Minimal linear σ-model for the Goldstone Higgs

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Mainly based on the following papers: F. Feruglio et al. JHEP 1606 (2016) 038
B. Gavela, et al. Eur.Phys.J. C76 (2016) no.12, 690

Contents

- **The (Higgs) Hierarchy Problem;**
  - The Higgs as a (p)NGB of a global symmetry breaking;
- **The Linear σ-model for a NGB-Higgs;**
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- **Linear vs non-Linear realisation (if time);**
  - Integrating out the heavy scalar d.o.f;
- **Conclusions & Outlooks**

The Higgs Hierarchy Problem

- If the resonance found @LHC is the SM Higgs then some NP@TeV should be present to stabilise its mass

$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$$

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88 GeV

0.13

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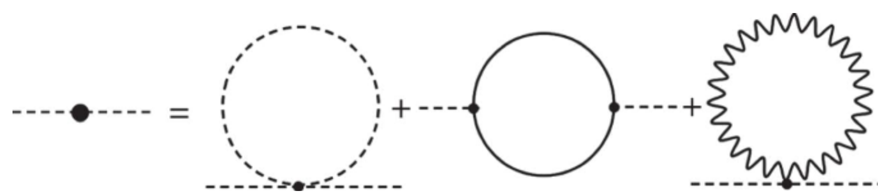
$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$$

- The Higgs mass receives quadratically divergent contributions from loop corrections:



$$\mu^2 = \mu^2_{tree} + \sum_i \delta\mu_i^2 \quad \Longrightarrow \quad \frac{\delta\mu_i^2}{\mu^2} \simeq \pm \frac{g_i^2}{16\pi^2}\left(\frac{\Lambda_i^2}{\mu^2}\right)$$

EW HIERARCHY PROBLEM: $\Lambda_{NP} \gg v$

The Higgs as a (p)NG Boson

[Georgi, Kaplan (1985), Agashe, Contino, Pomarol (2005)]

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$$\begin{tikzcd}
\boxed{\text{Global Symmetry} \; - \; \mathcal{G}} \arrow[r] \arrow[d] & \boxed{\text{Unbroken sector} \; - \; \mathcal{H}} \\
\boxed{\text{Spontaneous Breaking} - f} &
\end{tikzcd}$$

The Higgs as a (p)NG Boson

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Global Symmetry – $\mathcal{G}$ → Unbroken sector – $\mathcal{H}$

Spontaneous Breaking – f → dim($\mathcal{G}/\mathcal{H}$) m=0 GBs

π = (h, $\varphi_1, \varphi_2, \varphi_3$, ...)

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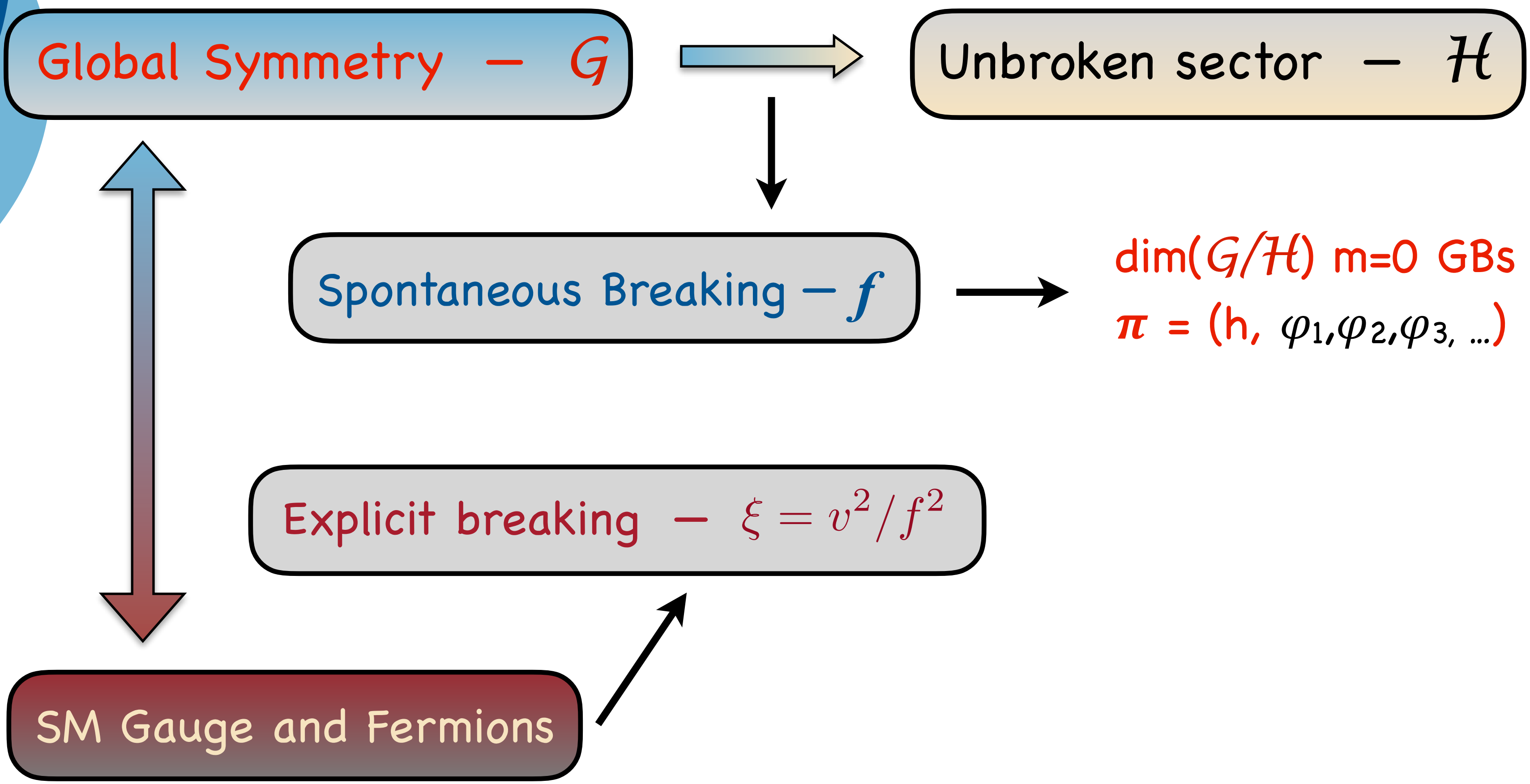

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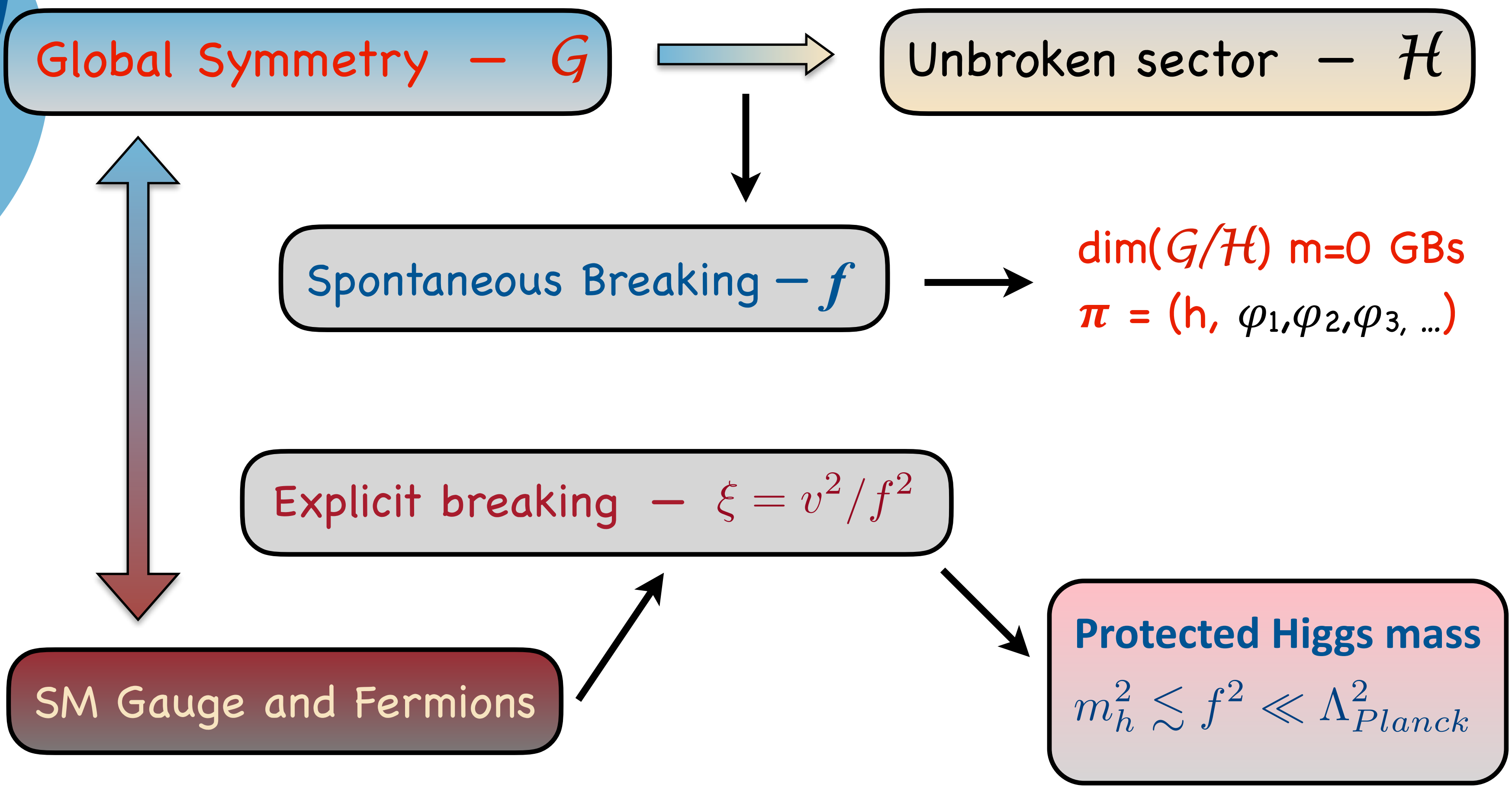

