Two-loop corrections to the Higgs trilinear coupling in models with extended scalar sectors

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based on

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Investigating the Higgs trilinear coupling λ_{hhh}

Probing the shape of the Higgs potential

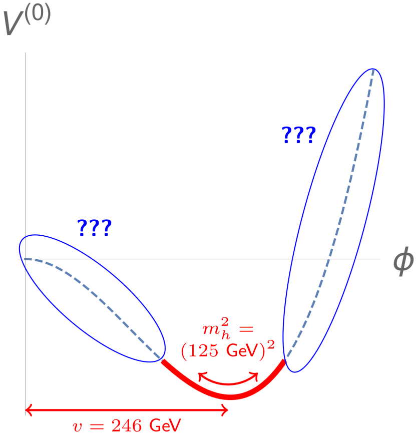
- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

  $\rightarrow$ the location of the EW minimum: $v \simeq 246$ GeV
  $\rightarrow$ the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

  However what we still don't know is the **shape** of the Higgs potential, which **depends on λ_{hhh}**

- λ_{hhh} determines the nature of the EWPT!

  $\Rightarrow \mathcal{O}(20-30\%)$ deviation of λ_{hhh} from its SM prediction needed to have a strongly first-order EWPT
  $\rightarrow$ necessary for EWBG
  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



Investigating the Higgs trilinear coupling λ_{hhh}

Alignment with or without decoupling

- Aligned scenarios already seem to be favoured $\rightarrow$ Higgs couplings are SM-like at **tree-level**
- Non-aligned scenarios ($e.g.$ in 2HDMs) could be almost entirely excluded in the close future using synergy of HL-LHC and ILC! [c.f. Prof. Kanemura's talk on Wednesday]

  $\rightarrow$ Alignment **through decoupling**? or alignment **without** decoupling?
- If alignment without decoupling, Higgs couplings like λ_{hhh} can still exhibit **large deviations** from SM predictions because of **BSM loop effects**
- Current best limit (at 95% CL): $-3.2 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 11.9$ [ATL-PHYS-PUB-2019-009]

Future measurement prospects

- HL-LHC with 3 ab^{-1} could reach $0.1 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 2.3$
- ILC-250 cannot measure λ_{hhh}, but 500-GeV and 1-TeV extensions could obtain measurements with precisions of 27% and 10% respectively
- CLIC 1.4 TeV + 3 TeV $\rightarrow$ 20% accuracy
- 100-TeV hadron collider with 30 ab^{-1} $\rightarrow$ 5-7% accuracy

see $e.g.$ [Di Vita et al. 1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Gonçalves et al. 1802.04319], [Chang et al. 1804.07130], etc.

Radiative corrections to the Higgs trilinear coupling and non-decoupling effects

The Two-Higgs-Doublet Model (2HDM)

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry $(\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2)$ to avoid tree-level FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2} = \begin{pmatrix} \Phi_{1,2}^+ \\ \Phi_{1,2}^0 \end{pmatrix}$ of hypercharge $1/2$

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) \\ + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

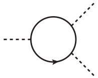
- 7 free parameters in scalar sector: m_3^2, $\lambda_i\ (i = 1\cdots 5)$, $\tan\beta \equiv \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$
  (m_1^2, m_2^2 eliminated with tadpole equations, and $\langle\Phi_1^0\rangle + \langle\Phi_2^0\rangle = v^2 = (246\text{ GeV})^2$)
- Doublets expanded in terms of mass eigenstates:
  h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs
- $\lambda_i\ (i = 1\cdots 5)$ traded for mass eigenvalues m_h, m_H, m_A, $m_{H^\pm}$ and CP-even mixing angle α
- m_3^2 replaced by a soft-breaking mass scale $M^2 = 2m_3^2/s_{2\beta}$

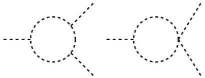
Non-decoupling effects in λ_{hhh} at one loop

First studies of the one-loop corrections to λ_{hhh} in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- Leading one-loop corrections to λ_{hhh} (for $s_{\beta-\alpha} = 1$)

$$\delta^{(1)}\lambda_{hhh} = \underbrace{-\frac{48m_t^4}{v^3}}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm} \underbrace{\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{BSM}}$$





(recall $\lambda_{hhh}^{(0)} = 3m_h^2/v$)

