Boltzmann equation for relativistic species and Hot Dark Matter

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Motivation

Hubble constant measurement discrepancy:

Planck CMB data: $67.8 \pm 0.9\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [arXiv:1502.01589]
Direct measurement: $73.24 \pm 1.74\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [arXiv:1604.01424]

Possible solution: additional effective number of relativistic degrees of freedom ΔN_{eff}:

$$\rho_{\mathrm{HDM}} = \Delta N_{\mathrm{eff}} \, \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

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Therefore, it is crucial to estimate ΔN_{eff} with better accuracy in models with HDM component

Boltzmann equation for relativistic particles

Dolgov & Kainulainen, Nucl. Phys. B **402** (1993) 349 [hep-ph/9211231]

$$E(\partial_t - pH\partial_p)f(p,t) = C_E(p,t) + C_I(p,t)$$

Pseudopotential method ($\bar{\psi}\psi \to \bar{N}N$, $x = \frac{m}{T}$, $y = \frac{E}{T}$, z):

$$f(p,t) \simeq \left(e^{xy+z} \mp 1\right)^{-1}$$

$$f_N(p,t) \simeq (e^{xy_N} \mp 1)^{-1}$$

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where:

$$A(z,x) = \frac{g}{2\pi^2} m^3 e^z J_2(z,x)$$

$$B(z,x) \equiv \frac{g}{2\pi^2} m^3 e^z x\, J_3(z,x)\left(H(T) + \frac{1}{T}\frac{dT}{dt}\right)$$

$$J_n(z,x) \equiv \int_0^\infty dy\, y^n \frac{e^{xy}}{(1 \mp e^{xy+z})^2}$$

Boltzmann equation for relativistic particles

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$$S_I(z,x) = \frac{m^4}{512\pi^6}\left(e^{2z}-1\right)\int \mathscr{D}\Phi \int_0^{2\pi} d\phi \sum_{\text{spin}} |M(u,v,t,\cos\theta,\phi)|^2$$

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For $|M|^2 = |M(s)|^2$:

$$\mathscr{D}\Phi = \frac{1}{x^4}\int_{2x}^{\infty} dp \int_0^{\sqrt{p^2-4x^2}} dq \, \frac{1}{(1-e^{-p})(e^{p+2z}-1)}$$

$$\times \ln\left[\frac{\cosh\left(\frac{1}{2}(p+q)+z\right)\mp 1}{\cosh\left(\frac{1}{2}(p-q)+z\right)\mp 1}\right] \ln\left[\frac{\cosh\left(\frac{1}{2}\left(p+qV(p,q)\right)\right)\mp 1}{\cosh\left(\frac{1}{2}\left(p-qV(p,q)\right)\right)\mp 1}\right]$$

where $V = \left(1-\frac{4x^2}{p^2-q^2}\right)^{1/2}$, $s = m^2\frac{p^2-q^2}{x^2}$

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Finally:

$$n(x) \equiv n_\psi(x) = g\frac{m^3}{2\pi^2}\int_0^\infty \frac{y^2 dy}{e^{xy+z}\mp 1}$$

Approximations

$$Y'(x) = -\sqrt{\frac{\pi}{45}}\,\frac{g(x)}{\sqrt{g_s(x)}}\,\frac{M_{\rm Pl}\,m}{x^2}\left(Y^2(x) - Y_{\rm eq}^2(x)\right)\langle\sigma v\rangle_{\rm MB}\,\frac{1}{\zeta^{\mp}}$$

$$Y_{\rm eq}(x) = g\,\frac{45}{2\pi^4}\,\frac{\zeta^{\mp}}{g_s(x)} \qquad\qquad \zeta^{\mp} \equiv \zeta(3)\begin{cases}1 & \text{BE } (-)\\ 3/4 & \text{FD } (+)\end{cases}$$

$$\langle\sigma v\rangle_{\rm MB} = \frac{1}{512\pi}\,\frac{x^5}{m^5}\int_{4m^2}^{\infty} ds\,\sqrt{s}\,\sqrt{1-\frac{4m^2}{s}}\,|M(s)|^2 K_1\left(\frac{x\sqrt{s}}{m}\right)$$

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- Pure MB: $\langle\sigma v\rangle_{\rm MB}$ with $\zeta^{\mp} = 1$ — artificial — MB
- Fractional BE/FD: $\langle\sigma v\rangle_{\rm MB} \times \zeta^{\mp}$ — used — fBE/FD
- Partial BE/FD: $\langle\sigma v\rangle_{\rm p}$ — used — pBE/FD

