Detecting Quantum Entanglement at Colliders

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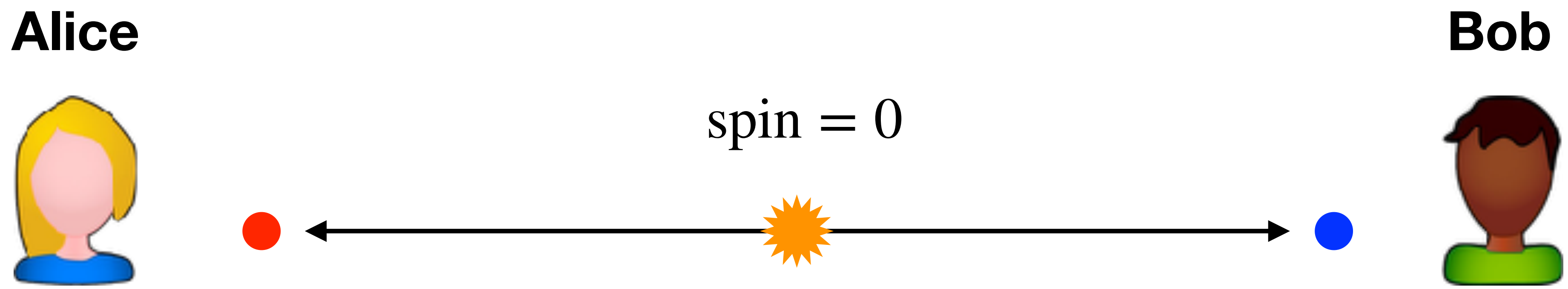
Plan

- **Introduction**
  - Entanglement, Non-locality, Bell inequality
- **Entanglement detection at Colliders**
  - $H \to \tau^+\tau^-$
  - $pp \to t\bar{t}$
  - $H \to WW^*, ZZ^*$, $pp \to WW, ZZ$

Entanglement



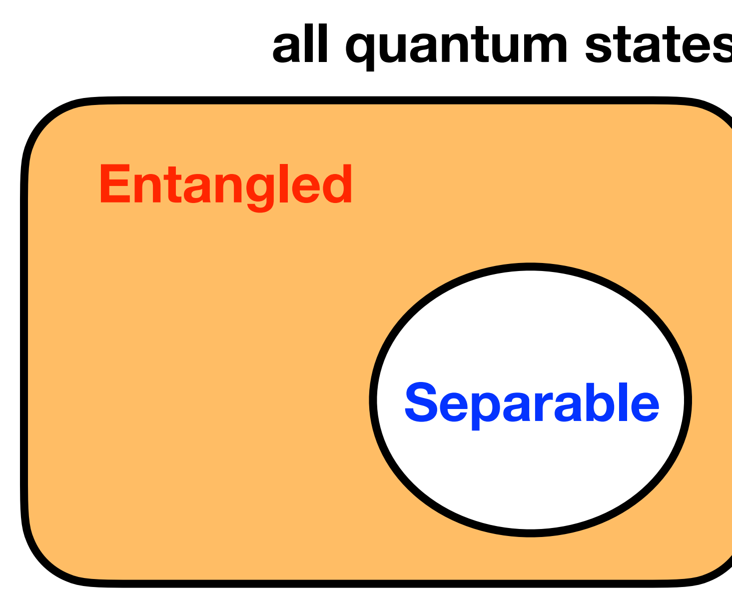
Alice Bob

$$|\Psi_{AB}^{(0,0)}\rangle \simeq |+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle$$

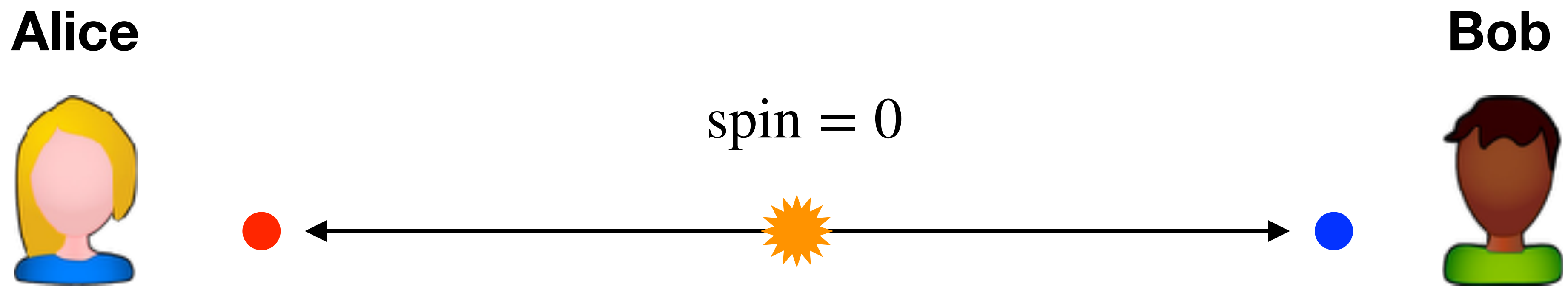
$$\neq |\Psi_A\rangle \otimes |\Psi_B\rangle \longleftarrow$$ **entangled**

$$|\Psi_{AB}^{\text{sep}}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle \longleftarrow$$ **separable**

$$\text{Entanglement} = \overline{\text{Separable}}$$



Entanglement



Alice ↓ Bob ↓

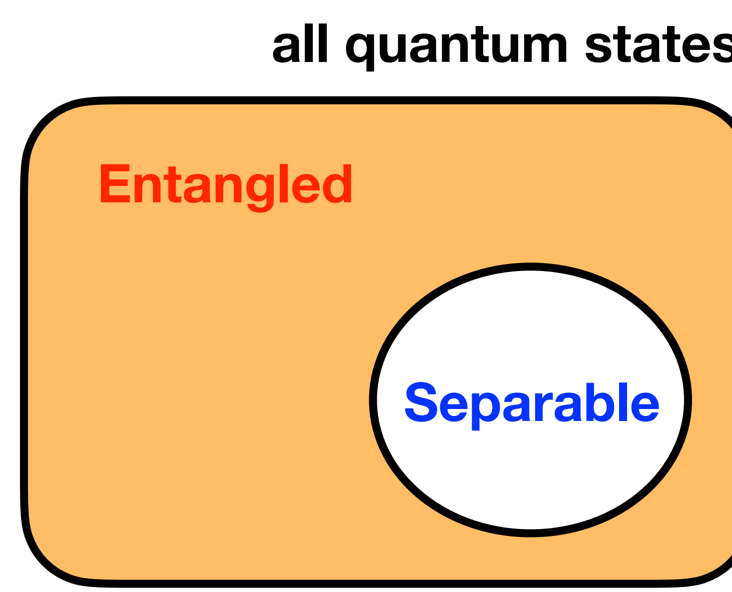
$$|\Psi^{(0,0)}_{AB}\rangle \simeq |+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle \xrightarrow{\text{Alice measures } s_z \text{ and found } +} |+,-\rangle$$

$$\neq |\Psi_A\rangle \otimes |\Psi_B\rangle \longleftarrow \textbf{entangled}$$

Alice’s local measurement changes the global state involving (spacelike separated) Bob’s particle.
~ **Nonlocality**

$$|\Psi^{\text{sep}}_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle \longleftarrow \textbf{separable}$$

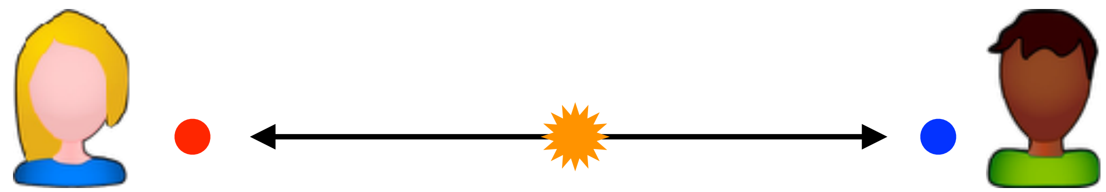
$$\text{Entanglement} = \overline{\text{Separable}}$$



measures axis: $x \in \{\vec{n}\}$ $\quad$ $y \in \{\vec{n}\}$

outcome: $a \in \{-1, +1\}$ $\quad$ $b \in \{-1, +1\}$

probability for a: $p_A(a \mid x)$ $\quad$ $p_B(b \mid y)$



Any data is expressed by the ***joint distribution*** $\quad p(a, b \mid x, y)$

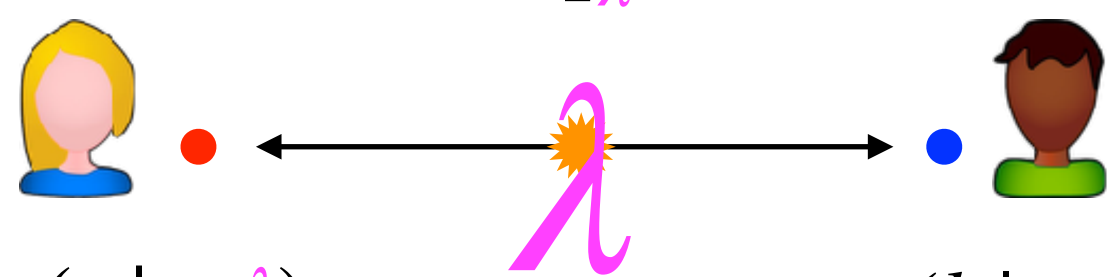
$\Rightarrow$ **Models describing the experiment can be classified by possible forms of** $p(a, b \mid x, y)$

measures axis: $x \in \{\vec{n}\}$ $\qquad$ $y \in \{\vec{n}\}$

outcome: $a \in \{-1, +1\}$ $\qquad$ $b \in \{-1, +1\}$

probability for a: $p_A(a\,|\,x)$ $\qquad$ $p_B(b\,|\,y)$

q_λ



$p_A(a\,|\,x, \lambda)$ $\qquad$ $p_B(b\,|\,y, \lambda)$

Any data is expressed by the ***joint distribution*** $\quad p(a, b\,|\,x, y)$

⇒ Models describing the experiment can be classified by possible forms of $p(a, b\,|\,x, y)$

probability for λ

$$\boxed{\textbf{Local theories}} \qquad p_L(a, b\,|\,x, y) = \sum_\lambda q_\lambda \cdot p_A(a\,|\,x, \lambda) \cdot p_B(b\,|\,y, \lambda)$$

measures axis: $x \in \{\vec{n}\}$ $\qquad$ $y \in \{\vec{n}\}$

outcome: $a \in \{-1, +1\}$ $\qquad$ $b \in \{-1, +1\}$

probability for a: $p_A(a|x)$ $\qquad$ $p_B(b|y)$

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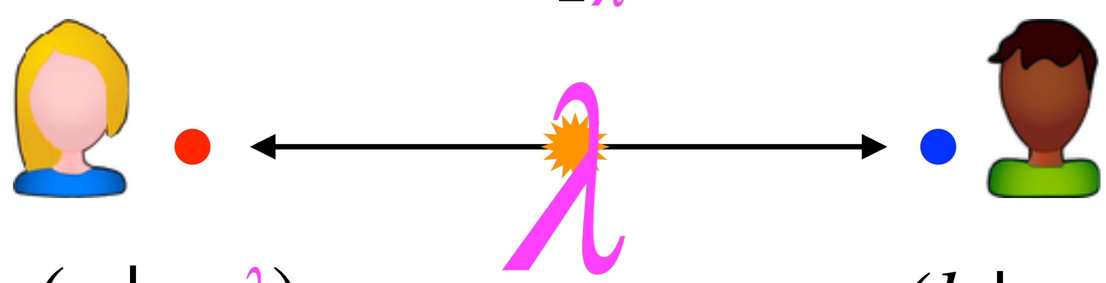


$p_A(a|x, \lambda)$ $\qquad$ $p_B(b|y, \lambda)$

Any data is expressed by the *joint distribution* $\quad p(a, b|x, y)$

⇒ Models describing the experiment can be classified by possible forms of $p(a, b|x, y)$

probability for λ

Local theories $\qquad p_L(a, b|x, y) = \sum_\lambda q_\lambda \cdot p_A(a|x, \lambda) \cdot p_B(b|y, \lambda)$

density operator

Quantum Mechanics $\qquad p_Q(a, b|x, y) = \mathrm{Tr}\left[\rho_{AB}\left(M_{a|x} \otimes M_{b|y}\right)\right]$

