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# From parameters to observables in the 2HDM

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Based on work with B. Grzadkowski and P. Osland

# The general 2HDM potential

$$\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\} \\
&+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+\frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}Z_{abcd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d)
\end{aligned}$$

- Standard parametrization(s) of the general 2HDM potential.
- Second form most convenient in the study of invariants.

$$Y_{11} = -\frac{m_{11}^2}{2}, \quad Y_{12} = -\frac{m_{12}^2}{2},$$
$$Y_{21} = -\frac{(m_{12}^2)^*}{2}, \quad Y_{22} = -\frac{m_{22}^2}{2},$$

$$\begin{aligned}
&Z_{1111} = \lambda_1, \quad Z_{2222} = \lambda_2, \quad Z_{1122} = Z_{2211} = \lambda_3, \\
&Z_{1221} = Z_{2112} = \lambda_4, \quad Z_{1212} = \lambda_5, \quad Z_{2121} = (\lambda_5)^*, \\
&Z_{1112} = Z_{1211} = \lambda_6, \quad Z_{1121} = Z_{2111} = (\lambda_6)^*, \\
&Z_{1222} = Z_{2212} = \lambda_7, \quad Z_{2122} = Z_{2221} = (\lambda_7)^*.
\end{aligned}$$

# Choice of basis is not unique

- Initial expression of potential is defined with respect to doublets $\Phi_1$ and $\Phi_2$.
- We may rotate to a new basis by the following transformation

$$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is any U(2) matrix.
- Potential parameters change under change of basis.
- Physics is the same regardless of our choice of basis.
- Observables cannot depend on choice of basis – they should be basis-independent, i.e. **invariant** under a change of basis.
- Most general U(2) matrix:

$$U = e^{i\psi}\begin{pmatrix} \cos\theta & e^{-i\xi}\sin\theta \\ -e^{i\chi}\sin\theta & e^{i(\chi-\xi)}\cos\theta \end{pmatrix}$$

# Parameters transform under change of basis

- All the parameters of the potential change under a U(2) basis transformation.
- Meaning: None of the parameters represent physical observables.
- Combinations of parameters can remain unchanged, for instance

$$\begin{aligned}
\bar{m}_{11}^2 + \bar{m}_{22}^2 &= m_{11}^2 + m_{22}^2, \\
\bar{\lambda}_1 + \bar{\lambda}_2 + 2\bar{\lambda}_3 &= \lambda_1 + \lambda_2 + 2\lambda_3, \\
\bar{\lambda}_1 + \bar{\lambda}_2 + 2\bar{\lambda}_4 &= \lambda_1 + \lambda_2 + 2\lambda_4.
\end{aligned}$$

- Meaning: These combinations represent physical observables.

