Real Scalar Dark Matter

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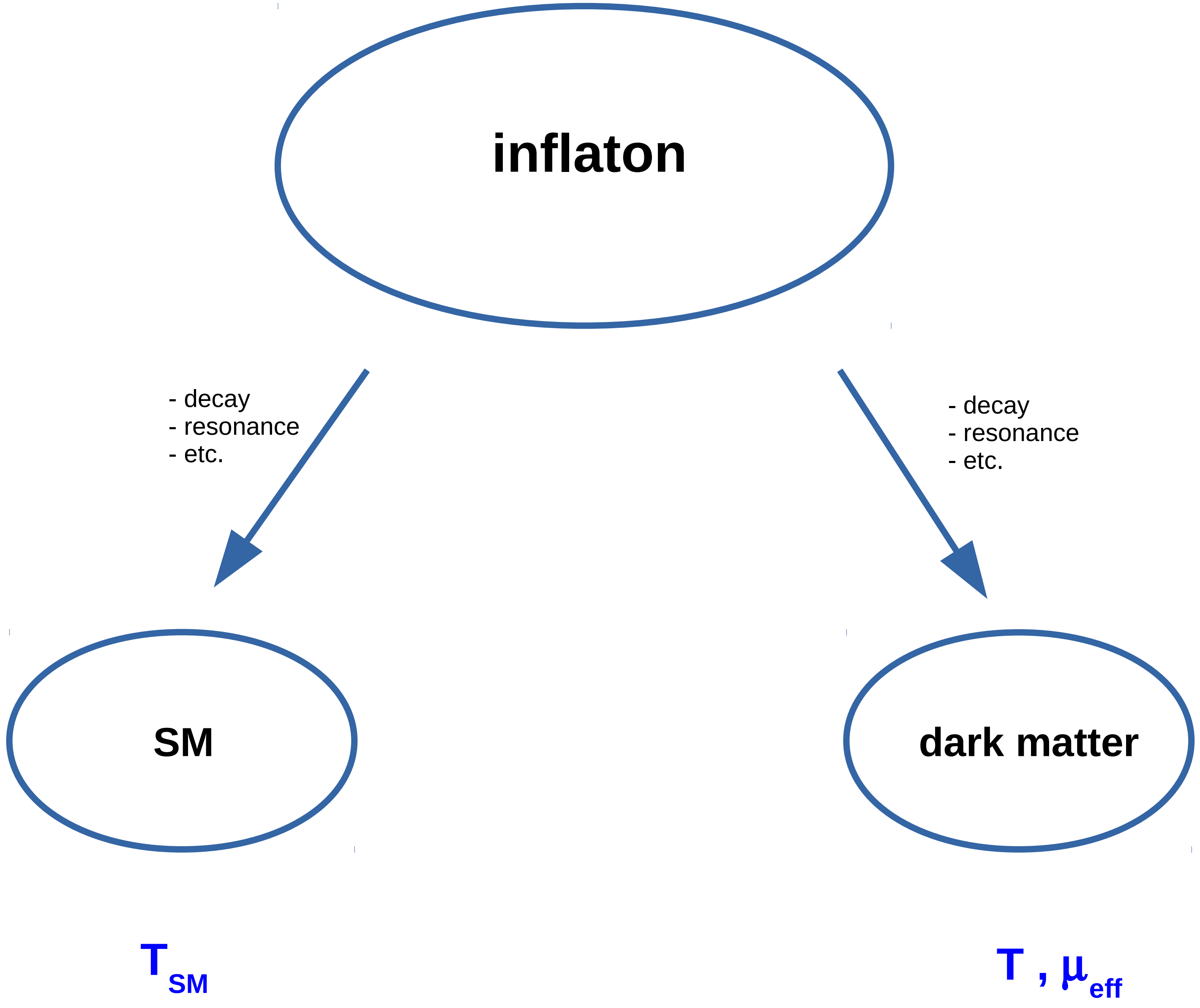
Based on

- 1906.07659 Arcadi, Lebedev, Pokorski, Toma
  “Real Scalar Dark Matter: Relativistic Treatment”
- 1908.05491 Lebedev, Toma
  “Relativistic Freeze-in”


inflaton
- decay
- resonance
- etc.
- decay
- resonance
- etc.
SM
dark matter
T_{SM}
T , μ_{eff}