- Masses of additional scalars $\Phi = H, A, H^\pm$ in 2HDM can be written as $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ ($\tilde{\lambda}_\Phi$: some combination of λ_i)
- Power-like dependence of BSM terms $\propto m_\Phi^4$, and

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

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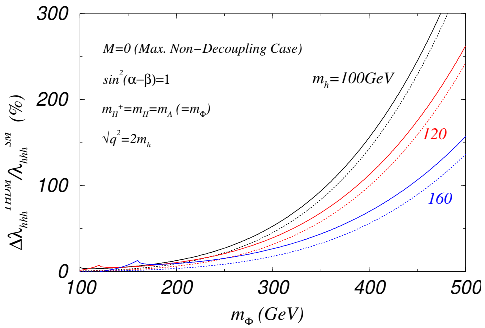


figure from [Kanemura, Okada, Senaha, Yuan '04]

- **Huge deviations possible, without violating unitarity! $\to$ non-decoupling effects**

State-of-the-art calculations of λ_{hhh}

At one loop

- ▷ Complete diagrammatic, OS-scheme, calculations been performed for a number of BSM models with extended sectors (with singlets, doublets, triplets)
- ▷ One-loop calculations available for 2HDMs, HSM, IDM in program `H-COUP` [Kanemura, Kikuchi, Sakurai, Yagyu '17], [Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu '19]

Non-decoupling effects found for a range of BSM models at one loop
$\Rightarrow$ **what happens at two loops?**

At two loops

Model [ref.]	Included Corrections	Eff. pot. approx.	Typical size
MSSM [Brucherseifer, Gavin, Spira '14]	$\mathcal{O}(\alpha_s\alpha_t)$	Yes	$\mathcal{O}(\sim 10\%)$
NMSSM [Mühlleitner, Nhung, Ziesche '15]	$\mathcal{O}(\alpha_s\alpha_t)$	Yes	$\mathcal{O}(\sim 10\%)$
IDM [Senaha '18]	$\mathcal{O}(\lambda_\Phi^3)$ (partial)	Yes	$\mathcal{O}(\sim 2\%)$

We also want to investigate the fate of non-decoupling effects at two loops
$\Rightarrow$ **we derive <u>dominant</u> two-loop corrections to λ_{hhh} in a 2HDM** [J.B., Kanemura 1903.05417]

Our two-loop calculation of λ_{hhh} in the Two-Higgs-Doublet Model

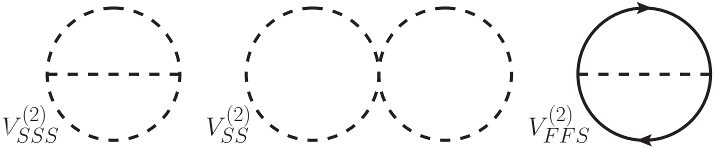
Setup of our effective-potential calculation

Step 1: calculate $\underbrace{V_{\text{eff}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 2:** $\underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\Big|_{\text{min.}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 3:** convert from $\overline{\text{MS}}$ to OS scheme

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \qquad \left(\kappa \equiv \frac{1}{16\pi^2}\right)$$

- $V^{(2)}$: 1PI vacuum bubble diags., and we want to study the leading two-loop BSM corrections from **additional scalars** and **top quark**, so we only need