$$\begin{aligned}
\bar{m}_{11}^2 &= m_{11}^2 \cos^2\theta + m_{22}^2 \sin^2\theta + \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \sin 2\theta \\
\bar{m}_{22}^2 &= m_{11}^2 \sin^2\theta + m_{22}^2 \cos^2\theta - \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \sin 2\theta \\
\bar{m}_{12}^2 &= \left[\tfrac{1}{2}(-m_{11}^2 + m_{22}^2) \sin 2\theta + \mathrm{Re}\,(m_{12}^2 e^{i\xi}) \cos 2\theta + i\mathrm{Im}\,(m_{12}^2 e^{i\xi})\right] e^{-i\chi} \\
\bar{\lambda}_1 &= \lambda_1 \cos^4\theta + \lambda_2 \sin^4\theta + \tfrac{1}{2}\lambda_{345} \sin^2 2\theta + 2\sin 2\theta[\cos^2\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \sin^2\theta \mathrm{Re}\,(\lambda_7 e^{i\xi})] \\
\bar{\lambda}_2 &= \lambda_1 \sin^4\theta + \lambda_2 \cos^4\theta + \tfrac{1}{2}\lambda_{345} \sin^2 2\theta - 2\sin 2\theta[\sin^2\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \cos^2\theta \mathrm{Re}\,(\lambda_7 e^{i\xi})] \\
\bar{\lambda}_3 &= \frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_3 - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] \\
\bar{\lambda}_4 &= \frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_4 - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] \\
\bar{\lambda}_5 &= \left(\frac{1}{4}\sin^2 2\theta(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \mathrm{Re}\,(\lambda_5 e^{2i\xi}) + i\cos 2\theta \mathrm{Im}\,(\lambda_5 e^{2i\xi})\right. \\
&\qquad \left. - \sin 2\theta \cos 2\theta \mathrm{Re}\,[(\lambda_6 - \lambda_7)e^{i\xi}] - i\sin 2\theta \mathrm{Im}\,[(\lambda_6 - \lambda_7)e^{i\xi}]\right) e^{-2i\chi} \\
\bar{\lambda}_6 &= \left(-\tfrac{1}{2}\sin 2\theta[\lambda_1 \cos^2\theta - \lambda_2 \sin^2\theta - \lambda_{345}\cos 2\theta - i\mathrm{Im}\,(\lambda_5 e^{2i\xi})]\right. \\
&\qquad + \cos\theta \cos 3\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \sin\theta \sin 3\theta \mathrm{Re}\,(\lambda_7 e^{i\xi}) \\
&\qquad \left. + i\cos^2\theta \mathrm{Im}\,(\lambda_6 e^{i\xi}) + i\sin^2\theta \mathrm{Im}\,(\lambda_7 e^{i\xi})\right) e^{-i\chi} \\
\bar{\lambda}_7 &= \left(-\tfrac{1}{2}\sin 2\theta[\lambda_1 \sin^2\theta - \lambda_2 \cos^2\theta + \lambda_{345}\cos 2\theta + i\mathrm{Im}\,(\lambda_5 e^{2i\xi})]\right. \\
&\qquad + \sin\theta \sin 3\theta \mathrm{Re}\,(\lambda_6 e^{i\xi}) + \cos\theta \cos 3\theta \mathrm{Re}\,(\lambda_7 e^{i\xi}) \\
&\qquad \left. + i\sin^2\theta \mathrm{Im}\,(\lambda_6 e^{i\xi}) + i\cos^2\theta \mathrm{Im}\,(\lambda_7 e^{i\xi})\right) e^{-i\chi}.
\end{aligned}$$

Ref: Gunion & Haber, Phys. Rev. D **72**, 095002 (2005)

# Vacuum expectation values (VEVs)

Most general form that conserves electric charge:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1} \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2} \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

- We demand that the VEVs should represent a minimum of the potential
- Electroweak Symmetry Breaking: Work out stationary-point equations by differentiating the potential with respect to the fields and put these to zero. [Ref. JHEP11 (2014) 084].
- Minimum enforced by demanding all physical scalars have positive squared masses (later).

- VEVs also change under basis transformations:

$$\begin{aligned} \bar{v}_1 &= \sqrt{v_1^2\cos^2\theta + v_2^2\sin^2\theta + v_1 v_2 \sin 2\theta \cos(\xi_{21}-\xi)}, \\ \bar{v}_2 &= \sqrt{v_1^2\sin^2\theta + v_2^2\cos^2\theta - v_1 v_2 \sin 2\theta \cos(\xi_{21}-\xi)}. \end{aligned} \qquad \xi_{21} \equiv \xi_2 - \xi_1$$

$$\begin{aligned} \cos\bar{\xi}_{21} &= \frac{\bar{v}_1\left(2v_1v_2(\cos 2\theta\cos(\xi_{21}-\xi)\cos\chi - \sin(\xi_{21}-\xi)\sin\chi) + (v_2^2-v_1^2)\sin 2\theta\cos\chi\right)}{\bar{v}_2\left(v_1^2+v_2^2-(v_2^2-v_1^2)\cos 2\theta + 2v_1v_2\cos(\xi_{21}-\xi)\sin 2\theta\right)}, \\ \sin\bar{\xi}_{21} &= \frac{\bar{v}_1\left(2v_1v_2(\cos 2\theta\cos(\xi_{21}-\xi)\sin\chi + \sin(\xi_{21}-\xi)\cos\chi) + (v_2^2-v_1^2)\sin 2\theta\sin\chi\right)}{\bar{v}_2\left(v_1^2+v_2^2-(v_2^2-v_1^2)\cos 2\theta + 2v_1v_2\cos(\xi_{21}-\xi)\sin 2\theta\right)}. \end{aligned}$$

- It is easy to show that

$$\bar{v}_1^2 + \bar{v}_2^2 = v_1^2 + v_2^2$$

- Meaning: $v_1^2 + v_2^2 = v^2$ is a basis-invariant quantity, hence a physical observable.