Carlson,Machacek,Hall ‘92
Bernal, Chu ‘16

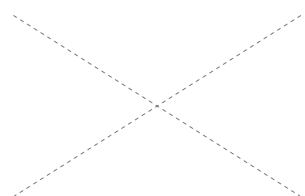
Simplest dark matter model

$$\boxed{V = \frac{m^2}{2} S^2 + \frac{\lambda}{4!} S^4}$$

Thermodynamics:

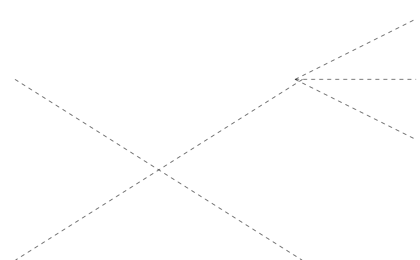
Bose-Einstein statistics $\quad f(p) = \frac{1}{e^{\frac{E-\mu}{T}} - 1}$

$2 \leftrightarrow 2$



$\Gamma_{22} > H \Rightarrow T\,,\ \mu_{eff}$

$2 \leftrightarrow 4$



$\Gamma_{24} > H \Rightarrow T$

Dynamics:

initial state → thermalization → freeze-out

Common approach:

$$\begin{cases} f(p) = \dfrac{1}{e^{\frac{E-\mu}{T}} - 1} & \rightarrow \qquad f(p) = e^{-(E-\mu)/T} \\ \mu \rightarrow 0 & \end{cases}$$

Dœs not work in the relativistic regime T >>m !

Relativistic reaction rates

$$\Gamma_{a\to b} = \int \left(\prod_{i\in a} \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} f(p_i) \right) \left(\prod_{j\in b} \frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j} (1 + f(p_j)) \right) |\mathcal{M}_{a\to b}|^2 \, (2\pi)^4 \delta^4(p_a - p_b)$$

2 → 4 :

$$\sigma(p_1,p_2) = \frac{1}{4F(p_1,p_2)} \int |\mathcal{M}_{2\to 4}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - \sum_i k_i) \prod_i \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_{k_i}} (1 + f(k_i))$$

can't neglect!

$$\Gamma_{2\to 4} = (2\pi)^{-6} \int d^3\mathbf{p_1} d^3\mathbf{p_2} \; f(p_1) f(p_2) \; \sigma(p_1,p_2) v_{\text{Møl}}$$

Issue: can only compute $\sigma(2\to 4)$ in the center-of-mass frame (CalcHEP, etc.)

Conversion to the CM frame:

$$p_1\,,p_2 \quad \rightarrow \quad p=\frac{p_1+p_2}{2}\,,\;k=\frac{p_1-p_2}{2}$$