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$$\times\int_{\sqrt{s}}^{\infty} dE_+\,\frac{e^{-\frac{x}{2m}E_+}}{\sinh\left(\frac{x}{2m}E_+\right)}\,\ln\left[\frac{\mathrm{fh}\left(\frac{x}{4m}\left(E_+ + \sqrt{E_+^2 - s}\right)\right)}{\mathrm{fh}\left(\frac{x}{4m}\left(E_+ - \sqrt{E_+^2 - s}\right)\right)}\right]$$

Weinberg's Higgs portal model

S. Weinberg, Phys. Rev. Lett. **110** (2013) no.24, 241301 [arXiv:1305.1971]

$$\mathcal{L}_{H,\phi} = (D_\mu H)^\dagger (D^\mu H) + \mu_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2$$

$$+\partial_\mu \phi^* \partial^\mu \phi + \mu_\phi (\phi^* \phi)^2 - \lambda_\phi (\phi^* \phi)^2 + \kappa (H^\dagger H)(\phi^* \phi)$$

$$H_0 = v_H + \frac{\tilde{h} + iG^0}{\sqrt{2}} \qquad \phi = v_\phi + \frac{\rho + i\sigma}{\sqrt{2}}$$

Diagonalization from $(\tilde{h}\ \tilde{\rho})$ to $(h\ \rho)$: $\tan 2\theta = \frac{\kappa\, v_H v_\phi}{\lambda_H v_H^2 - \lambda_\phi v_\phi^2}$

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Let's consider HDM annihilation into muons: $\sigma\sigma \to \bar{\mu}\mu$.

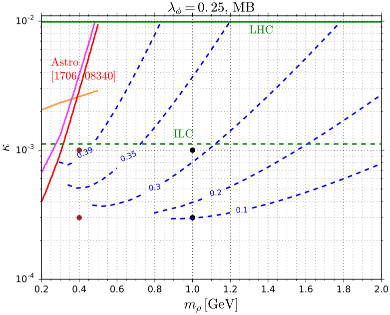
In narrow resonance approximation $(\Gamma_\rho / m_\rho)^2 \ll 1$:

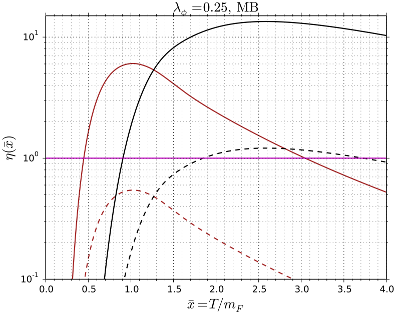
$$|M|^2 = \frac{2\pi\kappa^2}{m_\rho \Gamma_\rho} (m_\rho^2 - 4m_F^2) \frac{m_\rho^4 + \frac{m_\rho^2 m_h^2}{(m_\rho^2 - m_h^2)^2} (m_\rho \Gamma_h - m_h \Gamma_\rho)^2}{\left[(m_\rho^2 - m_h^2)^2 + \Gamma_h^2 m_h^2\right]}$$

Freeze-out approximation

$$\eta(x) = \frac{\Gamma}{H} = \left.\frac{n\langle\sigma v\rangle}{H}\right|_{x=x_f} = 1$$

$$\eta(x) \approx \frac{\kappa^2}{\lambda_\phi}\frac{\sqrt{45}\,\zeta(3)}{32\pi^{5/2}}\frac{m_\rho^5 M_{\rm Pl}}{m_h^4 m_F^2}\left(1-\frac{4m_F^2}{m_\rho^2}\right)^{3/2}\left.\frac{x^4}{\sqrt{g(x)}}K_1\left(\frac{x\,m_\rho}{m_F}\right)\right|_{x=x_f} = 1$$





Comparison of different statistics & approximations

$$\mathscr{D}\Phi_{\rm fBE} \approx \mathscr{D}\Phi_{\rm MB} \times \zeta^{\mp} \qquad \mathscr{D}\Phi_{\rm fFD} \approx \mathscr{D}\Phi_{\rm MB} \times \zeta^{\mp}$$

$$\mathscr{D}\Phi_{\rm pBE} \approx \mathscr{D}\Phi_{\rm MB} \times \coth\left(\frac{m_\rho x}{4 m_F}\right) \qquad \mathscr{D}\Phi_{\rm pFD} \approx \mathscr{D}\Phi_{\rm MB} \times \tanh\left(\frac{m_\rho x}{4 m_F}\right)$$

$$\mathscr{D}\Phi_{\rm BEFD} \approx \mathscr{D}\Phi_{\rm MB} \qquad \mathscr{D}\Phi_{\rm FDBE} \approx \mathscr{D}\Phi_{\rm MB}$$

