How to study models with an arbitrary number of Higgs-boson doublets

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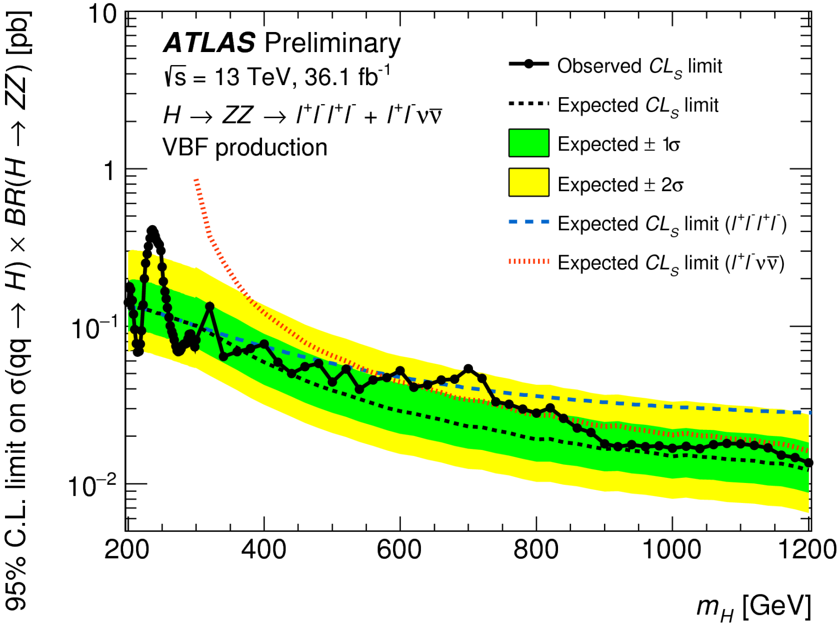
Why study nHDM's?

- T. D. Lee introduced 2HDM getting CP violation.
- Dark matter models realized by nHDM's.
- Neutrino mixing models realized by nHDM's
- Supersymmetry requieres at least two doublets.
- Number of doublets not restricted.


ATLAS Preliminary
√s = 13 TeV, 36.1 fb-1
H → ZZ → l+l-l+l- + l+l-νν̄
VBF production
Observed CLs limit
Expected CLs limit
Expected ± 1σ
Expected ± 2σ
Expected CLs limit (l+l-l+l-)
Expected CLs limit (l+l-νν̄)
95% C.L. limit on σ(qq → H) × BR(H → ZZ) [pb]
mH [GeV]
10
1
10−1
10−2
200
400
600
800
1000
1200

- Suppose we have nHDM; we want to study

  stability
  global minima
  electroweak symmetry breaking
  symmetries
  squared mass matrices

- Simple case one Higgs doublet

$$V_{SM} = -\mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2$$

- Electroweak symmetry breaking: $\mu^2 > 0$, stability: $\lambda > 0$,

- Vacuum expectation value: $v_0 = \sqrt{\frac{\mu^2}{\lambda}} \simeq$ 246 GeV.

Example of 3HDM

- 3 Higgs-boson doublets to generate ν masses and mixing.

$$\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}, \qquad i = 1, 2, 3$$

W. Grimus, L. Lavoura and D. Neubauer, JHEP **0807**, 051 (2008)

- Symmetry $O(2) \times \mathbb{Z}_2 \cong \mathbb{Z}_2' \times U(1) \times \mathbb{Z}_2$
- Potential

$$\begin{aligned} V_{O(2)\times\mathbb{Z}_2} = {} & \mu_0 \varphi_3^\dagger \varphi_3 + \mu_{12} \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) + \mu_m \left(\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1 \right) \\ & + a_1 (\varphi_3^\dagger \varphi_3)^2 + a_2 \varphi_3^\dagger \varphi_3 \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) + a_3 \left(\varphi_3^\dagger \varphi_1 \cdot \varphi_1^\dagger \varphi_3 + \varphi_3^\dagger \varphi_2 \cdot \varphi_2^\dagger \varphi_3 \right) \\ & + a_4 \varphi_3^\dagger \varphi_1 \cdot \varphi_3^\dagger \varphi_2 + a_4^* \varphi_1^\dagger \varphi_3 \cdot \varphi_2^\dagger \varphi_3 + a_5 \left((\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2 \right) \\ & + a_6 \varphi_1^\dagger \varphi_1 \cdot \varphi_2^\dagger \varphi_2 + a_7 \varphi_1^\dagger \varphi_2 \cdot \varphi_2^\dagger \varphi_1 \end{aligned}$$

Bilinears

- Higgs doublets $\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}$, $i = 1, \dots, n$,

  write

$$\phi = \begin{pmatrix} \varphi_1^{\mathrm{T}} \\ \vdots \\ \varphi_n^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \, \varphi_1^0 \\ \vdots \\ \varphi_n^+ \, \varphi_n^0 \end{pmatrix}$$

- Arrange all $SU(2)_L \times U(1)_Y$ invariants into hermitian $n \times n$ matrix

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \cdots & \varphi_n^\dagger\varphi_n \\ \vdots & \ddots & \vdots \\ \varphi_1^\dagger\varphi_n & \cdots & \varphi_n^\dagger\varphi_n \end{pmatrix}$$

- $\underline{K} = \phi\phi^\dagger$ is hermitian, positive semidefinite with rank ≤ 2.
- Basis for $\underline{K}$ are Gell-Mann matrices λ_α, $\lambda_0 = \sqrt{\frac{2}{n}}\mathbb{1}_n$,

$$\underline{K} = \frac{1}{2} K_\alpha \lambda_\alpha, \qquad \alpha = 0, 1, \dots, n^2 - 1$$

- One-to-one correspondance between Higgs-boson doublets and Hermitean matrix $\underline{K}$ with rank ≤ 2.