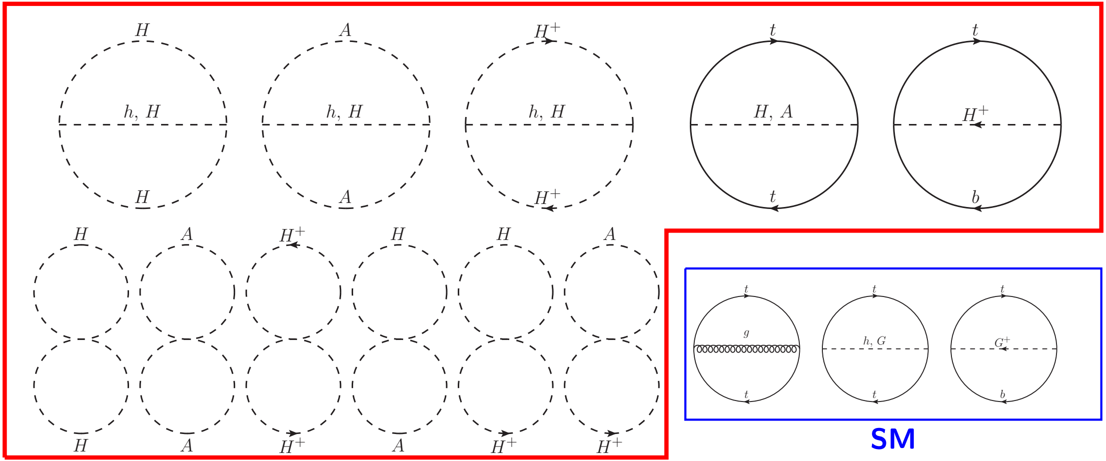


- Also, we **neglect subleading contributions** from h, $G, G^{\pm}$, and light fermions $\Rightarrow$ no need to specify type of 2HDM + greatly simplifies the $\overline{\text{MS}} \rightarrow$ OS scheme conversion (*details in backup*)
- **Scenarios without mixing**: aligned 2HDM ($s_{\beta-\alpha} = 1$) $\Rightarrow$ evade exp. constrains! (loop-induced deviations from alignment also neglected)

λ_{hhh} at two loops in the 2HDM

In [JB, Kanemura '19], we considered for the first time $\lambda_{hhh}^{(2)}$ in the 2HDM:
$\rightarrow$ 15 new BSM diagrams appearing in $V^{(2)}$ in the 2HDM w.r.t. the SM case

2HDM

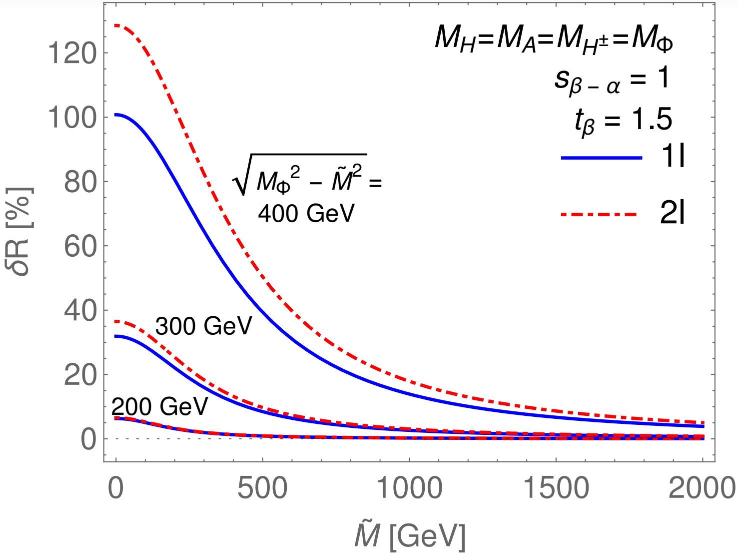


SM

In the following we show results for the BSM deviation δR:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1 = \frac{\Delta\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}}$$

Decoupling behaviour



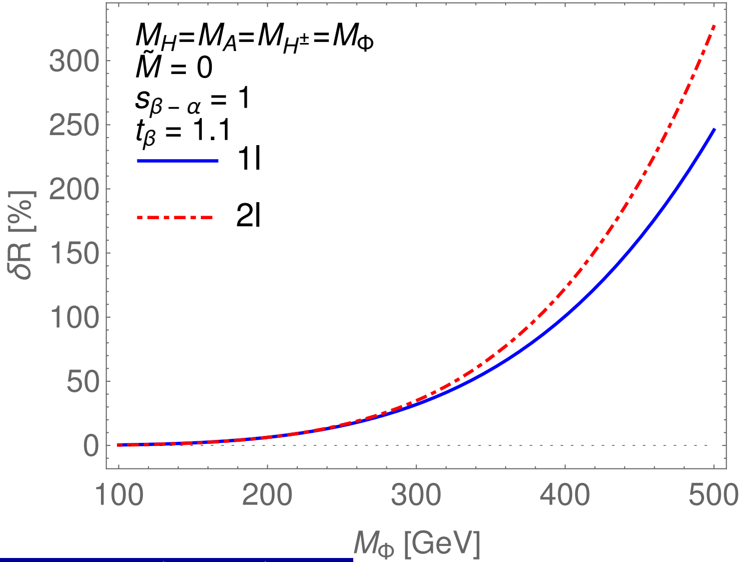
- δR size of BSM contributions to λ_{hhh}:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- $\tilde{M}$: "OS" version of M, defined so as to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$

