2 Higgs doublets as pseudo Nambu-Goldstone bosons



Kei Yagyu

INFN, U. of Florence



Collaboration with

Stefania De Curtis, Luigi Delle Rose, Stefano Moretti

Scalars 2017, 2nd Dec, U of Warsaw

Introduction

From LHC results

- □ At least 1 Higgs boson exists.
- **\square** It shows SU(2)_L doublet nature.
- □ Its mass is 125 GeV.

Questions

- □ Why the Higgs boson mass is such small with respect to NP scale?
- What is the true shape of the Higgs sector?

(Possible) answer

- □ Higgs is a pseudo Nambu-Goldstone boson (pNGB).
- □ Higgs sector has a multi-doublet structure (many motivations).

Let's combine **pNGB Higgs** and **multi-Higgs structure**!

- Suppose there is a global symmetry G at scale above f (~TeV), which is spontaneously broken down into a subgroup H.
- □ The structure of the Higgs sector is determined by the coset G/H.
- **\square** H should contain the custodial SO(4) \simeq SU(2)_L×SU(2)_R symmetry.
- □ The number of NGBs (dimG-dimH) should be 4 or lager.



□ Suppose there is a global symmetry G at scale above f (~TeV),

which is spontaneously broken down into a subgroup H.

 \square The structure of the Higgs sector is determined by the coset G/H.

G	Н	N_G	NGBs rep.[H] = rep.[SU(2) × SU(2)]
SO(5)	SO(4)	4	4 = (2, 2)
SO(6)	SO(5)	5	5 = (1, 1) + (2, 2)
SO(6)	$SO(4) \times SO(2)$	8	$4_{+2} + \bar{4}_{-2} = 2 \times (2, 2)$
SO(7)	SO (6)	6	$6 = 2 \times (1, 1) + (2, 2)$
SO(7)	G_2	7	7 = (1, 3) + (2, 2)
SO(7)	$SO(5) \times SO(2)$	10	$10_0 = (3, 1) + (1, 3) + (2, 2)$
SO(7)	$[SO(3)]^3$	12	$(2, 2, 3) = 3 \times (2, 2)$
Sp(6)	$Sp(4) \times SU(2)$	8	$(4, 2) = 2 \times (2, 2), (2, 2) + 2 \times (2, 1)$
SU(5)	$SU(4) \times U(1)$	8	$4_{-5} + \bar{4}_{+5} = 2 \times (2, 2)$
SU(5)	SO(5)	14	14 = (3, 3) + (2, 2) + (1, 1)

□ Suppose there is a global symmetry G at scale above f (~TeV),

which is spontaneously broken down into a subgroup H.

 \square The structure of the Higgs sector is determined by the coset G/H.

G	Н	N_G	NGBs rep.[H] = rep.[SU(2) × SU(2)]
SO(5)	SO(4)	4	4 = (2, 2)
SO(6)	SO(5)	5	
SO(6)	$SO(4) \times SO(2)$	₈ Agashe,	Contino, Pomarol (2005)
SO(7)	1 Doublet: Minimal C	Composite Higgs Mo	del
SO(7)	U 2		$I = (1, \mathbf{J}) \pm (\mathbf{Z}, \mathbf{Z})$
SO(7)	$SO(5) \times SO(2)$	10	$10_0 = (3, 1) + (1, 3) + (2, 2)$
SO(7)	$[SO(3)]^3$	12	$(2, 2, 3) = 3 \times (2, 2)$
Sp(6)	$Sp(4) \times SU(2)$	8	$(4, 2) = 2 \times (2, 2), (2, 2) + 2 \times (2, 1)$
SU(5)	$SU(4) \times U(1)$	8	$4_{-5} + \bar{4}_{+5} = 2 \times (2, 2)$
SU(5)	SO(5)	14	14 = (3, 3) + (2, 2) + (1, 1)

□ Suppose there is a global symmetry G at scale above f (~TeV),

which is spontaneously broken down into a subgroup H.

