukawas from flavons ϕ_i charged under A_4 and other "driving symmetries".

$$\begin{cases} \phi_T \to (v_T, 0, 0) \\ \phi_S \to (v_S, v_S, v_S) \end{cases}$$

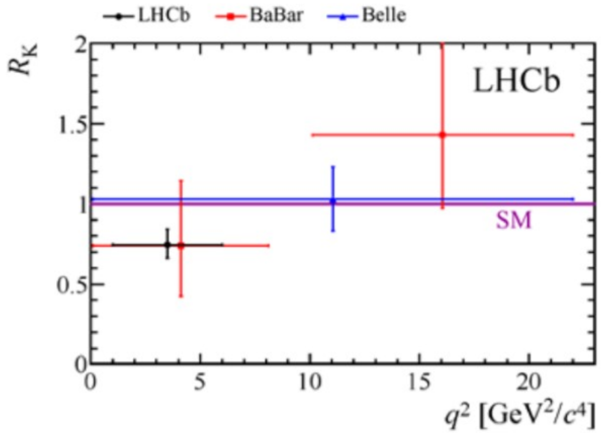
$T\langle\phi_T\rangle = \langle\phi_T\rangle \qquad S\langle\phi_S\rangle = \langle\phi_S\rangle$

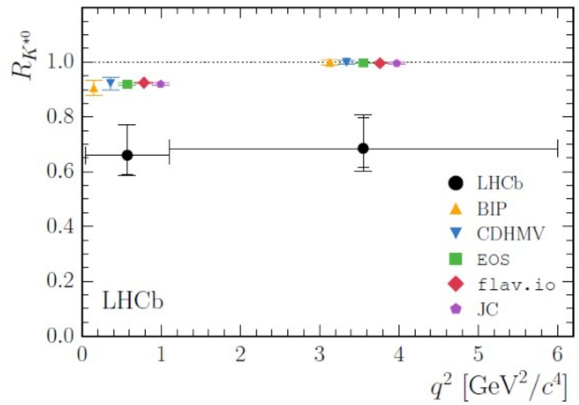
$\underbrace{m_\ell \text{ invariant under T}}_{(L\phi_T)[e^c + \mu^c + \tau^c]\ldots}$, m_ν invariant under S.

$\Longrightarrow U_{PMNS} = U_{TB}$

Hints of new physics in B decays?

$$R_H = \frac{\int \frac{d\Gamma(B \to H\mu^+\mu^-)}{dq^2} dq^2}{\int \frac{d\Gamma(B \to He^+e^-)}{dq^2} dq^2} \qquad R_H = \frac{\mathcal{B}(B \to H\mu^+\mu^-)}{\mathcal{B}(B \to HJ/\psi(\to \mu^+\mu^-))} \Bigg/ \frac{\mathcal{B}(B \to He^+e^-)}{\mathcal{B}(B \to HJ/\psi(\to e^+e^-))}$$



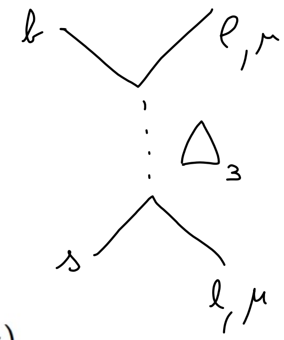


$$R_K = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$$

$$R_{K^*} = \begin{cases} 0.66 \ {}^{+\ 0.11}_{-\ 0.07}\,(\text{stat}) \pm 0.03\,(\text{syst}) & \text{for } 0.045 < q^2 < 1.1 \ \text{GeV}^2/c^4 \\ 0.69 \ {}^{+\ 0.11}_{-\ 0.07}\,(\text{stat}) \pm 0.05\,(\text{syst}) & \text{for } 1.1 \quad < q^2 < 6.0 \ \text{GeV}^2/c^4 \end{cases}$$

<u>Update Moriond 2019 :</u> $R_K = 0.846^{+0.060}_{-0.054}\ (stat.)\ ^{+0.016}_{-0.014}\ (syst.)$

Leptoquark extensions



Vector singlet, triplet and scalar triplet models

$$\Delta_3 \sim (\bar{3}, 3, 1/3)\,, \quad \Delta_1^\mu \sim (3, 1, 2/3)\,, \quad \Delta_3^\mu \sim (3, 3, 2/3)\,,$$

We will focus on the scalar triplet (very similar for the other extensions).

$$\Delta_3 : \qquad \mathcal{L} \supset y^{LL}_{3,ij}\, \bar{Q}_L^{C\,i,a} \epsilon^{ab} (\tau^k \Delta_3^k)^{bc} L_L^{j,c} + z^{LL}_{3,ij}\, \bar{Q}_L^{C\,i,a} \epsilon^{ab} ((\tau^k \Delta_3^k)^\dagger)^{bc} Q_L^{j,c} + \text{h.c.}$$

In the **fermion mass basis** (LQ redefinition + flavour rotations)

$$\begin{aligned}\mathcal{L}^{LQ}_{mass} \supset\ & \underbrace{-(U_d^T y_3^{LL} U_\nu)_{ij}}_{\lambda_{d\nu}}\, \bar{d}_L^{C\,i} \Delta_3^{1/3} \nu_L^j \underbrace{-\sqrt{2}(U_d^T y_3^{LL} U_l)_{ij}}_{\lambda_{dl}}\, \bar{d}_L^{C\,i} \Delta_3^{4/3} l_L^j \\ & + \underbrace{\sqrt{2}(U_u^T y_3^{LL} U_\nu)_{ij}}_{\lambda_{u\nu}}\, \bar{u}_L^{C\,i} \Delta_3^{-2/3} \nu_L^j \underbrace{-(U_u^T y_3^{LL} U_l)_{ij}}_{\lambda_{ul}}\, \bar{u}_L^{C\,i} \Delta_3^{1/3} l_L^j \\ & + \text{h.c.}\end{aligned}$$

[fermion mass basis]

$$\mathcal{L}^{Y}_{LQ} \supset \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{d}_L^C\, \lambda_{dl}\, l_L\, \Delta_3^{4/3} + \text{h.c.} \qquad -\sqrt{2}\, (U_d^T\, y_3^{LL}\, U_l) \equiv \lambda_{dl} = \lambda_0 \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