For projective measurement:

$M_{a|x} = |a_x\rangle\langle a_x|$

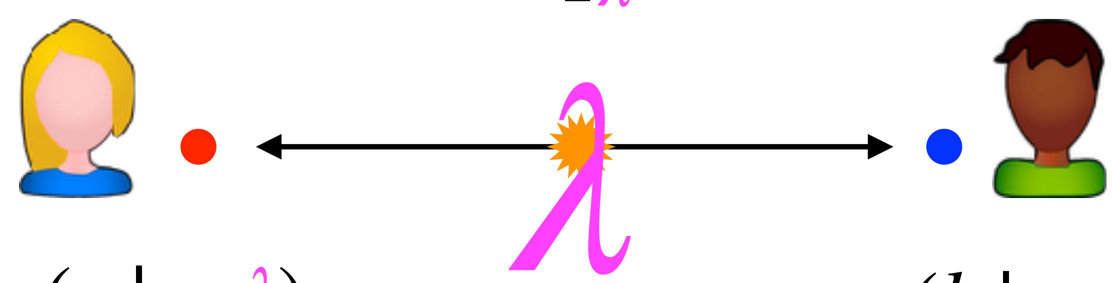
$\hat{s}_x|a_x\rangle = a_x|a_x\rangle$

measures axis: $x \in \{\vec{n}\}$

outcome: $a \in \{-1, +1\}$

probability for a: $p_A(a\,|\,x)$

q_λ



$y \in \{\vec{n}\}$

$b \in \{-1, +1\}$

$p_B(b\,|\,y)$

$p_A(a\,|\,x, \lambda)$ $\quad$ $p_B(b\,|\,y, \lambda)$

Any data is expressed by the ***joint distribution*** $\quad p(a, b\,|\,x, y)$

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probability for λ

Local theories

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Quantum Mechanics

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For projective measurement:

$$M_{a|x} = |a_x\rangle\langle a_x|$$

$$\hat{s}_x\,|a_x\rangle = a_x\,|a_x\rangle$$

For separable quantum states:

$$\rho_{AB} = \sum_\lambda q_\lambda \cdot \rho_A^\lambda \otimes \rho_B^\lambda \Longrightarrow p_{Q_{\mathrm{sep}}}(a, b\,|\,x, y) = \sum_\lambda q_\lambda \cdot \mathrm{Tr}\left[\rho_A^\lambda M_{a|x}\right] \cdot \mathrm{Tr}\left[\rho_B^\lambda M_{a|x}\right]$$

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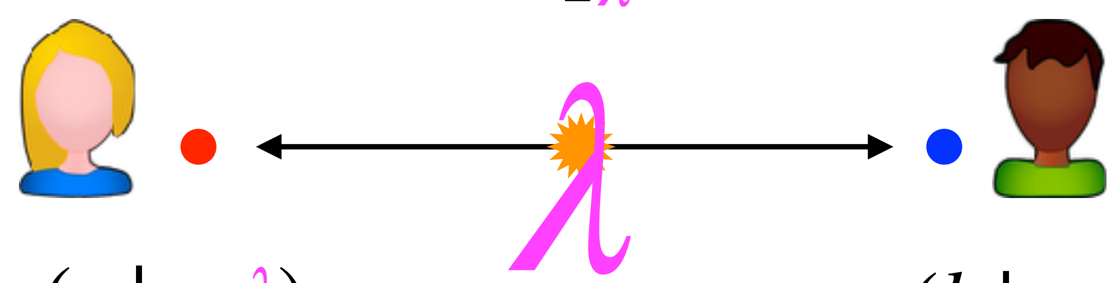

$y \in \{\vec{n}\}$

$b \in \{-1, +1\}$

$p_B(b \mid y)$

$p_A(a \mid x, \lambda)$ $\qquad$ $p_B(b \mid y, \lambda)$

Any data is expressed by the ***joint distribution*** $\quad p(a, b \mid x, y)$

$\Rightarrow$ Models describing the experiment can be classified by possible forms of $p(a, b \mid x, y)$

Local theories

$$p_L(a, b \mid x, y) = \boxed{\sum_\lambda q_\lambda \cdot p_A(a \mid x, \lambda) \cdot p_B(b \mid y, \lambda)}$$

Quantum Mechanics

density operator

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For separable quantum states:

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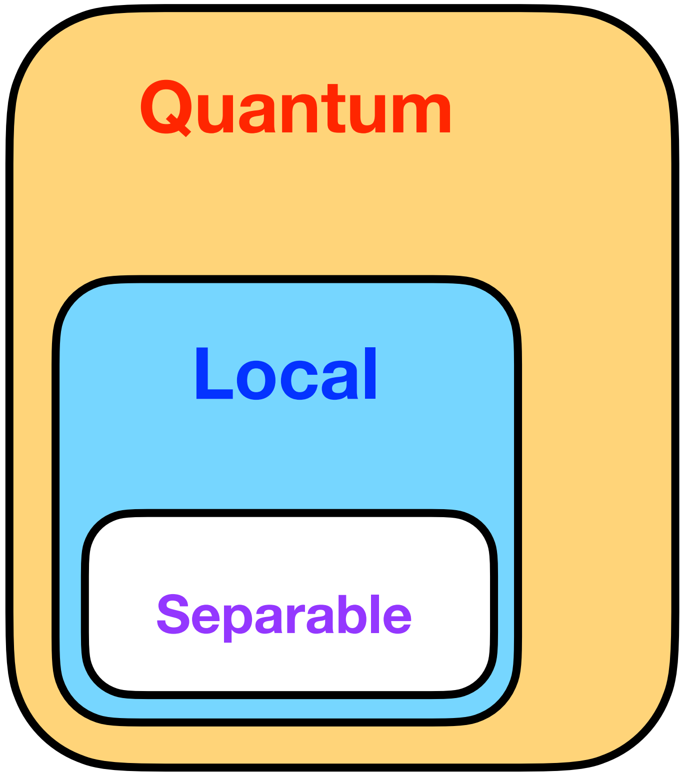

Quantum ⊃ Local ⊃ Separable

Local theories

$$p_L(a, b \,|\, x, y) = \boxed{\sum_{\lambda} q_{\lambda} \cdot p_A(a \,|\, x, \lambda) \cdot p_B(b \,|\, y, \lambda)}$$

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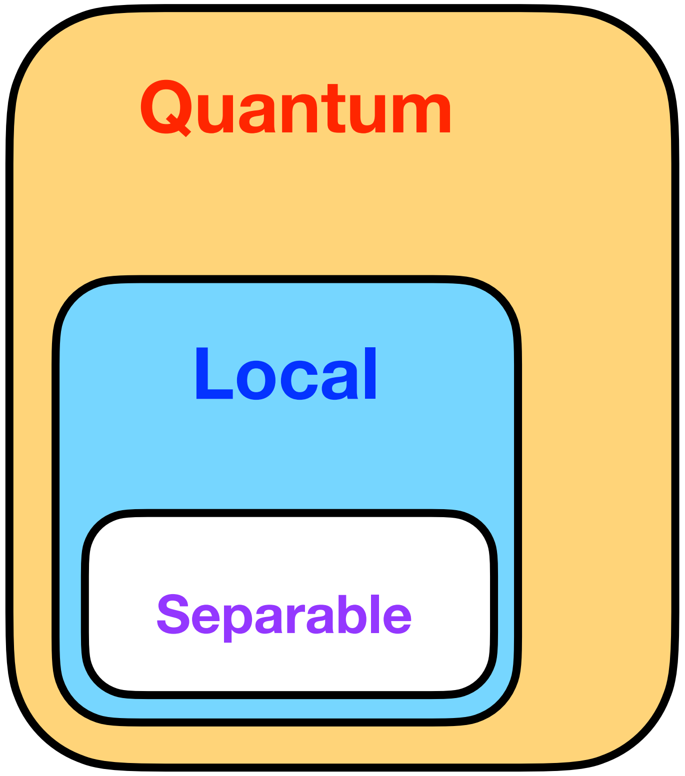
For separable quantum states:

Local

$$\rho_{AB} = \sum_{\lambda} q_{\lambda} \cdot \rho_A^{\lambda} \otimes \rho_B^{\lambda} \implies p_{Q_{\mathrm{sep}}}(a, b \,|\, x, y) = \boxed{\sum_{\lambda} q_{\lambda} \cdot \mathrm{Tr}\left[\rho_A^{\lambda} M_{a|x}\right] \cdot \mathrm{Tr}\left[\rho_B^{\lambda} M_{a|x}\right]}$$



$$\text{Quantum} \supset \text{Local} \supset \text{Separable}$$

$$\updownarrow \qquad \updownarrow$$

$$\text{Nonlocal} \subset \text{Entanglement}$$

Local theories

$$p_L(a,b\,|\,x,y) = \boxed{\sum_{\lambda} q_{\lambda} \cdot p_A(a\,|\,x,\lambda) \cdot p_B(b\,|\,y,\lambda)}$$

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- Nonlocal states in QM does not violate causality

$$p(a\,|\,x, y) \equiv \sum_b p(a, b\,|\,x, y)$$

Condition for no causality violation: No-Signalling [Cirel'son(1980), Popescu, Rohrlich(1994)]

$$^\forall a, b, x, x', y, y' \quad \begin{cases} p(a\,|\,x, y) = p(a\,|\,x, y') \\ p(b\,|\,x, y) = p(b\,|\,x', y) \end{cases}$$

Alice's dist. is indep. of Bob's choice for meas. axis

Bob's dist. is indep. of Alice's choice for meas. axis

No-signalling ⊃ Quantum ⊃ Local ⊃ Separable

Bell Inequalities

- Bell-type inequalities (in general) are the inequalities that separate different types of distributions (No-signalling, Quantum, Local).

- Define the correlator $C_{xy} = \langle A_x B_y \rangle \equiv \sum_{a,b} abp(a, b \,|\, x, y)$

- CHSH inequality [Clauser-Horne-Shimony-Holt(1969)]

For $a, b \in \{\pm 1\}$, $x \in \{\mathbf{n}_1, \mathbf{n}_2\}$, $y \in \{\mathbf{e}_1, \mathbf{e}_2\}$

$$S_{\mathrm{CHSH}} \equiv C_{\mathbf{n}_1,\mathbf{e}_1} + C_{\mathbf{n}_1,\mathbf{e}_2} + C_{\mathbf{n}_2,\mathbf{e}_1} - C_{\mathbf{n}_2,\mathbf{e}_2}$$

$$S_{\mathrm{CHSH}} \leq \begin{cases} 2 & \textbf{Local theories} \quad \text{[CHSH(1969)]} \\ 2\sqrt{2} & \textbf{Quantum Mechanics} \quad \text{[Tsirelson(1987)]} \\ 4 & \textbf{No-signalling} \quad \text{[Popescu, Rohrlich(1994)]} \end{cases}$$

Entanglement witness

- **Entanglement witness** is a function that distinguishes separable/entangled states

  - **Concurrence** (for bi-qubits) $C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$ [Wootters (1998)]

$$0 \leq C[\rho] \leq 1$$

λ_i are eigenvalues, in descendent order, of

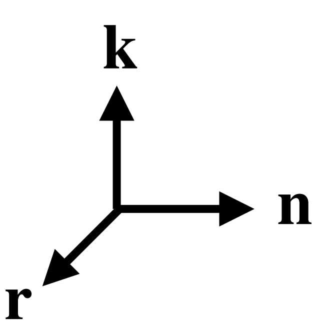
$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}} \text{ with } \tilde{\rho} = (\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$$

$C[\rho] > 0$ **iff ρ is entangled**

  - **Sufficient condition:** [Afik Nova (2021)]

$$C[\rho] > 0 \quad \Leftarrow \quad \begin{cases} \tilde{\Delta} \equiv |\,C_{kk} + C_{rr}\,| - C_{nn} > 1 \\ 3D \equiv \mathrm{Tr}[C] < -1 \end{cases}$$

[C_{xy} with $x, y \in \{\mathbf{r}, \mathbf{n}, \mathbf{k}\}$]



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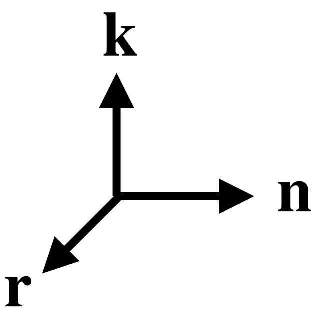