# Parametrization of the doublets and the charged fields

- Each doublet is parametrized as:

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

- Massless charged goldstone fields $G^\pm$ are extracted by introducing orthogonal states:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}$$

- $H^\pm$ represent the massive charged scalars

- We work out the mass of the charged scalars:

$$M_{H^\pm}^2 = \frac{v^2}{2v_1 v_2 \cos\xi_{21}} \mathrm{Re}\left(m_{12}^2 - v_1^2\lambda_6 - v_2^2\lambda_7 - v_1 v_2\left[\lambda_4 \cos\xi_{21} + \lambda_5 e^{i\xi_{21}}\right]\right)$$

- Performing a change of basis we find that

$$\bar{M}_{H^\pm}^2 = M_{H^\pm}^2$$

- telling us that $M_{H^\pm}^2$ is a basis invariant and therefore a physical observable (as it must be).

# Parametrization of the doublets and the neutral fields

- Massless neutral goldstone field $G^0$ is also extracted by introducing orthogonal states:

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

- We are left with three massive fields: $\eta_1$, $\eta_2$ and $\eta_3$, but these are not mass eigenstates.

- Mass terms given as

$$\frac{1}{2}\begin{pmatrix} \eta_1 & \eta_2 & \eta_3 \end{pmatrix} \mathcal{M}^2 \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}.$$

- Matrix elements are given in [JHEP11(2014)084].

- We rotate into the physical fields by diagonalizing $\mathcal{M}^2$ using an orthogonal matrix $R$:

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

- Physical fields are now given as

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

# Transformations of mass matrix elements and rotation matrix elements under change of basis

$$
\begin{aligned}
\bar{\mathcal{M}}^2_{11} &= P_{11}^2\mathcal{M}^2_{11} + P_{21}^2\mathcal{M}^2_{22} + P_{31}^2\mathcal{M}^2_{33} + 2P_{11}P_{21}\mathcal{M}^2_{12} + 2P_{11}P_{31}\mathcal{M}^2_{13} + 2P_{21}P_{31}\mathcal{M}^2_{23} \\
\bar{\mathcal{M}}^2_{22} &= P_{12}^2\mathcal{M}^2_{11} + P_{22}^2\mathcal{M}^2_{22} + P_{32}^2\mathcal{M}^2_{33} + 2P_{12}P_{22}\mathcal{M}^2_{12} + 2P_{12}P_{32}\mathcal{M}^2_{13} + 2P_{22}P_{32}\mathcal{M}^2_{23} \\
\bar{\mathcal{M}}^2_{33} &= P_{13}^2\mathcal{M}^2_{11} + P_{23}^2\mathcal{M}^2_{22} + P_{33}^2\mathcal{M}^2_{33} + 2P_{13}P_{23}\mathcal{M}^2_{12} + 2P_{13}P_{33}\mathcal{M}^2_{13} + 2P_{23}P_{33}\mathcal{M}^2_{23} \\
\bar{\mathcal{M}}^2_{12} &= P_{11}P_{12}\mathcal{M}^2_{11} + P_{21}P_{22}\mathcal{M}^2_{22} + P_{31}P_{32}\mathcal{M}^2_{33} \\
&\quad +(P_{12}P_{21} + P_{11}P_{22})\mathcal{M}^2_{12} + (P_{12}P_{31} + P_{11}P_{32})\mathcal{M}^2_{13} + (P_{22}P_{31} + P_{21}P_{32})\mathcal{M}^2_{23} \\
\bar{\mathcal{M}}^2_{13} &= P_{11}P_{13}\mathcal{M}^2_{11} + P_{21}P_{23}\mathcal{M}^2_{22} + P_{31}P_{33}\mathcal{M}^2_{33} \\
&\quad +(P_{13}P_{21} + P_{11}P_{23})\mathcal{M}^2_{12} + (P_{13}P_{31} + P_{11}P_{33})\mathcal{M}^2_{13} + (P_{23}P_{31} + P_{21}P_{33})\mathcal{M}^2_{23} \\
\bar{\mathcal{M}}^2_{23} &= P_{12}P_{13}\mathcal{M}^2_{11} + P_{22}P_{23}\mathcal{M}^2_{22} + P_{32}P_{33}\mathcal{M}^2_{33} \\
&\quad +(P_{13}P_{22} + P_{12}P_{23})\mathcal{M}^2_{12} + (P_{13}P_{32} + P_{12}P_{33})\mathcal{M}^2_{13} + (P_{23}P_{32} + P_{22}P_{33})\mathcal{M}^2_{23}
\end{aligned}
$$