$R_K^{(*)}$

Controlling LQ flavour couplings with symmetries

$$SM \;\sim\; q_L^i\,(3,2)_{+1/3}\,,\quad \overline{u}_L^i\,(\overline{3},1)_{-4/3}\,,\quad \overline{d}_L^i\,(\overline{3},1)_{+2/3}\,,\quad l_L^i\,(1,2)_{-1}\,,\quad \overline{e}_L^i\,(1,1)_{+2}$$

$$\mathbf{U(3)^5 \;\sim\; U(3)_q \;\times\; U(3)_u \;\times\; U(3)_d \;\times\; U(3)_l \;\times\; U(3)_e}$$

Yukawa / Mass terms have innate symmetries

$$\mathcal{L}^{SM}_{mass} \supset \frac{1}{2}\bar{\nu}^c_L\, m_\nu\, \nu_L + \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{u}_R\, m_u\, u_L + \text{h.c.}$$

$$\nu_L \to T_{\nu_i}\nu_L\,, \quad \text{with} \quad T_{\nu 1} = diag\,(1,-1,-1) \quad \text{and} \quad T_{\nu 2} = diag\,(-1,1,-1)\,,$$

$$f \to T_f f\,, \quad \text{with} \quad T_f = diag\left(e^{i\alpha_f}, e^{i\beta_f}, e^{i\gamma_f}\right) \quad \text{for} \quad f \in \{E_R, l_L, d_R, d_L, u_R, u_L\}.$$

we assume are remnants of the parent group

$$G_F \longrightarrow \begin{cases} G_L \longrightarrow \begin{cases} G_\nu \\ G_l \end{cases} \\ G_d \longrightarrow \begin{cases} G_u \\ G_d \end{cases} \end{cases}$$

$$\mathcal{L}_{mass}^{LQ} = \bar{d}_L^C\,\lambda_{dl}\,l_L\,\Delta_3^{4/3} + \bar{d}_L^C\,\lambda_{d\nu}\,\nu_L\,\Delta_3^{1/3} + \bar{u}_L^C\,\lambda_{ul}\,l_L\,\Delta_3^{1/3} + \bar{u}_L^C\,\lambda_{u\nu}\,\nu_L\,\Delta_3^{-2/3} + \text{h.c.}$$

Hypothesis: LQ terms invariant under the same residual symmetries as the fermion Yukawa

$$T_Q^{(T,\dagger)}\,\lambda_{QL}\,T_L \stackrel{!}{=} \lambda_{QL}$$

$$\begin{pmatrix} e^{i(\alpha_d+\alpha_l)}\,\lambda_{de} & e^{i(\alpha_d+\beta_l)}\,\lambda_{d\mu} & e^{i(\alpha_d+\gamma_l)}\,\lambda_{d\tau} \\ e^{i(\beta_d+\alpha_l)}\,\lambda_{se} & e^{i(\beta_d+\beta_l)}\,\lambda_{s\mu} & e^{i(\beta_d+\gamma_l)}\,\lambda_{s\tau} \\ e^{i(\gamma_d+\alpha_l)}\,\lambda_{be} & e^{i(\gamma_d+\beta_l)}\,\lambda_{b\mu} & e^{i(\gamma_d+\gamma_l)}\,\lambda_{b\tau} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

This constraint is fulfilled only by a few patterns

'Isolation patterns'

$$\lambda_{dl}^{[e]} \sim \begin{pmatrix} \lambda_{de} & 0 & 0 \\ \lambda_{se} & 0 & 0 \\ \lambda_{be} & 0 & 0 \end{pmatrix},\quad \lambda_{dl}^{[\mu]} \sim \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ 0 & \lambda_{s\mu} & 0 \\ 0 & \lambda_{b\mu} & 0 \end{pmatrix},\quad \lambda_{dl}^{[\tau]} \sim \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ 0 & 0 & \lambda_{s\tau} \\ 0 & 0 & \lambda_{b\tau} \end{pmatrix}$$

'Two column patterns'

$$\lambda_{dl}^{[e\mu]} \sim \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix},\quad \lambda_{dl}^{[e\tau]} \sim \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix},\quad \lambda_{dl}^{[\mu\tau]} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

'Three column patterns'

$$\lambda_{dl}^{[e\mu 1]} \sim \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix},\quad \lambda_{dl}^{[e1\tau]} \sim \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix},\quad \lambda_{dl}^{[1\mu\tau]} \sim \begin{pmatrix} \lambda_{de} & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

SU(2) relations

$$\mathcal{L}_{LQ}^{Y} \supset \frac{1}{2}\bar{\nu}_L^c\, m_\nu\, \nu_L + \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{u}_R\, m_u\, u_L + \bar{d}_L^C\, \lambda_{dl}\, l_L\, \Delta_3^{4/3} + \bar{d}_L^C\, \lambda_{d\nu}\, \nu_L\, \Delta_3^{1/3} + \bar{u}_L^C\, \lambda_{ul}\, l_L\, \Delta_3^{1/3} + \bar{u}_L^C\, \lambda_{u\nu}\, \nu_L\, \Delta_3^{-2/3} + \text{h.c.}$$

$$\lambda_{d\nu} = \frac{1}{\sqrt{2}}\lambda_{dl}\, U_{PMNS}, \quad \lambda_{ul} = \frac{1}{\sqrt{2}} U_{CKM}^{\star}\, \lambda_{dl}, \quad \lambda_{u\nu} = -U_{CKM}^{\star}\, \lambda_{dl}\, U_{PMNS}$$

To proceed, we considered two different frameworks, SE1 and SE2

`SE1'

$$T_Q^T\, \lambda_{QL}\, T_L \stackrel{!}{=} \lambda_{QL} \quad \forall \quad \{Q, L\}$$

Extremely constraining: 1 free parameter
Can be realized with flavons
Requires SU(2) breaking in the LQ terms

`SE2'

$$T_d^T\, \lambda_{dl}\, T_l \stackrel{!}{=} \lambda_{dl}$$

Easily realized with flavons
Mimics existing models
Less constraining

SE1 conclusions

Catalogued all patterns and matrices
(1 free parameter)