$$p=\Lambda(p)\begin{pmatrix}E\\0\\0\\0\end{pmatrix}$$

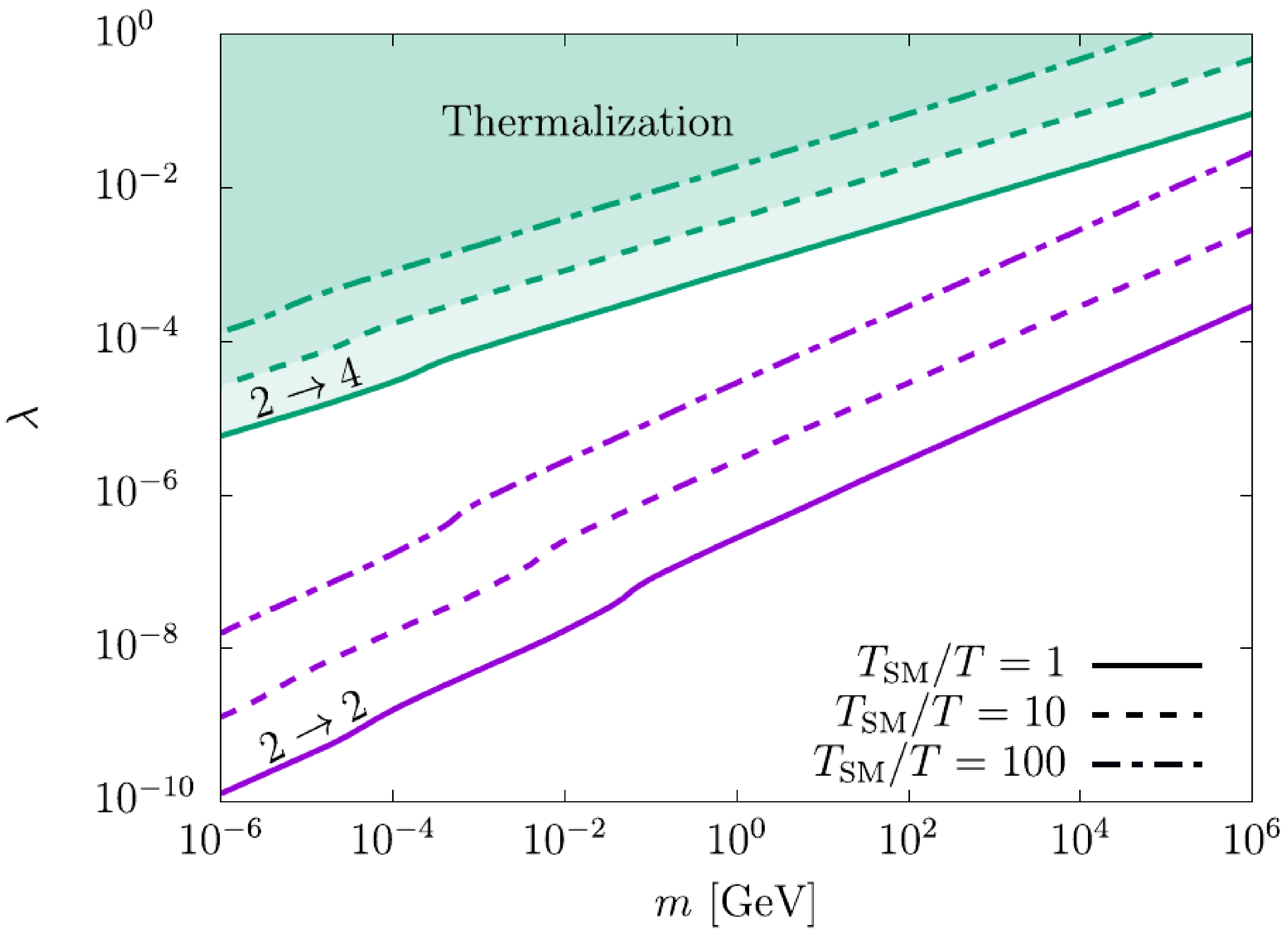
$$\Rightarrow \quad \int \frac{d^3\mathbf{p_1}}{2E_1}\frac{d^3\mathbf{p_2}}{2E_2}\,\ldots \;=\; 2\int_m^\infty dE\,\sqrt{E^2-m^2}\,E^2\int_0^\infty d\eta\,\sinh^2\eta\int d\Omega_p\,d\Omega_k\,\ldots$$

Relativistic analog of the Gelmini-Gondolo formula:

$$\Gamma_{2\to 4} = \frac{4T}{\pi^4}\int_m^\infty dE\;E^3\sqrt{E^2-m^2}\int_0^\infty d\eta\frac{\sinh\eta}{e^{2(E\cosh\eta-\mu)/T}-1}\;\ln\frac{\sinh\frac{E\cosh\eta+\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}}{\sinh\frac{E\cosh\eta-\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}}\;\sigma_{\rm CM}(E,\eta)$$

includes BE final state factors

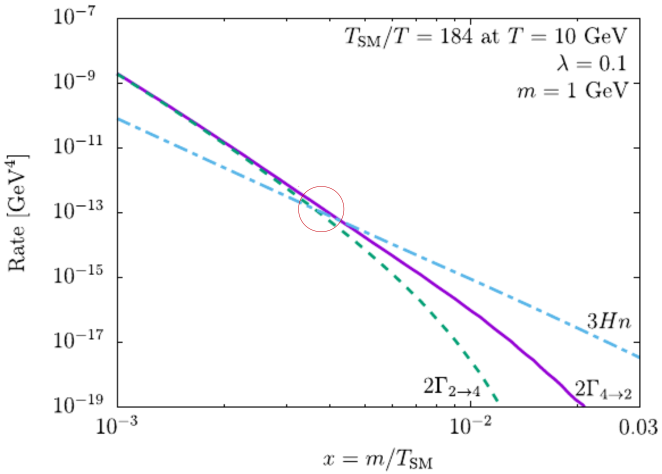
Thermal or kinetic equilibrium: Γ_{24} or Γ_{22} > n H