No-signalling $\supset$ **Quantum** $\supset$ **Local** $\supset$ **Separable**

$S_{\rm CHSH} = 2\sqrt{2}$ $\qquad$ $S_{\rm CHSH} = 2$ $\qquad$ $C[\rho] = 0_+$ $\qquad$ $C[\rho] = 0$

for all separable states

✤ Violation of Bell inequality, $S_{\mathrm{CHSH}} > 2$, has been observed at energies $\ll \mathrm{TeV}$

- **Entangled photon pairs** (from decays of Calcium atoms)

  Clauser, Horne, Shimony, Holt (1969), Freedman and Clauser (1972), A. Aspect et. al. (1981, 1982), Y. H. Shih, C. O. Alley (1988), L. K. Shalm et al. (2015) [5σ]

- **Entangled proton pairs** (from decays of ^{2}He)

  M. M. Lamehi-Rachti, W. Mitting (1972), H. Sakai (2006)

- $K^0\overline{K^0}$, $B^0\overline{B^0}$ **flavour oscillation** CPLEAR (1999), Belle (2004, 2007)

- $B^0 \to J/\psi + K^*(892)^0$ **spin correlation,** $S_{\mathrm{CGLMP}} > 2$, **[36σ]**

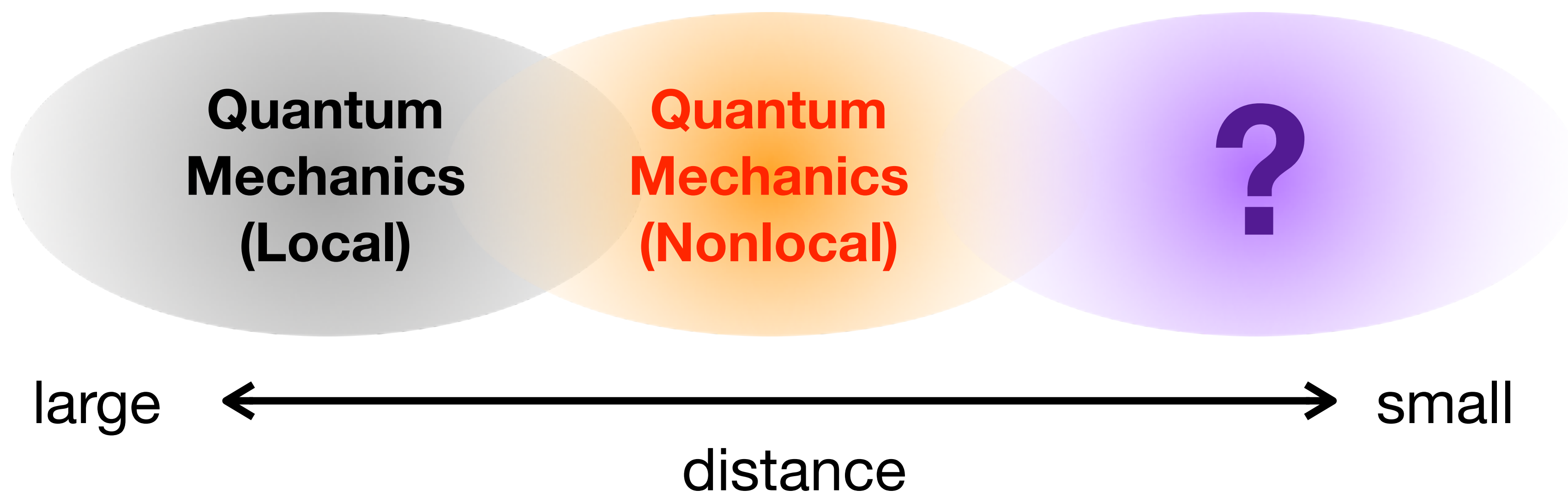
  Fabbrichesi, Floreanini, Gabrielli, Marzola (2023)

Normalised helicity amplitude for $B^0 \to J/\psi + K^*(892)^0$

$$
\begin{aligned}
|A_{||}|^2 &= 0.227 \pm 0.004\ (\text{stat.}) \pm 0.011\ (\text{syst.}),\\
|A_{\perp}|^2 &= 0.201 \pm 0.004\ (\text{stat.}) \pm 0.008\ (\text{syst.}),\\
\delta_{||}\ [\text{rad}] &= -2.94 \pm 0.02\ (\text{stat.}) \pm 0.03\ (\text{syst.}),\\
\delta_{\perp}\ [\text{rad}] &= 2.94 \pm 0.02\ (\text{stat.}) \pm 0.02\ (\text{syst.}).
\end{aligned}
$$

LHCb [1307.2782]

Testing QM at high energy colliders



Motivation

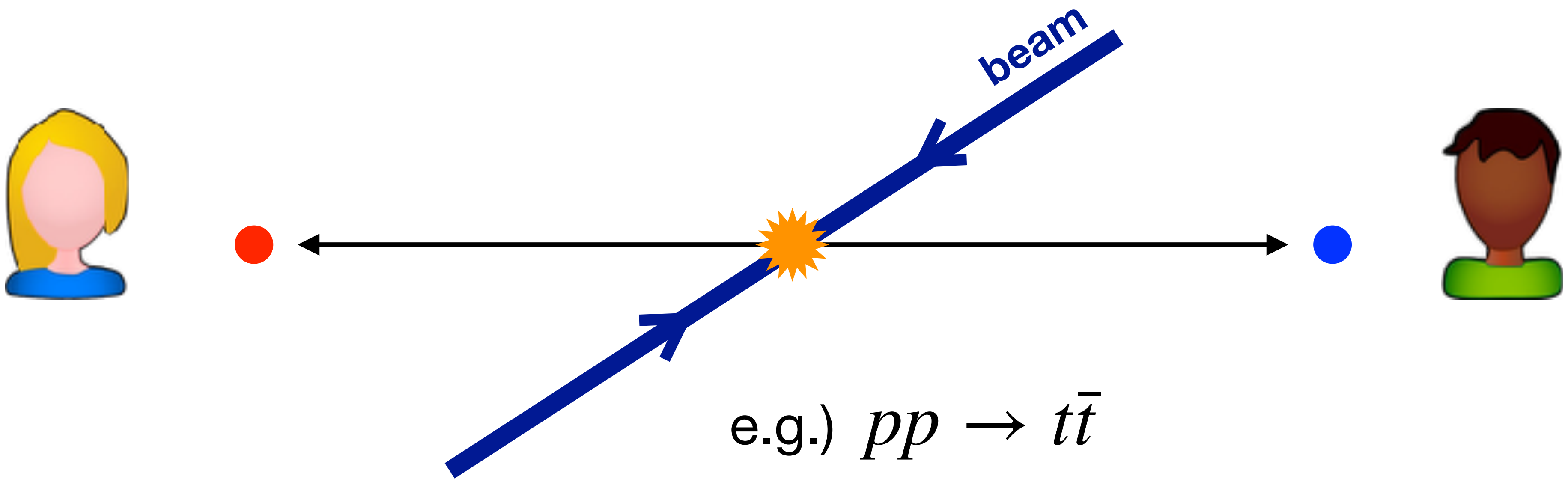
✤ Bell inequalities/Entanglement have not been tested at the TeV energy scale:

➡ LHC (and $\mathrm{FCC}_{ee/hh}$) provides the unique opportunity for this test

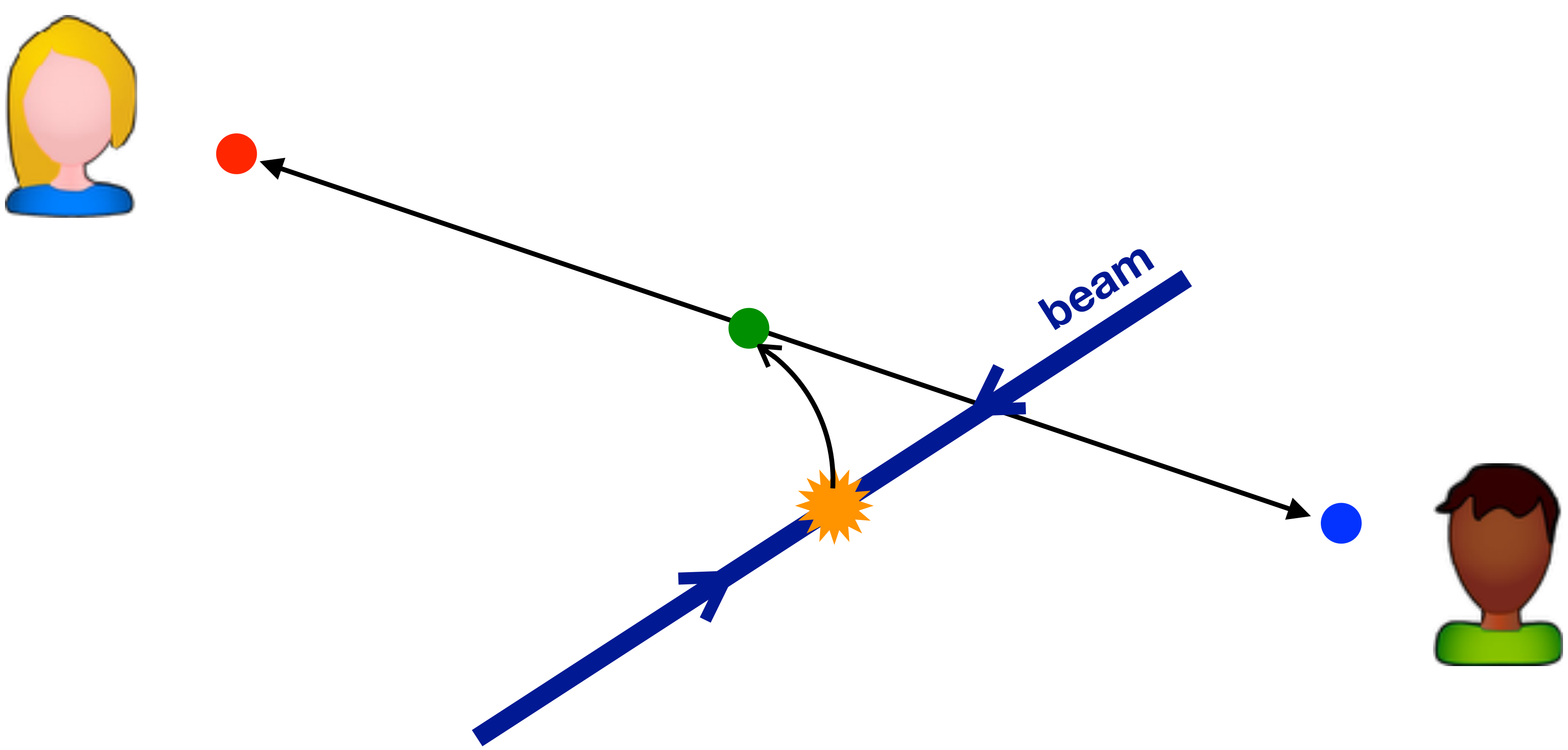
✤ Detection of Entanglement/Bell violation requires a detailed analysis of spin correlation:

➡ provides a very good test for the Standard Model (sensitive to BSM)

