Mateusz Duch
University of Warsaw

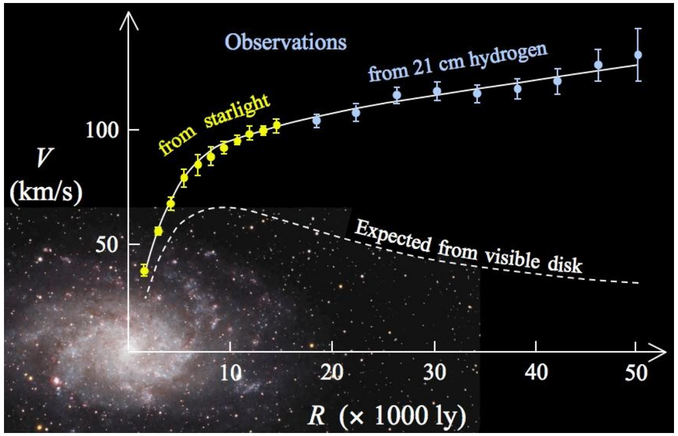
Gauge independent approach to resonant annihilation of vector dark matter

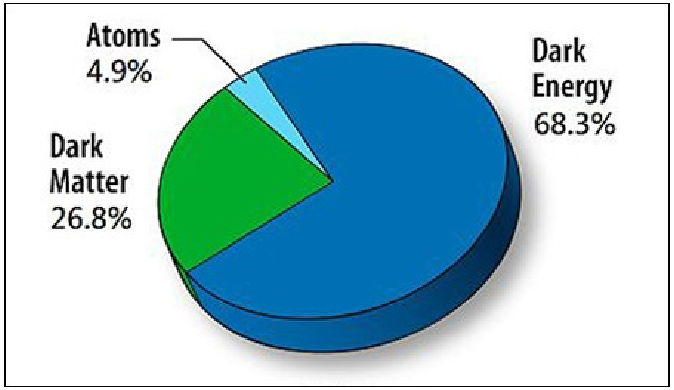
Scalars Conference
13 September 2019

MD, Bohdan Grządkowski, Apostolos Pilaftsis, *Gauge-Independent Approach to Resonant Dark Matter Annihilation,* JHEP 1902 (2019) 141 [1812.11944]

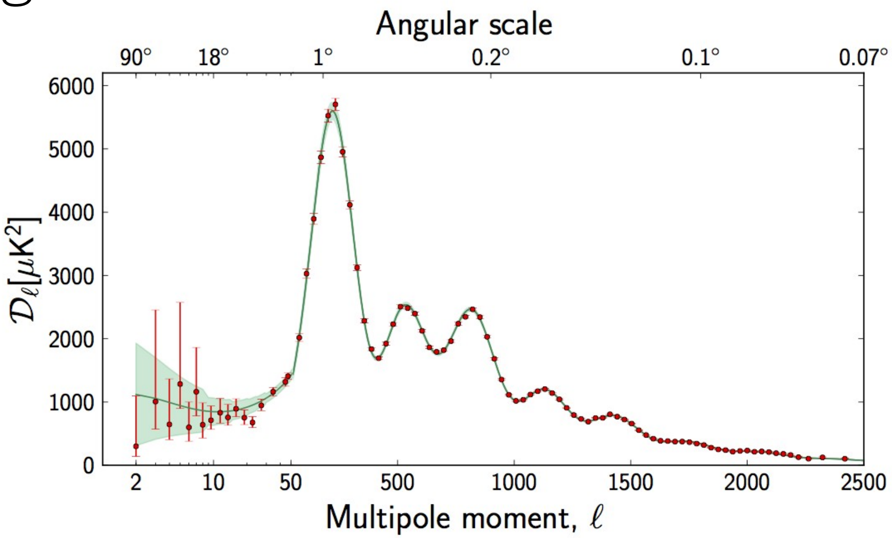
MD, Bohdan Grządkowski, *Resonance enhancement of dark matter interactions,* JHEP 1709 (2017) 159 [1705.10777]