$$
\begin{aligned}
\bar{R}_{11} &= (P_{11}R_{11} + P_{21}R_{12} + P_{31}R_{13}) \\
\bar{R}_{12} &= (P_{12}R_{11} + P_{22}R_{12} + P_{32}R_{13}) \\
\bar{R}_{13} &= (P_{13}R_{11} + P_{23}R_{12} + P_{33}R_{13}) \\
\bar{R}_{21} &= (P_{11}R_{21} + P_{21}R_{22} + P_{31}R_{23}) \\
\bar{R}_{22} &= (P_{12}R_{21} + P_{22}R_{22} + P_{32}R_{23}) \\
\bar{R}_{23} &= (P_{13}R_{21} + P_{23}R_{22} + P_{33}R_{23}) \\
\bar{R}_{31} &= (P_{11}R_{31} + P_{21}R_{32} + P_{31}R_{33}) \\
\bar{R}_{32} &= (P_{12}R_{31} + P_{22}R_{32} + P_{32}R_{33}) \\
\bar{R}_{33} &= (P_{13}R_{31} + P_{23}R_{32} + P_{33}R_{33})
\end{aligned}
$$

$$
\begin{aligned}
P_{11} &= \frac{\cos\theta(v_1\cos\theta + v_2\sin\theta\cos(\xi_{21}-\xi))}{\bar{v}_1}, \\
P_{12} &= -\frac{\sin\theta(v_2\cos\theta\cos(\xi_{21}-\xi) - v_1\sin\theta)}{\bar{v}_2}, \\
P_{13} &= \frac{vv_2\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}, \\
P_{21} &= \frac{\sin\theta(v_1\cos\theta\cos(\xi_{21}-\xi) + v_2\sin\theta)}{\bar{v}_1}, \\
P_{22} &= \frac{\cos\theta(v_2\cos\theta - v_1\sin\theta\cos(\xi_{21}-\xi))}{\bar{v}_2}, \\
P_{23} &= -\frac{vv_1\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}, \\
P_{31} &= -\frac{v\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_1}, \\
P_{32} &= \frac{v\sin 2\theta\sin(\xi_{21}-\xi)}{2\bar{v}_2}, \\
P_{33} &= \frac{2v_1v_2\cos 2\theta + (v_2^2 - v_1^2)\sin 2\theta\cos(\xi_{21}-\xi)}{2\bar{v}_1\bar{v}_2}.
\end{aligned}
$$

**None of the squared mass matrix elements or rotation matrix elements are invariants, and therefore they are not observables:**

# Invariance of the neutral masses

- Combinations of squared mass matrix elements that are invariant are the trace, the sum of principal cofactors and the determinant, i.e.

$$
\begin{aligned}
b &= -(\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 + \mathcal{M}_{33}^2), \\
c &= \mathcal{M}_{11}^2\mathcal{M}_{22}^2 + \mathcal{M}_{11}^2\mathcal{M}_{33}^2 + \mathcal{M}_{22}^2\mathcal{M}_{33}^2 \\
&\quad -(\mathcal{M}_{12}^2)^2 - (\mathcal{M}_{13}^2)^2 - (\mathcal{M}_{23}^2)^2, \\
d &= \mathcal{M}_{11}^2(\mathcal{M}_{23}^2)^2 + \mathcal{M}_{22}^2(\mathcal{M}_{13}^2)^2 + \mathcal{M}_{33}^2(\mathcal{M}_{12}^2)^2 \\
&\quad -\mathcal{M}_{11}^2\mathcal{M}_{22}^2\mathcal{M}_{33}^2 - 2\mathcal{M}_{12}^2\mathcal{M}_{13}^2\mathcal{M}_{23}^2.
\end{aligned}
$$