Entangled pairs at Colliders

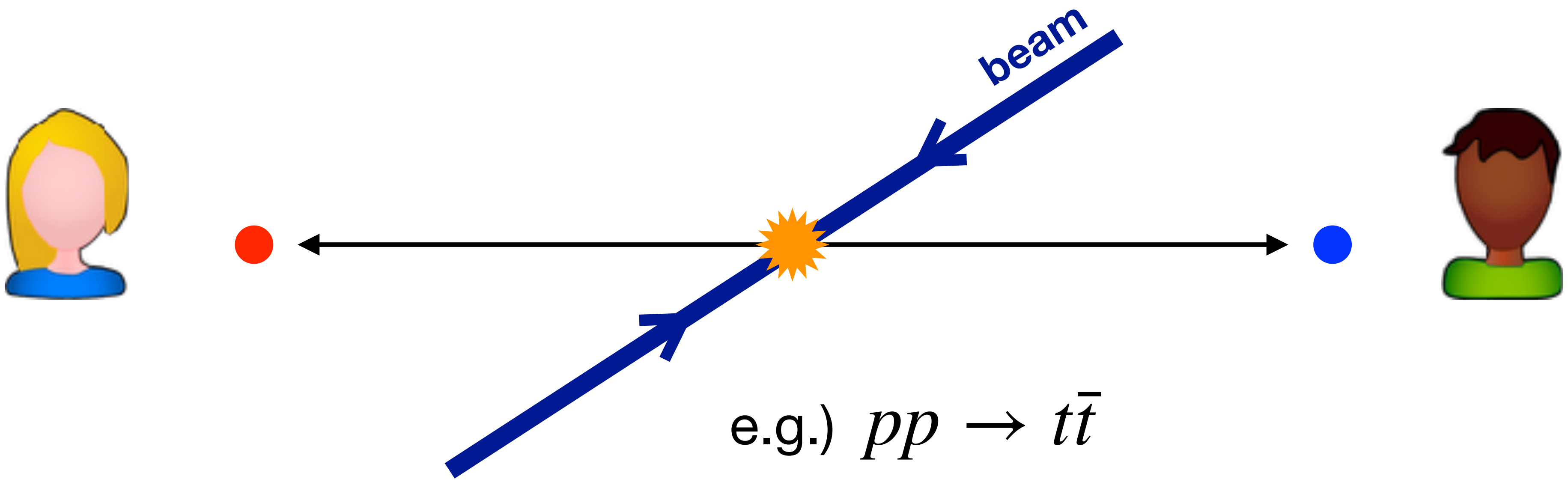


e.g.) $pp \to t\bar{t}$



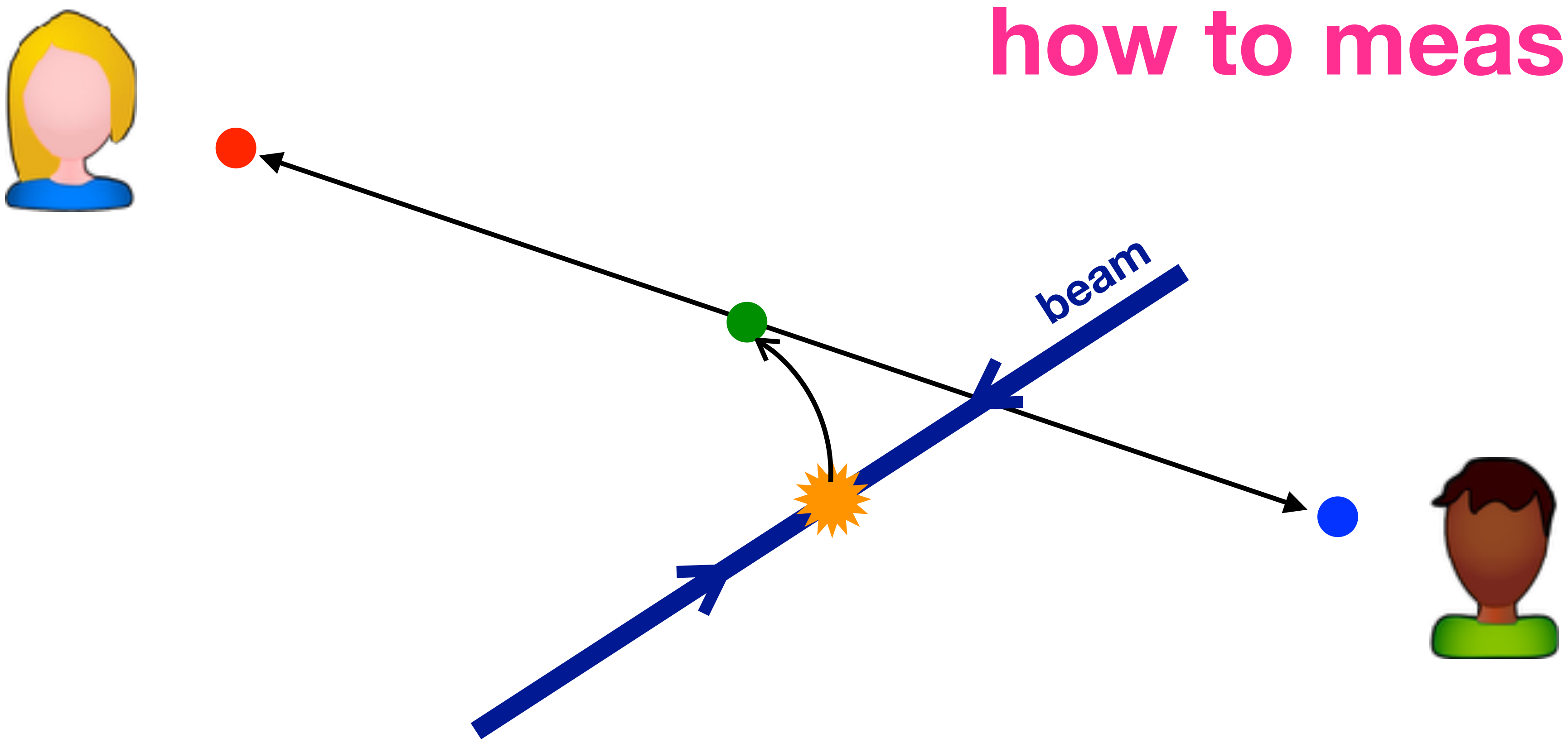
e.g.) $H \to \tau^+\tau^-, WW^*, ZZ^*$

Entangled pairs at Colliders



e.g.) $pp \to t\bar{t}$

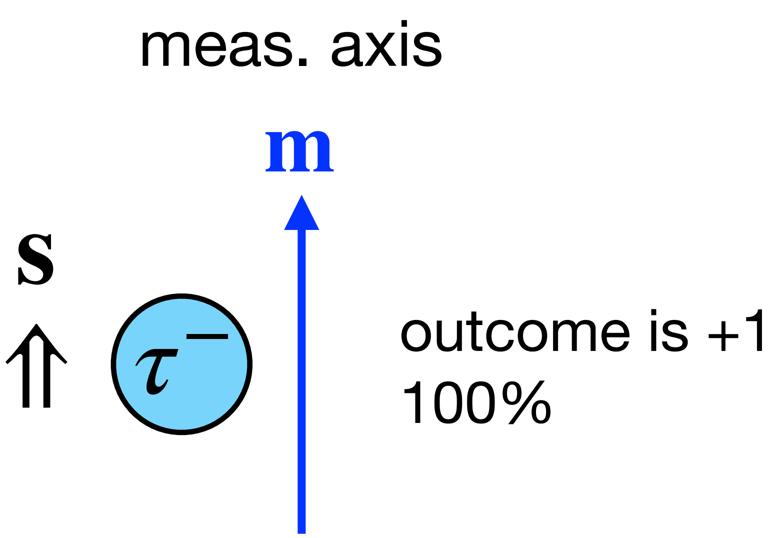
how to measure spin?

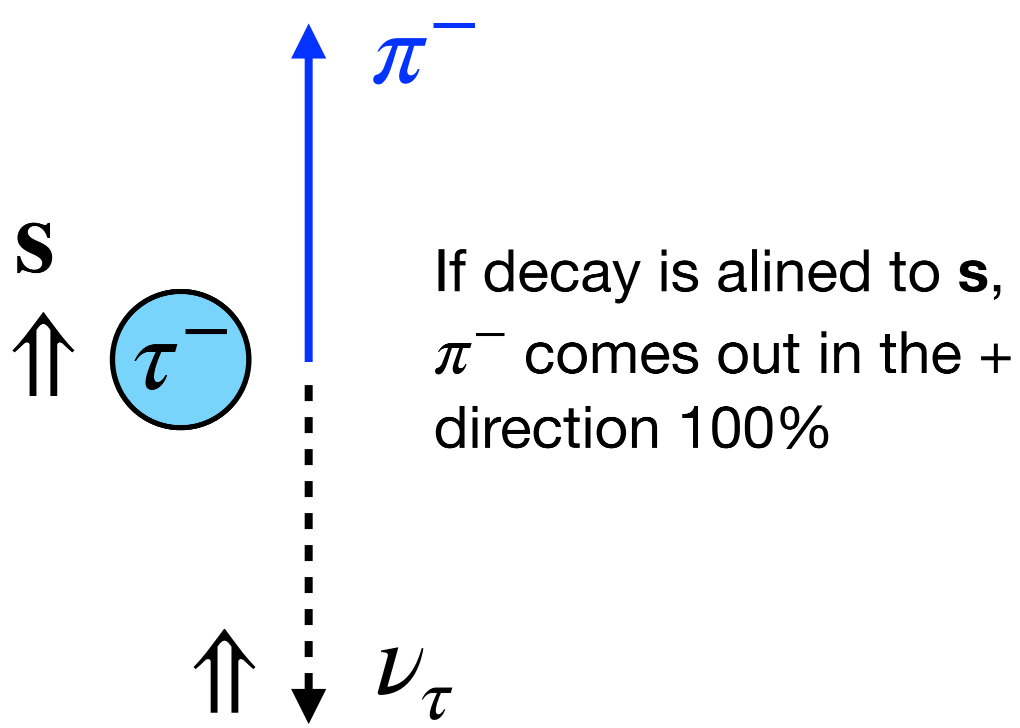


e.g.) $H \to \tau^+\tau^-, WW^*, ZZ^*$

Particles with weak decays are their own polarimeters

e.g.) For $\tau^- \to \pi^- + \nu_\tau$ (τ^- rest frame), the spin of τ^- is measured in the direction of π^- ($\vec{\pi}$) and the outcome is +1.





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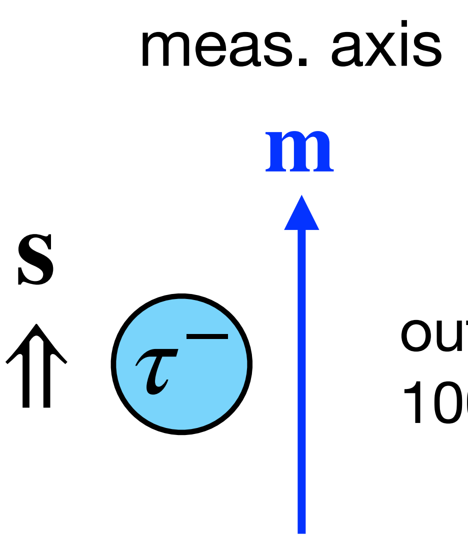
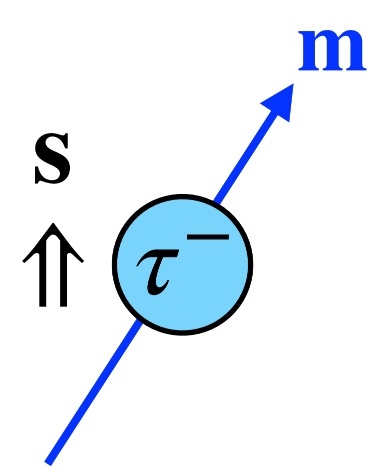
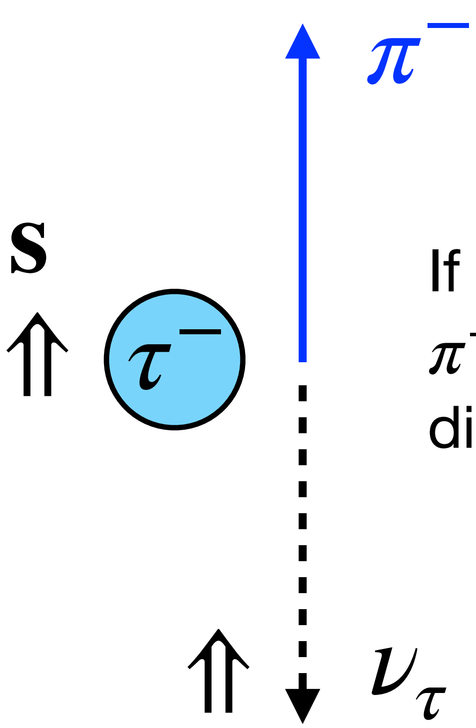


outcome is +1
100%

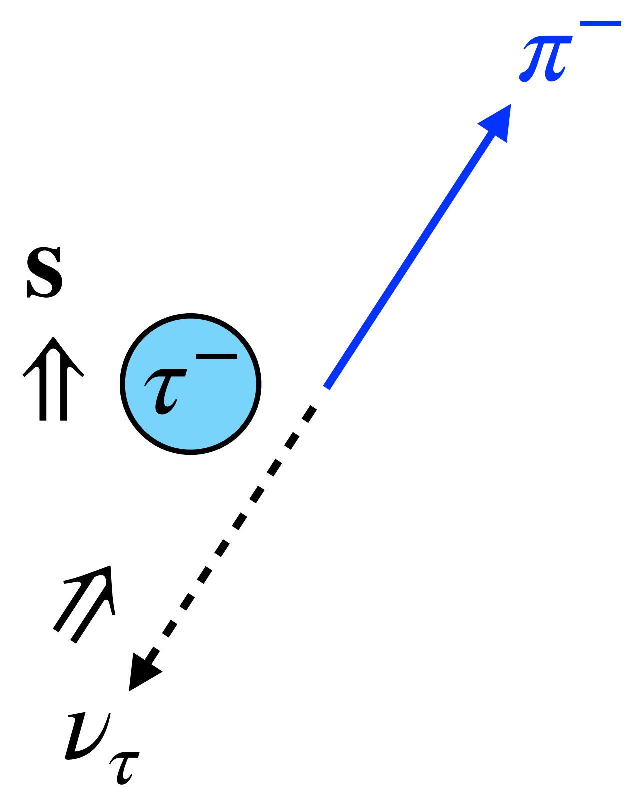


$$p(+\,|\,\mathbf{m}) = |\langle +_{\mathbf{m}} \,|\, +_{\mathbf{s}} \rangle|^2$$

$$= \frac{1 + (\mathbf{m}\cdot\mathbf{s})}{2}$$



If decay is alined to **s**,
π^- comes out in the +
direction 100%



$$\frac{d\Gamma}{d\Omega} = \frac{1 + (\vec{\pi}\cdot\mathbf{s})}{2}$$

Particles with weak decays are their own polarimeters

More generally, $\dfrac{d\Gamma}{d\Omega} = \dfrac{1 + \alpha_x \cdot (\vec{x} \cdot \mathbf{s})}{2}$ $\alpha_x \in [-1, +1]$: spin analyzing power