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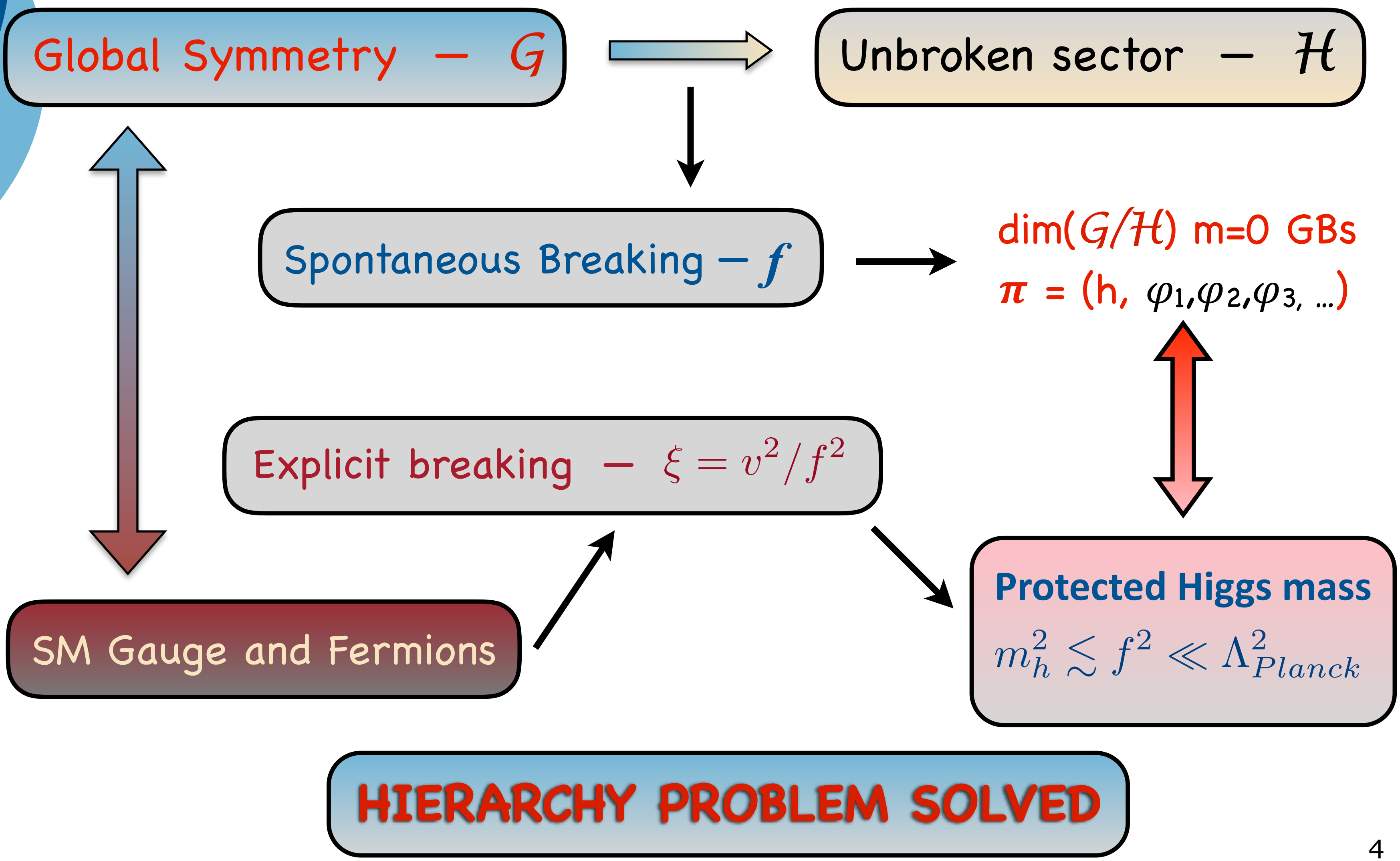

The Higgs as a (p)NG Boson

The Composite Higgs framework (but not only ...)

- Several coset configurations (4 or plus pGBs): SO(5)/SO(4), SU(5)/SO(5), SO(6)/SO(5), SU(4)/Sp(4), SU(4)/SU(3), ...;
- Often in the context of non-linear (HEFT) realisation of the symmetry breaking: [Contino (2011), Panico (2012), Redi (2012), Carena (2014), Carmona (2014), ...]

+ (Mostly) Model Independent approach, providing a parameterisation of all possible UV completions;

− Contains polynomial dependence on GBs, non renormalisable: limited energy validity;

Following chiral QCD example it may be enlightening to analyse a renormalisable **(Minimal) Linear σ-model**

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$$\phi = (\pi_1, \pi_2, \pi_3, h, \sigma)$$

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$$\begin{array}{ccc} \psi \sim 5 & \rightarrow & M_5 \\ \chi \sim 1 & \rightarrow & M_1 \end{array}$$

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$$y_1 \, \bar{\psi}_L \, \phi \, \chi_R + y_2 \, \bar{\psi}_R \, \phi \, \chi_L$$

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- Partial compositeness heavy-SM fermion couplings (breaks explicitly global SO(5)): [Kaplan '91]

$$\Lambda_1 \, (\bar{q}_L \Delta_{2\times 5}) \, \psi_R \, + \, \Lambda_2 \, \bar{\psi}_L \, (\Delta_{5\times 1} t_R) \, + \, \Lambda_3 \, \bar{\chi}_L \, (\Delta_{1\times 1} t_R)$$

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$\cancel{SO(5)}$ SM mass term

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The scalar potential

The most general renormalisable scalar potential SO(4) invariant contains 8 parameters, but only 4 are needed at 1-loop level:

[Barbieri (2007)]

$$V(h,\sigma) = \lambda\left(h^2+\sigma^2-f^2\right)^2 + \alpha f^3\sigma - \beta f^2 h$$

SO(5) invariant term induces SB to SO(4)

Explicit SO(5) breaking induced by fermionic loops

One obtains the following expressions for vevs:

$$v_\sigma^2 = f^2\frac{\alpha^2}{4\beta^2} \qquad , \qquad v_h^2 = f^2\left(1-\frac{\alpha^2}{4\beta^2}+\frac{\beta}{2\lambda}\right)$$

and masses for the physical light and heavy scalar states ($\tilde{h},\tilde{\sigma}$)

$$m^2_{h,\sigma} = 4\lambda f^2\left\{\left(1+\frac{3}{4}\frac{\beta}{\lambda}\right)\mp\left[1+\frac{\beta}{2\lambda}\left(1+\frac{\alpha^2}{4\beta^2}+\frac{\beta}{8\lambda}\right)\right]^{1/2}\right\}$$

rotated with respect to the original fields (h,σ):

$$\begin{pmatrix} h \\ \sigma \end{pmatrix} = \begin{pmatrix} \tilde{h}\cos\gamma + \tilde{\sigma}\sin\gamma \\ \tilde{\sigma}\cos\gamma - \tilde{h}\sin\gamma \end{pmatrix} \qquad \tan 2\gamma = \frac{4v_h v_\sigma}{3v_\sigma^2 - v_h^2 - f^2}$$