 \square The structure of the Higgs sector is determined by the coset G/H.

G	Н	N_G	NGBs rep.[H] = rep.[SU(2) × SU(2)]
SO(5)	SO(4)	4	4 = (2, 2)
SO(6)	SO(5)	5	5 = (1, 1) + (2, 2)
SO(6)	$SO(4) \times SO(2)$	n	
SO(7)		Gripaios, Pomarol,	, Riva, Serra (2009)
SO(7)	1 Doublet + 1 Singlet	Redi, Tesi (2012)	
SO(7)	$SO(5) \times SO(2)$	10	$10_0 = (3,1) + (1,3) + (2,2)$
SO(7)	$[SO(3)]^3$	12	$(2, 2, 3) = 3 \times (2, 2)$
Sp(6)	$Sp(4) \times SU(2)$	8	$(4, 2) = 2 \times (2, 2), (2, 2) + 2 \times (2, 1)$
SU(5)	$SU(4) \times U(1)$	8	$4_{-5} + \bar{4}_{+5} = 2 \times (2, 2)$
SU(5)	SO(5)	14	14 = (3, 3) + (2, 2) + (1, 1)

□ Suppose there is a global symmetry G at scale above f (~TeV),

which is spontaneously broken down into a subgroup H.

 \square The structure of the Higgs sector is determined by the coset G/H.

G	Н	N_G	NGBs rep.[H] = rep.[SU(2) × SU(2)]				
SO(5)	SO(4)	4	4 = (2, 2)				
SO(6)	SO(5)	5	5 = (1, 1) + (2, 2)				
SO(6)	$SO(4) \times SO(2)$	8	$4_{+2} + \bar{4}_{-2} = 2 \times (2, 2)$				
SO(7)	SO(6)	6	$6 = 2 \times (1, 1) + (2, 2)$				
$\frac{SO(7)}{SO(7)}$ 2 Doublets $\frac{2}{O(5)}$ Mrazek, Pomarol, Rattazi, Redi, Serra, Wulzer (2011)							
SO(7)	[SO(3) Bertuzzo, I	Ray, Sandes, Sav	voy (2013)				
Sp(6)	$Sp(4) \times SU(2)$	8	$(4, 2) = 2 \times (2, 2), (2, 2) + 2 \times (2, 1)$				
SU(5)	$SU(4) \times U(1)$	8	$4_{-5} + \bar{4}_{+5} = 2 \times (2, 2)$				
SU(5)	SO(5)	14	14 = (3, 3) + (2, 2) + (1, 1)				

□ Suppose there is a global symmetry G at scale above f (~TeV),

which is spontaneously broken down into a subgroup H.

 \square The structure of the Higgs sector is determined by the coset G/H.

G	Н	N_G	NGBs rep.[H] = rep.[SU(2) × SU(2)]
SO(5)	SO(4)	4	4 = (2, 2)
SO(6)	SO(5)	5	5 = (1, 1) + (2, 2)
SO(6)	$SO(4) \times SO(2)$	8	$4_{+2} + \bar{4}_{-2} = 2 \times (2, 2)$
SO(7)	SO(6)	6	$6 = 2 \times (1, 1) + (2, 2)$
SO(7)	G_2	7	7 = (1, 3) + (2, 2)
SO(7)	$SO(5) \times SO(2)$	10	$10_0 = (3, 1) + (1, 3) + (2, 2)$
SO(7)	$[SO(3)]^3$	12	$(2, 2, 3) = 3 \times (2, 2)$
Sp(6)	$Sp(4) \times SU(2)$	8	$(4, 2) = 2 \times (2, 2), (2, 2) + 2 \times (2, 1)$
SU(5)	$SU(4) \times U(1)$	8	$4_{-5} + \bar{4}_{+5} = 2 \times (2, 2)$
SU(5)	SO(5)	14	14 = (3, 3) + (2, 2) + (1, 1)