Dark matter – motivation



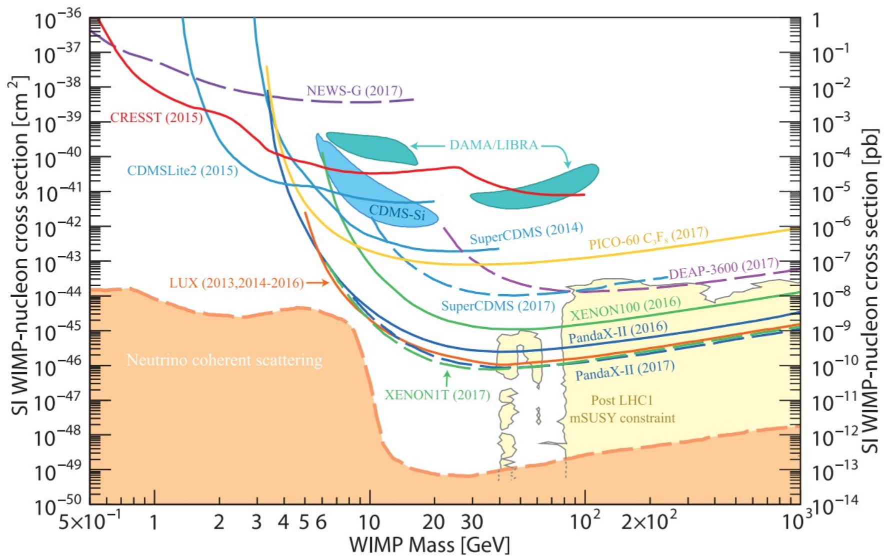


Convincing evidence on various astrophysical and cosmological scales



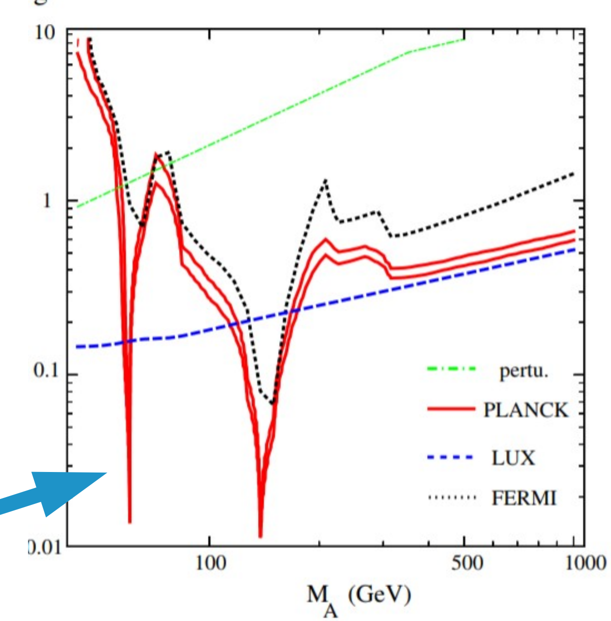
leading hypothesis → new, unknown particle