The SO(5)/SO(4) scalar-gauge sector

Setting the notation useful in the next slides:

$$H = \frac{(h+v)}{\sqrt{2}} \mathbf{U} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \longrightarrow \mathbf{U}(x) = e^{i\frac{\pi(x)}{f}}$$

$$\mathbf{D}_\mu \mathbf{U}(x) \equiv \partial_\mu \mathbf{U} + i\frac{g}{2} W_\mu^a \sigma_a \mathbf{U} - i\frac{g'}{2} B_\mu \mathbf{U} \sigma_3 \longrightarrow \mathbf{V}_\mu = (\mathbf{D}_\mu \mathbf{U})\, \mathbf{U}^\dagger$$

The SO(5)/SO(4) scalar-gauge sector reads (σ is a SM singlet)

$$\mathcal{L}_{g,s} \equiv (D_\mu H)^\dagger (D_\mu H) \supset \frac{v_h^2}{4} \langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle + \frac{v_h}{2} \left(\tilde{h} \cos\gamma + \tilde{\sigma} \sin\gamma \right) \langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle$$
$$+ \frac{1}{4} \left(\tilde{h}^2 \cos^2\gamma + 2\,\tilde{h}\,\tilde{\sigma} \sin\gamma \cos\gamma + \tilde{\sigma}^2 \sin^2\gamma \right) \langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle$$

- The first term identify the Gauge Boson masses:

$$M_W^2 = \frac{g^2\, v_h^2}{4} \quad , \quad M_Z^2 = \frac{(g^2 + g'^2)\, v_h^2}{4} \quad \rightarrow \quad v_h \equiv v = 246\,\text{GeV}$$

- The scalar-gauge couplings are "SM like" but with a $\cos\gamma$ suppression for $\tilde{h}$ (and a $\sin\gamma$ suppression for $\tilde{\sigma}$)

Parameters renormalisation

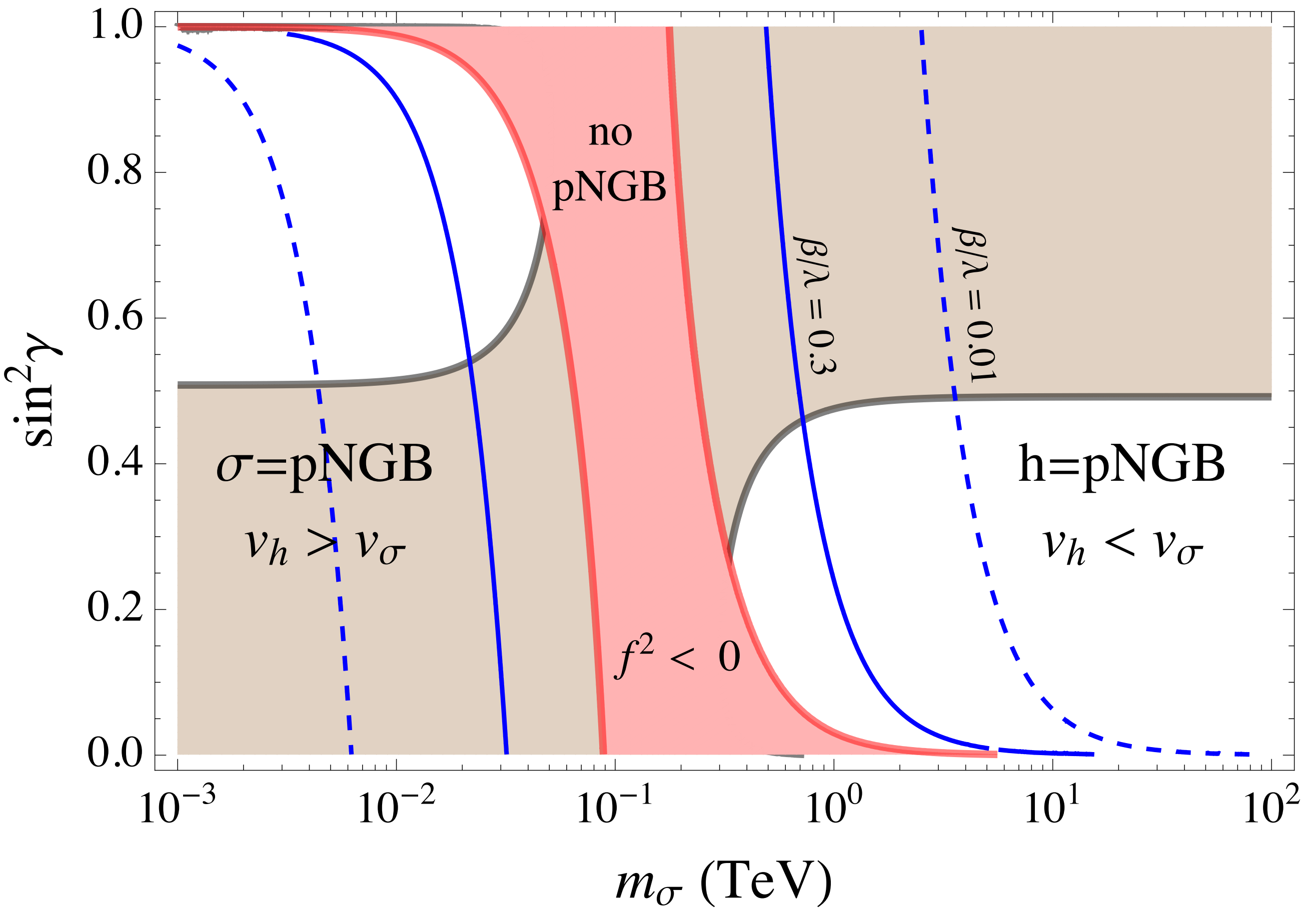
The 4 parameters appearing in the scalar Lagrangian can be expressed in terms of the following 2+2 observables:

$$\left\{ G_F \equiv (\sqrt{2}v^2)^{-1}, \qquad m_h, \qquad m_\sigma, \qquad \sin^2\gamma \right\}$$

by the following exact relation:

$$\begin{aligned}
\lambda &= \frac{\sin^2\gamma m_\sigma^2}{8v^2}\left(1+\cot^2\gamma\frac{m_h^2}{m_\sigma^2}\right),\\
\frac{\beta}{4\lambda} &= \frac{m_h^2 m_\sigma^2}{\sin^2\gamma m_\sigma^4+\cos^2\gamma m_h^4-2m_h^2 m_\sigma^2},\\
\frac{\alpha^2}{4\beta^2} &= \frac{\sin^2(2\gamma)(m_\sigma^2-m_h^2)^2}{4(\sin^2\gamma m_\sigma^4+\cos^2\gamma m_h^4-2m_h^2m_\sigma^2)},\\
f^2 &= \frac{v^2(\sin^2\gamma m_\sigma^4+\cos^2\gamma m_h^4-2m_h^2m_\sigma^2)}{(\sin^2\gamma m_\sigma^2+\cos^2\gamma m_h^2)^2}.
\end{aligned}$$