We consider 2 Higgs doublets as pNGB from SO(6) \rightarrow SO(4)×SO(2)

Construction of 2 pNGB Doublets

D 15 SO(6) generators: $T^A = \{ \underline{T^a_{L,R}}, \underline{T_S}, \underline{T^{\hat{a}}_{1,2}} \}$ (A=1-15, a=1-3, \hat{a} =1-4) 6 SO(4) 1 SO(2) 8 Broken

Construction of 2 pNGB Doublets

 $\Box \text{ 15 SO(6) generators: } T^{A} = \{ \underline{T_{L,R}^{a}, T_{S}, \underline{T_{1,2}^{a}} \} \text{ (A=1-15, a=1-3, \hat{a}=1-4)} \\ 6 \text{ SO(4) } 1 \text{ SO(2) 8 Broken} \\ \Phi \equiv (\phi_{1}^{\hat{a}}, \phi_{2}^{\hat{a}}) \\ \Box \text{ pNGB matrix: } U(\phi_{1}^{\hat{a}}, \phi_{2}^{\hat{a}}) \equiv \exp \left[\sqrt{2}i \left(T_{1}^{\hat{a}} \frac{\phi_{1}^{\hat{a}}}{f} + T_{2}^{\hat{a}} \frac{\phi_{2}^{\hat{a}}}{f} \right) \right] = \exp \left[\begin{pmatrix} 0_{4 \times 4} & \Phi \\ -\Phi^{T} & 0_{2 \times 2} \end{pmatrix} \right]$

U is transformed non-linearly under SO(6): $U \rightarrow g U h^{-1}(g, \phi_{1,2}^{\hat{a}})$

Construction of 2 pNGB Doublets

 $\Box \text{ 15 SO(6) generators: } T^{A} = \{ \underline{T_{L,R}^{a}, T_{S}, \underline{T_{1,2}^{a}} \} \text{ (A=1-15, a=1-3, \hat{a}=1-4)} \\ 6 \text{ SO(4) } 1 \text{ SO(2) 8 Broken} \\ \Phi \equiv (\phi_{1}^{\hat{a}}, \phi_{2}^{\hat{a}}) \\ \Box \text{ pNGB matrix: } U(\phi_{1}^{\hat{a}}, \phi_{2}^{\hat{a}}) \equiv \exp\left[\sqrt{2}i \left(T_{1}^{\hat{a}} \frac{\phi_{1}^{\hat{a}}}{f} + T_{2}^{\hat{a}} \frac{\phi_{2}^{\hat{a}}}{f} \right) \right] = \exp\left[\begin{pmatrix} 0_{4 \times 4} & \Phi \\ -\Phi^{T} & 0_{2 \times 2} \end{pmatrix} \right] \\ \end{bmatrix}$

U is transformed non-linearly under SO(6): $U \rightarrow g U h^{-1}(g, \phi_{1,2}^{\hat{a}})$

□ Linear rep. **Σ(15): 15** = (**6**,**1**) \oplus (**4**,**2**) \oplus (**1**,**1**) under SO(4)×SO(2)

$$\Sigma = U \Sigma_0 U^T = egin{pmatrix} \Sigma(6,1) & \Sigma(4,2) \ -\Sigma^T(4,2) & \Sigma(1,1) \end{pmatrix} \quad \Sigma_0 = i \sqrt{2} T_S = egin{pmatrix} 0_{4 imes 4} & 0_{4 imes 2} \ 0_{2 imes 4} & i \sigma_2 \end{pmatrix}$$

 Σ is transformed linearly under SO(6): $\Sigma \rightarrow g \Sigma g^{-1}$

Higgs Potential

□ The potential becomes 0 because of the shift symmetry of the NGB.

 \rightarrow Higgs mass also becomes 0.

Higgs Potential

□ The potential becomes 0 because of the shift symmetry of the NGB.