Resonance region

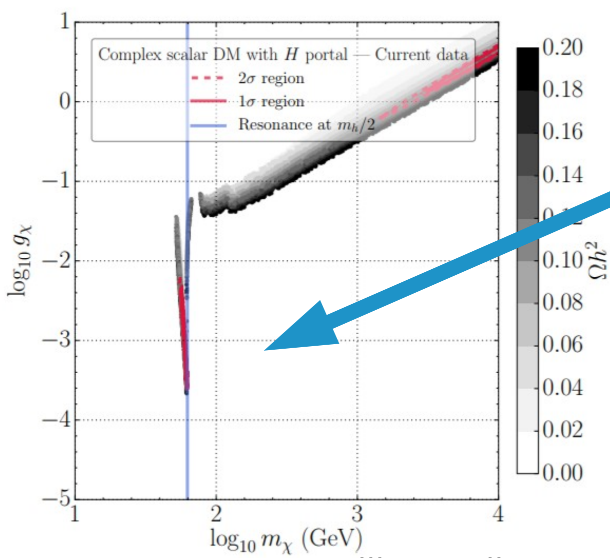


STRONG CONSTRAINTS FROM DIRECT DETECTION

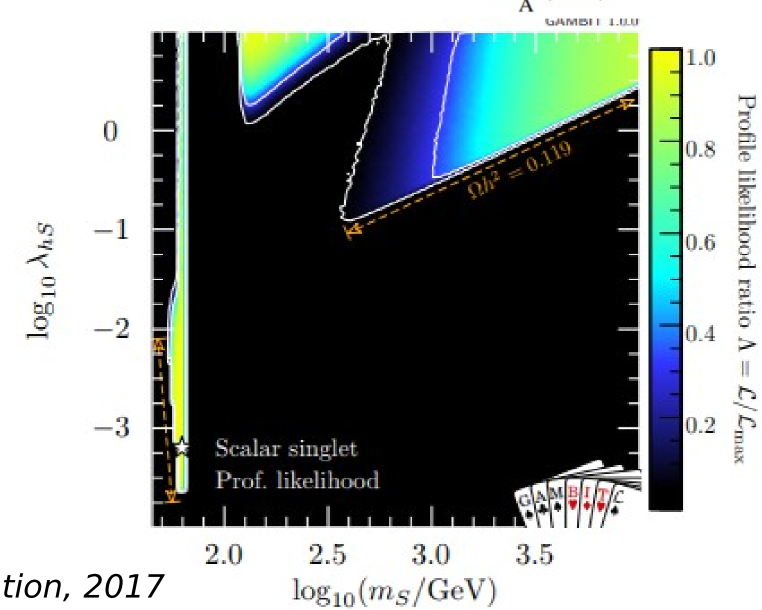
Gross, Lebedev, Mambrini, 2015



SMALL COUPLING REGIONS ARE VIABLE



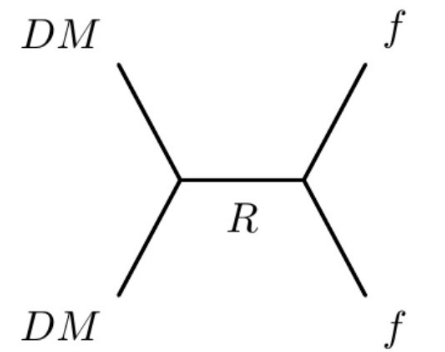
Ellis, Fowlie, Marzola, Raidal, 2017



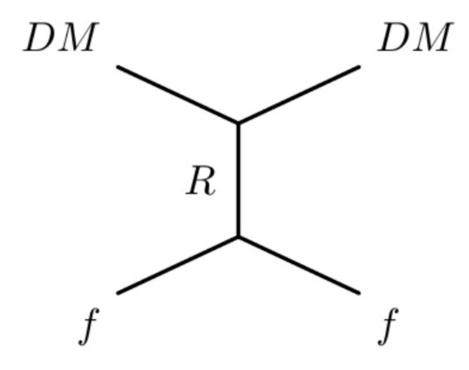
GAMBIT Collaboration, 2017

Breit-Wigner resonance

Breit-Wigner resonance $2M_{\mathrm{DM}} \approx M_{\mathrm{R}}$

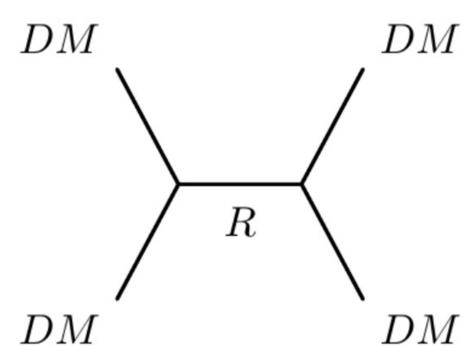


enhanced annihilation → suppressed coupling

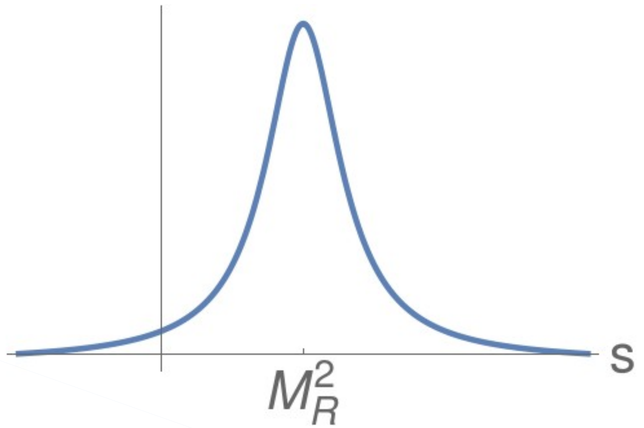


- low sensitivity to direct detection
- velocity dependent cross-section → possibility of enhanced indirect detection signals
- kinetic decoupling $T_{\mathrm{DM}} \neq T_{\mathrm{SM}}$
- large self-interaction cross-section constrained by indirect detection
- proper description of annihilation amplitudes

  Is Breit-Wigner approximation applicable?



Breit-Wigner resonance

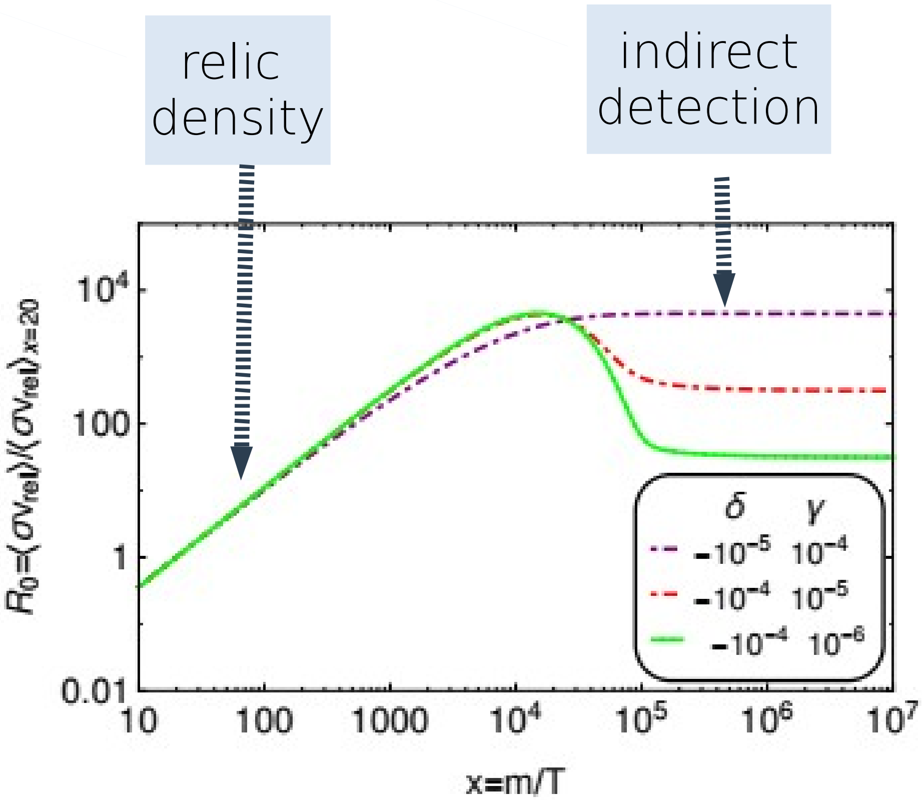


Resonant cross-section

$$\sigma \simeq \frac{1}{s}\sum_{f\neq i}\frac{M_R^2\Gamma_R^2 B_i B_f}{(s-M_R^2)^2+M_R^2\Gamma_R^2}$$

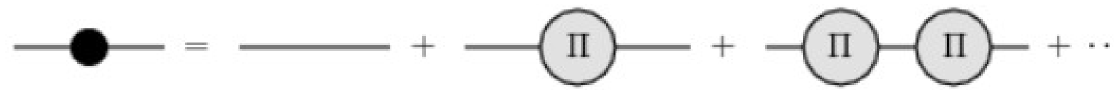
$$\delta = \frac{4M_{DM}^2}{M_R^2}-1, \qquad \gamma = \frac{\Gamma_R}{M_R}$$

resonance position , width



- strong temperature dependence
- thermally averaged cross section grows with falling temperature