- nHDM potential can be written with K_0, $\boldsymbol{K} = \begin{pmatrix} K_1 \\ \vdots \\ K_{n^2-1} \end{pmatrix}$

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

avoid unphysical gauge degrees of freedom.
real parameters.
reduce power of potential.

C. Nishi **PRD 74** (2006)
MM, A. Manteuffel, O. Nachtmann, F. Nagel **EPJC 48** (2006)

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1'(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n'(x)^{\mathrm{T}} \end{pmatrix} = U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$K_0' = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

with $U^\dagger \lambda_a U = R_{ab}(U)\lambda_b$, $R \in SO(n^2-1)$, proper rotations in $\boldsymbol{K}$-space.

- Under change of basis $K_0' = K_0$, $\boldsymbol{K}' = R(U)\boldsymbol{K}$ potential remains invariant if

$$\xi_0' = \xi_0, \quad \eta_{00}' = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

Symmetries

- Symmetry desirable to restrict nHDM.
- Symmetries easily formulated in terms of bilinears.

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0 \, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to \bar{R} \boldsymbol{K}$, $\bar{R} \in O(n^2 - 1)$ is symmetry of potential iff

$$\boldsymbol{\xi} = \bar{R} \, \boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R} \, \boldsymbol{\eta}, \qquad E = \bar{R} \, E \, \bar{R}^{\mathrm{T}}$$

- $\bar{R} \in O(n^2 - 1)$, keeping kinetic terms invariant.

I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
MM, O. Nachtmann, JHEP 11 151 (2011)
V. Keus, S.F. King, S. Moretti, JHEP 1401 (2014)
B. Grzadkowski, MM, J. Wudka, JHEP 1111 (2011)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

CP symmetry

- CP transformation of the doublet fields

$$\varphi_i(x) \longrightarrow \varphi_i^*(x'), \qquad i = 1, \dots n, \quad x = (t, \boldsymbol{x})^{\mathrm{T}}, \quad x' = (t, -\boldsymbol{x})^{\mathrm{T}}$$

- In terms of bilinears

$$K_0(x) \longrightarrow K_0(x'), \qquad \boldsymbol{K}(x) \longrightarrow \bar{R}\, \boldsymbol{K}(x')$$

- $\bar{R}$ is defined by the (generalized) Gell-Mann matrices

$$\lambda_a^{\mathrm{T}} = \bar{R}_{ab}\lambda_b, \qquad a, b \in \{1, \dots, n^2 - 1\}.$$

THDM: $\bar{R} = \mathrm{diag}(1, -1, 1),$

3HDM: $\bar{R} = \mathrm{diag}(1, -1, 1, 1, -1, 1, -1, 1)$

- CP symmetry conditions

$$\xi = \bar{R}\,\xi, \qquad \eta = \bar{R}\,\eta, \qquad E = \bar{R}\,E\,\bar{R}^{\mathrm{T}}.$$

- CP symmetry respected by vacuum if

$$\bar{R}\langle \boldsymbol{K} \rangle = \langle \boldsymbol{K} \rangle$$

- Basis invariance condition, generalized CP studied in THDM.

C. Nishi **PRD 74** (2006),
MM, A. Manteuffel, O. Nachtmann, EPJC57 (2008)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

Study of the nHDM

- Concise conditions for stability given.

- Stationarity equations with different sets coresponding to electroweak symmetry breaking behavior.

- Squared mass matrices given for the general case.

MM, O. Nachtmann, PR D92 7 (2015)

Conclusion

- Bilinears are powerful tool in the nHDM.

Electroweak symmetry breaking

- EW symmetry breaking given by global minimum

$$\langle\phi\rangle = \left\langle \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \vdots & \\ \varphi_n^+ & \varphi_n^0 \end{pmatrix} \right\rangle = \begin{pmatrix} v_1^+ & v_1^0 \\ \vdots & \\ v_3^+ & v_3^0 \end{pmatrix}, \quad \underline{K} = \langle\phi\rangle\langle\phi\rangle^\dagger$$

- Fully broken EW symmetry corresponds to $\langle\phi\rangle$ rank 2

$$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) > 0, \qquad \det(\underline{K}) = 0.$$

- $SU(2)_L \times U(1)_Y \ \to U(1)_{em}$ corresponds to $\langle\phi\rangle$ rank 1,

$$\mathrm{tr}(\underline{K}) > 0, \qquad (\mathrm{tr}(\underline{K}))^2 - \mathrm{tr}(\underline{K}^2) = 0, \qquad \det(\underline{K}) = 0.$$

- Unbroken $SU(2)_L \times U(1)_Y$ corresponds to $\langle\phi\rangle$ rank 0,

$$\underline{K} = 0, \qquad \varphi_i = 0, \qquad V = 0$$