- \rightarrow Higgs mass also becomes 0.
- □ We need to introduce the explicit breaking of G.



Higgs Potential

□ The potential becomes 0 because of the shift symmetry of the NGB.

- \rightarrow Higgs mass also becomes 0.
- □ We need to introduce the explicit breaking of G.



Non-zero potential appears at 1-loop level via the CW mechanism.
 Mrazek, Pomarol, Rattazi, Redi, Serra, Wulzer

 (1) Spurion method,
 (2) Explicit model





$${\cal L}_{
m ele} = -rac{1}{4g_W^2} W^a_{\mu
u} W^{a\,\mu
u} + rac{1}{y_L^2} ar q_L i D \!\!\!\!/ q_L + rac{1}{y_R^2} ar t_R i D \!\!\!\!/ t_R$$



$$\begin{aligned} & U(\phi_1^{\hat{a}},\phi_2^{\hat{a}}) \rightarrow C_2 U(\phi_1^{\hat{a}},\phi_2^{\hat{a}})C_2 = U(\phi_1^{\hat{a}},-\phi_2^{\hat{a}})\\ & \Sigma \rightarrow -C_2 \Sigma C_2\\ & \Psi^6 \rightarrow C_2 \Psi^6 \end{aligned} \qquad \begin{array}{l} & C_2 = \operatorname{diag}(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})\\ & \mathbf{Elementary\ Sector} \end{aligned} \qquad \begin{array}{l} & \mathbf{Strong\ Sector}\\ & \mathbf{W}_{\mu}^{a},\ q_L,\ t_R\\ & g_W,\ y_L,\ y_R \end{aligned} \qquad \begin{array}{l} & \mathsf{Mixing}\\ & \rho_{\mu}^{A},\ \Psi^6,\ \Sigma\\ & m_{\rho},\ m_{\Psi},\ Y_1,\ Y_2 \end{aligned}$$

$$\begin{aligned} & \mathcal{L}_{\mathrm{str}} = \bar{\Psi}^6(i D - m_{\Psi}) \Psi^6 - \bar{\Psi}_L^6(Y_1 \Sigma + Y_2 \Sigma^2) \Psi_R^6 + \mathrm{h.c.}\\ & -\frac{1}{4} \operatorname{tr} \rho_{\mu\nu}^{A} \rho^{A\,\mu\nu} + \frac{m_{\rho}^2}{2} (\rho^A)_{\mu} (\rho^A)^{\mu} + \Sigma \cdot (\rho,\ W)\ \text{interaction} \end{aligned}$$

Based on the 4DCHM, De Curtis, Redi, Tesi, JHEP04 (2012) 042

Embeddings into SO(6) multiplets :

$$W^a_\mu \in W^A_\mu ~~ t^6_R = (\Delta_{t_R})^I \, t_R ~~ q^6_L = (\Delta_{q_L})^{lpha I} \, q^lpha_L$$



$$\mathcal{L}_{ ext{mix}} = m_
ho^2 W^A_\mu
ho^{A\mu} + \Delta (ar q_L^{\ 6} \Psi^6 + ar t_R^{\ 6} \Psi^6)$$

□ Integrating out the heavy degrees of freedom (ρ^A and ψ^6), we obtain the effective low energy Lagrangian

$$\begin{split} \mathcal{L}_{\text{eff}} &= (W_{\mu}, B_{\mu}) \begin{pmatrix} G_{LL} & G_{LR} \\ G_{LR} & G_{RR} \end{pmatrix} \begin{pmatrix} W^{\mu} \\ B^{\mu} \end{pmatrix} + (\bar{q}_{L}, \bar{t}_{R}) \begin{pmatrix} \not q & K_{LL} & K_{LR} \\ K_{LR} & \not q & K_{RR} \end{pmatrix} \begin{pmatrix} q_{L} \\ t_{R} \end{pmatrix} \\ &\coloneqq \mathsf{G} & \coloneqq \mathsf{K} \end{split}$$