Resummed propagator

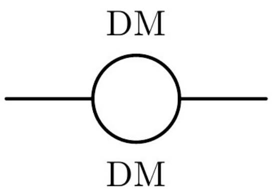


Dyson resummed propagator

$$\frac{1}{s - M_R^2 \; + \; i\mathrm{Im}\Pi_R(s)}$$

In the resonant region: $s \gtrsim 4M_{\mathrm{DM}}^2 \approx M_R^2$

$$\Pi_R(s) \;=\; \Pi_{\mathrm{non-DM}}(s) + \Pi_{\mathrm{DM}}(s)$$



other SM or dark sector fields

DM contribution

no nearby thresholds ⇨ Breit-Wigner approximation

$$\mathrm{Im}\,\Pi_{\mathrm{non-DM}}(s) \approx \mathrm{Im}\,\Pi_{\mathrm{non-DM}}(M_R^2) = M_{DM}\Gamma_{\mathrm{non-DM}}$$

nearby threshold $s \gtrsim 4M_{DM}^2$

$$\mathrm{Im}\,\Pi_{DM}(s) \sim \sqrt{1 - 4M_{DM}^2/s}$$

problem with Breit-Wigner approximation

Abelian vector dark matter

Additional complex scalar field S

- singlet of $U(1)_Y \times SU(2)_L \times SU(3)_c$, charged under $U(1)_X$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + (D_\mu S)^* D^\mu S + \tilde{V}(H, S)$$

$$V(H,S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2$$

Vacuum expectation values: $\langle H \rangle = \frac{v_{SM}}{\sqrt{2}}$, $\langle S \rangle = \frac{v_x}{\sqrt{2}}$

Dark $U(1)_X$ vector gauge boson X_μ

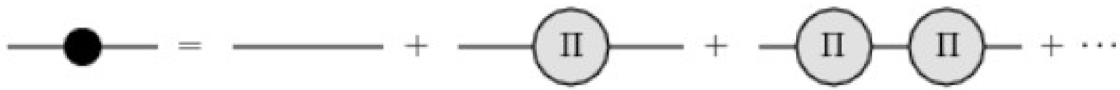
- Stability condition - no mixing of $U(1)_X$ with $U(1)_Y$ $\quad \cancel{B_{\mu\nu} V^{\mu\nu}}$

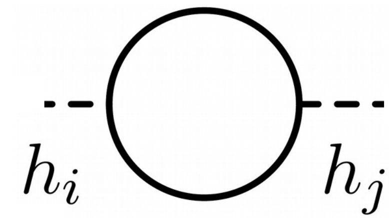
  $\mathcal{Z}_2 : V_\mu \to -V_\mu$, $\quad S \to S^*$, $\quad S = \phi e^{i\sigma}$: $\phi \to \phi$, $\sigma \to -\sigma$
- Higgs mechanism in the hidden sector $M_X = g_x v_x$

Higgs couplings – mixing angle α, $\quad M_{h_1} = 125$ GeV

$$\mathcal{L} \supset \frac{h_1 c_\alpha + h_2 s_\alpha}{v} \left(2 M_W W_\mu^+ W^{\mu-} + M_Z^2 Z_\mu Z^\mu - m_f \bar{f} f \right) + \frac{h_1 s_\alpha - h_2 c_\alpha}{v_x} M_X^2 X_\mu X^\mu$$

Problems with resummation in R_ξ gauge





Dark vector contribution to the Higgs self-energy in R_ξ gauge

$$\Pi_{ij}^{(XX)}(s) = \frac{g_x^2 R_{2i} R_{2j}}{32\pi^2 M_X^2}\Big[\left(s^2 - 4M_X^2 s + 12M_X^4\right) B_0(s, M_X^2, M_X^2) \\ - (s^2 - m_i^2 m_j^2) B_0(s, \xi_X M_X^2, \xi_X M_X^2)\Big]$$

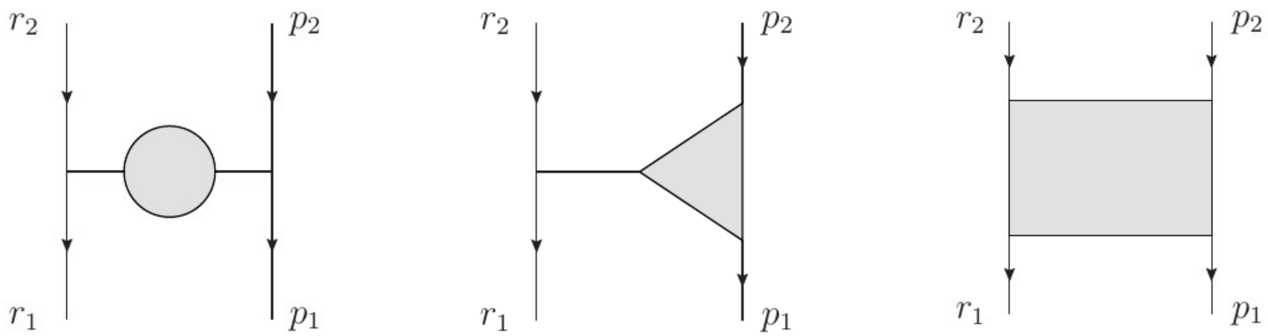
Problems with self-energy:

$$\mathrm{Im} B_0(s, M_X^2, M_X^2) \sim \sqrt{1 - 4M_X^2/s}$$

- explicit dependence on gauge fixing parameter
- presence of s^2 term – modification of high-energy behavior
- unphysical threshold at $s = \xi_X M_X^2$