- Are all found to be basis invariant, hence observable

- The eigenvalues of the squared mass matrix gives us the three neutral masses.
- Characteristic equation for eigenvalues:

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0$$

- Eigenvalues (masses) are found to be

$$
\begin{aligned}
M_1^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) + \frac{2\pi}{3}\right], \\
M_2^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - \frac{2\pi}{3}\right], \\
M_3^2 &= \frac{-b}{3} + 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right)\right].
\end{aligned}
$$

**All masses are basis invariant, hence observable**

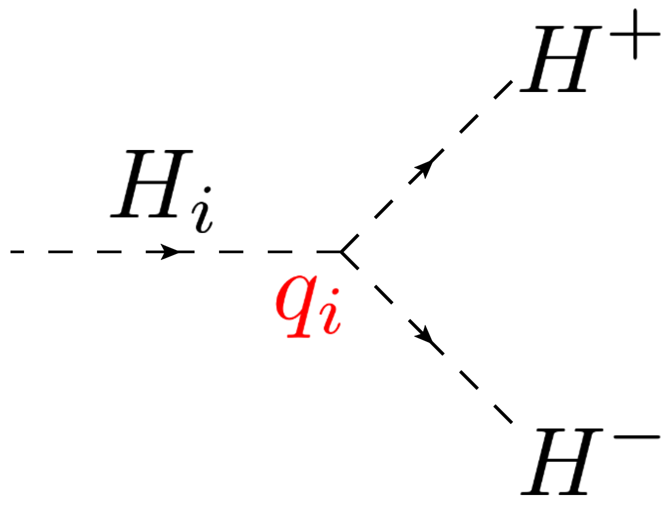
$$
\begin{aligned}
p &= c - b^2/3 \\
q &= \frac{2b^3 - 9bc + 27d}{27}
\end{aligned}
$$