Th-available parameter space



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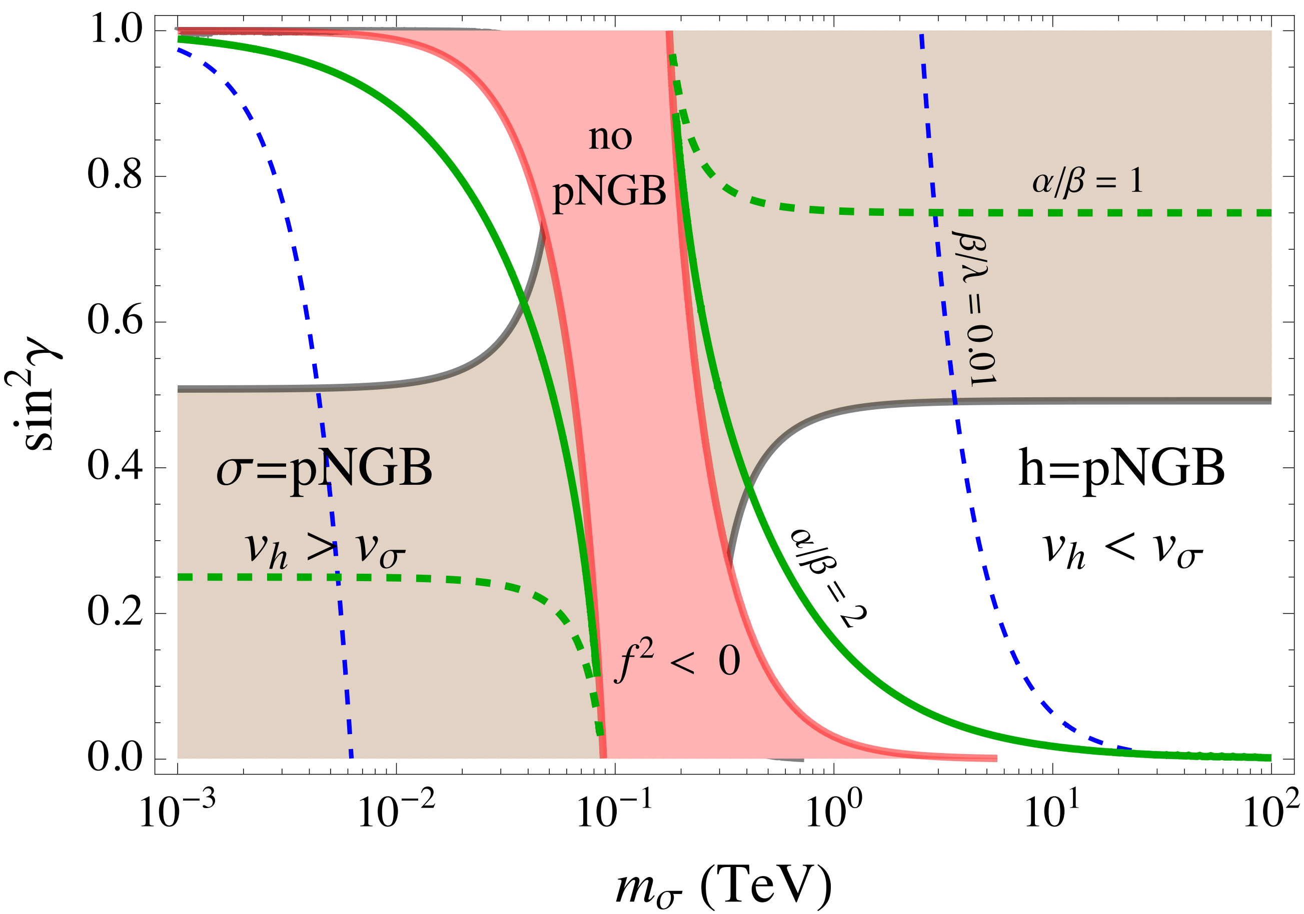
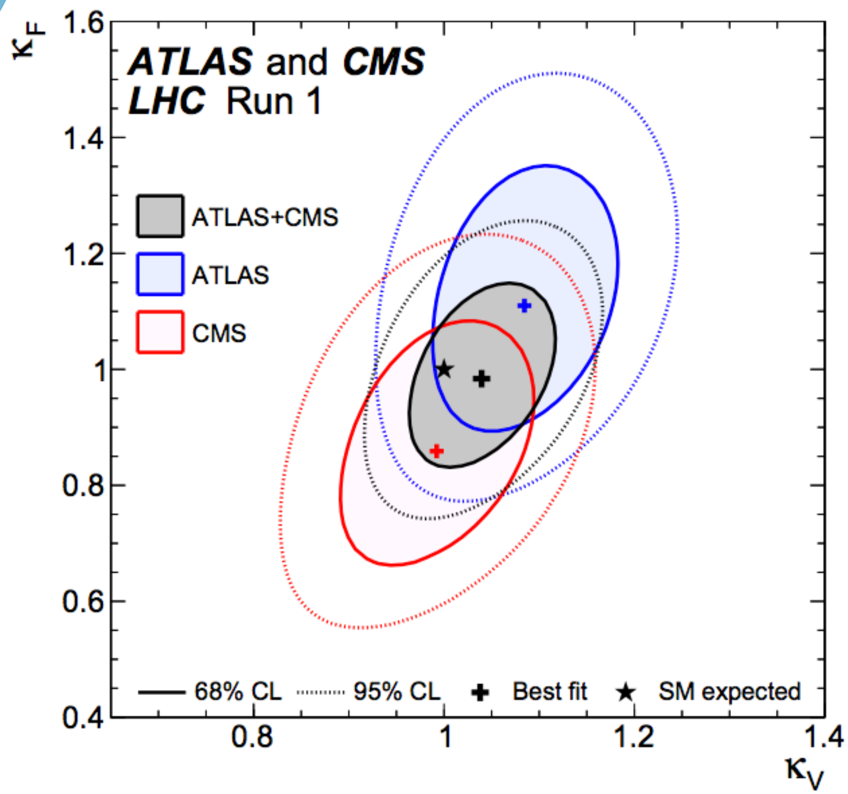


Exp-available parameter space

Higgs coupling to gauge bosons + effective coupling to gluons give bounds on the mixing angle γ

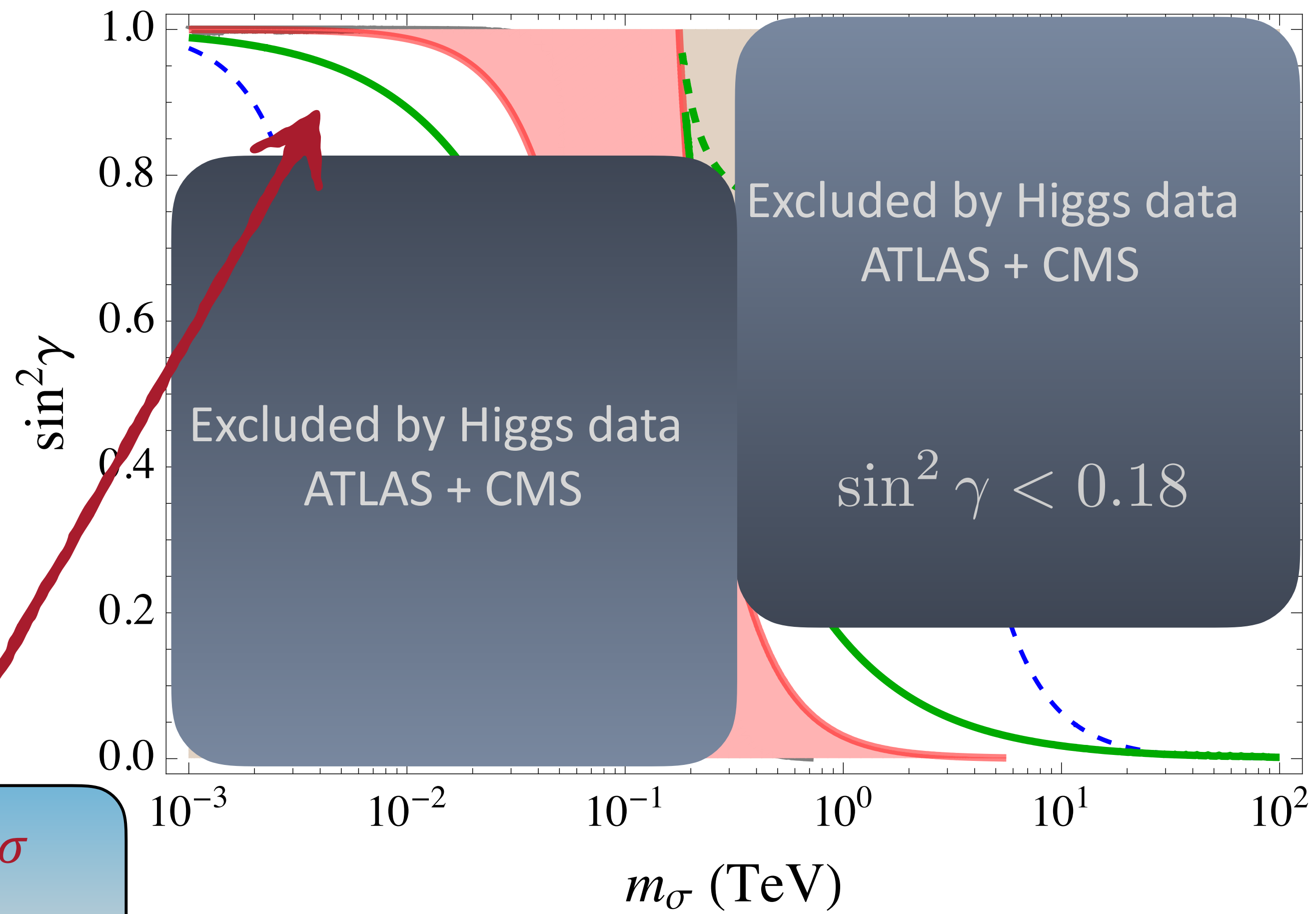


$$\sin^2\gamma < 0.18 \text{ at } 2\sigma$$

$$\cos^2\gamma > 0.82 \text{ at } 2\sigma$$

$$\left(\kappa_F = \frac{g_{hFF}}{g_{HFF}^{SM}} \qquad \kappa_V = \frac{g_{hVV}}{g_{HVV}^{SM}}\right)$$