Pinch Technique

Reorganization of the sub-amplitudes that have the same kinematical properties



$$T(s,t,m_i) \;=\; \widehat{T}_1(s) \;+\; \widehat{T}_2(s,m_i) \;+\; \widehat{T}_3(s,t,m_i)$$

Individually gauge invariant

We have to look for the propagator-like pieces inside vertex and box diagrams

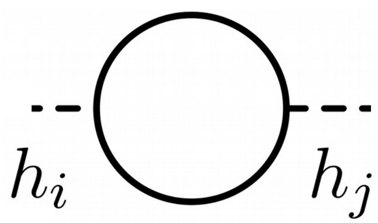
- PT algorithm: employ Ward identities
- equivalent to calculation in Background Field Method ($\xi_Q = 1$)

Cornwall 1989
Denner+ 1994,
Papavasilliou,
Pilaftsis 1995
Binosi+ 2002

Model with mixed scalars

Contributions to Higgs self-energy X, Z, W, f, h

$$
\begin{aligned}
\widehat{\Pi}^{(XX)}_{ij}(s) &= \frac{g_x^2 R_{2i} R_{2j}}{8\pi^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2+m_j^2}{2} - (2s-3M_X^2)\right] B_0(s, M_X^2, M_X^2)\,,\\
\widehat{\Pi}^{(ZZ)}_{ij}(s) &= \frac{g^2 R_{1i} R_{1j} M_Z^2}{32\pi^2 M_W^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2+m_j^2}{2} - (2s-3M_Z^2)\right] B_0(s, M_Z^2, M_Z^2)\,,\\
\widehat{\Pi}^{(WW)}_{ij}(s) &= \frac{g^2 R_{1i} R_{1j}}{32\pi^2}\left[\frac{(m_i m_j)^2}{4M_X^2} + \frac{m_i^2+m_j^2}{2} - (2s-3M_W^2)\right] B_0(s, M_W^2, M_W^2)\,,\\
\widehat{\Pi}^{(tt)}_{ij}(s) &= \frac{3g^2 R_{1i} R_{1j} m_t^2}{32\pi^2 M_W^2}\left(s-4m_t^2\right) B_0(s, m_t^2, m_t^2)\,,\\
\widehat{\Pi}^{(h_k h_l)}_{ij}(s) &= \frac{-V^h_{ikl} V^h_{jkl}}{32\pi^2} B_0(s, m_{h_k}^2, m_{h_l}^2)\,.
\end{aligned}
$$



no fictitious thresholds

no s^2 terms

Resummation of the propagator with scalar mixing

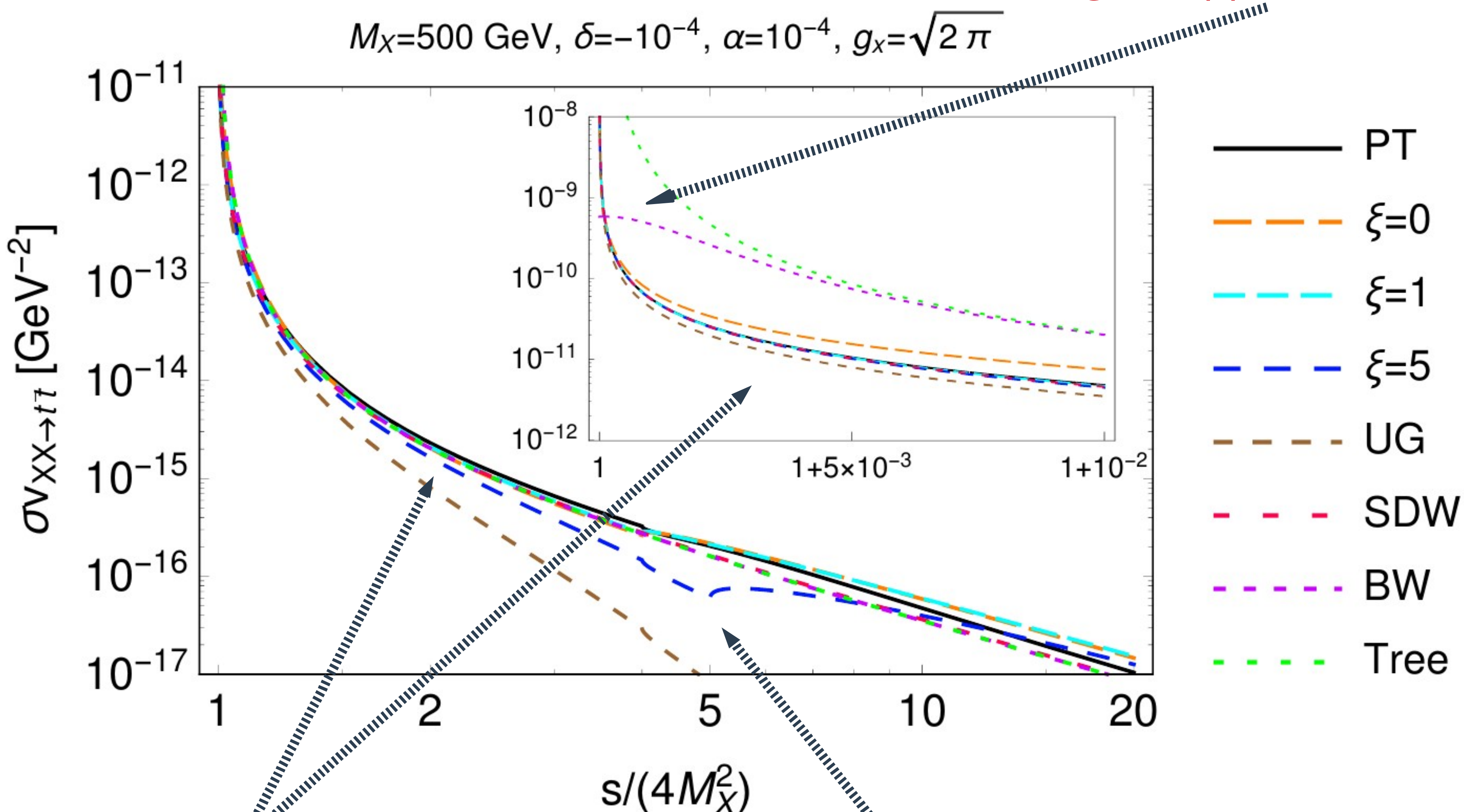
$$
i\widehat{\Delta} \;=\; i\Delta_0 \;+\; i\Delta_0\, i\widehat{\Pi}\, i\Delta_0 \;+\; i\Delta_0\,(i\widehat{\Pi}\, i\Delta_0)^2 \;+\; \ldots
$$