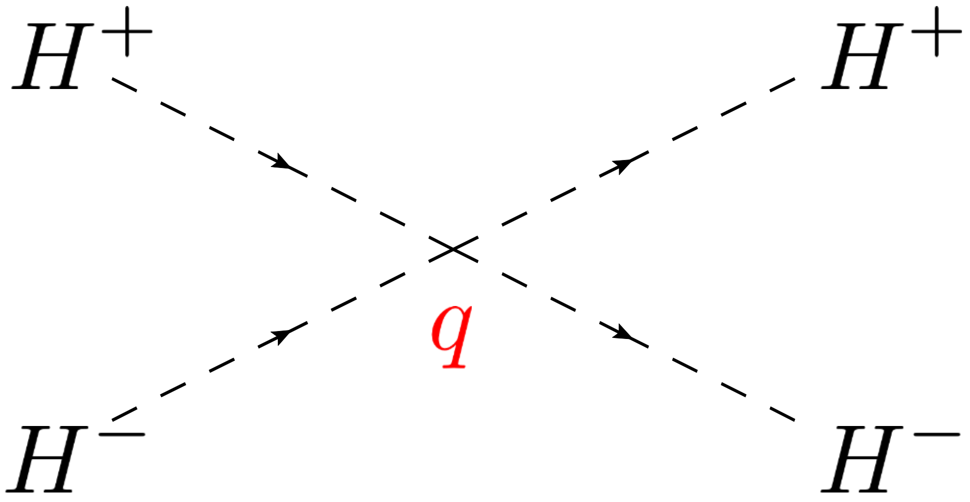
# Invariance of scalar couplings

- Some important scalar couplings

$$
\begin{aligned}
q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\
&= \frac{v_1 v_2^2}{v^2} R_{i1}\lambda_1 + \frac{v_1^2 v_2}{v^2} R_{i2}\lambda_2 + \frac{v_1^3 R_{i1} + v_2^3 R_{i2}}{v^2}\lambda_3 \\
&\quad - \frac{v_1 v_2 (v_2 R_{i1} + v_1 R_{i2})}{v^2}(\lambda_4 + \text{Re}\,\lambda_5) + \frac{v_1 v_2}{v} R_{i3} \text{Im}\,\lambda_5 \\
&\quad + \frac{v_2(v_2^2 - 2v_1^2) R_{i1} + v_1 v_2 R_{i2}}{v^2} \text{Re}\,\lambda_6 - \frac{v_2^2}{v} R_{i3} \text{Im}\,\lambda_6 \\
&\quad + \frac{v_1(v_1^2 - 2v_2^2) R_{i2} + v_1 v_2 R_{i1}}{v^2} \text{Re}\,\lambda_7 - \frac{v_1^2}{v} R_{i3} \text{Im}\,\lambda_7, \\
q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+) \\
&= \frac{v_2^4}{2v^4}\lambda_1 + \frac{v_1^4}{2v^4}\lambda_2 + \frac{v_1^2 v_2^2}{v^4}(\lambda_3 + \lambda_4 + \text{Re}\,\lambda_5) - \frac{2 v_1 v_2^3}{v^4}\text{Re}\,\lambda_6 - \frac{2 v_1^3 v_2}{v^4}\text{Re}\,\lambda_7.
\end{aligned}
$$

- These also turn out to be basis invariant, hence observables.



# Invariance of gauge couplings

- Gauge couplings

$$H_i H_j Z_\mu : \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i - p_j)_\mu,$$

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W}e_i\, g_{\mu\nu},$$

$$H_i W_\mu^+ W_\nu^- : \quad \frac{ig^2}{2}e_i\, g_{\mu\nu}.$$

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

$$e_1^2 + e_2^2 + e_3^3 = v^2 = (246\,\mathrm{GeV})^2$$

$$\bar{v}_1 \bar{R}_{i1} + \bar{v}_2 \bar{R}_{i2} = v_1 R_{i1} + v_2 R_{i2}$$

- Showing that these gauge couplings are invariant under a change of basis, hence they are observables.
- Most couplings are invariants. Some (the complex ones) are pseudo-invariants (their absolute value is invariant).
- **No surprise**: Masses and couplings are invariants and possible to measure in experiments.

# Systematic construction of invariants by use of tensors.

- Introduce $V_{ab}$ tensor as:

$$V_{ab} = \frac{v_a v_b^*}{v^2} = \frac{1}{v^2}\begin{pmatrix} v_1^2 & v_1 v_2 e^{-i\xi_{21}} \\ v_1 v_2 e^{i\xi_{21}} & v_2^2 \end{pmatrix}$$

- Transformation rules of $Y_{ab}$, $Z_{abcd}$ and $V_{ab}$ tensors under change of basis:

$$\begin{aligned} \bar{Y} &= UYU^\dagger, \\ \bar{V} &= UVU^\dagger, \\ \bar{Z}_{abcd} &= U_{ae}U_{cg}Z_{efgh}U^\dagger_{fb}U^\dagger_{hd} \end{aligned}$$

- We may now put together an arbitrary number of *Y*-, *Z*- and *V*-tensors and contract the indices with an odd-numbered position with the indices with an even-numbered position to get an invariant quantity.
- Examples

$$\begin{aligned} V_{aa} &= 1, \\ Y_{aa} &= -\frac{1}{2}(m_{11}^2 + m_{22}^2), \\ Z_{aabb} &= \lambda_1 + \lambda_2 + 2\lambda_3, \\ Z_{abba} &= \lambda_1 + \lambda_2 + 2\lambda_4. \end{aligned}$$

- **We already know these to be invariant!**

# Systematic construction of CP-violating invariants by use of tensors

- Invariants are observables and so they must be expressible in terms of observable couplings and masses
- How do we translate an invariant expression consisting of potential parameters/VEVs to an expression containing only masses and couplings?
- Choose to work in a particular basis (the Higgs-basis) and establish identities between observable quantities in this basis.