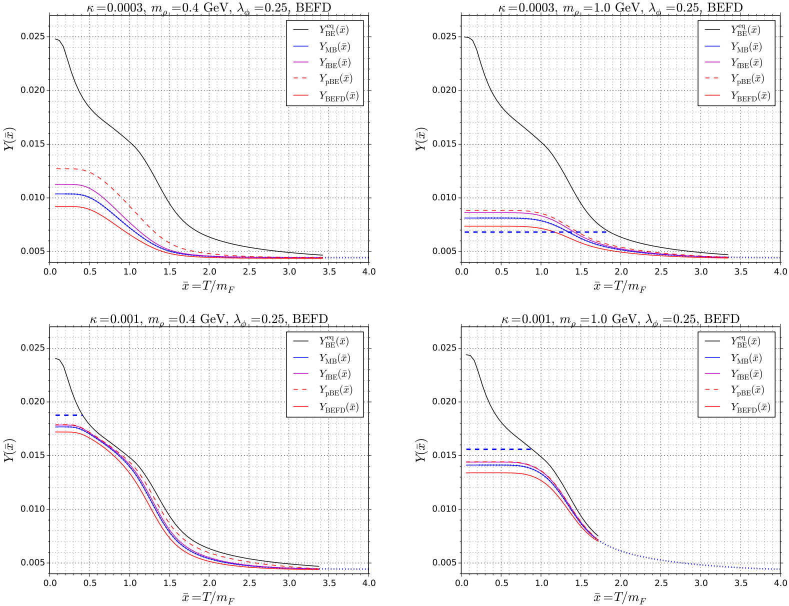
$$\mathscr{D}\Phi_{\rm BEBE} \approx \mathscr{D}\Phi_{\rm MB} \times \coth^2\left(\frac{m_\rho x}{4 m_F}\right) \qquad \mathscr{D}\Phi_{\rm FDFD} \approx \mathscr{D}\Phi_{\rm MB} \times \tanh^2\left(\frac{m_\rho x}{4 m_F}\right)$$

where

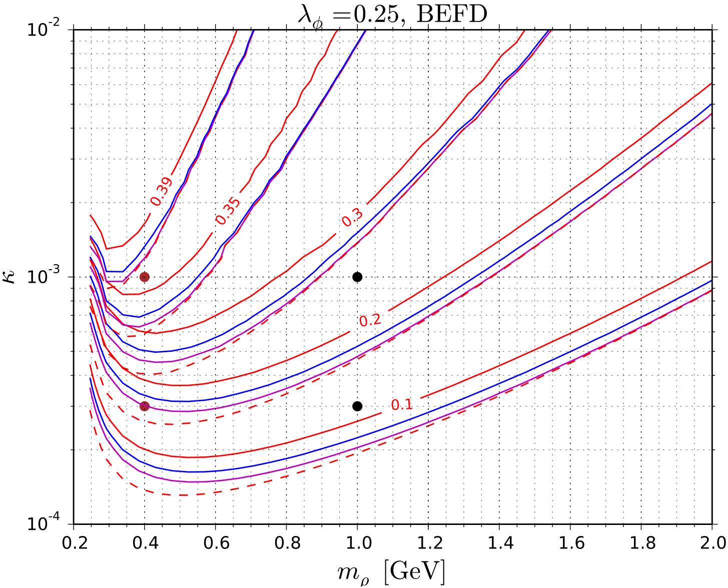
$$\mathscr{D}\Phi_{\rm MB} = e^{-2z} \frac{|M|^2}{2x} \sqrt{\frac{m_\rho^2}{m_F^2} - 4}\; K_1\left(\frac{x\, m_\rho}{m_F}\right)$$