□ Integrating out the heavy degrees of freedom (ρ^A and ψ^6), we obtain the effective low energy Lagrangian

$$\begin{split} \mathcal{L}_{\text{eff}} &= (W_{\mu}, B_{\mu}) \begin{pmatrix} G_{LL} & G_{LR} \\ G_{LR} & G_{RR} \end{pmatrix} \begin{pmatrix} W^{\mu} \\ B^{\mu} \end{pmatrix} + (\bar{q}_{L}, \bar{t}_{R}) \begin{pmatrix} \not q & K_{LL} & K_{LR} \\ K_{LR} & \not q & K_{RR} \end{pmatrix} \begin{pmatrix} q_{L} \\ t_{R} \end{pmatrix} \\ &\coloneqq \mathsf{G} & \coloneqq \mathsf{K} \end{split}$$

□ All the strong sector info. are packaged in these form factors G and K.

□ Integrating out the heavy degrees of freedom (ρ^A and ψ^6), we obtain the effective low energy Lagrangian

$$\begin{split} \mathcal{L}_{\text{eff}} &= (W_{\mu}, B_{\mu}) \begin{pmatrix} G_{LL} & G_{LR} \\ G_{LR} & G_{RR} \end{pmatrix} \begin{pmatrix} W^{\mu} \\ B^{\mu} \end{pmatrix} + (\bar{q}_{L}, \bar{t}_{R}) \begin{pmatrix} \not q & K_{LL} & K_{LR} \\ K_{LR} & \not q & K_{RR} \end{pmatrix} \begin{pmatrix} q_{L} \\ t_{R} \end{pmatrix} \\ &\coloneqq \mathsf{G} & \coloneqq \mathsf{K} \end{split}$$

□ All the strong sector info. are packaged in these form factors G and K.

$$egin{aligned} K_{LL} &= I_{2 imes 2} + \sum_{i,j}^{1,2} K_{LL}^{ij} \Phi_i^c \Phi_j^{c\dagger}(\cdots) \ (\cdots): ext{function of SU(2) invariants e.g., } \Phi_1^\dagger \Phi_1, \ (\Phi_1^\dagger \Phi_1)^2, \ldots \end{aligned}$$

□ Integrating out the heavy degrees of freedom (ρ^A and ψ^6), we obtain the effective low energy Lagrangian

$$\begin{split} \mathcal{L}_{\text{eff}} &= (W_{\mu}, B_{\mu}) \begin{pmatrix} G_{LL} & G_{LR} \\ G_{LR} & G_{RR} \end{pmatrix} \begin{pmatrix} W^{\mu} \\ B^{\mu} \end{pmatrix} + (\bar{q}_{L}, \bar{t}_{R}) \begin{pmatrix} \not q & K_{LL} & K_{LR} \\ K_{LR} & \not q & K_{RR} \end{pmatrix} \begin{pmatrix} q_{L} \\ t_{R} \end{pmatrix} \\ &\coloneqq \mathsf{G} & \coloneqq \mathsf{K} \end{split}$$

□ All the strong sector info. are packaged in these form factors G and K.

□ The Higgs potential can be straightforwardly calculated as

$$V = rac{1}{f^4} \int rac{d^4k}{(2\pi)^4} \left(rac{3}{2} \ln \det G - 2N_c \ln \det K
ight)$$

 \square After int. and expansion by $\Phi_{1,2}$, we can extract the potential parameters.

