Unstable particles in physical processes.

Solved and unsolved problems

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1. The t–channel singularity in small angle scattering (almost solved).
2. The s–channel singularity near threshold (a way for solution is seen).
3. The perturbative QFT with unstable particles for the observable processes (unsolved)

1. t-channel singularity

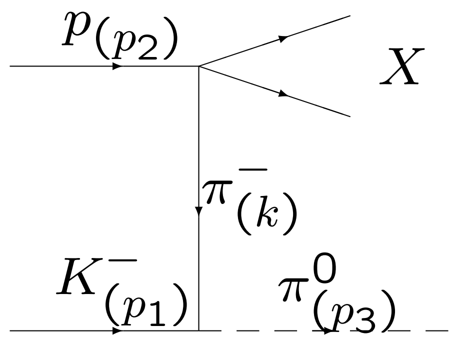
The process: $K^- p \to \pi^0 + X$

with no strange particles in the final state, at $s \gg M_K^2$ and small momentum transfer k $(M_X^2 = sx)$.

$k^2 \equiv (p_1 - p_3)^2 = t^m(x) - 2|\mathbf{p_1}||\mathbf{p_3}| \sin^2(\theta/2)$, $\quad t^m = \frac{x}{1-x}\left[M_K^2(1-x) - m_\pi^2\right]$,

$\left(t^m_{max} = (M_K - M_\pi)^2 \text{ at } x = \frac{M_n}{M_K}\right)$.

The diagram gives the factor $\left(\frac{1}{k^2 - m_\pi^2}\right)^2$ in the matrix element $\mathcal{M}$ squared. Since $t^m > m_\pi^2$, the integration over k^2 results in a divergent cross section $\int_{t_{min}}^{t_{max}} |\mathcal{M}|^2 dk^2$.



This paradox originates from the instability of the kaon decaying into the $\pi^0\pi^-$ system: the point $k^2 = m_\pi^2$ corresponds to a real decay.

R.F. Peierls, PRL **6** (1961) 641 first considered such a problem for processes $\pi\rho \to \rho\pi$, etc.

Recent analyses (for $\mu^+\mu^- \to We\nu$ with $M_{e\nu} < m_\mu$):

I.F. Ginzburg, DESY 95-168 (1995); Nucl. Phys. B (Proc. Suppl.) **51A** (1996) 85:
K. Melnikov, V.G. Serbo. Phys. Rev. Lett. **76** (1996) 3263; Nucl. Phys. **B 483** (1997) 67; & G.L. Kotkin. Phys. Rev. **D54** (1996) 3289;
...

$$\boxed{\text{The divergence is eliminated if one takes into account the fact that, because the kaon is unstable, the kaon initial state differs from the standard plane wave.}}$$

The result depends on the relation between two lengths, a and $c\tau$:
Here $\boldsymbol{a}$ – transverse size of the beam, $\boldsymbol{\tau = 1/\Gamma}$ – particle lifetime.

We consider two asymptotic cases.

• The case $a \gg c\tau$

For demonstration of idea, we start with the kaon rest frame, where we set its 4–momentum as $p = (M_K - i\Gamma/2, 0, 0, 0)$. If we fix k^2, the energy of the produced pion $p^0_{3,r} = (M_K^2 + M_\pi^2 - k^2)/2M_K$.
The new value $k^2_{new} \equiv (p_1 - p_3)^2 = M_K^2 - iM_K\Gamma + M_\pi^2 + i\Gamma p^0_{3,r}$
$\Rightarrow k^2 - i\gamma$; $\gamma = \frac{\Gamma(M_K^2 - M_\pi^2 + k^2)}{2M_K}$. One can now calculate the cross section of the process in the standard way. At $s \gg M_K^2$:

$$d\sigma = \frac{|\mathcal{M}|^2 dk^2}{4(4\pi)^3 s^2}; \;\; |\mathcal{M}|^2 \sim \frac{|\mathcal{M}_{K\pi\pi}|^2 |\mathcal{M}_{\pi p}|^2}{(k^2 - m_\pi^2)^2 + \gamma^2} \Rightarrow$$

$$\sigma \sim \frac{|\mathcal{M}_{K\pi\pi}|^2 |\mathcal{M}_{\pi p}|^2}{\gamma} \left(\propto \frac{\Gamma_K |\mathcal{M}_{\pi p}|^2}{\gamma} \right) \approx \sigma_{\pi p} \, .$$

If we start from the lab frame, where the kaon has a definite 3–momentum $\vec{p}$ and a complex energy $E = \sqrt{M_K^2 - iM_K\Gamma + \vec{p}^2}$, we obtain a similar result but with another energy distribution of produced pions.

Final result depends on the method of kaon beam preparation!