$U_{13}=0$

$$\lambda_{dl}^{[e3B]} = \lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -\frac{V_{cb}}{V_{cs}} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \simeq \lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -A\lambda^2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_{d\nu}^{[e3B]} = \frac{\lambda_{be}}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ -U_{11}\frac{V_{cb}}{V_{cs}} & -U_{12}\frac{V_{cb}}{V_{cs}} & 0 \\ U_{11} & U_{12} & 0 \end{pmatrix} \simeq \lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -A\lambda^2(0.58) & -A\lambda^2(0.39) & 0 \\ 0.58 & 0.39 & 0 \end{pmatrix}$$

$$\lambda_{ul}^{[e3B]} = \frac{\lambda_{be}}{\sqrt{2}}\begin{pmatrix} \left(-\frac{V_{us}V_{cb}}{V_{cs}} + V_{ub}\right) & 0 & 0 \\ 0 & 0 & 0 \\ \left(-\frac{V_{cb}V_{ts}}{V_{cs}} + V_{tb}\right) & 0 & 0 \end{pmatrix} \simeq \lambda_{be}\begin{pmatrix} -A\lambda^3(0.71)(1-\rho-i\eta) & 0 & 0 \\ 0 & 0 & 0 \\ 0.71 & 0 & 0 \end{pmatrix}$$

$$\lambda_{u\nu}^{[e3B]} = \lambda_{be}\begin{pmatrix} U_{11}\left(\frac{V_{us}V_{cb}}{V_{cs}} - V_{ub}\right) & U_{12}\left(\frac{V_{us}V_{cb}}{V_{cs}} - V_{ub}\right) & 0 \\ 0 & 0 & 0 \\ U_{11}\left(\frac{V_{cb}V_{ts}}{V_{cs}} - V_{tb}\right) & U_{12}\left(\frac{V_{cb}V_{ts}}{V_{cs}} - V_{tb}\right) & 0 \end{pmatrix}$$

$$\simeq \lambda_{be}\begin{pmatrix} A\lambda^3(0.82)(1-\rho-i\eta) & A\lambda^3(0.55)(1-\rho-i\eta) & 0 \\ 0 & 0 & 0 \\ -0.82 & -0.55 & 0 \end{pmatrix}$$

hep-ph/1901.10484

λ_{QL}	Phase Equalities		λ_{dl}
λ_{QL}^{e3A}	Δ_3	$\{\beta_d, \gamma_d, -\alpha_\nu, -\beta_\nu, -\alpha_l, \beta_u, \gamma_u\}$	$\lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -\frac{V_{ub}}{V_{us}} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \alpha_\nu, \beta_\nu, \alpha_l, \beta_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l\}\ \{\alpha_\nu, \beta_\nu, \beta_u, \gamma_u\}$	
λ_{QL}^{e3B}	Δ_3	$\{\beta_d, \gamma_d, -\alpha_\nu, -\beta_\nu, -\alpha_l, \alpha_u, \gamma_u\}$	$\lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -\frac{V_{cb}}{V_{cs}} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \alpha_\nu, \beta_\nu, \alpha_l, \alpha_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l\}\ \{\alpha_\nu, \beta_\nu, \alpha_u, \gamma_u\}$	
λ_{QL}^{e3C}	Δ_3	$\{\beta_d, \gamma_d, -\alpha_\nu, -\beta_\nu, -\alpha_l, \alpha_u, \beta_u\}$	$\lambda_{be}\begin{pmatrix} 0 & 0 & 0 \\ -\frac{V_{tb}}{V_{ts}} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \alpha_\nu, \beta_\nu, \alpha_l, \alpha_u, \beta_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l\}\ \{\alpha_\nu, \beta_\nu, \alpha_u, \beta_u\}$	
$\lambda_{QL}^{e\mu 1A}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\alpha_l, -\beta_l, \beta_u, \gamma_u\}$	$\lambda_{b\mu}\begin{pmatrix} 0 & 0 & 0 \\ \frac{V_{ub}}{V_{us}}\frac{U_{21}}{U_{11}} & -\frac{V_{ub}}{V_{us}} & 0 \\ -\frac{U_{21}}{U_{11}} & 1 & 0 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \alpha_l, \beta_l, \beta_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l, \beta_l\}\ \{\beta_\nu, \gamma_\nu, \beta_u, \gamma_u\}$	
$\lambda_{QL}^{e\mu 1B}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\alpha_l, -\beta_l, \alpha_u, \gamma_u\}$	$\lambda_{b\mu}\begin{pmatrix} 0 & 0 & 0 \\ \frac{U_{21}}{U_{11}}\frac{V_{cb}}{V_{cs}} & -\frac{V_{cb}}{V_{cs}} & 0 \\ -\frac{U_{21}}{U_{11}} & 1 & 0 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \alpha_l, \beta_l, \alpha_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l, \beta_l\}\ \{\beta_\nu, \gamma_\nu, \alpha_u, \gamma_u\}$	
$\lambda_{QL}^{e\tau 1A}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\alpha_l, -\gamma_l, \beta_u, \gamma_u\}$	$\lambda_{b\tau}\begin{pmatrix} 0 & 0 & 0 \\ \frac{U_{31}}{U_{11}}\frac{V_{ub}}{V_{us}} & 0 & -\frac{V_{ub}}{V_{us}} \\ -\frac{U_{31}}{U_{11}} & 0 & 1 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \alpha_l, \gamma_l, \beta_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l, \gamma_l\}\ \{\beta_\nu, \gamma_\nu, \beta_u, \gamma_u\}$	
$\lambda_{QL}^{e\tau 1B}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\alpha_l, -\gamma_l, \alpha_u, \gamma_u\}$	$\lambda_{b\tau}\begin{pmatrix} 0 & 0 & 0 \\ \frac{U_{31}}{U_{11}}\frac{V_{cb}}{V_{cs}} & 0 & -\frac{V_{cb}}{V_{cs}} \\ -\frac{U_{31}}{U_{11}} & 0 & 1 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \alpha_l, \gamma_l, \alpha_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \alpha_l, \gamma_l\}\ \{\beta_\nu, \gamma_\nu, \alpha_u, \gamma_u\}$	
$\lambda_{QL}^{\mu\tau 1A}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\beta_l, -\gamma_l, \beta_u, \gamma_u\}$	$\lambda_{b\tau}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{U_{31}}{U_{21}}\frac{V_{ub}}{V_{us}} & -\frac{V_{ub}}{V_{us}} \\ 0 & -\frac{U_{31}}{U_{21}} & 1 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \beta_l, \gamma_l, \beta_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \beta_l, \gamma_l\}\ \{\beta_\nu, \gamma_\nu, \beta_u, \gamma_u\}$	
$\lambda_{QL}^{\mu\tau 1B}$	Δ_3	$\{\beta_d, \gamma_d, -\beta_\nu, -\gamma_\nu, -\beta_l, -\gamma_l, \alpha_u, \gamma_u\}$	$\lambda_{b\tau}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{U_{31}}{U_{21}}\frac{V_{cb}}{V_{cs}} & -\frac{V_{cb}}{V_{cs}} \\ 0 & -\frac{U_{31}}{U_{21}} & 1 \end{pmatrix}$
	Δ_3^μ	$\{\beta_d, \gamma_d, \beta_\nu, \gamma_\nu, \beta_l, \gamma_l, \alpha_u, \gamma_u\}$	
	Δ_1^μ	$\{\beta_d, \gamma_d, \beta_l, \gamma_l\}\ \{\beta_\nu, \gamma_\nu, \alpha_u, \gamma_u\}$	