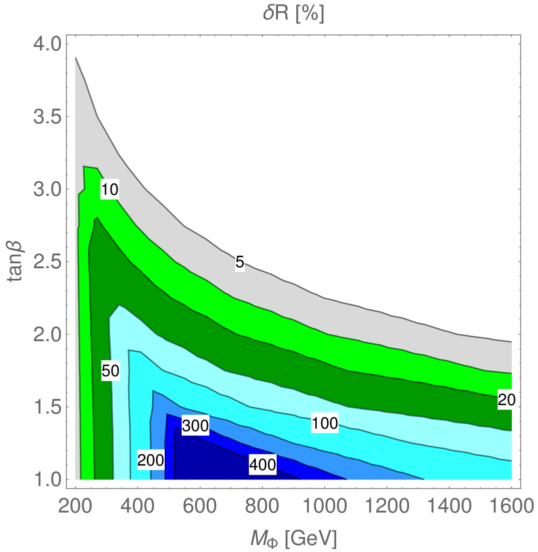
Non-decoupling effects



$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- Other limit of interest: $\tilde{M} = 0 \to$ maximal non-decoupling effects
- $\delta^{(1)}\hat{\lambda}_{hhh} \to \propto M_\Phi^4$
- $\delta^{(2)}\hat{\lambda}_{hhh} \to \propto M_\Phi^6$
- For $\tilde{M} = 0$, $\tan\beta = 1.1$, tree-level unitarity is lost around $M_\Phi \approx 600$ GeV [Kanemura, Kubota, Takasugi '93]

Maximal BSM allowed deviations



$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

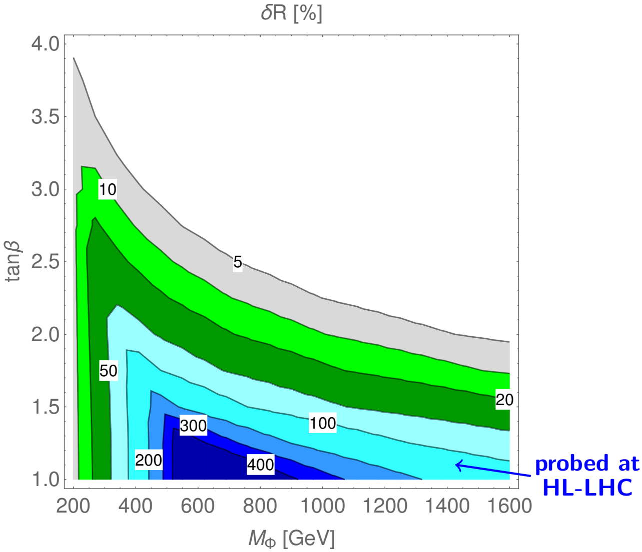
- Here: Maximal deviation δR (1ℓ+2ℓ) while fulfilling perturbative unitarity, in $(\tan\beta, M_\Phi)$ plane

$$M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$$

- One cannot take $M_\Phi \to \infty$ with $\tilde{M} = 0$ without breaking unitarity
- At some point $\tilde{M}$ must be non-zero $\to$ reduction factor

$$\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^n < 1$$

Maximal BSM allowed deviations



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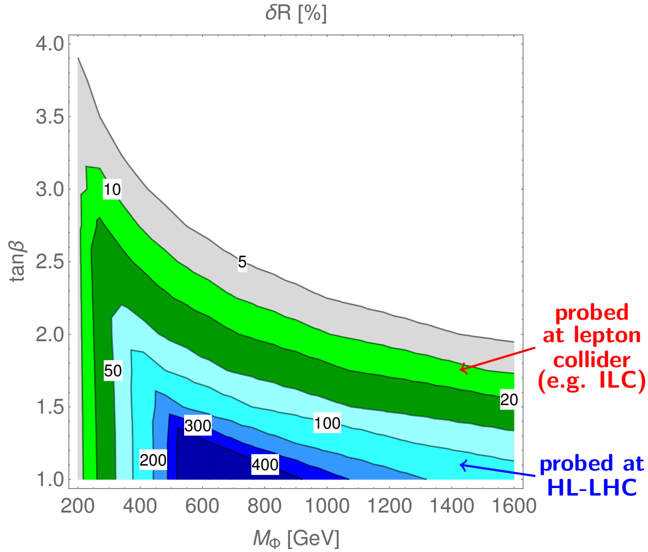

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Summary

- **First two-loop calculation of λ_{hhh} in 2HDM**, in a scenario with alignment

- Two-loop corrections to λ_{hhh} remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained $\rightarrow$ **typical size $10-20\%$ of one-loop contributions**

$\Rightarrow$ non-decoupling effects found at one loop are **not drastically changed**

$\Rightarrow$ in the future perspective of a precise measurement of λ_{hhh}, computing corrections beyond one loop will be **necessary**

- Precise calculation of Higgs couplings (λ_{hhh}, etc.) can allow distinguishing aligned scenarios **<u>with</u> or <u>without</u> decoupling**

Thank you for your attention!