This very result can be obtained by considering initial kaon as a superposition of plane waves corresponding to stable particles of different masses with the Breit–Wigner spectral density. In this way denominators of the matrix element and its conjugate under the integral correspond to different masses, and the second order pole disappear. The result looks as a complete decay of the kaon prior to interaction with the target BUT

It goes in parallel with the standard interactions (conserving strangeness). We integrated here over the entire space–time irrespective to the size of the interaction region. The space scale of the phenomena is $c\tau$, where $\tau = \hbar/\Gamma$ is the kaon time of life.

• The case $a \ll c\tau$

Melnikov, Serbo, Kotkin (prepared for $\mu^+\mu^- \to We\nu$ with $M_{e\nu} < m_\mu$)

At $a \ll c\tau$ the kaon width is inessential in calculations.
The kaons and protons in the initial states are not plane waves but wave packets with some distribution over momenta

$$|p_i\rangle \;\to\; \int \frac{d^3P_i}{(2\pi)^{3/2}} \Phi_i(\vec{P}_i)|P_i\rangle \quad (i = 1, 2).$$

When we calculate the cross section, we sum over final states. One can use an arbitrary complete set of states. We use plane waves $|p_3\rangle, ...$

$$|\mathcal{M}|^2 = \frac{1}{(2\pi)^6}\int\prod_{i=1,2} d^3P_i d^3P_i'\Phi(P_i)\Phi^*(P_i')\times$$
$$\mathcal{M}(P_1,P_2;p_3,...)\mathcal{M}^*(P_1',P_2';p_3,...)\times$$
$$\delta(P_1+P_2-P_1'-P_2')\delta(P_1+P_2-p_3-...).$$

The same final state is obtained from different initial states.

Next we write the identity ($\varepsilon_i \equiv P_i^0$):

$$2\pi\delta(\sum P_i - \sum P_i') = \delta(\sum \vec{P}_i - \sum \vec{P}_i')\int dt e^{it(\sum p_i^0 - \sum p_i'^0)}.$$

The phase averaging results in density matrices for the kaons and protons in the beams: $\langle \Phi(P_i)\Phi(P_i')\exp[it(\varepsilon_i - \varepsilon_i')]\rangle = \rho(\vec{P}_i, \vec{P}_i', t)$.

After the change of variables $p_i = (P_i + P_i')/2$, $\ell_i = (P_i - P_i')/2$ we switch to the mixed representation of the density matrix — Wigner function $n(p,r,t)$: $\rho(\vec{P}_i, \vec{P}_i', t)d^3P_i d^3P_i' = \int n(\vec{p_i}, \vec{r_i}, t)e^{2i\vec{\ell}_i\vec{r}_i}\frac{d^3p_i d^3\ell_i d^3r_i}{(2\pi)^{3/2}}$.

In the quasi–classical limit the Wigner function coincides with the density in the phase space. This is the point when the known distributions of particles within the beams enter the result.

Near the pole

$$\langle|\mathcal{M}|^2\rangle \sim \int n_1(\mathbf{p_1},\mathbf{r},\mathbf{t})\mathbf{n_2}(\mathbf{p_2},\mathbf{r},\mathbf{t})\mathbf{e^{2i\ell r}}\mathbf{d^3r}\,\mathbf{d^3}\ell\,\mathbf{d^3p}$$
$$\times\frac{|\mathcal{M}_{K\pi n}|^2|\mathcal{M}_{p\pi}|^2}{[(k-\ell)^2-m_\pi^2][(k+\ell)^2-m_\pi^2]}\,.$$

We have
$n_1(\mathbf{r},\mathbf{p},t)=n_{1z}(z-v_1t)n_{1\perp}(\mathbf{r}_\perp)n_{1p}(\mathbf{p})$;
$n_2(\mathbf{r},\mathbf{p},t)=n_{2z}(z+v_2t)n_{2\perp}(\mathbf{r}_\perp)n_{2p}(\mathbf{p})$;
(v_i are velocities of the colliding particles).
The integration over the longitudinal coordinates and time results in δ–functions. In the integration over transverse variables we have only linear form in $\ell_\perp$ near the pole.

The final result is written via the transverse size of the beam a and the kaon lifetime

$$\sigma_{eff} = \frac{\pi a}{2}\Gamma K_{\pi p} = \frac{\pi a}{2c\tau}\sigma_{\pi p}\,.$$

The discussed divergence is regularized. For large bunches the regulator is the proper width of unstable particle, for small bunches the size of bunch becomes the regulator of divergence.

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One can ask: Perhaps, rescatterings with, e.g., two–pion exchange are essential?

The answer is: **NO**.

In this case we have integration over loop virtuality, and the pion momenta in the diagram and its conjugate enter with uncorrelated. It gives regularization of the initial divergence.