SE2 conclusions

Model-independent patterns (how many generations distinct)

Example: 3 charged leptons distinguished and at least 2 generations in the other sectors

Quarks and Leptons

$$\lambda_{dl}^{[e0]} = \lambda_{be} \begin{pmatrix} 0 & 0 & 0 \\ x_e & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\mu 0]} = \lambda_{b\mu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_\mu & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{with} \quad x_X = \frac{\lambda_{sX}}{\lambda_{bX}}.$$

Pattern not allowed in SE1 due to additional symmetry in d-nu sector

Leptons Only

Top row entry now allowed thanks to absence of quark symmetry

$$\lambda_{dl}^{[e]} = \lambda_{be} \begin{pmatrix} y_e & 0 & 0 \\ x_e & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\mu]} = \lambda_{b\mu} \begin{pmatrix} 0 & y_\mu & 0 \\ 0 & x_\mu & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{with} \quad x_X = \frac{\lambda_{sX}}{\lambda_{bX}}, \quad y_X = \frac{\lambda_{dX}}{\lambda_{bX}}$$

Quarks Only

Multiple columns now allowed thanks to absence of lepton symmetry

Still need bottom two rows for R_K(*)

$$\lambda_{dl}^{[bs]} = \lambda_{b\tau} \begin{pmatrix} 0 & 0 & 0 \\ x_s & y_s & z_s \\ x_b & y_b & 1 \end{pmatrix}, \quad \text{with} \quad x_X = \frac{\lambda_{Xe}}{\lambda_{b\tau}}, \quad y_X = \frac{\lambda_{X\mu}}{\lambda_{b\tau}}, \quad z_s = \frac{\lambda_{s\tau}}{\lambda_{b\tau}}$$

Reconstructing the parent group: Leptoflavour basis

$$G_F \leftarrow \begin{cases} G_L \leftarrow \begin{cases} G_\nu \\ G_l \end{cases} \\ G_q \leftarrow \begin{cases} G_u \\ G_d \end{cases} \end{cases}$$

$$\underbrace{\{\mathcal{G}_u,\ \mathcal{G}_d,\ \mathcal{G}_\nu, \mathcal{G}_l\}}_{\text{mass basis}} \quad \underbrace{\longrightarrow}_{\text{SM mixing rotations}} \quad \underbrace{\{\mathcal{G}'_u,\ \mathcal{G}'_d,\ \mathcal{G}'_\nu, \mathcal{G}'_l\}}_{\text{"flavour" basis}} \quad \underbrace{\longrightarrow}_{\text{Close algebra}} \quad \mathcal{G}_\mathcal{F}$$

In the flavour basis, generators have the information about the mixing; we need also information from LQ

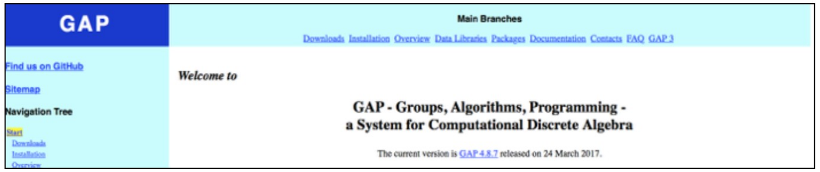
Diagonal basis (d-l coupling) $\lambda'_{dl} = \Lambda_d^* \lambda_{dl} \Lambda_l^\dagger$

$$\begin{aligned} &l_L \to \Lambda_l^\dagger l'_L, && d_L \to \Lambda_d^\dagger d'_L, && \nu_L \to U^\dagger_{PMNS} \Lambda_l^\dagger \nu'_L, && u_L \to U_{CKM} \Lambda_d^\dagger u'_L, \\ &E_R \to \Lambda_E^\dagger E'_R, && d_R \to \Lambda_D^\dagger d'_R, && \nu_R \to \Lambda_R^\dagger \nu'_R, && u_R \to \Lambda_U^\dagger u'_R, \end{aligned}$$

Generators in the Leptoflavour basis have the information from respective LQ patterns

$$T'_l = \Lambda_l T_l \Lambda_l^\dagger, \quad T'_\nu = \Lambda_l U_{PMNS} T_\nu U^\dagger_{PMNS} \Lambda_l^\dagger, \quad T'_d = \Lambda_d T_d \Lambda_d^\dagger, \quad T'_u = \Lambda_d U^\dagger_{CKM} T_u U_{CKM} \Lambda_d^\dagger,$$

Bottom-up scans



(mind the GAP)

Residual symmetry with degenerate phases: can't reproduce entirely the associated mixing

$$T_{aU} = U_a\, T_a^{ii=jj}\, U_a^\dagger = U_a\, R_{ij}\, T_a^{ii=jj}\, R_{ij}^\dagger\, U_a^\dagger, \quad \text{with} \quad R_{ij} \equiv \begin{pmatrix} \cos\theta_{ij} & \sin\theta_{ij}\, e^{-i\delta_{ij}} \\ -\sin\theta_{ij}\, e^{i\delta_{ij}} & \cos\theta_{ij} \end{pmatrix}$$

We aim to reproduce leading order mixing matrices

PMNS

$$U_{PMNS} \simeq U_{\mu\tau} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta_{\mu\tau} & \sqrt{2}\sin\theta_{\mu\tau} & 0 \\ -\sin\theta_{\mu\tau} & \cos\theta_{\mu\tau} & 1 \\ \sin\theta_{\mu\tau} & -\cos\theta_{\mu\tau} & 1 \end{pmatrix} + \mathcal{O}\left(\theta_{13}^l\right)$$