$\alpha_x = 1$ e.g. for $(x = \pi^-$ in $\tau^- \to \pi^- \nu)$, $(x = \ell^+, \bar{d}$ in $t \to b \ell^+ \nu,\ b \bar{d} u)$

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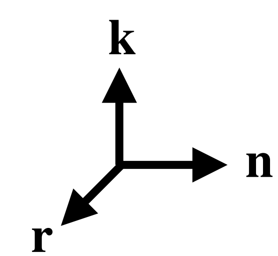
Spin correlation: $C_{\mathbf{n}_x,\mathbf{n}_y} = \dfrac{9}{\alpha_x \alpha_y} \langle (\vec{x} \cdot \mathbf{n}_x)(\vec{y} \cdot \mathbf{n}_y) \rangle$

$\mathbf{n}_x, \mathbf{n}_y$: spin measurement axes

$\vec{x}, \vec{y}$: direction of decay products

$S_{\mathrm{CHSH}} = C_{\mathbf{n}_1,\mathbf{e}_1} + C_{\mathbf{n}_1,\mathbf{e}_2} + C_{\mathbf{n}_2,\mathbf{e}_1} - C_{\mathbf{n}_2,\mathbf{e}_2} > 2$ Bell inequality violation

$\tilde{\Delta} = |C_{\mathbf{kk}} + C_{\mathbf{rr}}| - C_{\mathbf{nn}} > 0$ Entanglement detection



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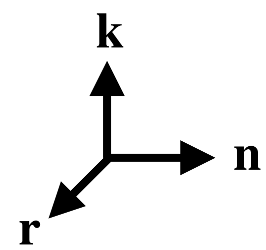
$S_{\mathrm{CHSH}} = C_{\mathbf{n}_1, \mathbf{e}_1} + C_{\mathbf{n}_1, \mathbf{e}_2} + C_{\mathbf{n}_2, \mathbf{e}_1} - C_{\mathbf{n}_2, \mathbf{e}_2} > 2$ Bell inequality violation

$\tilde{\Delta} = |\, C_{\mathbf{kk}} + C_{\mathbf{rr}} \,| - C_{\mathbf{nn}} > 0$ Entanglement detection



For spins to be measurable, one must focus on entangled pairs of weakly decaying particles

$$\tau,\ t,\ W^{\pm},\ Z^0$$

Particles with weak decays are their own polarimeters

More generally, $\frac{d\Gamma}{d\Omega} = \frac{1 + \alpha_x \cdot (\vec{x} \cdot \mathbf{s})}{2}$ $\alpha_x \in [-1, +1]$: spin analyzing power

$\alpha_x = 1$ e.g. for $(x = \pi^- \text{ in } \tau^- \to \pi^- \nu)$, $(x = \ell^+, \bar{d} \text{ in } t \to b\ell^+\nu,\, b\bar{d}u)$

Spin correlation: $C_{\mathbf{n}_x,\mathbf{n}_y} = \frac{9}{\alpha_x \alpha_y} \langle (\vec{x} \cdot \mathbf{n}_x)(\vec{y} \cdot \mathbf{n}_y) \rangle$

$\mathbf{n}_x, \mathbf{n}_y$: spin measurement axes

$\vec{x}, \vec{y}$: direction of decay products

$S_{\mathrm{CHSH}} = C_{\mathbf{n}_1,\mathbf{e}_1} + C_{\mathbf{n}_1,\mathbf{e}_2} + C_{\mathbf{n}_2,\mathbf{e}_1} - C_{\mathbf{n}_2,\mathbf{e}_2} > 2$ Bell inequality violation

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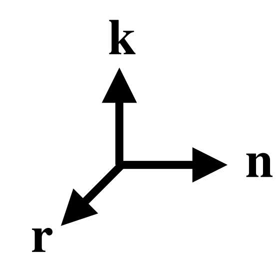


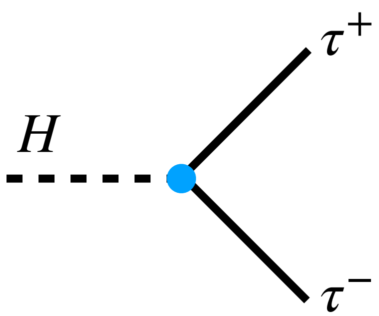
For spins to be measurable, one must focus on entangled pairs of weakly decaying particles

$$\tau,\ t,\ W^{\pm},\ Z^0$$

$H \to \tau^+\tau^-$

SM: $(\kappa, \delta) = (1,0)$

$$\mathscr{L}_{\rm int} = -\frac{m_\tau}{v_{\rm SM}}\kappa H\,\bar{\psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\,\psi_\tau$$



$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}}|\mathcal{M}^{m\bar{m}}|^2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle}$$

[Fabbrichesi, Floreanini, Gabrielli (2023)]

$\delta = 0$
(CP even)

$$|\Psi^{(s=1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ \boxed{|+-\rangle + |-+\rangle} \\ |--\rangle \end{pmatrix}$$

$\delta = \pi/2$ (CP odd)

$$|\Psi^{(0,0)}\rangle \propto \boxed{|+-\rangle - |-+\rangle}$$

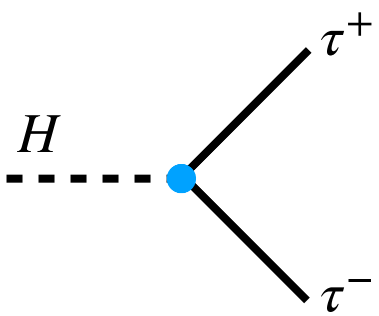
Parity: $P = (\eta_f\eta_{\bar{f}})\cdot(-1)^l$ with $\eta_f\eta_{\bar{f}} = -1$:

$$J^P = \begin{cases} 0^+ \Longrightarrow & l = s = 1 \\ 0^- \Longrightarrow & l = s = 0 \end{cases}$$

$H \to \tau^+\tau^-$

SM: $(\kappa, \delta) = (1,0)$

$$\mathscr{L}_{\rm int} = -\frac{m_\tau}{v_{\rm SM}}\kappa H \bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\,\psi_\tau$$



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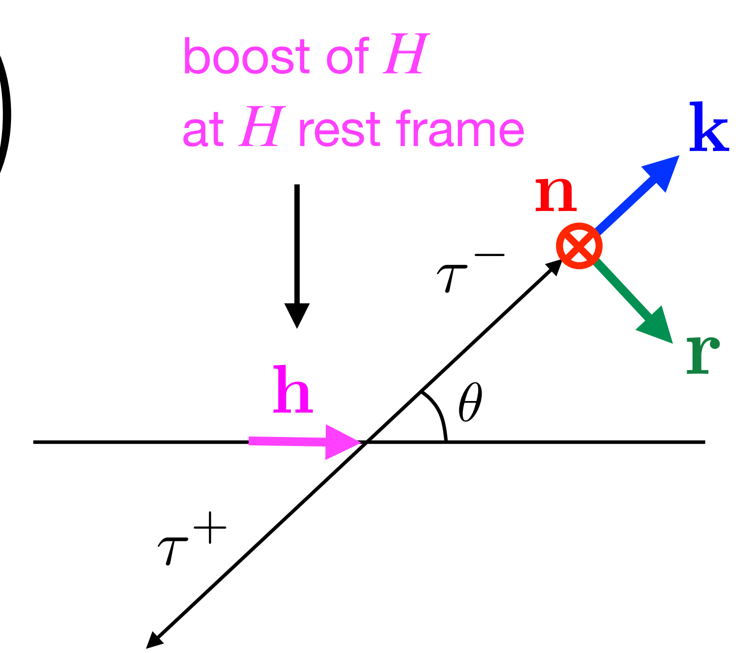
$$\boxed{|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle}$$

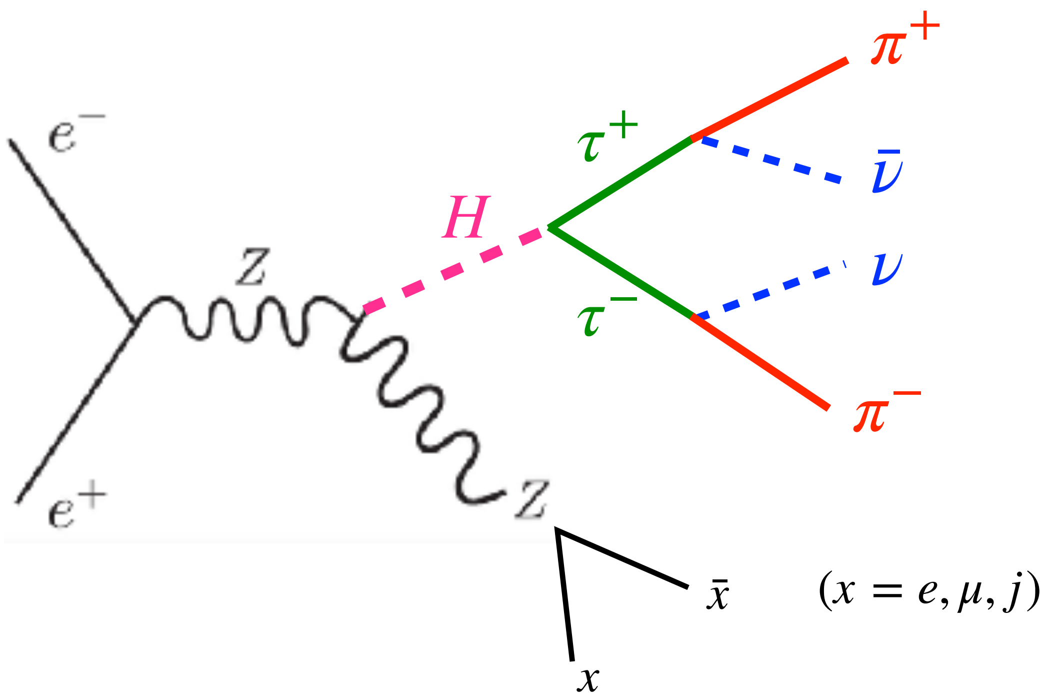
[Fabbrichesi, Floreanini, Gabrielli (2023)]

Four measurement axes in $S_{\rm CHSH}$: $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{e}_1, \mathbf{e}_2) = \left(\mathbf{r}, \mathbf{n}, \frac{\mathbf{n}+\mathbf{r}}{\sqrt{2}}, \frac{\mathbf{n}-\mathbf{r}}{\sqrt{2}}\right)$



$$\begin{cases} S_{\rm CHSH} = 2\sqrt{2} & \text{(saturating QM bound)} \\ \mathscr{C}[\rho] = 1 & \text{(maximally entangled)} \end{cases}$$