$$\begin{split} V(\Phi_1, \Phi_2) &= m_1^2 \Phi_1^{\dagger} \Phi_1 + m_2^2 \Phi_2^{\dagger} \Phi_2 - \left[m_3^2 \Phi_1^{\dagger} \Phi_2 + \text{h.c.} \right] \\ &+ \frac{\lambda_1}{2} (\Phi_1^{\dagger} \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^{\dagger} \Phi_2)^2 + \lambda_3 (\Phi_1^{\dagger} \Phi_1) (\Phi_2^{\dagger} \Phi_2) + \lambda_4 (\Phi_1^{\dagger} \Phi_2) (\Phi_2^{\dagger} \Phi_1) \\ &+ \frac{\lambda_5}{2} (\Phi_1^{\dagger} \Phi_2)^2 + \lambda_6 (\Phi_1^{\dagger} \Phi_1) (\Phi_1^{\dagger} \Phi_2) + \lambda_7 (\Phi_2^{\dagger} \Phi_2) (\Phi_1^{\dagger} \Phi_2) + \text{h.c.} \end{split}$$

$$m_i^2 = m_i^2(g_
ho, f, \dots) \quad \lambda_i = \lambda_i(g_
ho, f, \dots)$$

□ The Higgs potential can be straightforwardly calculated as

$$V = rac{1}{f^4} \int rac{d^4k}{(2\pi)^4} \left(rac{3}{2} \ln \det G - 2N_c \ln \det K
ight)$$

 \square After int. and expansion by $\Phi_{1,2}$, we can extract the potential parameters.

Matching Conditions

□ We need to reproduce the top mass and the weak boson mass.

$$\begin{split} m_W^2 &= \langle G_{LL} \rangle_{q^2 \to 0} = \frac{1}{4} \begin{bmatrix} g_W^2 g_\rho^2 \\ g_W^2 + g_\rho^2 \end{bmatrix} f^2 \sin^2 \frac{v}{f} \\ \mathbf{g}^2 & \mathbf{v}_{sm}^2 = [\mathbf{sqrt}(2) \ \mathbf{G}_{\mathsf{F}}]^{-1} \\ m_t &= \langle K_{LR} \rangle_{q^2 \to 0} \sim \frac{v}{\sqrt{2}} \underbrace{\frac{m_\Psi \Delta_L \Delta_R}{Y\sqrt{\Delta_L^2 + m_\Psi^2} \sqrt{\Delta_R^2 + Y^2}} \frac{Y_1 s_\beta + Y_2 c_\beta}{f} \\ \mathbf{v}_t \\ v^2 &= v_1^2 + v_2^2 & \tan \beta = v_2/v_1 \end{split}$$

 $\Delta_{L,R}=y_{L,R}\Delta$ $Y=\sqrt{Y_1^2+Y_2^2}$

Typical Prediction of Mass Spectrum



Typical Prediction of Mass Spectrum



Correlation b/w f and M

 $\tan\beta = 2, m_{\rho} = f, mt = 173 \text{ GeV}, 124 < m_{h} < 126 \text{ GeV}$



Correlation b/w tanß and m(extra)

f = 1.2 TeV,
$$m_{\rho}$$
 = f, mt = 173 GeV, 124 < m_{h} < 126 GeV



Correlation b/w extra Higgs masses

f = 1.2 TeV, $m_o = f$, mt = 173 GeV, 124 < $m_h < 126$ GeV



 $m_{H^+} \gtrsim m_A \gtrsim m_H$

Correlation b/w m_A and m_{ψ}

f = 1.2 TeV,
$$m_{\rho}$$
 = f, mt = 173 GeV, 124 < m_{h} < 126 GeV





- Higgs as pNGB scenario gives natural explanation for a light Higgs.
 - Taking a lager global sym., we obtain a multi-Higgs structure.
- □ Giving the strong sector Lagrangian, we can explicitly calculate all the potential parameters in terms of the strong parameters.
- We found a strong/characteristic prediction for the Higgs mass spectrum and a correlation to the mass of heavy fermion resonance.

Spurion Method

 \blacksquare The Higgs potential is calculated only by using the spurion VEV Δ_{ψ} and U.

$$\mathcal{L}_{ ext{mix}} = (\Delta_{\psi})^{lpha I} \, \psi^{lpha}_{ ext{ele}} \, \mathcal{O}^{I}_{ ext{str}}$$

Merit: Quite General (but still we need to assume fermion rep.)