- Promote your observables to invariant couplings.
- The identities established must then be valid in any basis.
- All observables (invariants) can be expressed in terms of the 11 masses/couplings:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

# From parameters to masses and couplings via Higgs-basis

- Only one VEV is non-zero.

$$v_1 = v, \quad v_2 = 0, \quad \xi_{21} = 0$$

$$\langle \Phi_1 \rangle_{\mathrm{HB}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Phi_2 \rangle_{\mathrm{HB}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Not unique, as one may still perform a U(1) transform on $\Phi_2$ without giving $\Phi_2$ a non-zero VEV.
- Stationary-point equations

$$\begin{aligned} Y_{11} &= -\frac{v^2}{2} Z_{1111}, \\ \mathrm{Re}\, Y_{12} &= -\frac{v^2}{2} \mathrm{Re}\, Z_{1112}, \\ \mathrm{Im}\, Y_{12} &= -\frac{v^2}{2} \mathrm{Im}\, Z_{1112}, \end{aligned}$$

$$\begin{aligned} m_{11}^2 &= v^2 \lambda_1, \\ \mathrm{Re}\, m_{12}^2 &= v^2 \mathrm{Re}\, \lambda_6, \\ \mathrm{Im}\, m_{12}^2 &= v^2 \mathrm{Im}\, \lambda_6, \end{aligned}$$

- Charged scalar mass:

$$M_{H^\pm}^2 = Y_{22} + \frac{v^2}{2} Z_{1122}$$

- Neutral mass matrix:

$$\begin{aligned} \mathcal{M}^2 &= R^T \mathrm{diag}(M_1^2, M_2^2, M_3^2) R \\ &= v^2 \begin{pmatrix} Z_{1111} & \mathrm{Re}\, Z_{1112} & -\mathrm{Im}\, Z_{1112} \\ \mathrm{Re}\, Z_{1112} & \frac{1}{2}(Z_{1122} + Z_{1221} + \mathrm{Re}\, Z_{1212}) + \frac{Y_{22}}{v^2} & -\frac{1}{2}\mathrm{Im}\, Z_{1212} \\ -\mathrm{Im}\, Z_{1112} & -\frac{1}{2}\mathrm{Im}\, Z_{1212} & \frac{1}{2}(Z_{1122} + Z_{1221} - \mathrm{Re}\, Z_{1212}) + \frac{Y_{22}}{v^2} \end{pmatrix} \end{aligned}$$

- Treat the above as seven equations and solve to get

$$\begin{aligned} Y_{22} &= M_{H^\pm}^2 - \frac{v^2}{2} Z_{1122}, \\ Z_{1111} &= \frac{R_{11}^2 M_1^2 + R_{21}^2 M_2^2 + R_{31}^2 M_3^2}{v^2}, \\ Z_{1221} &= \frac{-2M_{H^\pm}^2 + (R_{12}^2 + R_{13}^2) M_1^2 + (R_{22}^2 + R_{23}^2) M_2^2 + (R_{32}^2 + R_{33}^2) M_3^2}{v^2}, \\ \mathrm{Re}\, Z_{1112} &= \frac{R_{11} R_{12} M_1^2 + R_{21} R_{22} M_2^2 + R_{31} R_{32} M_3^2}{v^2}, \\ \mathrm{Im}\, Z_{1112} &= -\frac{R_{11} R_{13} M_1^2 + R_{21} R_{23} M_2^2 + R_{31} R_{33} M_3^2}{v^2}, \\ \mathrm{Re}\, Z_{1212} &= \frac{(R_{12}^2 - R_{13}^2) M_1^2 + (R_{22}^2 - R_{23}^2) M_2^2 + (R_{32}^2 - R_{33}^2) M_3^2}{v^2}, \\ \mathrm{Im}\, Z_{1212} &= -2 \frac{R_{12} R_{13} M_1^2 + R_{22} R_{23} M_2^2 + R_{32} R_{33} M_3^2}{v^2}. \end{aligned}$$

# From parameters to masses and couplings via Higgs-basis

- Scalar couplings in the Higgs-basis.

$$\begin{aligned} q_i &= v(R_{i1} Z_{1122} + R_{i2}\mathrm{Re}\, Z_{1222} - R_{i3}\mathrm{Im}\, Z_{1222}), \\ q &= \frac{1}{2} Z_{2222}. \end{aligned}$$