$H \to \tau^+\tau^-$ @ lepton colliders

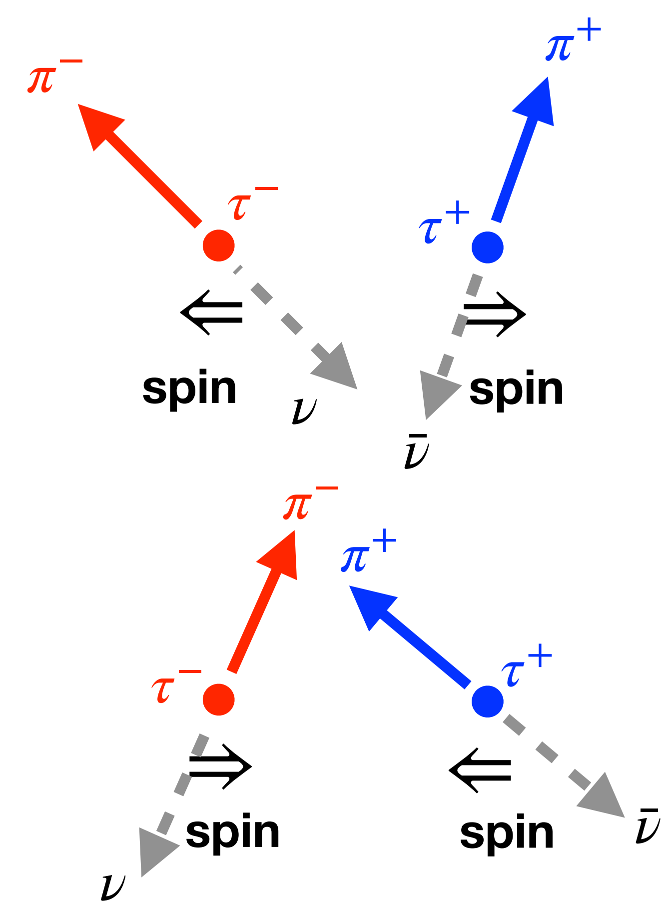


[Altakach, Lamba, Maltoni, Mawatari, Sakurai (2023)]

The main challenge:

$$C_{\mathbf{n}_+,\mathbf{n}_-} = -9\langle \underbrace{(\vec{\pi}^+ \cdot \mathbf{n}_+)}(\underbrace{\vec{\pi}^- \cdot \mathbf{n}_-})\rangle$$

measured in τ^+ and τ^- rest frames, respectively



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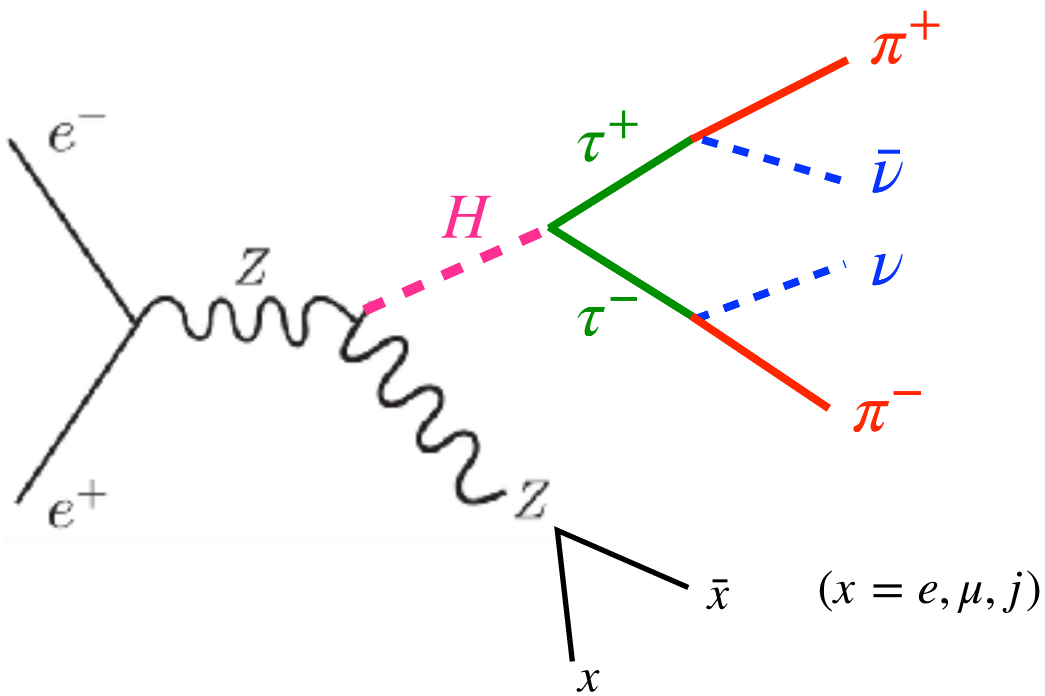

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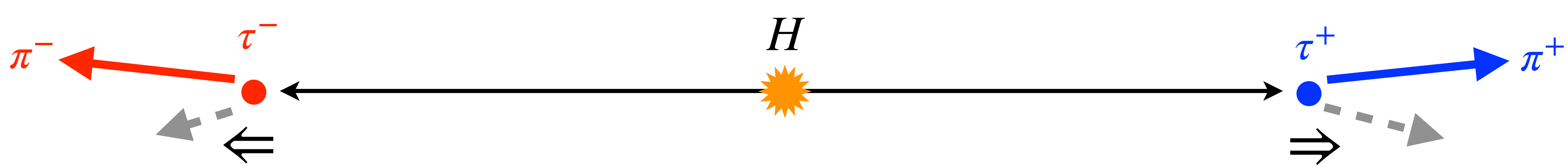
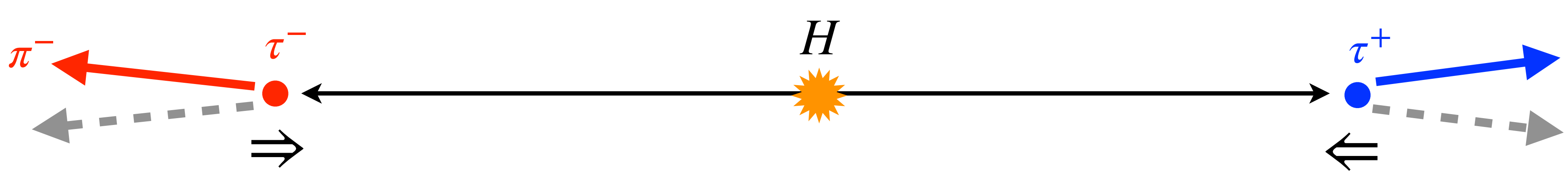
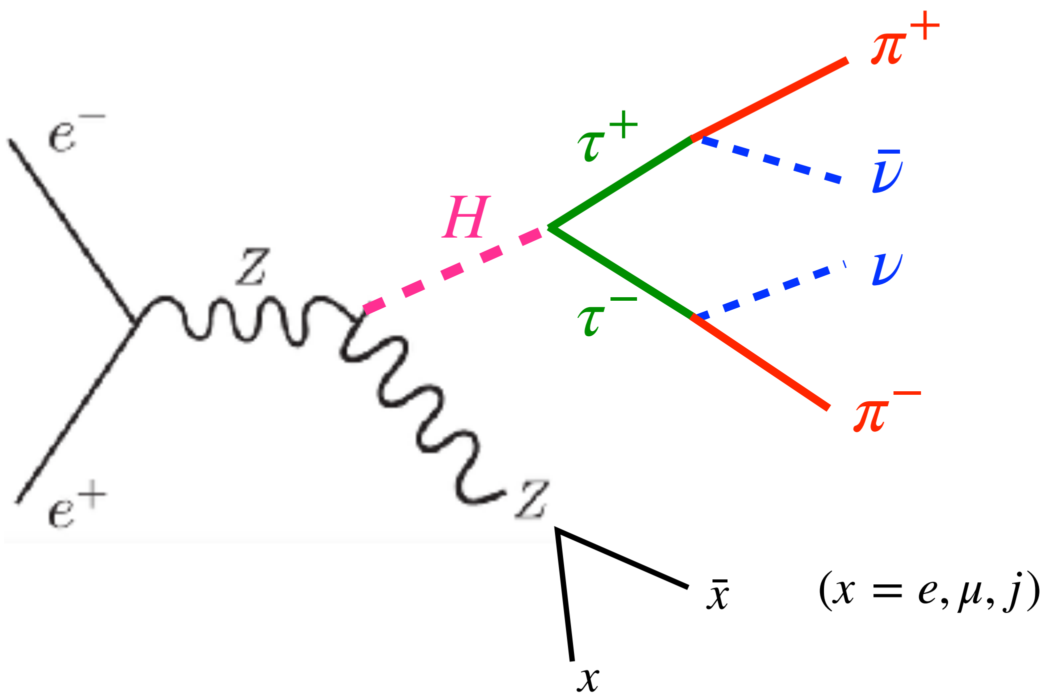


$$\boxed{m_\tau \ll m_H}$$



$H \to \tau^+\tau^-$ @ lepton colliders



[Altakach, Lamba, Maltoni, Mawatari, Sakurai (2023)]

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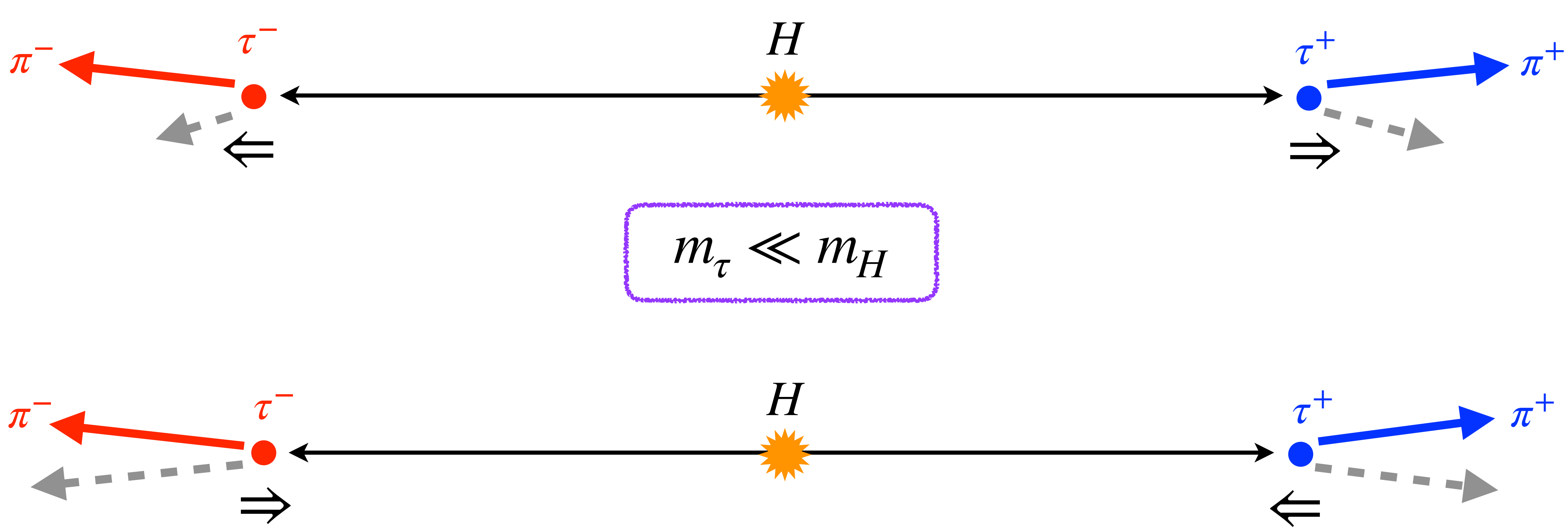
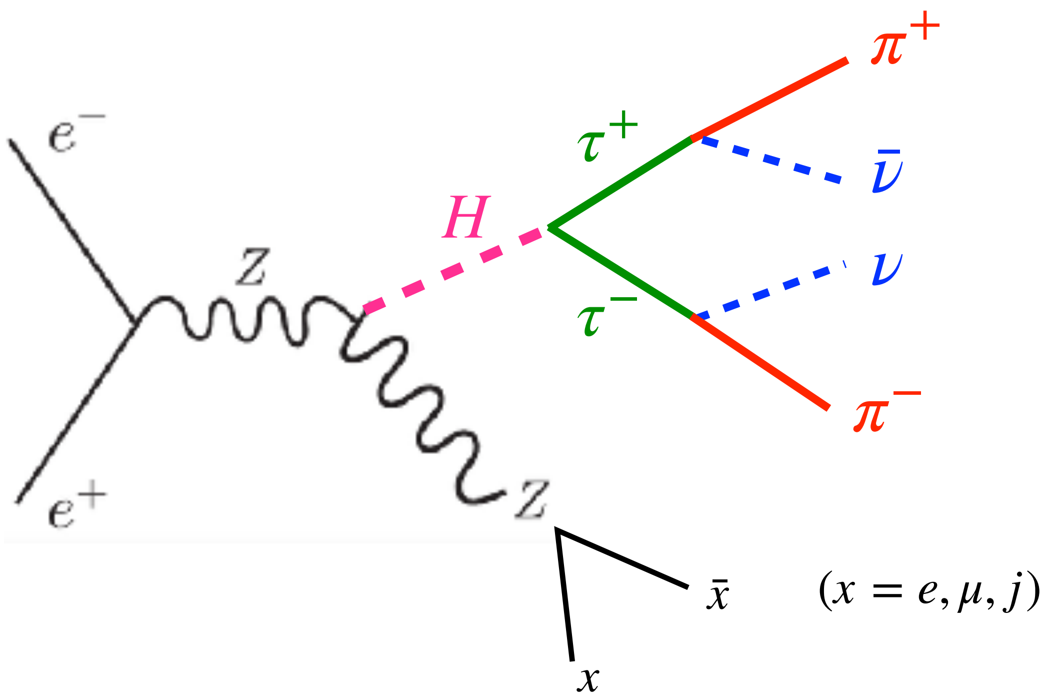
$\tau^\pm$ rest frames must be reconstructed very precisely