Backup

Current experimental limits

- Current limits on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ are (at 95% CL)

  $-3.2 < \kappa_\lambda < 11.9$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)

  see [ATL-PHYS-PUB-2019-009] (ATLAS), [CMS-HIG-17-008] (CMS)

Future prospects

- HL-LHC with 3 ab^{-1} could reach $0.1 < \kappa_\lambda < 2.3$, and a 27-TeV HE-LHC with 15 ab^{-1} $0.58 < \kappa_\lambda < 1.45$
- ILC-250 cannot measure λ_{hhh}, but 500-GeV and 1-TeV extensions could obtain measurements with precisions of 27% and 10% respectively
- CLIC 1.4 TeV + 3 TeV → 20% accuracy
- 100-TeV hadron collider with 30 ab^{-1} → 5-7% accuracy

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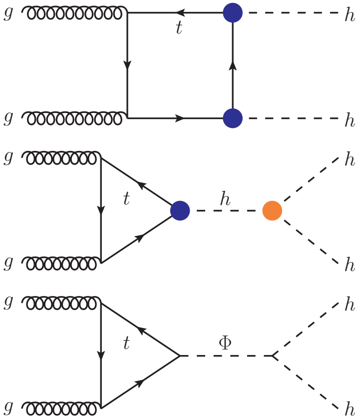
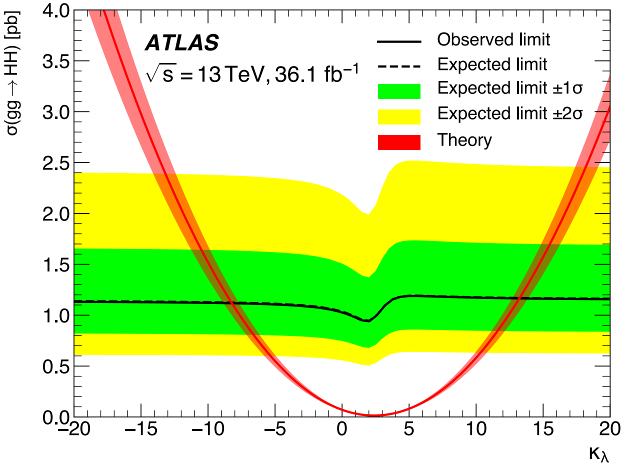


Double Higgs production

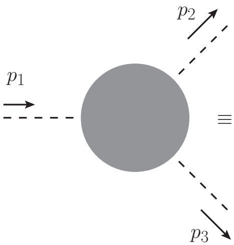
An example of experimental limits on λ_{hhh}



Example of current limits on κ_λ from the ATLAS search of $hh \to b\bar{b}\gamma\gamma$
(*taken from* [ATLAS collaboration 1807.04873])

Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2,p_2^2,p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum
- Instead it is much more convenient to work with an effective Higgs trilinear coupling λ_{hhh}



$\equiv \Gamma_{hhh}(p_1^2,p_2^2,p_3^2)$

$$\mathcal{L} \supset -\frac{1}{6}\lambda_{hhh}h^3 \quad \rightarrow \quad \underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}}}_{\overline{\text{MS}}\text{ result}}$$

$V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}}$: effective potential (calculated in $\overline{\text{MS}}$ scheme)

- In effective-potential calculations, one should usual fix conditions for the lower derivatives of V_{eff}

$$\underbrace{\frac{\partial V_{\text{eff}}}{\partial h}\bigg|_{\text{min.}} = 0,}_{\text{tadpole condition}} \qquad \underbrace{[M_h^2]_{V_{\text{eff}}} = \frac{\partial^2 V_{\text{eff}}}{\partial h^2}\bigg|_{\text{min.}} - \frac{1}{v}\frac{\partial V_{\text{eff}}}{\partial h}\bigg|_{\text{min.}}}_{\text{curvature mass of the Higgs}}$$