This mu-tau symmetric form captures many popular model-building starting points:

$$U_{\mu\tau}\left(\theta_{\mu\tau}\right) \to \begin{cases} U_{TBM} & \rightleftarrows \tan\theta_{\mu\tau} = \frac{1}{\sqrt{2}} \\ U_{BM} & \rightleftarrows \tan\theta_{\mu\tau} = 1 \ \text{ or } \ \theta_{\mu\tau} = \frac{\pi}{4} \\ U_{GR_i} & \rightleftarrows \tan\theta_{\mu\tau} = \frac{2}{(1+\sqrt{5})},\ \theta_{\mu\tau} = \frac{\pi}{5} \\ U_{HM} & \rightleftarrows \tan\theta_{\mu\tau} = \frac{1}{\sqrt{3}} \ \text{ or } \ \theta_{\mu\tau} = \frac{\pi}{6} \end{cases}$$

CKM

$$U_{CKM} \simeq U_C \equiv \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}\left(\theta_c^2, \theta_c^3\right)$$

Cabibbo mixing and null (13,23) angles good first approximation as well…

$$\sin\theta_c \simeq .225$$

Algorithm

$$G_F \leftarrow \begin{cases} G_L \leftarrow \begin{cases} G_\nu \\ G_l \end{cases} \\ G_q \leftarrow \begin{cases} G_u \\ G_d \end{cases} \end{cases}$$

1. Assign residual group (Zn)
2. Discretize mixing angles (apply experimental constraints)
3. Form generators in appropriate basis (Leptoflavour basis for LQ scans)
4. Close group (mind the GAP)

Mixing angles

$$\Theta_i \stackrel{!}{=} \pi \left(\frac{n}{m}\right)_i \quad \text{or}$$

$$\tan(\Theta_i) \stackrel{!}{=} \left(\sqrt{\frac{n/m}{1-n/m}}\right)_i$$

Phases

$$\Theta_i \stackrel{!}{=} 2\pi \left(\frac{n}{m}\right)_i$$

LQ couplings

$$\Theta_i \stackrel{!}{=} \left(\pm\sqrt{\frac{n}{m}}\right)_i$$

Matrices necessary to go to 'Leptoflavour' basis...

$$T_l' = \Lambda_l T_l \Lambda_l^\dagger, \quad T_\nu' = \Lambda_l U_{PMNS} T_\nu U_{PMNS}^\dagger \boxed{\Lambda_l^\dagger}, \quad T_d' = \Lambda_d T_d \boxed{\Lambda_d^\dagger}, \quad T_u' = \Lambda_d U_{CKM}^\dagger T_u U_{CKM} \Lambda_d^\dagger$$

SE1 and SE2:

$$\mathcal{G}_F \sim \{T_d', T_l', T_u', T_\nu'\}$$

$$\mathcal{G}_F \sim \{T_d', T_u'\} \times \{T_l', T_\nu'\}$$

SE2 only:

$$\mathcal{G}_F \sim \{T_l', T_\nu'\}$$

$$\mathcal{G}_F \sim \{T_u', T_d'\}$$

$$\Sigma(3N^2) \cong (Z_N \times Z_N) \rtimes Z_2,$$

$$\Delta(3N^2) \cong (Z_N \times Z_N) \rtimes Z_3,$$

$$\Delta(6N^2) \cong ((Z_N \times Z_N) \rtimes Z_3) \rtimes Z_2,$$

$$\Sigma(3N^3) \cong Z_N \times \Delta\left(3N^2\right) \quad \text{for} \quad N/3 \neq Integer,$$

$$\Sigma(3 \cdot 3^3) \cong (Z_3 \times Z_3 \times Z_3) \rtimes Z_3.$$

SE2 results

Electron Isolation and Fermionic Mixing in SE2					
$\{x_e, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$
$\{\frac{1}{2}, 1, \star\}$	[-1,1,-1]	[-1,-1,1]	[1,-1,-1]	[12,4]	D_{12}
$\{1, \star, \star\}$	[-1,1,-1]	[-1,-1,1]	[-1,-1,1]	[24,12]	S_4
$\{1, \star, \star\}$	[-1,ω_4,ω_4]	[-1,-1,1]	[-1,-1,1]	[96,64]	$\Delta(96)$
$\{x_e, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[6,1])	$D_N \times S_3$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,1,-1]	[-1,1,-1]	[ω_4,-ω_4,-1]	([N,d],[24,12])	$D_N \times S_4$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,ω_4,ω_4]	[-1,1,-1]	[1,-1,-1]	([N,d],[32,11])	$D_N \times \Sigma(32)$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,ω_4,ω_4]	[-1,1,-1]	[1,ω_4,-ω_4]	([N,d],[96,67])	$D_N \times (SL_3^2 \rtimes Z_4)$

Table 7: Flavour symmetries controlling $\lambda_{dl}^{[e0]}$, U_c, and $U_{\mu\tau}$ in SE2. In all cases T_d = diag$(1,-1,-1)$, and the filtered results we present here hold for all three leptoquarks. The variables $\{c,d\} = \{N/2, 3\}$ for $D_{N=(28,30)}$, and $\{c,d\} = \{N,1\}$ for $D_{N=14}$. The phase alignments shown are for D_{14} — send $T_u^{ii} \to [1,-1,-1]$ for $D_{28,30}$. $M \in \{1..5\}$.

Lepton Isolation and Lepton Mixing in SE2					
$\lvert\tan\theta_{\mu\tau}\rvert$	T_l^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{L}$	Electron/Muon
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[-1, 1, -1]$	$[12, 3]$	A_4	✓/✓
1	$[1, \omega_4, -\omega_4]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/✗
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/✗
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_4, -1, \omega_4]$	$[48, 3]$	$\Delta(48)$	✓/✓
1	$[\omega_4, 1, -1]$	$[1, -1, \omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	✗/✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, -\omega_4, -\omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	✓/✗
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, -1, -1]$	$[72, 42]$	$Z_3 \times S_4$	✗/✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_5, \omega_5^3, \omega_5]$	$[75, 2]$	$\Delta(75)$	✓/✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, \omega_3, 1]$	$[81, 7]$	$\Sigma(81)$	✓/✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/✗
1	$[\omega_4, 1, -1]$	$[1, -1, 1]$	$[96, 186]$	$Z_4 \times S_4$	✓/✓

Table 10: Flavour symmetries $\mathcal{G}_\mathcal{L}$ controlling electron and/or muon isolation patterns $\lambda_{dl}^{[e,\mu]}$ alongside of $U_{\mu\tau}$ lepton mixing in SE2. Note that the phase configurations for $T_{l,\nu}$ are not necessarily equivalent between the electron and muon isolation patterns. When both are applicable (two ✓), we show the phase configurations associated to $\lambda_{dl}^{[\mu]}$.