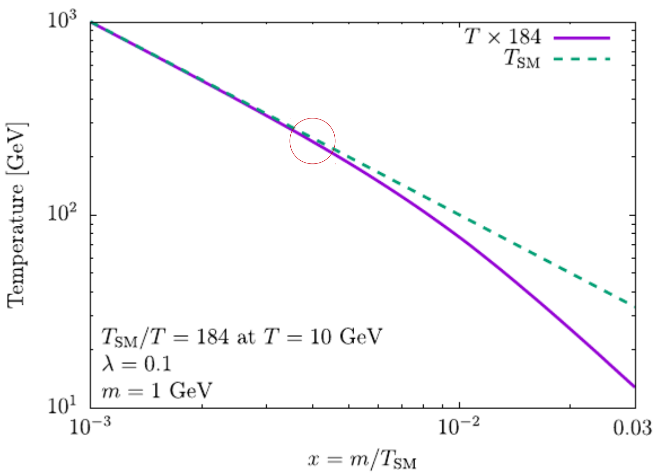


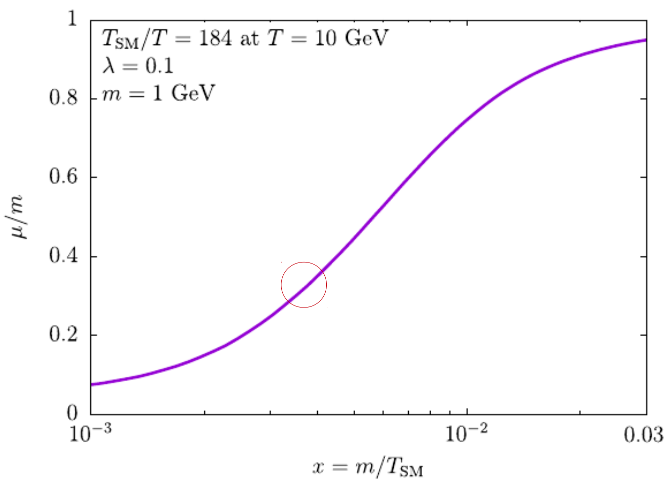
Boltzmann equation + entropy conservation:

$$\begin{cases} \dfrac{dn}{dt} + 3Hn = 2\,(\Gamma_{2\to4} - \Gamma_{4\to2}) \\[2ex] \dfrac{s}{s_{\rm SM}} = \text{const} \end{cases} \quad \Rightarrow \quad \mathbf{T(t)\,,\ \mu(t)}$$

Freeze-out $\quad 2\,\Gamma_{24} < 3\,n\,H$:







Thermal mass effect at high T:

$$m^2 \to m^2 + \frac{\lambda}{24} T^2$$

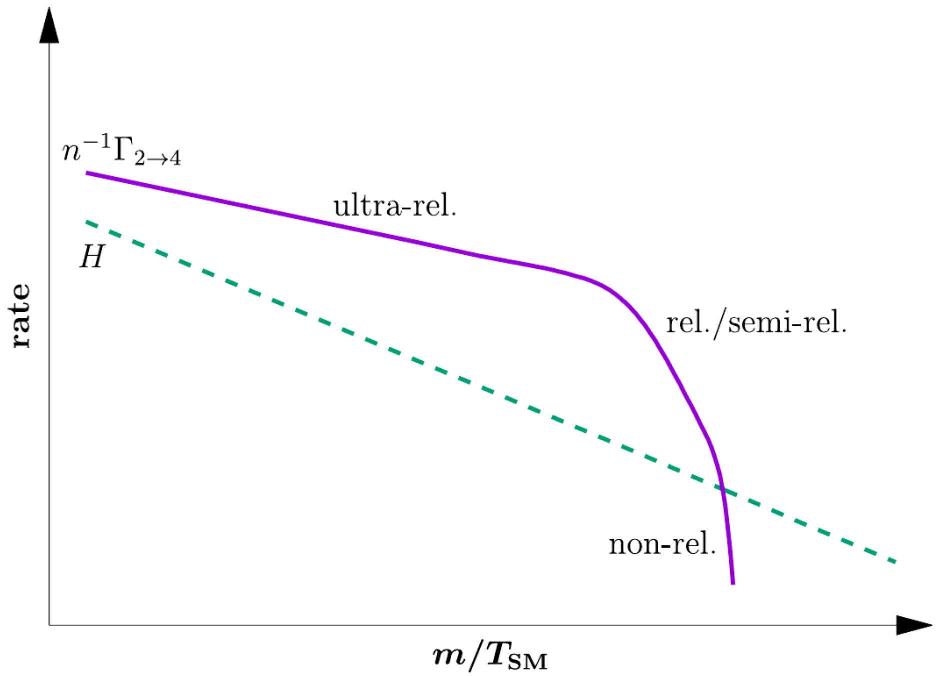
$$\Gamma_{2\to 2} \propto T^4 \ln \frac{T}{m_{\rm eff}} \;\; \to c\, T^4$$

IR divergent without
thermal mass

Freeze-out can't be ultra-relativistic:

$\Gamma_{24} \sim T^4$

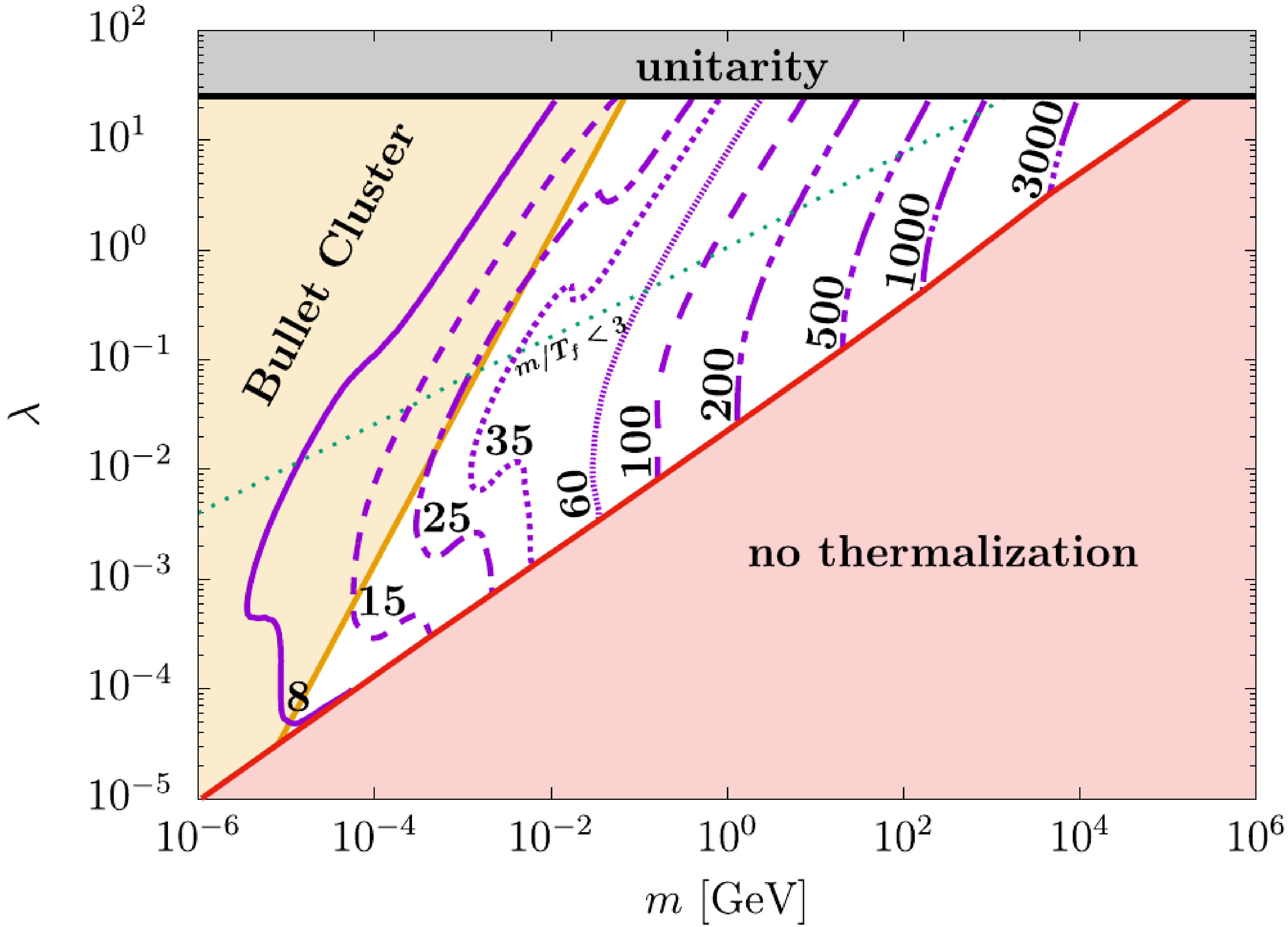
$n\,H \sim T^5$



Correct DM relic abundance:

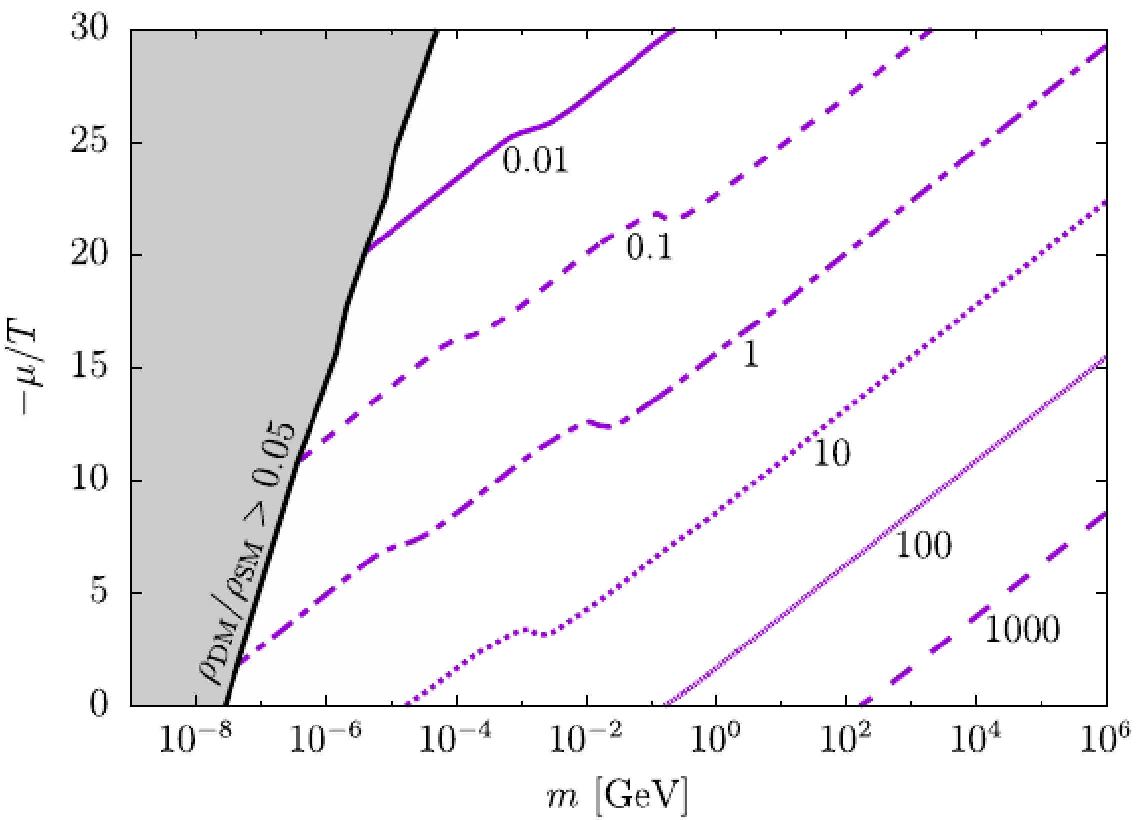
= *boundary of relativistic freeze-out*

8, 15, 25, ... = T / T_{SM}



Only kinetic equilibrium:

$$n = \frac{T^3}{\pi^3}\,\mathrm{Li}_3(e^{\mu/T}) \qquad , \qquad \mu \propto T \propto T_{\rm SM}$$



white = correct relic density

0.01, 0.1, 1, ... = T / T_{SM}

NB: $|\mu| < m$ if there are "non-trivial" anti-particles (Haber, Weldon '81)

Relativistic freeze-in

$$V_{hs} = \frac{1}{2}\lambda_{hs} H^\dagger H s^2$$

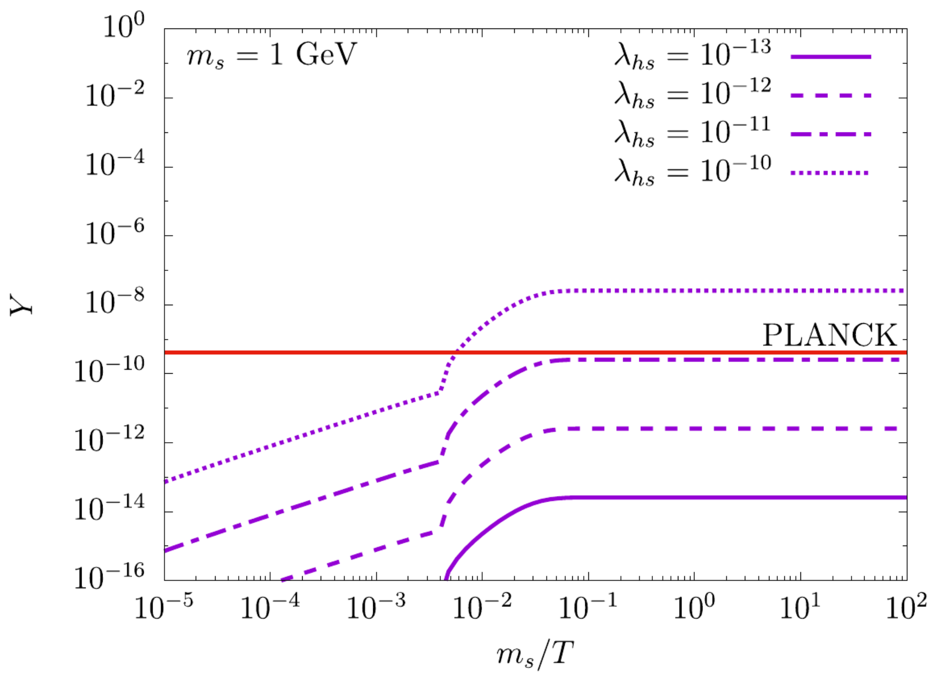
T > T$_{EW}$ regime: $h_i h_i \rightarrow ss$

$$\Gamma_{2\to 2} = \frac{1}{2!2!}\frac{\lambda_{hs}^2 T}{16\pi^5}\int_{m_h}^{\infty} dE\, E\sqrt{E^2 - m_s^2}\int_0^{\infty} d\eta \frac{\sinh\eta}{e^{\frac{2E}{T}\cosh\eta} - 1}\ \ln\frac{\sinh\frac{E\cosh\eta + \sqrt{E^2 - m_h^2}\sinh\eta}{2T}}{\sinh\frac{E\cosh\eta - \sqrt{E^2 - m_h^2}\sinh\eta}{2T}}$$

$$m_h^2 \simeq m_{h0}^2 + \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_1^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda_h\right)T^2$$

$$m_s > m_h \quad \rightarrow \quad \lambda_{hs} \simeq 2.2 \times 10^{-11}$$

$$m_s \ll m_h \quad \rightarrow \quad \lambda_{hs} \simeq 1.2 \times 10^{-11}\sqrt{\frac{\text{GeV}}{m_s}}$$



Conclusion:

- scalar DM evolution in the relativistic regime
- effective chemical potential
- relativistic effects in freeze-in