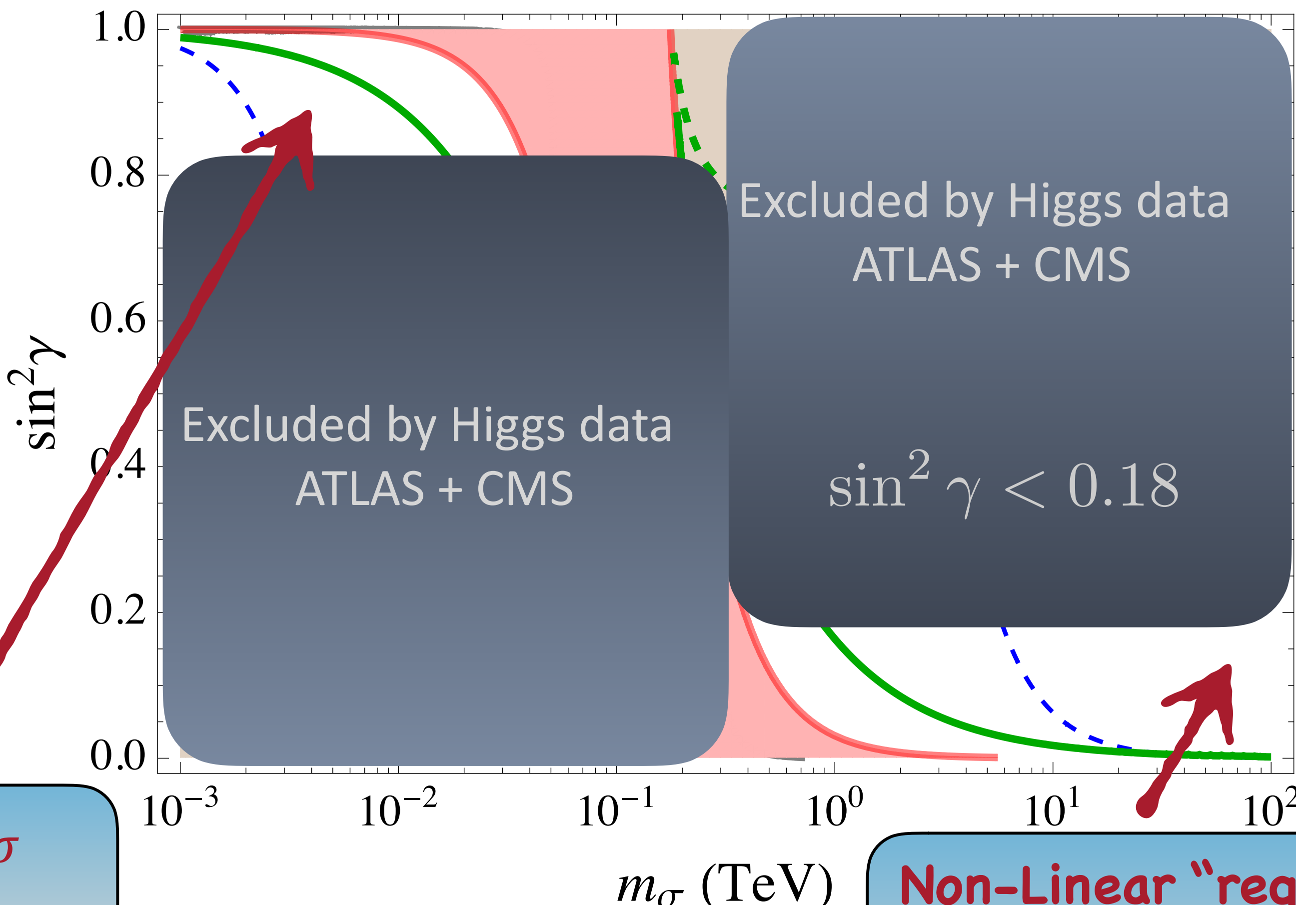
Available parameter space



Light σ

$m_\sigma \ll m_h \approx v$

Available parameter space



Light σ

$m_\sigma \ll m_h \approx v$

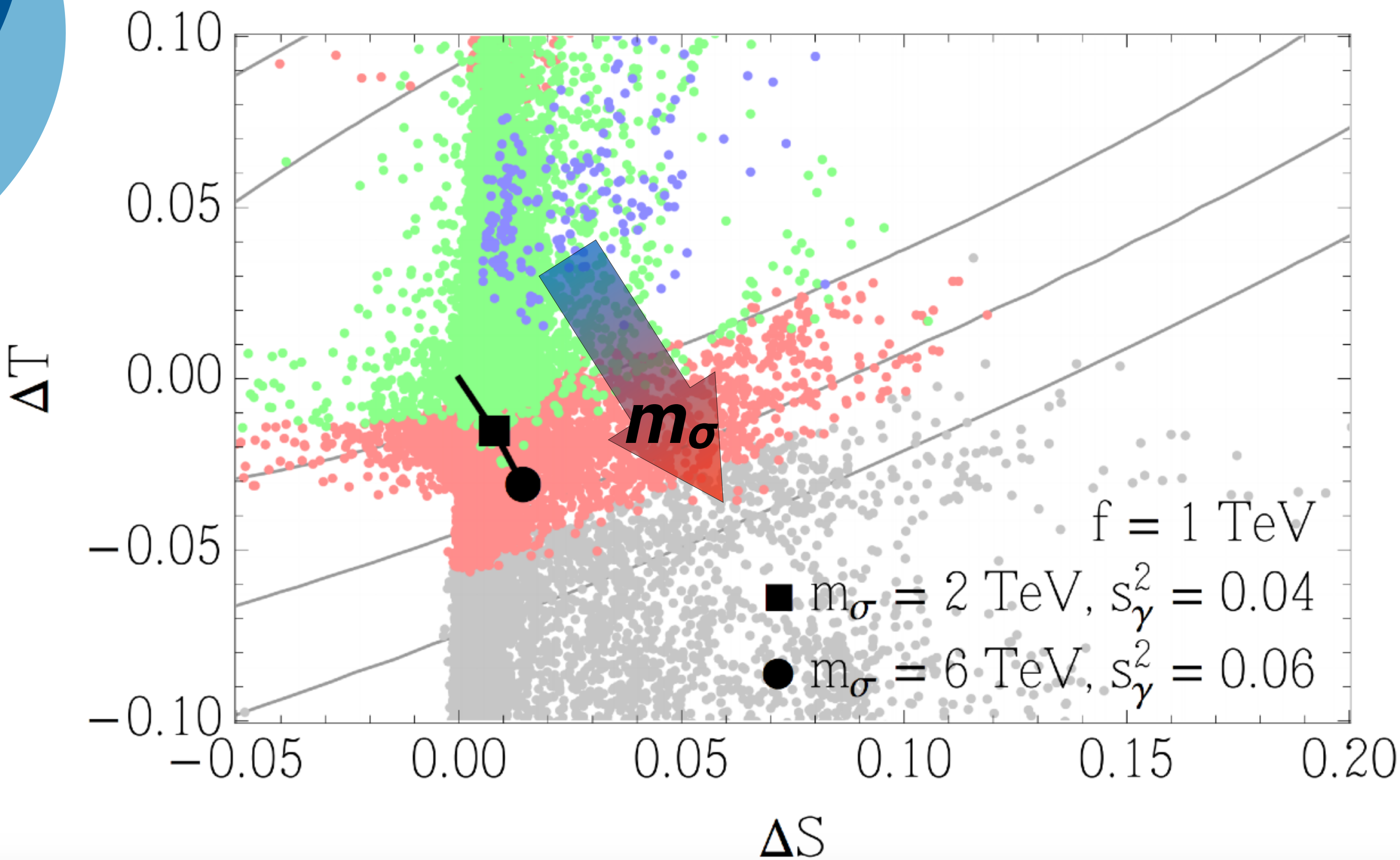
Non-Linear "regime"