Conclusions

Residual flavour symmetries framework used to find prospective parent flavour groups

The presence of leptoquark couplings subject to residual flavour symmetries severely restricts patterns

Predictive patterns have well-defined predictions in other flavour observables

Bottom-up numerical scan revealed several predictive finite groups

Thanks!

SE1 results

Electron Isolation and Fermionic Mixing in SE1							
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$	A/B
$\{\star, \frac{\pi}{14}\}$	[1,1,-1]	[-1,1,1]	[-1,1,1]	[1,1,-1]	[56, 5]	D_{56}	✓/ ✓
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$	A/B
$\{\star, \frac{\pi}{c}\}$	[1,ω_3,ω_3^2]	[-1,1,1]	[-1,1,1]	[1,1,-1]	([N,d],[6,1])	$D_N \times S_3$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,1,-1]	[-1,1,1]	[-1,1,1]	[1,1,-1]	([N,d],[8,3])	$D_N \times D_8$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,ω_5,ω_5^4]	[-1,1,1]	[-1,1,1]	[1,1,-1]	([N,d],[10,1])	$D_N \times D_{10}$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,1,-1]	[-1,1,1]	[-1,1,1]	[1,1,ω_4]	([N,d],[32,11])	$D_N \times \Sigma(32)$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,ω_4,-ω_4]	[-1,1,1]	[-1,1,1]	[1,1,ω_3]	([N,d],[36,6])	$D_N \times (Z_3 \times (Z_3 \rtimes Z_4))$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,1,-1]	[-1,1,1]	[-1,1,1]	[1,1,ω_5]	([N,d],[50,3])	$D_N \times (Z_5 \times D_{10})$	✓/✓
$\{\star, \frac{\pi}{c}\}$	[1,1,ω_4]	[-1,1,1]	[-1,1,1]	[1,1,ω_4]	([N,d],[96,67])	$D_N \times (SL_3^2 \rtimes Z_4)$	✓/✓*
$\{\star, \frac{\pi}{c}\}$	[1,ω_4,-ω_4]	[-1,1,1]	[-1,1,1]	[1,1,ω_5]	([N,d],[100,6])	$D_N \times (Z_5 \times (Z_5 \rtimes Z_4))$	✓/✓

Table 3: Flavour symmetries controlling $\lambda_{dl}^{[e3X]}$, U_c, and portions of $U_{\mu\tau}$ in SE1. NOTES: $N \in \{28, 30\}$ for all leptoquarks in Pattern A, $N = 14$ for $\Delta_3^{(\mu)}$ in Pattern B, and $N \in \{14, 28\}$ for Δ_1^μ in Pattern B. The corresponding phase alignments are those of $\Delta_3^{(\mu)}$ in Pattern A. Also, $\{c, d\} = \{N/2, 3\}$ for $D_{N=(28,30)}$, and $\{c, d\} = \{N, 1\}$ for $D_{N=14}$. Finally, the ✓* notation indicates that the result does not appear for $N = 28$, for Δ_1^μ in Pattern B.

$e-\mu$ **Patterns and Fermionic Mixing in SE1**							
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$	A/B
$\{1, \frac{\pi}{15}\}$	[1, 1, -1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	[60, 12]	D_{60}	✓/✓
$\{1, \frac{\pi}{14}\}$	[1, 1, -1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	[84, 14]	D_{84}	✓/✓
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$	A/B
$\{1, \frac{\pi}{14}\}$	[1, 1, -1]	[-1, 1, 1]	[1, -1, 1]	[-1, 1, 1]	([14,1],[6,1])	$D_{14} \times S_3$	✗/✓
$\{1, \frac{\pi}{14}\}$	[1, 1, -1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	([28,3],[6,1])	$D_{28} \times S_3$	✓/✗
$\{1, \frac{\pi}{15}\}$	[1, 1, -1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	([30,3],[6,1])	$D_{30} \times S_3$	✓/✗
$\{1, \frac{\pi}{14}\}$	[1, 1, -1]	[-1, 1, 1]	[1, -1, -1]	[1, -1, -1]	([28,3],[12,4])	$D_{28} \times D_{12}$	✓*/✓*

Table 4: Flavour symmetries controlling $\lambda_{dl}^{[e\mu X]}$, U_C, and $U_{\mu\tau}$ in SE1. When a group is found for both Patterns A and B, the phase assignments given are for Pattern A. These results hold for all three leptoquarks, spare the final row, which only appears for Δ_1^μ (hence the ✓*).

$e-\tau$ **Patterns and Fermionic Mixing in SE1**							
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$	A/B
$\{1, \frac{\pi}{15}\}$	[1, -1, 1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	[30,3]	D_{30}	See caption.
$\{1, \frac{\pi}{14}\}$	[1, -1, 1]	[-1, 1, 1]	[1, -1, 1]	[-1, 1, 1]	[42,5]	D_{42}	
$\{1, \frac{\pi}{15}\}$	[1, -1, 1]	[-1, 1, 1]	[1, -1, 1]	[-1, 1, 1]	[60,12]	D_{60}	
$\{1, \frac{\pi}{14}\}$	[1, -1, 1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	[84,14]	D_{84}	
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$	A/B
$\{1, \frac{\pi}{14}\}$	[1, -1, 1]	[-1, 1, 1]	[1, -1, 1]	[-1, 1, 1]	([14,1],[6,1])	$D_{14} \times S_3$	✗/✓
$\{1, \frac{\pi}{14}\}$	[1, -1, 1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	([28,3],[6,1])	$D_{28} \times S_3$	✓/✗
$\{1, \frac{\pi}{14}\}$	[1, -1, 1]	[-1, 1, 1]	[1, -1, -1]	[1, -1, -1]	([28,3],[12,4])	$D_{28} \times D_{12}$	✓*/✓*
$\{1, \frac{\pi}{15}\}$	[1, -1, 1]	[-1, 1, 1]	[-1, 1, 1]	[-1, 1, 1]	([30,3],[6,1])	$D_{30} \times S_3$	✓/✗

Table 5: The same as Table 4, but for $\lambda_{dl}^{[e\tau X]}$. For $\mathcal{G}_\mathcal{F}$, D_{30} is only found for Pattern A, and D_{42} is only found for Pattern B. The same is respectively true for D_{84} and D_{60} when considering $\Delta_3^{(\mu)}$, but both are found in both patterns for Δ_1^μ (we show triplet phases). The ✓* notation implies that this group is only found for Δ_1^μ, and the phases correspond to Pattern A.