$H \to \tau^+\tau^-$ @ lepton colliders



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The main challenge:

$$C_{\mathbf{n}_+,\mathbf{n}_-} = -9\langle \underbrace{(\vec{\pi}^+ \cdot \mathbf{n}_+)} \underbrace{(\vec{\pi}^- \cdot \mathbf{n}_-)} \rangle$$

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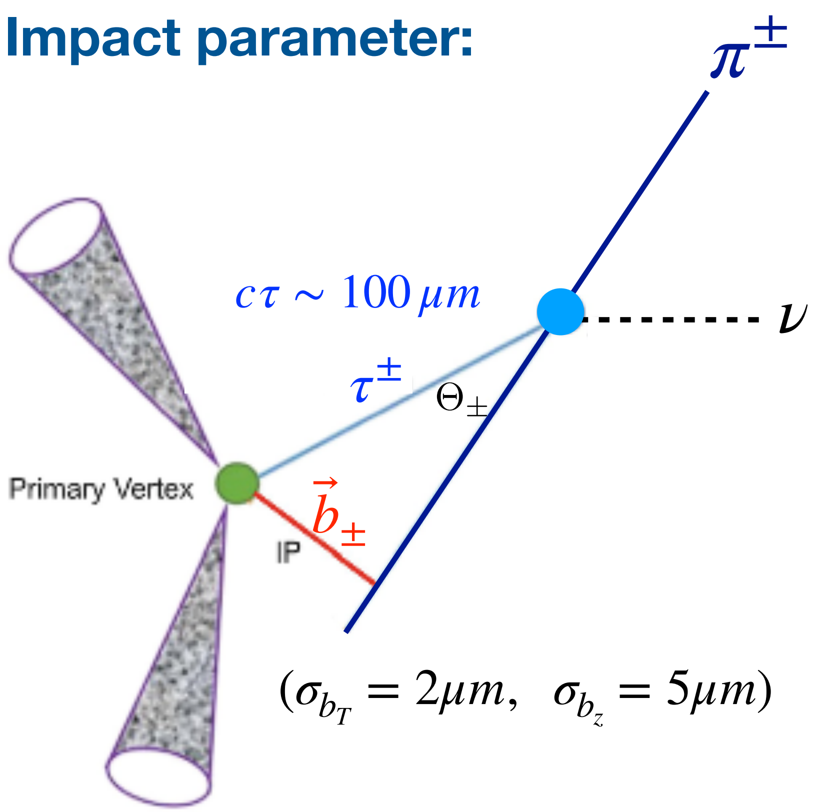
➡ $\tau^\pm$ rest frames must be reconstructed very precisely

Impact parameter:



We overcome this by incorporating the **impact parameter** info in the event reconstruction with the log-likelihood

➡ Talk by Priyanka Lamba (tomorrow)

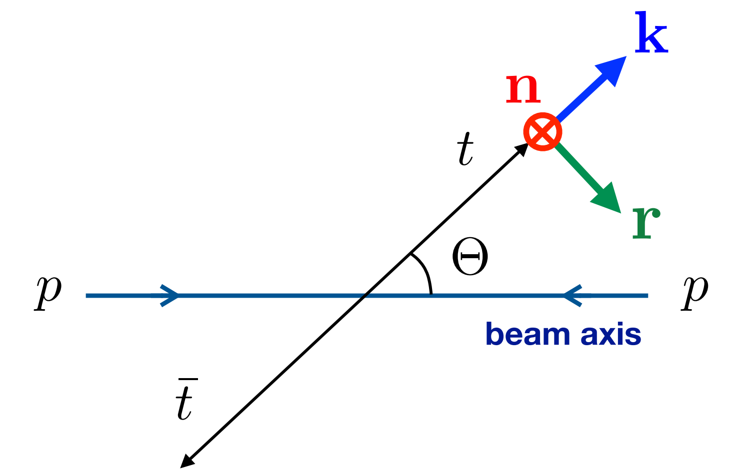
MC-sim for FCCee: $L = 5\,\mathrm{ab}^{-1}$, $E_{\mathrm{CM}} = 240\,\mathrm{GeV}$

$C[\rho] = 0.871 \pm 0.084$ $\Rightarrow$ Entanglement $\gg 5\sigma$

$S_{\mathrm{CHSH}}/2 = 1.276 \pm 0.094$ $\Rightarrow$ Bell nonlocality $\sim 3\sigma$

[Afik, Nova (2021, 2022)]

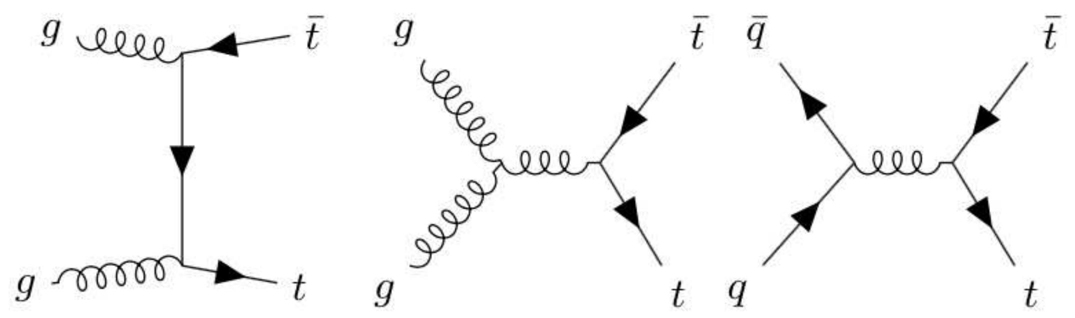
$pp \to t\bar{t}$ @ LHC



- At the rest frame of $t\bar{t}$, the kinematics is determined by:

  Θ : the angle between t and the beam line ($0 \le \Theta \le \pi/2$)

  $M_{t\bar{t}}$: the inv. mass of $t\bar{t}$



- gg and $q\bar{q}$ initial states contribute stochastically
  $\Rightarrow$ the $t\bar{t}$ spin state is necessarily **mixed**

$$\rho(M_{t\bar{t}}, \Theta) = \sum_{I=gg,q\bar{q}} w_I(M_{t\bar{t}}, \Theta) \cdot \rho^I(M_{t\bar{t}}, \Theta)$$

$$w_I(M_{t\bar{t}}, \Theta) = \frac{L_I(M_{t\bar{t}})\tilde{A}^I(M_{t\bar{t}}, \Theta)}{\sum_J L_J(M_{t\bar{t}})\tilde{A}^J(M_{t\bar{t}}, \Theta)}$$

$\tilde{A}^I(M_{t\bar{t}}, \Theta)$: partonic differential x-section

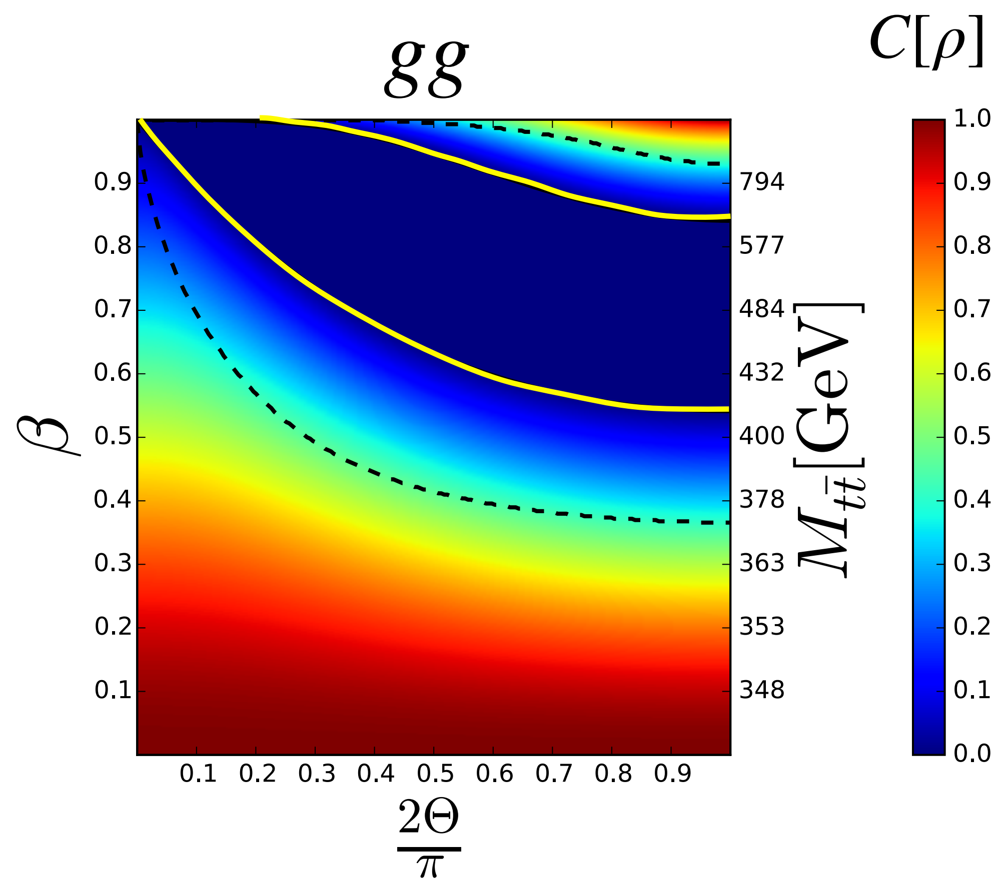
$L_I(M_{t\bar{t}})$: luminosity function