$\alpha \approx \beta \ll \lambda$

$m_h \approx v \ll m_\sigma$

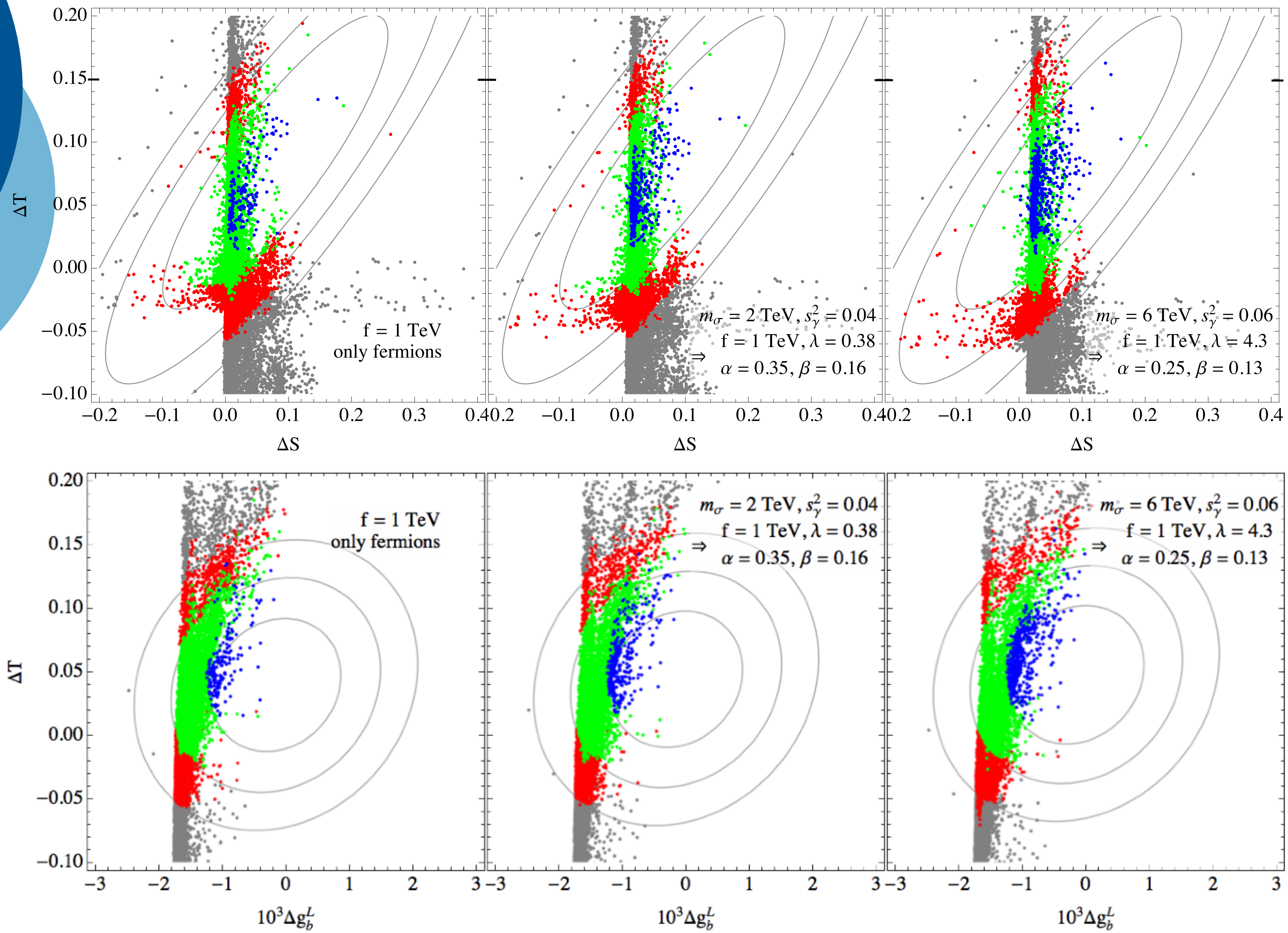
Bounds from Precision @ LEP

Lower m_σ is better, but fermion can always compensate

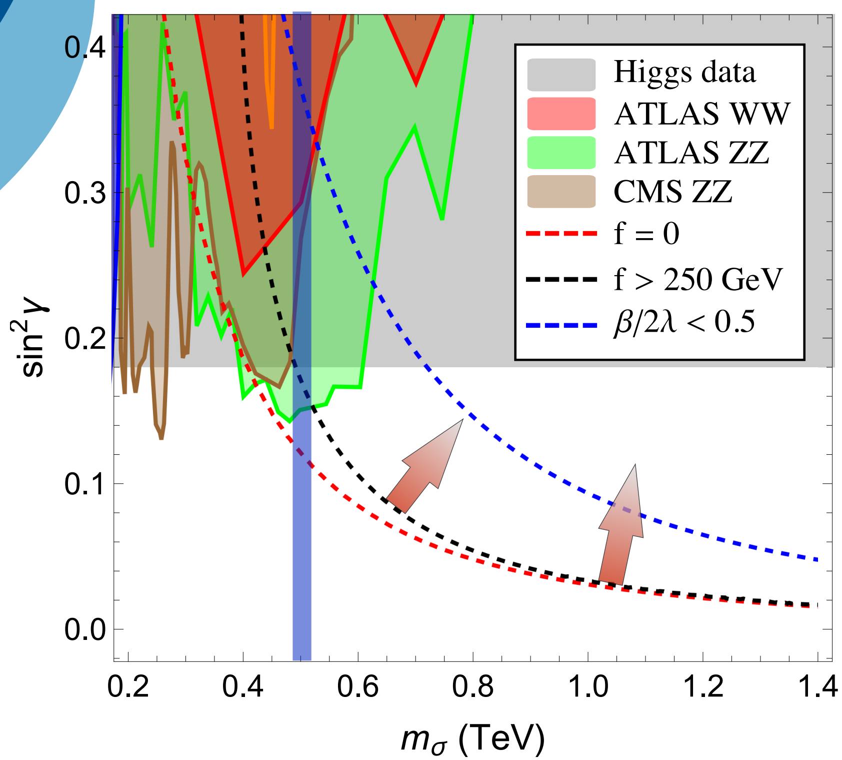


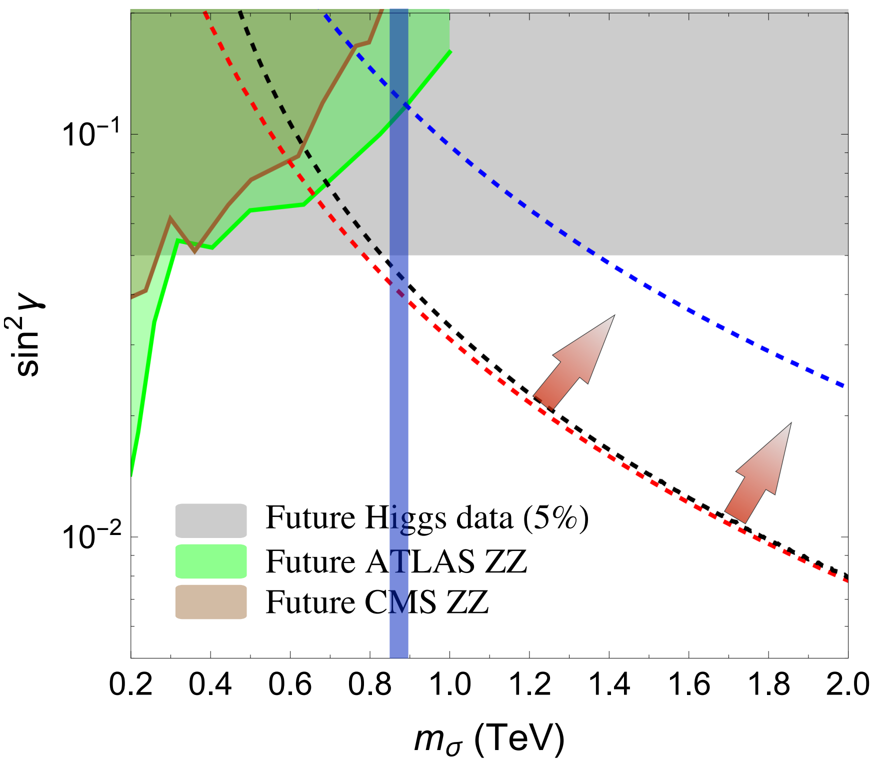
[see also Barbieri (2007) and Gertov (2015)]