This effect was considered in detail for the process $\mu^-\mu^+ \to e\bar{\nu}W$ for the case when the effective mass of $e\bar{\nu}$ system is less than muon mass. These results could be applied to the process $Kp \to \pi X$ ($a \ll c\tau$ in both cases). For the processes like $\rho\pi \to \pi\rho$ (discussed in 1960's) $a \gg c\tau$.

	$c\tau$
$\mu^\pm$	800m
$\pi^\pm$	7.8m
$K^\pm$	3.7m

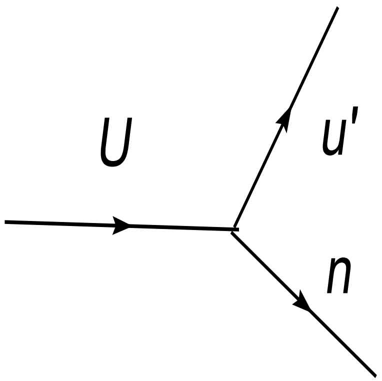
	$c\tau$
Σ^-	4.3cm
Σ^+	2.4cm
K_S^0	2.7cm

For $\mu^\pm$, $\pi^\pm$, $K^\pm$ size of beam $a \ll c\tau$.

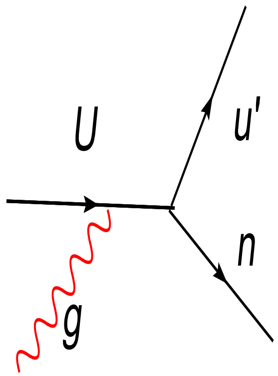
For former beams of Σ^- at CERN size of beam $a = 3.7cm$ was close to $c\tau$. It is interesting to study this case.

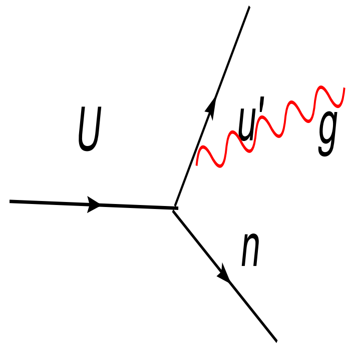
2. The s-channel singularity

Consider a toy example: decay of scalar nucleus U to scalar U' + scalar neutron, $U \to U' + n$, with some lifetime T.



Then consider a scattering process $\gamma + U \to U' + n$ at small photon energy ω. Denoting photon momentum as k, we





have an amplitude of the process:

$$\mathcal{M} = \frac{A}{(p_U + k)^2 - M_U^2} + \frac{B}{(p_{U'} - k)^2 - M_U'^2} = \frac{A}{2p_U \cdot k} - \frac{B}{2p_{U'} \cdot k} \propto \frac{1}{\omega}$$

The cross section $d\sigma = \frac{1}{I}|\mathcal{M}|^2 d\Gamma \propto \frac{1}{\omega^3} d\Gamma$.

Here $I \propto \omega$ is the total photon flux and Γ is the final phase space.

For Compton scattering at $\omega \to 0$, $\Gamma \to \omega^3$ $\Rightarrow$ cross section is finite.

In our case, to the contrary, at $\omega \to 0$ we have $\Gamma \to \Gamma_{decay}$ $\Rightarrow$ total cross section diverges as $1/\omega^3$!

At the first glance, this is terrible.

The atmosphere is opaque due to small admixture of C^{14}!

Idea for the solution of the paradox

Since $\mathcal{M} \propto 1/\omega$ is large, one must to take into account processes with 2 photons,

$$\gamma + \gamma + U \to U' + n,\ \gamma + U \to U' + n + \gamma, \text{ etc.}$$

In the contribution of additional photons, an extra factor appears, which is proportional their energy per the characteristic size of process $\propto \mathcal{F}^2$ ($\mathcal{F}$ is electromagnetic field strength). Summation over multiphoton contributions must regularize the amplitude as

$$\frac{1}{\omega} \to \frac{1}{\omega^2 + c\mathcal{F}^2}$$

It is not done up to now.

These effects can be essential at very low X, process of type of $g + t \to W + b$ and in earliear Universe.

The experience with nonlinear QED (large coherent $\mathcal{F}$) can be useful here.

3. Unstable particles in the final state or in loops

An example: process $e^+e^- \to W^+W^-$ cannot be observed in pure form. The observable is, e.g. $e^+e^- \to W^+W^- \to (\mu^+\nu)(\mu^-\bar{\nu})$. Its description contains integration over the lepton phase space. With the standard propagators of EW theory, this integral diverges since it includes the region where the denominator in the integrand $|k^2 - M_W^2 + i\varepsilon|^2$ is 0. To avoid this divergence, the W propagator is usually changed by inserting the full W-width

$$\frac{1}{k^2 - M_W^2 + i\varepsilon} \to \frac{1}{k^2 - M_W^2 + i\Gamma M_W}.$$

In more refined approaches the entire polarization operator is added (having in mind partial summation of perturbation series).