Demerit: Losing the correlation, O(1) uncertainties in pot. parameters.



Spurion Method

Mrazek, Pomarol, Rattazi, Redi, Serra, Wulzer NPB 853 (2011) 1-48

\Box Fermionic contribution assuming $\mathbf{r} = \mathbf{6}$ -plet of SO(6).

Operator	$\mathcal{I}^1_{(0,1)}$	${\cal I}^1_{(1,0)}$	${\cal I}^1_{(2,0)}$	$\mathcal{I}^{4}_{(2,0)}$	$\mathcal{I}^{5}_{(2,0)}$	$\mathcal{I}^{1}_{(1,1)}$	$\mathcal{I}_{(1,1)}^{5}$	$\mathcal{I}_{(1,1)}^{6}$	$\mathcal{I}^{1}_{(0,2)}$	$\mathcal{I}^{4}_{(0,2)}$
$\frac{1}{16\pi^2} \times$	$-\frac{y_L^2 g_\rho^2}{2}$	$y_R^2 g_\rho^2$	$\frac{y_R^4}{4}$	$\frac{y_R^4}{4}(\frac{g_\rho}{4\pi})^2$	$\frac{y_R^4}{4}(\frac{g_\rho}{4\pi})^2$	$\frac{y_R^2 y_L^2}{4}$	$y_R^2 y_L^2 (\frac{g_\rho}{4\pi})^2$	$-y_R^2 y_L^2 (\frac{g_\rho}{4\pi})^2$	$-\frac{y_{L}^{4}}{2}$	$-y_L^4(\frac{g_\rho}{4\pi})^2$
m_{11}^2/f^2	1	$\cos^2 \theta$	0	0	0	$\cos^2 \theta$	$\cos^2 \theta$	0	1	1
m_{22}^2/f^2	1	$\sin^2 \theta$	0	0	0	$\sin^2 \theta$	$\sin^2 \theta$	0	1	1
m_{12}^2/f^2	0	0	0	0	$\sin 4\theta$	0	0	0	0	0
\tilde{m}_{12}^2/f^2	0	0	0	0	0	$-\sin 2\theta$	0	$\frac{1}{2}\sin 2\theta$	0	0
λ_1	$-\frac{1}{3}$	$-\frac{1}{3}\cos^2\theta$	$2\cos^4\theta$	$2\cos^4\theta$	0	$-\frac{4}{3}\cos^2\theta$	$-\frac{7}{12}\cos^2\theta$	0	$-\frac{7}{12}$	$-\frac{11}{24}$
λ_2	$-\frac{1}{3}$	$-\frac{1}{3}\sin^2\theta$	$2\sin^4\theta$	$2\sin^4\theta$	0	$-\frac{4}{3}\sin^2\theta$	$-\frac{7}{12}\sin^2\theta$	0	$-\frac{7}{12}$	$-\frac{11}{24}$
λ3	0	0	$\sin^2 \theta$	$-\sin^2\theta$	0	0	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$
λ_4	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$2\sin^2 2\theta$	0	$-\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{7}{6}$	$-\frac{2}{3}$
$\tilde{\lambda}_4$	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0
λ_5	0	0	0	0	0	0	0	0	0	0
λ6	0	0	0	0	$-\frac{1}{3}\sin 4\theta$	0	0	0	0	0
$\tilde{\lambda}_6$	0	0	0	0	0	$\frac{2}{3}\sin 2\theta$	0	$-\frac{1}{12}\sin 2\theta$	0	0
λ7	0	0	0	0	$-\frac{1}{3}\sin 4\theta$	0	0	0	0	0
$\tilde{\lambda}_7$	0	0	0	0	0	$\frac{2}{3}\sin 2\theta$	0	$-\frac{1}{12}\sin 2\theta$	0	0

 \Box Arbitral O(1) parameters appear in front of each operator.