- Using these, we obtain

$$\lambda_{hhh} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \mathcal{D}_3 \Delta V_{\text{eff}}\Big|_{\text{min.}}, \qquad \text{with } \mathcal{D}_3 \equiv \frac{\partial^3}{\partial h^3} - \frac{3}{v}\left[-\frac{1}{v}\frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2}\right]$$

Radiative corrections to the Higgs trilinear coupling (detailed)

- Γ_{hhh} and λ_{hhh} can be related as

$$-\Gamma_{hhh}(0,0,0) = \underbrace{\hat{\lambda}_{hhh}}_{\text{OS result}} = \left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}\text{ result}} = \left(1 + \frac{3}{2}\frac{d}{dp^2}\Pi_{hh}(p^2)\Big|_{p^2=M_h^2}\right)\lambda_{hhh}$$

$\delta Z_h^{\text{OS},\overline{\text{MS}}} = Z_h^{\text{OS},\overline{\text{MS}}} - 1$: wave-function renormalisation counterterms in OS/$\overline{\text{MS}}$ scheme,
$\Pi_{hh}(p^2)$: finite part of Higgs self-energy at ext. momentum p^2

- Taking $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \Gamma_{hhh}(0,0,0)$ is a good approximation
  - $\to$ shown for λ_{hhh} at one loop in [Kanemura, Okada, Senaha, Yuan '04] (difference is only a few %)
  - $\to$ no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading

Setup of our effective-potential calculation (detailed)

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\text{MS}}}) + \kappa f^{(1)}(x^{\overline{\text{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right] + \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

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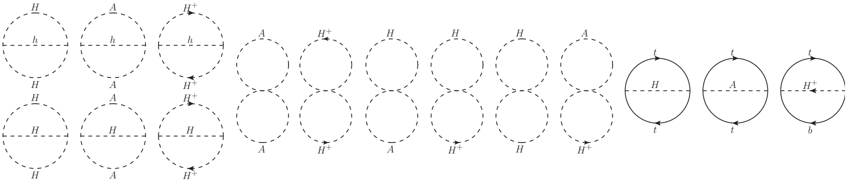
$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x} + \cancel{\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2}\right]$$

because we neglect m_h in the loop corrections and $\lambda^{(0)}_{hhh} = 3m_h^2/v$ (in absence of mixing)

λ_{hhh} at two loops in the 2HDM



- In the $\overline{\mathrm{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{96m_\Phi^4m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\,m_\Phi^2\right]+\mathcal{O}\left(\frac{m_\Phi^2m_t^4}{v^5}\right)$$

(Recall: aligned scenario, degenerate masses, dominant corrections only)

Decoupling behaviour of the $\overline{\mathrm{MS}}$ expressions

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(1)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3$$

$$\begin{aligned}\delta^{(2)}\lambda_{hhh} = {} & \frac{16 m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2+2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right] \\ & + \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\, m_\Phi^2\right] \\ & + \frac{96 m_\Phi^4 m_t^2 \cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)\end{aligned}$$

where $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

- To have $m_\Phi \to \infty$, then we must take $M \to \infty$, otherwise the quartic couplings grow out of control
- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2 = M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M\to\infty} 0$$

Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

- To obtain $\hat{\lambda}_{hhh} = -\Gamma_{hhh}(0,0,0)$, we must express our results in terms of physical parameters

$$\overline{\text{MS}}\text{ scheme:}\Big\{\underbrace{m_H, m_A, m_{H^\pm}}_{m_\Phi}, m_t, v\Big\} \longrightarrow \text{OS scheme:}\Big\{\underbrace{M_H, M_A, M_{H^\pm}}_{M_\Phi}, M_t, v_{\text{phys}} = (\sqrt{2}G_F)^{-1/2}\Big\}$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable
  ... but then, two-loop expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4\left\{4+3\cot^2 2\beta\left[3-\frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2}+2\right)\right]\right\} + \frac{576M_\Phi^6\cot^2 2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$
$$+\frac{288M_\Phi^4M_t^2\cot^2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3 + \frac{168M_\Phi^4M_t^2}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3 - \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^5 + \mathcal{O}\left(\frac{M_\Phi^2M_t^4}{v_{\text{phys}}^5}\right)$$