diagonal tree-level propagator

$$
\widehat{\Delta}(s) = \frac{1}{D(s)}\begin{pmatrix} s - m_2^2 + \widehat{\Pi}_{22}(s) & -\widehat{\Pi}_{12}(s) \\ -\widehat{\Pi}_{21}(s) & s - m_1^2 + \widehat{\Pi}_{11}(s) \end{pmatrix}
$$

Cross-section for XX->bb process

standard Breit-Wigner approximation fails



Results in PT and Feynman gauge are similar

amplitude distorted by unphysical threshold

Energy dependent width

No SM thresholds near the resonance → BW approximation applicable

$$\Gamma_{h_i \to \mathrm{SMSM}} = \mathrm{Im}\,\Pi_{ii}^{(\mathrm{SMSM})}(m_i^2)/m_i$$

For DM contribution, we cannot use the constant width, but

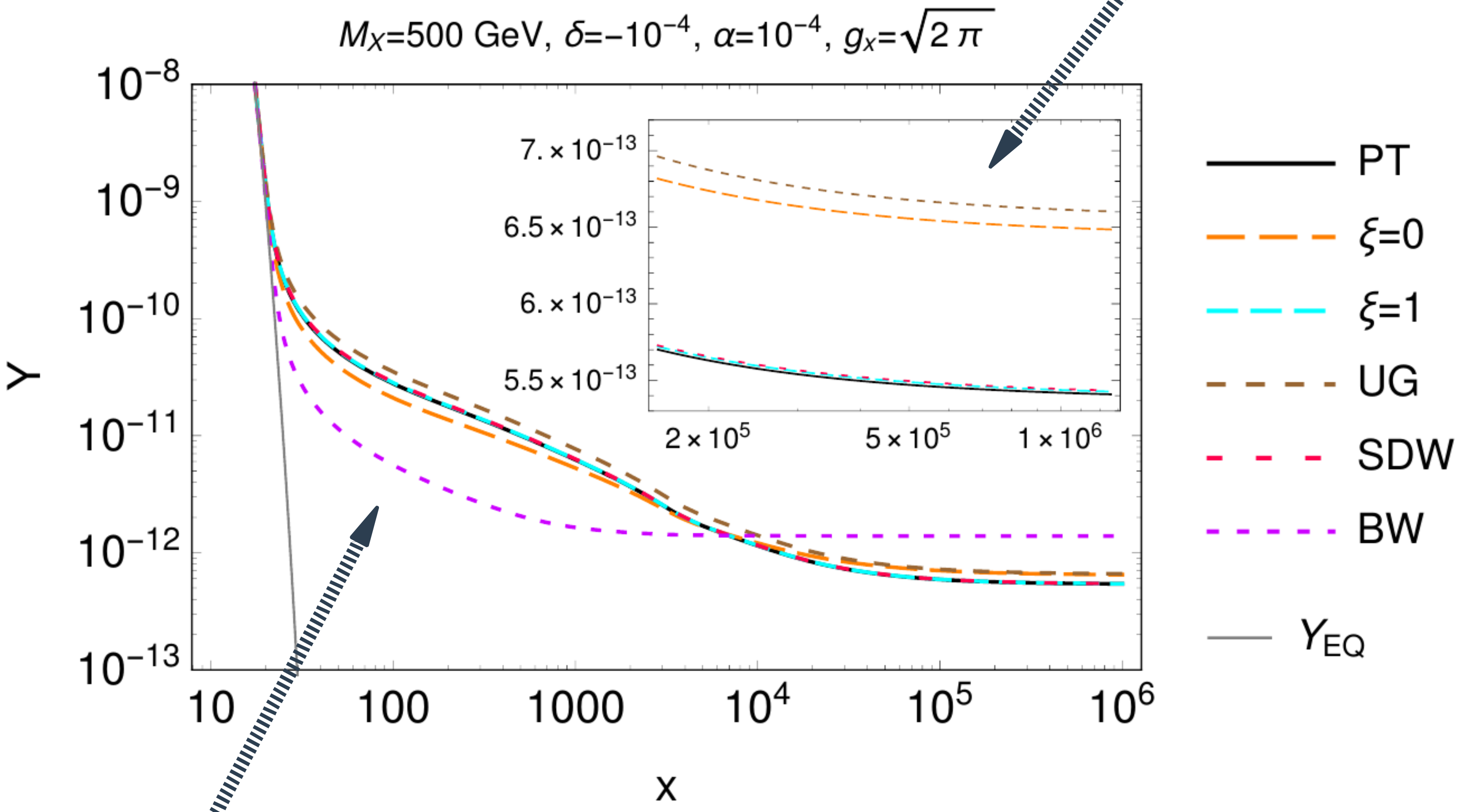
$$\Gamma_{h_i \to XX} = \sqrt{\frac{1-4M_X^2/s}{1-4M_X^2/m_i^2}}\,\frac{\mathrm{Im}\,\Pi_{ii}^{(XX)}(m_i^2)}{m_i}\,\theta(s-4M_X^2)$$