$\mu-\tau$ **Patterns and Fermionic Mixing in SE1**							
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$	A/B
$\{1, \frac{\pi}{14}\}$	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[56,5]	D_{56}	✓/✓
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{15}\}$	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[60,12]	D_{60}	✓/✓
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{14}\}$	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[1, -1, -1]	[84,14]	D_{84}	✓/✓
$\{t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$	A/B
$\{1, \frac{\pi}{c}\}$	[1,-1,-1]	[1,-1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[8,3])	$D_N \times D_8$	See caption below.
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{c}\}$	[1,-1,-1]	[1,-1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[12,4])	$D_N \times D_{12}$	
$\{1, \frac{\pi}{c}\}$	[-1,1,1]	[-1,1,1]	[1,-1,1]	[ω_3,1,1]	([N,d],[18,3])	$D_N \times (Z_3 \times S_3)$	
$\{1, \frac{\pi}{c}\}$	[1,-1,-1]	[1,-1,-1]	[-1,1,-1]	[ω_4,-1,-1]	([N,d],[32,11])	$D_N \times \Sigma(32)$	
$\{1, \frac{\pi}{c}\}$	[-1,1,1]	[-1,1,1]	[1,-1,1]	[ω_5,1,1]	([N,d],[50,3])	$D_N \times (Z_5 \times D_{10})$	
$\{\frac{1}{\sqrt{2}}, \frac{\pi}{c}\}$	[ω_3,1,1]	[-1,1,1]	[1,-1,1]	[ω_3,1,1]	([N,d],[72,25])	$D_N \times (Z_3 \times SL_3^2)$	
$\{1, \frac{\pi}{c}\}$	[ω_4,1,1]	[-1,1,1]	[1,-1,1]	[ω_4,1,1]	([N,d],[96,67])	$D_N \times (SL_3^2 \rtimes Z_4)$	

Table 6: The same as Table 4, but for $\lambda_{dl}^{[\mu\tau X]}$. Here $N \in \{14, 28, 30\}$, with $N = 14$ holding for Pattern B only and $N = 28, 30$ holding only for Pattern A, except when considering Δ_1^μ, which also realizes Pattern B when $N = 28$, when $\mathcal{G}_\mathcal{L}$ is contained in the first five rows of the $\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$ results. $\{c, d\} = \{N/2, 3\}$ for $D_{N=(28,30)}$, and $\{c, d\} = \{N, 1\}$ for $D_{N=14}$. The phase alignments in the $\mathcal{G}_\mathcal{F}$ section correspond to Pattern A, while those given for $\mathcal{G}_\mathcal{Q}$ in the $\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$ section are for D_{14}.

SE2 results

Electron Isolation and Fermionic Mixing in SE2					
$\{x_e, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{F}$
$\{\frac{1}{2}, 1, \star\}$	[-1,1,-1]	[-1,-1,1]	[1,-1,-1]	[12,4]	D_{12}
$\{1, \star, \star\}$	[-1,1,-1]	[-1,-1,1]	[-1,-1,1]	[24,12]	S_4
$\{1, \star, \star\}$	[-1,ω_4,ω_4]	[-1,-1,1]	[-1,-1,1]	[96,64]	$\Delta(96)$
$\{x_e, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{Q} \times \mathcal{G}_\mathcal{L}$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[6,1])	$D_N \times S_3$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,1,-1]	[-1,1,-1]	[ω_4,-ω_4,-1]	([N,d],[24,12])	$D_N \times S_4$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,ω_4,ω_4]	[-1,1,-1]	[1,-1,-1]	([N,d],[32,11])	$D_N \times \Sigma(32)$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,ω_4,ω_4]	[-1,1,-1]	[1,ω_4,-ω_4]	([N,d],[96,67])	$D_N \times (SL_3^2 \rtimes Z_4)$

Table 7: Flavour symmetries controlling $\lambda_{dl}^{[e0]}$, U_c, and $U_{\mu\tau}$ in SE2. In all cases T_d = diag$(1,-1,-1)$, and the filtered results we present here hold for all three leptoquarks. The variables $\{c,d\} = \{N/2, 3\}$ for $D_{N=(28,30)}$, and $\{c,d\} = \{N,1\}$ for $D_{N=14}$. The phase alignments shown are for D_{14} — send $T_u^{ii} \to [1,-1,-1]$ for $D_{28,30}$. $M \in \{1..5\}$.

Muon Isolation and Fermionic Mixing in SE2 ($\mathcal{G}_{\mathcal{F}}$ Case)					
$\{x_\mu, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_{\mathcal{F}}$
$\{1, \star, \star\}$	[-1,-1,1]	[-1,-1,1]	[-1,-1,1]	[8,3]	D_8
$\{1, 1, \star\}$	[1,-1,-1]	[-1,-1,1]	[1,-1,-1]	[8,3]	D_8
$\{1, \frac{1}{\sqrt{3}}, \star\}$	[1,-1,-1]	[-1,-1,1]	[1,-1,-1]	[12,4]	D_{12}
$\{1, 1, \star\}$	[1,-1,-1]	[-1,-1,1]	[ω_3,ω_3^2,1]	[12,4]	D_{12}
$\{1, \frac{1}{\sqrt{3}}, \star\}$	[1,-1,-1]	[-1,-1,1]	[ω_4,-ω_4,1]	[24,5]	$Z_4 \times S_3$
$\{\frac{1}{3}, \star, \star\}$	[1,-1,-1]	[-1,-1,1]	[-1,-1,1]	[24,6]	D_{24}
$\{1, 1, \star\}$	[-1,-1,1]	[-1,-1,1]	[1,-1,-1]	[24,12]	S_4
$\{1, \star, \frac{\pi}{14}\}$	[1,-1,-1]	[1,-1,-1]	[-1,-1,1]	[28,3]	D_{28}
$\{1, \frac{1}{\sqrt{3}}, \frac{\pi}{15}\}$	[1,-1,-1]	[1,-1,-1]	[-1,1,-1]	[30,3]	D_{30}
$\{1, \star, \star\}$	[-1,-1,1]	[-1,-1,1]	[ω_4,ω_4,-1]	[32,11]	$\Sigma(32)$
$\{1, 1, \star\}$	[1,-1,-1]	[-1,-1,1]	[1,ω_4,-ω_4]	[32,11]	$\Sigma(32)$
$\{1, 1, \star\}$	[1,-1,-1]	[-1,-1,1]	[1,ω_3,ω_3^2]	[36,12]	$Z_6 \times S_3$
$\{1, \frac{1}{\sqrt{3}}, \frac{\pi}{14}\}$	[1,-1,-1]	[-1,1,-1]	[-1,1,-1]	[42,5]	D_{42}
$\{1, \star, \frac{\pi}{14}\}$	[1,-1,-1]	[1,-1,-1]	[ω_4,ω_4,-1]	[56,4]	$Z_4 \times D_{14}$
$\{1, 1, \frac{\pi}{14}\}$	[1,-1,-1]	[1,-1,-1]	[1,-1,-1]	[56,5]	D_{56}
$\{1, 1, \frac{\pi}{14}\}$	[1,-1,-1]	[1,-1,-1]	[ω_4,-ω_4,1]	[56,7]	$(Z_{14} \times Z_2) \rtimes Z_2$
$\{\frac{1}{5}, 1, \star\}$	[1,-1,-1]	[-1,-1,1]	[ω_3,ω_3^2,1]	[60,5]	A_5
$\{\frac{1}{5}, \frac{1}{\sqrt{3}}, \star\}$	[1,-1,-1]	[-1,-1,1]	[-1,1,-1]	[60,5]	A_5
$\{1, \star, \frac{\pi}{15}\}$	[1,-1,-1]	[1,-1,-1]	[-1,-1,1]	[60,12]	D_{60}
$\{1, \frac{1}{\sqrt{3}}, \frac{\pi}{15}\}$	[1,-1,-1]	[1,-1,-1]	[1,-1,-1]	[60,12]	D_{60}
$\{1, \frac{1}{\sqrt{3}}, \frac{\pi}{14}\}$	[1,-1,-1]	[1,-1,-1]	[1,-1,-1]	[84,14]	D_{84}
$\{1, \star, \star\}$	[ω_4,-1,ω_4]	[-1,-1,1]	[ω_4,ω_4,-1]	[96,67]	$SL_3^2 \rtimes Z_4$