- Cancellation for mixed statistics
- Amplification for BE
- Suppression for FD

Example: $m_\rho = 0.4$ GeV $m_\rho = 1$ GeV



Numerical scan



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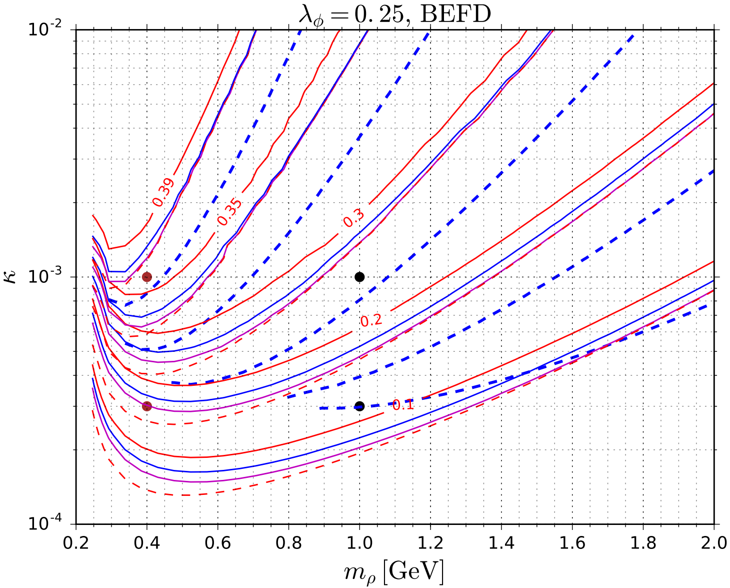
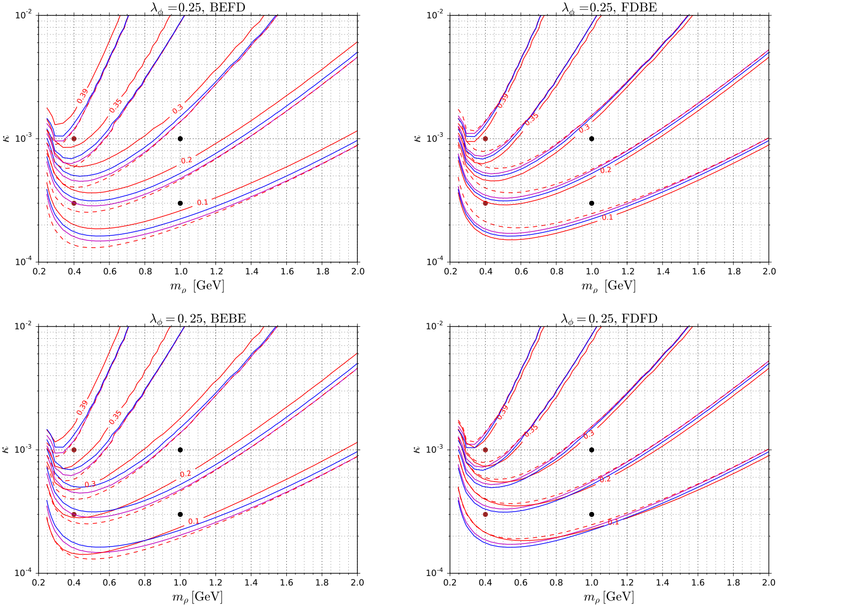


Conclusions

- We have compared a few approaches for estimating ΔN_{eff} of the Goldstone boson in Weinberg's model
- Freeze-out approximation breaks down for small Higgs portal coupling κ
- Pure Maxwell-Boltzmann approximation works better than others because of cancellation effect between mixed statistics
- Both the statistics of incoming and outgoing particles influence the results
- In order to obtain higher accuracy Boltzmann equation with full statistics should be considered
- Work in progress

Backup slides



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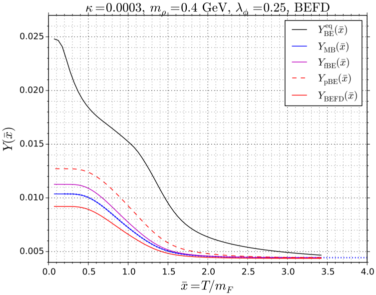

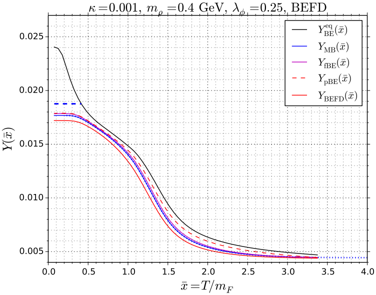

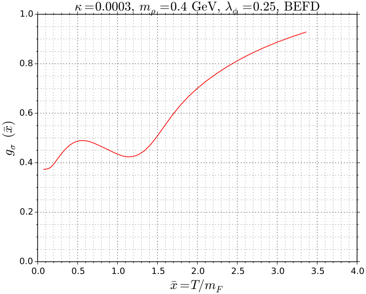

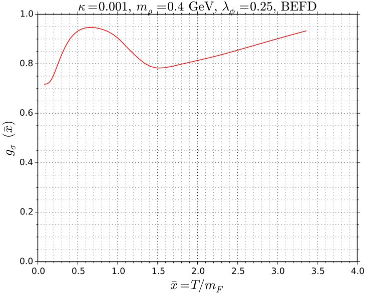

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