leading energy-dependent contribution

gauge-independent quantity

Relic density calculation

Landau and unitary gauge

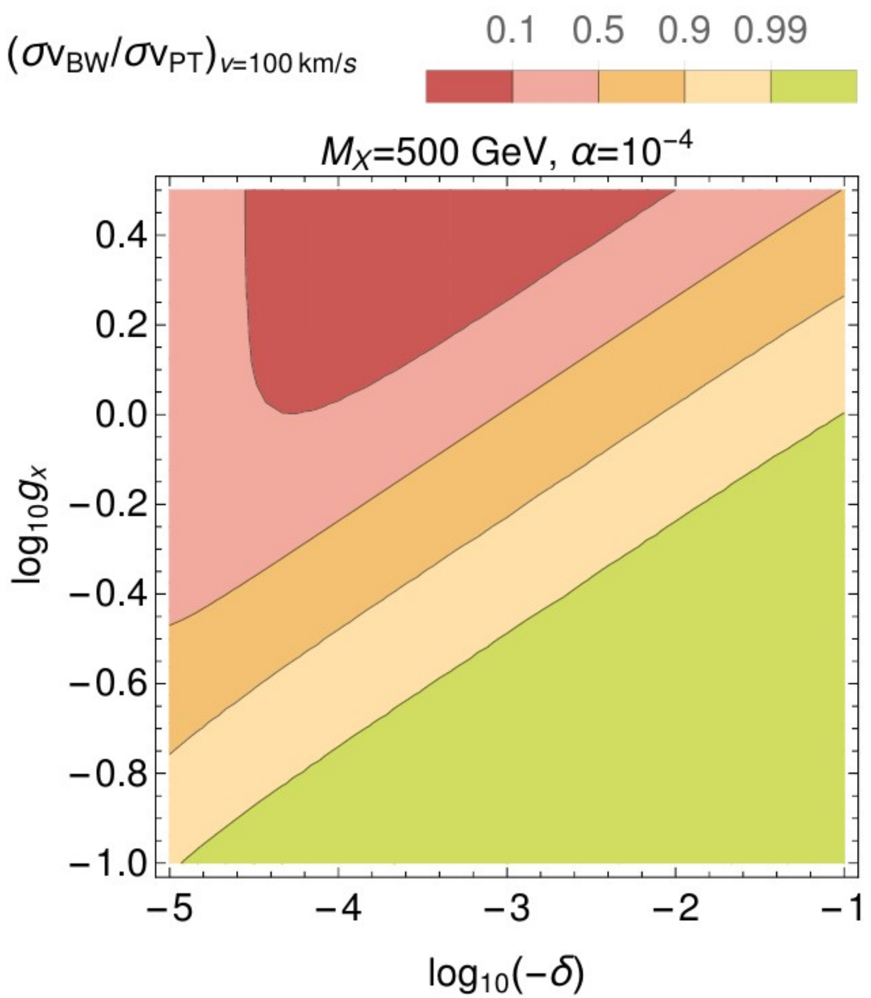


standard Breit-Wigner approximation

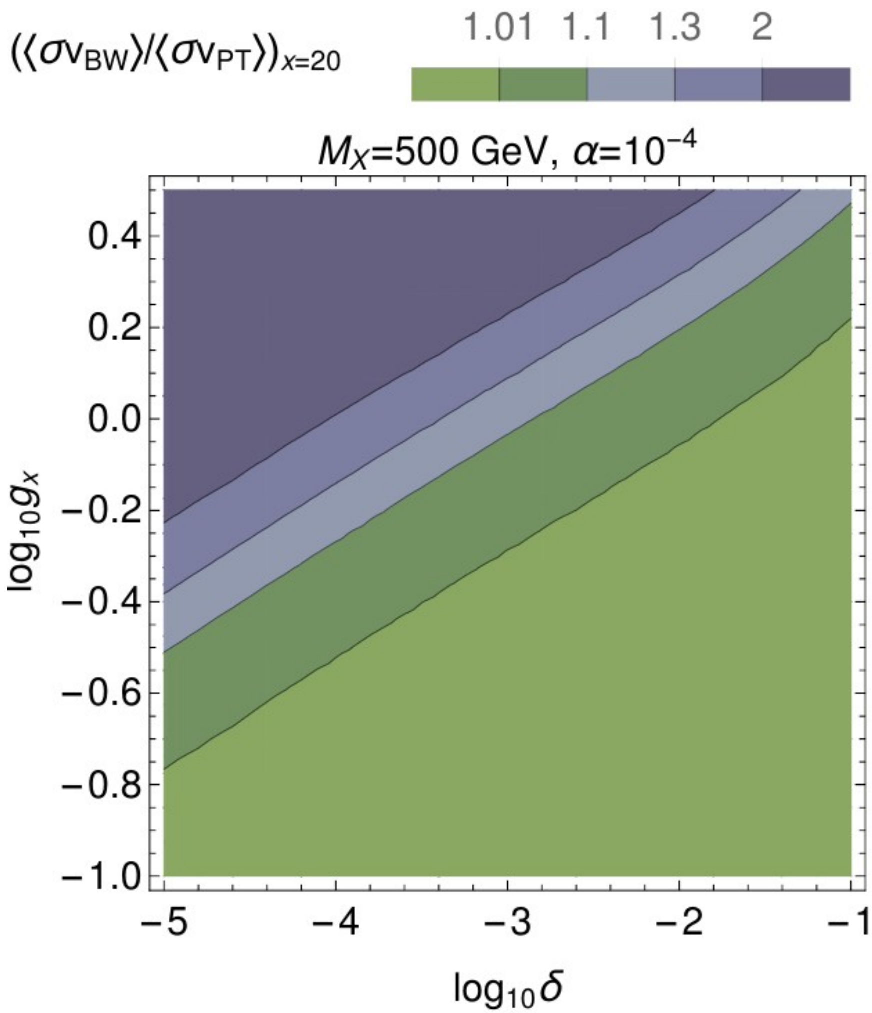
effects of **early kinetic decoupling** included [1705.10777]

standard Breit-Wigner vs. PT resummation

underestimated indirect detection signal



overestimated annihilation rate



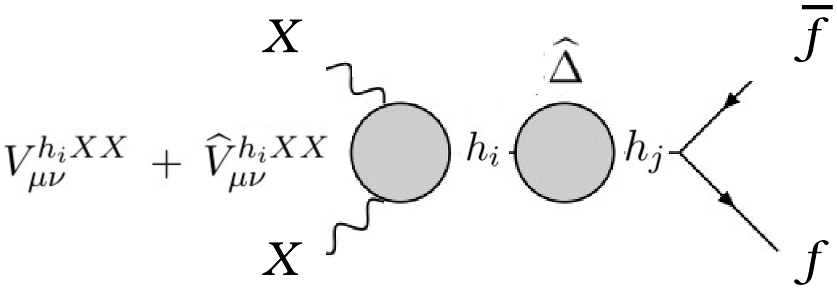
Summary

- resonance region is a viable part of many strongly constrained dark matter models
- the Breit-Wigner approximation may fail if mediator couples dominantly to the dark matter state
- relativistic treatment of resonant amplitudes requires proper resummation technique
- pinch technique provides a method respecting the gauge invariance and unitarity what results in the proper behavior near the resonance and in the high energy limit
- in the phenomenological analyses one can use properly approximated energy-dependent width

BACKUP

Born-improved amplitude

Pinch technique self-energy and one-loop corrected vertices:



$$iA^{XX\to\bar{f}f}_{\mu\nu} = \sum_{ij}(V^{XXh_i}_{\mu\nu} + \hat{V}^{XXh_i}_{\mu\nu})i\widehat{\Lambda}_{ij}V^{h_j\bar{f}f}$$

Tree-level like Ward identities are satisfied by the PT self-energies and vertices

$$\begin{aligned}
p_2^\nu \widehat{V}^{h_i XX}_{\mu\nu}(q,p_1,p_2) + iM_X \widehat{V}^{h_i X G_X}_{\mu} &= -g_x R_{2i}\widehat{\Pi}^{XG_X}_{\mu}(p_1)\\
p_1^\mu \widehat{V}^{h_i X G_X}_{\mu} + iM_X \widehat{V}^{h_i G_X G_X} &= -g_x\Big[R_{2j}\widehat{\Pi}_{ji}(q^2) + R_{2i}\widehat{\Pi}^{G_X G_X}(p_2)\Big],\\
p_1^\mu p_2^\nu \widehat{V}^{h_i XX}_{\mu\nu} + M_X^2 \widehat{V}^{h_i G_X G_X} &= ig_x M_X\Big[R_{2j}\widehat{\Pi}_{ji}(q^2) + R_{2i}\Big(\widehat{\Pi}^{G_X G_X}(p_1) + \widehat{\Pi}^{G_X G_X}(p_2)\Big)\Big]\\
\widehat{\Pi}^{XG_X}_{\mu}(p) &= -\frac{iM_X p_\mu}{p^2}\widehat{\Pi}^{G_X G_X}(p^2)
\end{aligned}$$

Generalized equivalence theorem satisfied

Proper high-energy behaviour as required by unitarity

$$p_1^\mu p_2^\nu \widehat{\Gamma}^{h_i XX}_{\mu\nu}(q,p_1,p_2) = ig_x M_X\, R_{2j}\widehat{\Delta}^{-1}_{ji}(q^2) + \mathcal{O}\big[\ln(s/M_X^2)\big]$$