Precision Bounds and Zbb



LHC Bounds on σ





$m_\sigma \geq 500$ GeV

$m_\sigma \geq 900$ GeV

Integrating out the heavy scalar

LHC bounds are going to push higher and higher the σ scale, so it may useful to integrate out the heavy field;

The mass of the heavy scalar is controlled by the self coupling λ

$$m_\sigma^2 = 8\lambda f^2 + O(\lambda^0)$$
$$m_h^2 = 2\beta v^2 + O(1/\lambda) \longrightarrow \frac{m_h^2}{m_\sigma^2} \simeq \frac{\beta\xi}{4\lambda}$$

It is useful to redefine the scalar fields in "polar" coordinates:

$$\begin{aligned} \sigma &= \rho\cos\varphi \\ h &= \rho\sin\varphi \end{aligned} \longrightarrow \begin{aligned} \rho &= \langle\rho\rangle + \sigma \\ \varphi &= \langle\varphi\rangle + h \end{aligned}$$

Solving perturbatively the ρ equation of motion:

$$\rho = \rho_0 + \rho_1/\lambda + \rho_2/\lambda^2 + \ldots$$

gives the effective Lagrangian exapanded in powers of $1/\lambda$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1/\lambda + \mathcal{L}_2/\lambda^2 + \ldots$$

Integrating out the heavy scalar

- The LO Lagrangian is the 2∂ custodial preserving SO(5)/SO(4) HEFT Lagrangian ($V_\mu = (D_\mu U)U^\dagger$) with the SSB breaking potential term induced by α and β parameters: [Agashe, Contino, Pomarol (2005)]

$$\mathcal{L}_0 = \underbrace{\frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{v^2}{4}\frac{s_\phi^2}{\xi}\langle V^\mu V_\mu\rangle}_{\text{SO(5) invariant terms}} - \underbrace{f^4\left(\alpha c_\varphi - \beta s_\varphi^2\right)}_{\text{SO(5) breaking term}}$$

- The NLO terms contains 4∂ operators, corrections to the 2∂ LO Lagrangian and correction to the SSB breaking potential:

$$\mathcal{L}_1 = \left[\underbrace{\frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{v^2}{4}\frac{s_\varphi^2}{\xi}\langle V_\mu V^\mu\rangle}_{\text{SO(5) invariant terms}} - \underbrace{\frac{f^4}{2}\left(\alpha c_\varphi - 2\beta s_\varphi^2\right)}_{\text{SO(5) breaking term}}\right]^2$$

NOTE: Interactions with SM and heavy fermions are here neglected but they can easily be included [Gavela et al. (2016)]

Only **5 derivative operators are generated** at tree level at O(1/λ)
(the Higgs potential terms at LO and NLO are not reported here)

	Operator	$\mathcal{F}_k(\varphi)$	$1/\lambda^n$
$\mathcal{L}_{2\partial}$	$\mathcal{P}_H = \frac{1}{2}(\partial_\mu h)^2 \mathcal{F}_H(\varphi)$	$1 - \frac{1}{4\lambda}(\alpha c_\varphi - 2\beta s_\varphi^2)$	0
	$\mathcal{P}_C = -\frac{v^2}{4}\langle V_\mu V^\mu \rangle \mathcal{F}_C(\varphi)$	$\frac{1}{\xi}\left[1 - \frac{1}{4\lambda}\left(\alpha c_\varphi - 2\beta s_\varphi^2\right)\right] s_\varphi^2$	0
$\mathcal{L}_{4\partial}$	$\mathcal{P}_{DH} = \frac{1}{v^4}(\partial_\mu h)^4 \mathcal{F}_{DH}(\varphi)$	$\frac{\xi^2}{16\lambda}$	1
	$\mathcal{P}_6 = \langle V_\mu V^\mu \rangle^2 \mathcal{F}_6(\varphi)$	$\frac{s_\varphi^4}{64\lambda}$	1
	$\mathcal{P}_{20} = \frac{1}{v^2}\langle V_\mu V^\mu \rangle (\partial_\nu h)^2 \mathcal{F}_{20}(\varphi)$	$-\frac{\xi}{16\lambda} s_\varphi^2$	1
[redacted]			

$$\varphi = \langle \varphi \rangle + h \quad c_\varphi = \cos(\varphi/f) \quad s_\varphi = \sin(\varphi/f)$$

Integrating out the heavy scalar

- The Higgs potential triggering the EW SB is proportional to the explicit SO(5) breaking parameters α and β:

- At NLO the Higgs kinetic term ($\mathcal{P}_H$) gets non-canonical and the Higgs field has to renormalised: $h \to \left(1 - \beta/(4\lambda)\right)$

- From $\mathcal{P}_C$ one defines $\mathrm{M_W}$ and the Higgs to Gauge Boson couplings

$$\kappa_V \equiv \frac{g_{hVV}}{g_{hVV}^{SM}} = \sqrt{1-\xi} + \frac{\beta\xi}{2\lambda}\frac{1-\xi/2}{\sqrt{1-\xi}} + O(1/\lambda^2)$$

- The 3 operators with 4∂, at O(1/λ), are SO(5) preserving; they contribute to 4h, 4V (+hs) and 2V+2h (+hs) couplings.

- Only few of the possible non-linear operators appears at tree level (at any order 1/λ). For example operators containing $\langle V_\mu V_\nu \rangle$ would be loop-induced and expected to be sub-leading.

Σ decomposition of G/H coset

- It is interesting to point out the connection with the Σ decomposition of the chiral Lagrangian of a generic G/H model;

[Alonso et al. JHEP 1412 (2014) 034]

- Let's denote with $\Theta(x)$ the GB field of the G/H coset, and with $\tilde{S}_\mu$ the gauge fields (subgroup of G)

$$\Theta(x) = e^{i\frac{\pi(x)}{f}} \longrightarrow \mathbf{D}_\mu \Theta(x) = \partial_\mu \Theta(x) + ig\left(\tilde{S}_\mu \Theta(x) - \Theta(x)\tilde{S}^{\mathcal{R}}_\mu\right)$$

- Following ALF, one can write the most general effective chiral Lagrangian (scale f) compatible with the SM gauge symmetry, (custodial preserving) using as building blocks:

[Appelquist, Carazzone (1980); Longhitano (1980), Feruglio (1993)]

$$\tilde{\mathbf{V}}_\mu = (\mathbf{D}_\mu \Theta)\,\Theta^\dagger$$

$$\mathcal{D}_\mu \tilde{\mathbf{V}}^\mu = \partial_\mu \tilde{\mathbf{V}}^\mu + ig_S\left[\tilde{S}_\mu, \tilde{\mathbf{V}}_\mu\right]$$

$$\tilde{S}_{\mu\nu} = \left(\tilde{W}_{\mu\nu}, \tilde{B}_{\mu\nu}\right)$$

SO(5)/SO(4) EW chiral Lagrangian

- Basis of CP-even gauge-Goldstone operators up to 4∂, assuming no extra-SM custodial symmetry breaking terms:

$$\widetilde{\mathcal{A}}_C = -\frac{f^2}{4}\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right)$$

$$\widetilde{\mathcal{A}}_B = -\frac{1}{4}\mathrm{Tr}\left(\widetilde{\mathbf{B}}_{\mu\nu}\widetilde{\mathbf{B}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_W = -\frac{1}{4}\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\widetilde{\mathbf{W}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_{B\Theta} = g'^2\mathrm{Tr}\left(\Theta\widetilde{\mathbf{B}}_{\mu\nu}\Theta^\dagger\widetilde{\mathbf{B}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_{W\Theta} = g^2\mathrm{Tr}\left(\Theta\widetilde{\mathbf{W}}_{\mu\nu}\Theta^\dagger\widetilde{\mathbf{W}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_1 = g\, g'\,\mathrm{Tr}\left(\Theta\widetilde{\mathbf{B}}_{\mu\nu}\Theta^\dagger\widetilde{\mathbf{W}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_2 = i\, g'\,\mathrm{Tr}\left(\widetilde{\mathbf{B}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right)$$

$$\widetilde{\mathcal{A}}_3 = i\, g\,\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right)$$

$$\widetilde{\mathcal{A}}_4 = \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right)\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right)$$

$$\widetilde{\mathcal{A}}_5 = \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}_\nu\right)\mathrm{Tr}\left(\widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}^\nu\right)$$

$$\widetilde{\mathcal{A}}_6 = \mathrm{Tr}\left((\mathcal{D}_\mu\widetilde{\mathbf{V}}^\mu)^2\right)$$

$$\widetilde{\mathcal{A}}_7 = \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\nu\right)$$

$$\widetilde{\mathcal{A}}_8 = \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}^\nu\right)$$

$$\widetilde{\mathcal{A}}_9 = \mathrm{Tr}\left((\mathcal{D}_\mu\widetilde{\mathbf{V}}^\mu)\widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\nu\right)$$