However, this procedure is not harmless.

Using the experimental width Γ^{exp} in this ansatz violates unitarity in the tree approximation.

With Γ^{exp}, the cross section calculated for the W bosons in the final state can differs from the sum over all partial channels. To avoid this difficulty, e.g. in the tree-level calculations one should use the value of width obtained in this very approximation.

The simple insertion of width in the propagator violates gauge invariance.

This very final state can be obtained from another intermediate state, e.g.

$$e^+e^- \to \gamma Z \to (\mu^+\mu^-)(\nu\bar{\nu})$$

Example: In the standard SM calculations (no width in denominator) with evident dependence on the gauge parameter ξ, separate diagrams give some fractions depending on ξ. In the entire amplitude these fractions are joined in one fraction with common denominator, the ξ dependence disappear in this sum. Changing some denominators by adding different widths (Γ_W or Γ_Z, etc.) destroys this compensation.

We meet here a series of fundamental difficulties

1. The standard perturbation theory contains new type of divergences in addition to the UV and IR. Perhaps, it breaks its self-consistency?

The answer is: NO (F.Tkachov)

The idea: The observable quantities are not amplitudes but their squares integrated with some weight. Therefore, one can consider amplitude as generalized function (distribution) and define what e.g. $1/(k^2 - M^2)^2$ means near the pole. With this definition, the perturbation theory becomes well-defined and gauge invariant. Therefore, this theory can be considered as self-consistent.

Unfortunately, this approach gives nothing for the practical solution of the problem.

2. In the perturbation theory we have two parameters — coupling constant g, assumed to be small, and parameter $g_r^{eff} \approx g(|p_i|/\Delta E)$ (where $\Delta E \propto (Q^2 - M^2)$ is the distance to peak). Near the resonance peak g_r^{eff} become large. That give inaccuracies $\sim \Gamma/M$ in the quantities like total cross sections and strong inaccuracies in the description of the process near peak. Therefore,

The new form of perturbation theory is necessary that gives regular description both far from resonance and near the resonance peak.

In the standard QFT language the goal is to obtain the gauge invariant resummation of the standard perturbation theory. This problem at tree and one loop levels was considered by
Veltman, Sirlin, Stuart, Oldenborgh, Denner, Dittmayer, Papavassiliou,....
The complete solution at the tree and 1-loop level is given by
W. Beenakker, F.A. Berends, A.P. Chapovsky. hep-ph/9909472.

However, the obtained recipes become extremely complex and different from each other at the multiloop level. However, the answer will be necessary for description of future experiments with high statistics, e.g., processes like $\gamma\gamma \to W^+W^-$ at $s \gg M_W^2$, well-observable at photon colliders.

I don't hope that an unambiguous recipe for the two loops can be constructed in this way.

3. The standard EW theory is the QFT, based on the complete set of the asymptotical states for the fundamental particles. It is the base for the construction of perturbation theory with the standard particle propagators. BUT THE FUNDAMENTAL PARTICLES OF THEORY (W, Z, H) ARE UNSTABLE.

THE QFT WITH UNSTABLE FUNDAMENTAL PARTICLES HAS NOT BEEN CONSTRUCTED SO FAR

In particular, the space of states is covered entirely by all states of stable particles. Adding unstable particles overfills this space. However, when we consider higher order diagrams, their imaginary parts contain unstable intermediate W–bosons, for example. They should not contribute to the unitarity in the fundamental approach.

Without such a theory, a precise description of EW processes is impossible.

For me, in solving this problem

breaking of gauge invariance in calculations is not the main effect but is the signal on an unsatisfactory state of the theory. This signal should be used as a test when constructing a satisfactory scheme.

My goal is to inform the physical community that this problem transforms from a theoretical problem of pure QFT to the problem whose solution will be necessary for precise description of the EW data.

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Note: experiments with production of gauge bosons or t–quarks offer the first domain in particle physics where this problem becomes very important. It is clear that the small parameter here is Γ/M. For muons this parameter is too small to speak of observable effects. In hadron physics some phenomenological ansatz is necessary, which would hide a possible effect.

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I hope that a specific way of constructing the EW theory together with gauge invariance would help in solving the problem for this specific case.

FANTASY

• The proper procedure of quantization is considered now as an algebraic problem independent of stability of the objects (Faddeev).

• When constructing S–matrix, we use some integrations by parts. We usually omit the surface items arising in this procedure (at $t, x \to \infty$). With unstable particles these terms cannot be neglected (since wave function grows at $t \to -\infty$ — in the opposite case the analyticity in x–space is broken).

Perhaps, some residual surface items should be added into the effective Lagrangian of theory *a la* ghosts etc. items in the Faddeev—Popov—De Witt method.

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