- Treat as four equations and solve to get

$$\begin{aligned} Z_{1122} &= \frac{R_{11}q_1 + R_{21}q_2 + R_{31}q_3}{v}, \\ \mathrm{Re}\, Z_{1222} &= \frac{R_{12}q_1 + R_{22}q_2 + R_{32}q_3}{v}, \\ \mathrm{Im}\, Z_{1222} &= -\frac{R_{13}q_1 + R_{23}q_2 + R_{33}q_3}{v}, \\ Z_{2222} &= 2q. \end{aligned}$$

- All parameters of the potential has now been replaced by scalar couplings and masses (and elements from the rotation matrix).
- Gauge couplings in the Higgs-basis

$$e_i = vR_{i1}$$

- Combinations of rotation matrix elements appearing in the invariants can all be expressed in terms of the three $e_i$ by utilizing the orthogonality of $R$.

# Applying the technique to CP-even invariants

- We already know

$$\begin{aligned} Y_{aa} &= -\frac{1}{2}(m_{11}^2 + m_{22}^2), \\ Z_{aabb} &= \lambda_1 + \lambda_2 + 2\lambda_3, \\ Z_{abba} &= \lambda_1 + \lambda_2 + 2\lambda_4. \end{aligned}$$

- Applying the technique outlined we arrive at

$$\begin{aligned} Y_{aa} &= M_{H^\pm}^2 - \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2) - \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3), \\ Z_{aabb} &= 2q + \frac{1}{v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2) + \frac{2}{v^2}(e_1 q_1 + e_2 q_2 + e_3 q_3), \\ Z_{abba} &= 2q - \frac{4}{v^2} M_{H^\pm}^2 - \frac{1}{v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2) + \frac{2}{v^2}(M_1^2 + M_2^2 + M_3^2). \end{aligned}$$

# Applying the technique to CP-odd invariants

- CP-properties given by three CP-odd invariants

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[V_{da}Y_{ab}Z_{bccd}\right],$$
$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[V_{ab}V_{dc}Y_{be}Y_{cf}Z_{eafd}\right],$$
$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[V_{ab}V_{dc}Z_{bgge}Z_{chhf}Z_{eafd}\right].$$

- Applying the technique outlined we immediately arrive at

$$\begin{aligned}\mathrm{Im}\, J_1 &= \frac{1}{v^5}[e_1e_3q_2(M_1^2-M_3^2)+e_2e_1q_3(M_2^2-M_1^2)+e_3e_2q_1(M_3^2-M_2^2)],\\ \mathrm{Im}\, J_2 &= 2\frac{e_1e_2e_3}{v^9}(M_1^2-M_2^2)(M_2^2-M_3^2)(M_3^2-M_1^2).\end{aligned}$$

- Im $J_3$ is a little more complicated:

$$\mathrm{Im}\, J_3 = c_1\mathrm{Im}\, J_1 + c_2\mathrm{Im}\, J_2 + c_{11}\mathrm{Im}\, J_{11} + c_{30}\mathrm{Im}\, J_{30}$$

Vanishes when
Im $J_1$ = Im $J_2$ = 0

$$\begin{aligned}\mathrm{Im}\, J_{30} &= [e_2q_1q_3(M_1^2-M_3^2)+e_3q_2q_1(M_2^2-M_1^2)\\ &\quad +e_1e_3q_2(M_3^2-M_2^2)],\end{aligned}$$

# CP properties of the 2HDM and Summary

- Put Im $J_1$ = Im $J_2$ = Im $J_{30}$ = 0 and solve

**6 distinct cases:**

- Case 1: $M_1=M_2=M_3$. Full mass degeneracy.
- Case 2: $M_1=M_2$ and $e_1q_2 = e_2q_1$
- Case 3: $M_2=M_3$ and $e_2q_3 = e_3q_2$
- Case 4: $e_1=0$ and $q_1=0$
- Case 5: $e_2=0$ and $q_2=0$
- Case 6: $e_3=0$ and $q_3=0$

***If none of the above occur, then CP is broken!***

- **Summary**
- All observables can be written in terms of invariant quantities (masses and couplings).
- Simple, yet powerful method presented for translating invariants from parameters to observables.