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$$\widetilde{\mathcal{A}}_W = -\frac{1}{4}\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\widetilde{\mathbf{W}}^{\mu\nu}\right)$$

$$\widetilde{\mathcal{A}}_{B\Theta} = g'^2\mathrm{Tr}\left(\Theta\widetilde{\mathbf{B}}_{\mu\nu}\Theta^\dagger\widetilde{\mathbf{B}}^{\mu\nu}\right)$$

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$$\widetilde{\mathcal{A}}_3 = i\,g\,\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right)$$

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Σ decomposition of SO(5)/SO(4) coset

- The building blocks for the SO(5)/SO(4) Lagrangian are defined:

$$\Theta(x) = e^{i\frac{\varphi(x)}{f}\mathcal{X}(x)} = \mathbf{1} + i\sin\left(\varphi/f\right)\mathcal{X} + \left(\cos\left(\varphi/f\right) - \mathbf{1}\right)\mathcal{X}^2 \qquad (\varphi(x) = h(x) + \langle\varphi\rangle)$$

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3 SM wBGBs

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"Higgs" scalar

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- Once the heavy scalar is integrated out, a particular subset of operators of the SO(5)/SO(4) effective chiral Lagrangian is obtained;

Backup Slides

The Heavy Fermion content

Two type of fermions with different charge under an extra $U(1)_X$ can be defined (X=2/3 and X=-1/3) for the 5 and the 1 of SO(5):

$$\psi^{(5)}_{+2/3} \sim (X, Q, T^{(5)}), \qquad \psi^{(1)}_{+2/3} \sim T^{(1)}$$

$$\psi^{(5)}_{-1/3} \sim (Q', X', B^{(5)}), \qquad \psi^{(1)}_{-1/3} \sim B^{(1)}$$

The decomposition of fermions under $SU(2)_L$ x $U(1)_Y$ group is shown

Charge/Field	X	Q	$T_{(1,5)}$	Q'	X'	$B_{(1,5)}$
$\Sigma_R^{(3)}$	$+1/2$	$-1/2$	0	$+1/2$	$-1/2$	0
$SU(2)_L \times U(1)_Y$	$(2,+7/6)$	$(2,+1/6)$	$(1,+2/3)$	$(2,+1/6)$	$(2,-5/6)$	$(1,-1/3)$
x	$+2/3$	$+2/3$	$+2/3$	$-1/3$	$-1/3$	$-1/3$
q_{EM}	$X^u = +5/3$ $X^d = +2/3$	$Q^u = +2/3$ $Q^d = -1/3$	$+2/3$	$Q'^u = +2/3$ $Q'^d = -1/3$	$X'^u = -1/3$ $X'^d = -4/3$	$-1/3$
		q_L	t_R	q_L		b_R

The Heavy Fermion Lagrangian

- The SO(5) preserving part of the Lagrangian includes the proto-Yukawa interactions with the scalar multiplet:

$$\begin{aligned}\mathcal{L}_{SO(5)} &= \bar{\psi}^{(2/3)}\left(i\not{D} - M_5\right)\psi^{(2/3)} + \bar{\psi}^{(-1/3)}\left(i\not{D} - M_5'\right)\psi^{(-1/3)} \\ &+ \bar{\chi}^{(2/3)}\left(i\not{D} - M_1\right)\chi^{(2/3)} + \bar{\chi}^{(-1/3)}\left(i\not{D} - M_1'\right)\chi^{(-1/3)} \\ &- y_1\,\bar{\psi}_L^{(2/3)}\phi\,\chi_R^{(2/3)} - y_2\,\bar{\psi}_R^{(2/3)}\phi\,\chi_L^{(2/3)} \\ &- y_1'\,\bar{\psi}_L^{(-1/3)}\phi\,\chi_R^{(-1/3)} - y_2'\,\bar{\psi}_R^{(-1/3)}\phi\end{aligned}$$

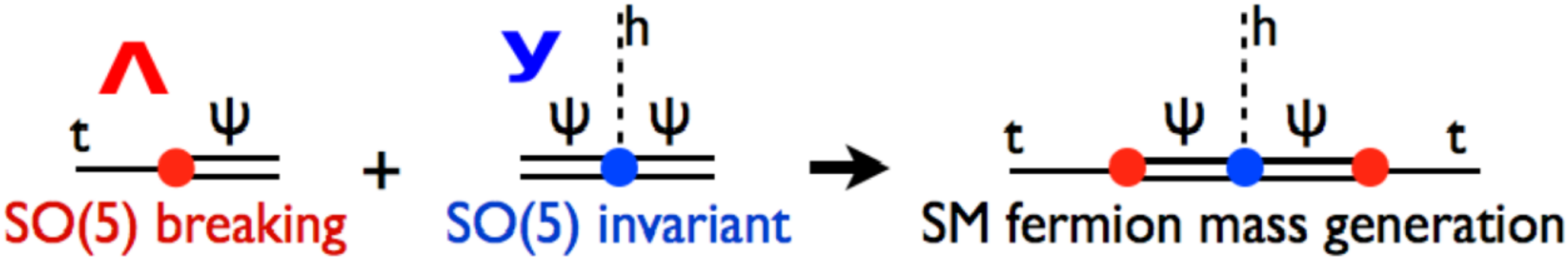
- The SO(5) breaking part of the fermionic Lagrangian is given by partial-compositeness couplings with SM (massless) fermion

[Kaplan '91]

$$\begin{aligned}\mathcal{L}_{\cancel{SO(5)}} &= -\left[\Lambda_1\bar{q}_L Q_R + \Lambda_2\bar{T}_L^{(5)}t_R + \Lambda_3\bar{T}_L^{(1)}t_R + h.c.\right] \\ &\quad -\left[\Lambda_1'\bar{q}_L Q_R' + \Lambda_2'\bar{B}_L^{(5)}b_R + \Lambda_3'\bar{B}_L^{(1)}b_R + h.c.\right]\end{aligned}$$

SM fermion mass generation

- Combining the SO(5) invariant proto-Yukawas with the SO(5) breaking partial composite interactions:



one gives rise to a see-saw mechanism for generating the SM fermion masses. The leading order can be obtained schematically

$$q_L \xrightarrow[\Lambda_1]{} Q_R \xrightarrow[M_5]{} Q_L \xrightarrow[y_1\langle H\rangle]{} T_R^{(1)} \xrightarrow[M_1]{} T_L^{(1)} \xrightarrow[\Lambda_3]{} t_R$$

$$y_t \sim y_1 \frac{\Lambda_1 \Lambda_3}{M_1 M_5} v \qquad \text{and} \qquad y_b \sim y_1' \frac{\Lambda_1' \Lambda_3'}{M_1' M_5'} v$$

Fermion loops and Scalar Potential

- The generalised mass matrix of heavy vector-like and SM fermions contains SO(5) breaking terms (Λ_1, Λ_2, Λ_3)

$$-\bar{\Psi}_L \mathcal{M}(h,\sigma)\Psi_R$$

Through Coleman–Weinberg mechanism, at one loop, the SO(5) breaking propagates to the scalar potential. Among other terms it induces two divergent SO(5) breaking terms:

$$V_{\rm CW} \supset \frac{1}{64\pi^2}\mathrm{Tr}\left[(\mathcal{M}\mathcal{M}^\dagger)^2\right]\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\mathrm{Tr}[(\mathcal{M}\mathcal{M}^\dagger)^2] = [\mathrm{SO}(5)_{\rm inv}] + A\sigma + Bh^2$$

One need to introduce such terms at tree-level in the potential in order to ensure the renormalisability to the model

$$V(h,\sigma) \supset \alpha f^3\sigma - \beta f^2 h^2$$