Table 8: The same as in Table 7 but for the muon isolation pattern. Here we only show reconstructed $\mathcal{G}_{\mathcal{F}}$, i.e. those groups formed from the closure of all four RFS generators.

Muon Isolation and Fermionic Mixing in SE2 ($\mathcal{G}_{\mathcal{Q}} \times \mathcal{G}_{\mathcal{L}}$ Case)					
$\{x_\mu, t_{\theta_{\mu\tau}}, \theta_C\}$	T_l^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_{\mathcal{Q}} \times \mathcal{G}_{\mathcal{L}}$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[1,-1,-1]	[-1,1,-1]	$[\omega_3,\omega_3^2,1]$	([N,d],[6,1])	$D_N \times S_3$
$\{\frac{1}{M}, \frac{1}{\sqrt{3}}, \frac{\pi}{c}\}$	[1,-1,-1]	[-1,1,-1]	[-1,1,-1]	([N,d],[6,1])	$D_N \times S_3$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[1,-1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[8,3])	$D_N \times D_8$
$\{\frac{1}{M}, \frac{1}{\sqrt{3}}, \frac{\pi}{c}\}$	[1,-1,-1]	[-1,1,-1]	[1,-1,-1]	([N,d],[12,4])	$D_N \times D_{12}$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[-1,-1,1]	[-1,1,-1]	$[\omega_4,-\omega_4,1]$	([N,d],[24,12])	$D_N \times S_4$
$\{\frac{1}{M}, 1, \frac{\pi}{c}\}$	[1,-1,-1]	[-1,1,-1]	$[1,\omega_4,-\omega_4]$	([N,d],[32,11])	$D_N \times \Sigma(32)$

Table 9: The same as in Table 8 but for $\mathcal{G}_{\mathcal{Q}} \times \mathcal{G}_{\mathcal{L}}$ group structures. $\{c, d\} = \{N/2, 3\}$ for $D_{N=(28,30)}$, and $\{c, d\} = \{N, 1\}$ for $D_{N=14}$. Again, $M \in \{1..5\}$ and the phase alignments shown are for D_{14} — send $T_u^{ii} \to [1, -1, -1]$ for $D_{28,30}$.

Lepton Isolation and Lepton Mixing in SE2					
$\lvert\tan\theta_{\mu\tau}\rvert$	T_l^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_\mathcal{L}$	Electron/Muon
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[-1, 1, -1]$	$[12, 3]$	A_4	✓/✓
1	$[1, \omega_4, -\omega_4]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/✗
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/✗
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_4, -1, \omega_4]$	$[48, 3]$	$\Delta(48)$	✓/✓
1	$[\omega_4, 1, -1]$	$[1, -1, \omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	✗/✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, -\omega_4, -\omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	✓/✗
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, -1, -1]$	$[72, 42]$	$Z_3 \times S_4$	✗/✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_5, \omega_5^3, \omega_5]$	$[75, 2]$	$\Delta(75)$	✓/✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, \omega_3, 1]$	$[81, 7]$	$\Sigma(81)$	✓/✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/✗
1	$[\omega_4, 1, -1]$	$[1, -1, 1]$	$[96, 186]$	$Z_4 \times S_4$	✓/✓

Table 10: Flavour symmetries $\mathcal{G}_\mathcal{L}$ controlling electron and/or muon isolation patterns $\lambda_{dl}^{[e,\mu]}$ alongside of $U_{\mu\tau}$ lepton mixing in SE2. Note that the phase configurations for $T_{l,\nu}$ are not necessarily equivalent between the electron and muon isolation patterns. When both are applicable (two ✓), we show the phase configurations associated to $\lambda_{dl}^{[\mu]}$.

$\lambda^{[bs0]}$ and Quark Mixing in SE2		
$\{\theta_C\}$	$\{y_b\}$	$\mathcal{G}_Q \sim D_N$
$\pi/14$	$\{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1\}$	$N \in 14, 28$
$\pi/15$	$\{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1\}$	$N \in 30$
T_d^{ii}	[-1, 1, 1]	
T_u^{ii} $(N = 14)$	[1, -1, 1]	
T_u^{ii} $(N = 28)$	[1, -1, -1]	
T_u^{ii} $(N = 30)$	[-1, 1, 1]	

GAP-ID	$\mathcal{G}_Q$
[14, 1]	D_{14}
[28, 3]	D_{28}
[30, 3]	D_{30}

Table 11: Flavour symmetries $\mathcal{G}_Q$ controlling the simplified $\lambda^{[bs0]}$ pattern and U_c quark mixing in SE2.