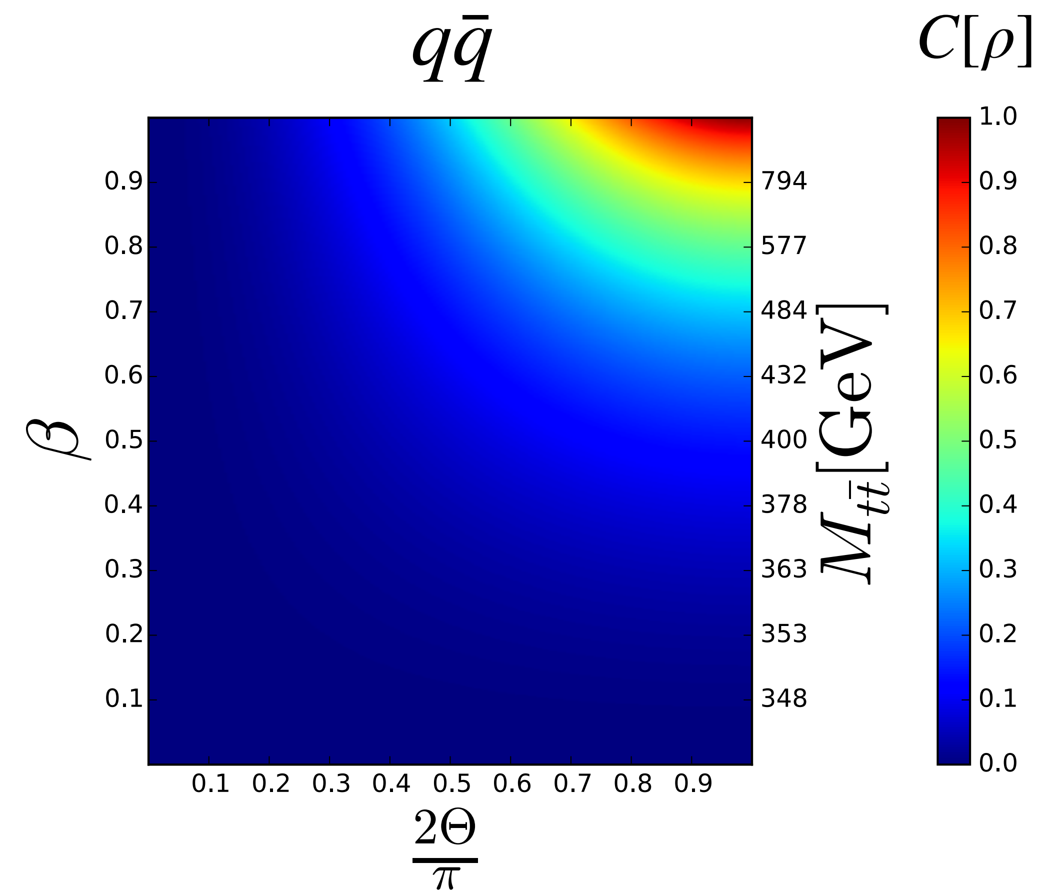
Yellow solid: $C[\rho] \geq 0$

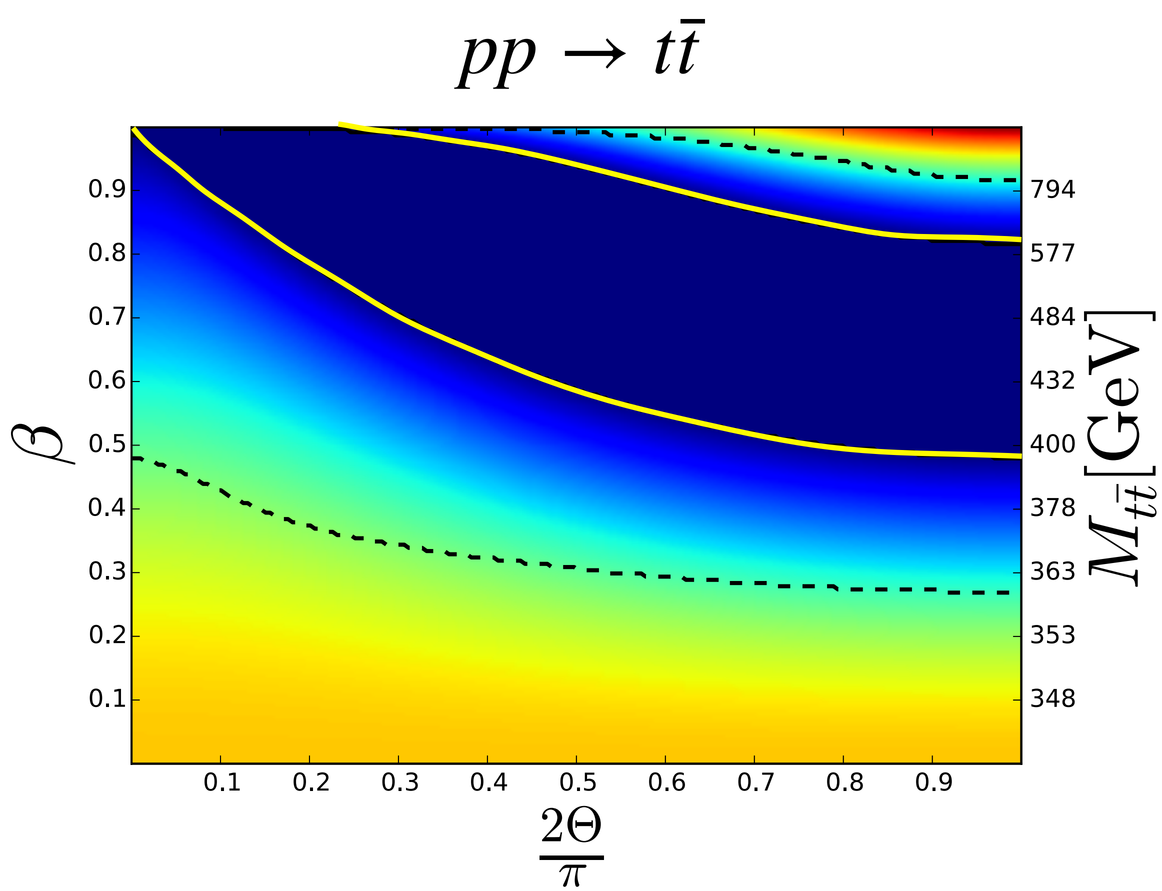
Black dashed: $S_{\mathrm{CHSH}} \geq 2$

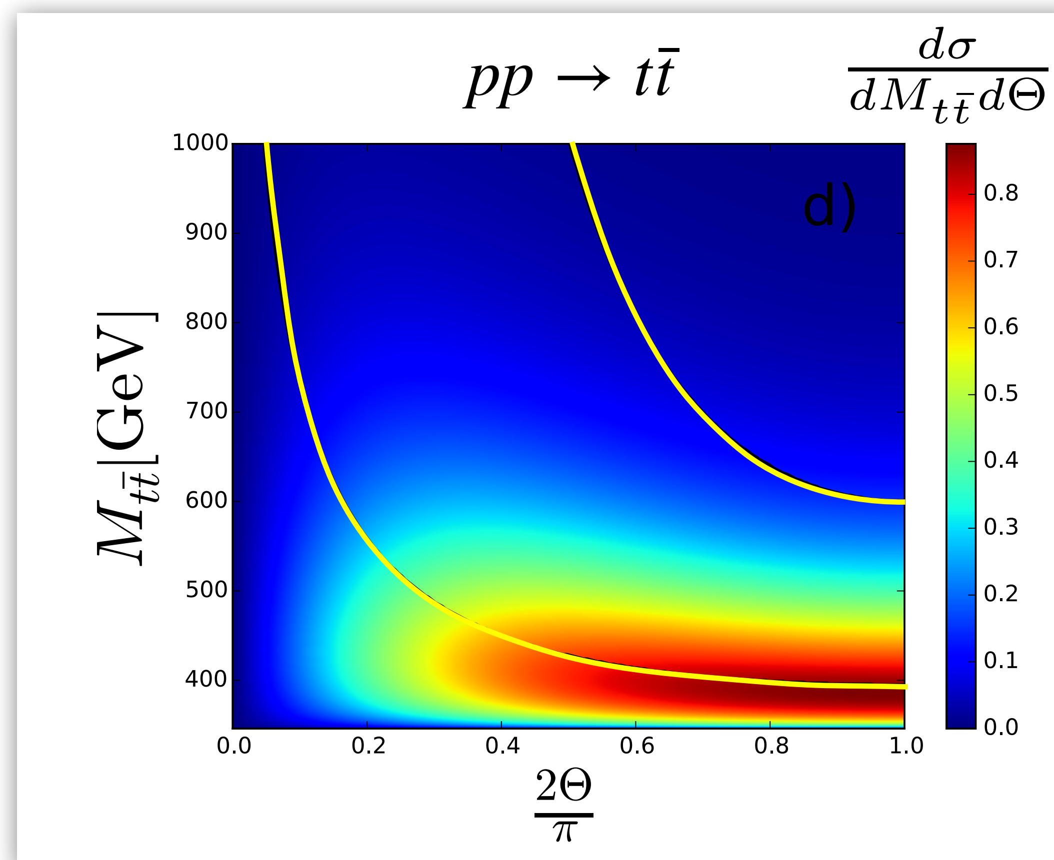
Concurrence

[Afik, Nova (2021, 2022)]

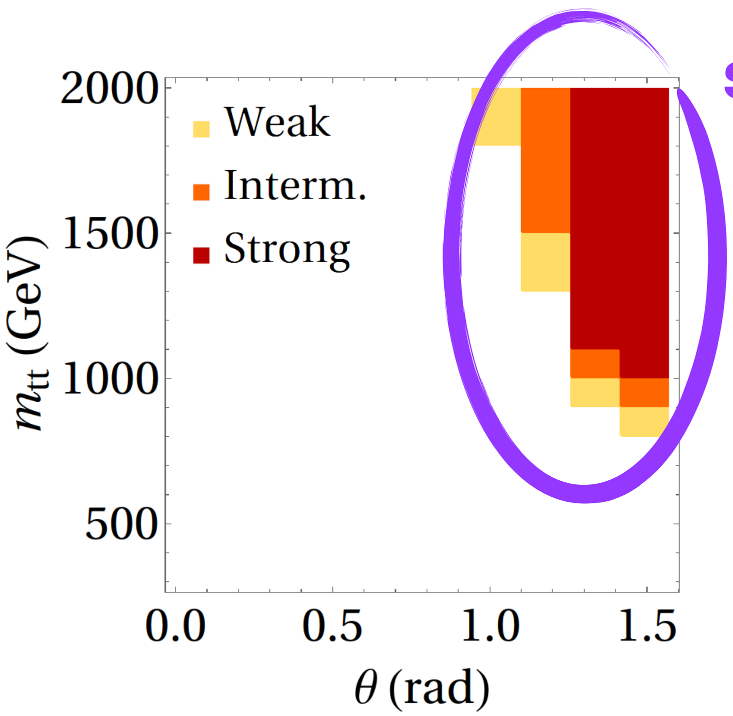








MC-sim: di-leptonic decay, $pp \to t\bar{t} \to (b\ell^+\nu)(\bar{b}\ell^-\bar{\nu})$



selecting events here

HL-LHC $(L = 3\,\mathrm{ab}^{-1})$ [Severi, Boschi, Maltoni, Sioli (2022)]

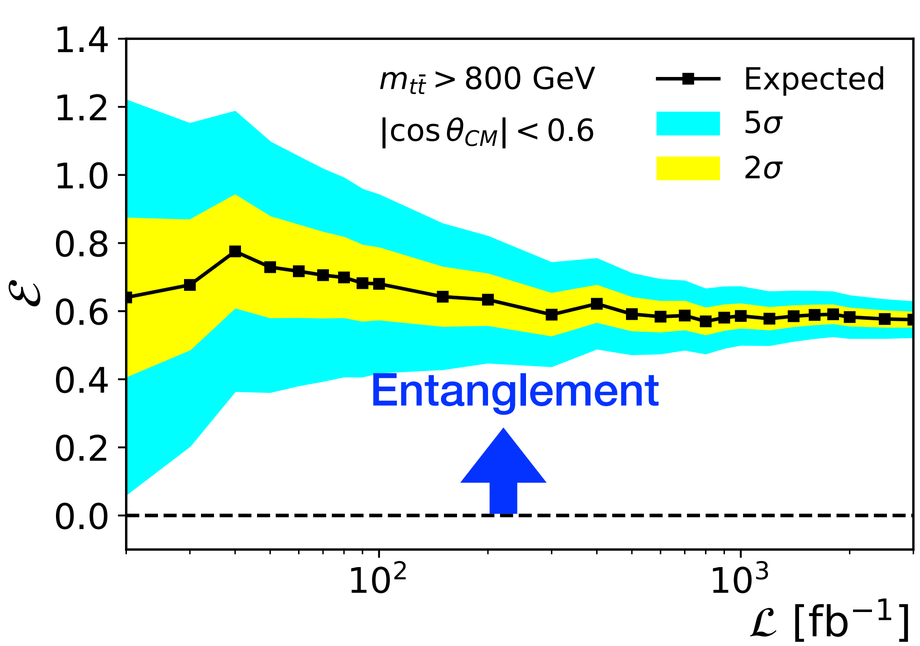
$$|C_{kk} + C_{rr}| - C_{nn} = 1.36 \pm 0.07 > 1$$ $\Rightarrow$ Entanglement $\gg 5\sigma$

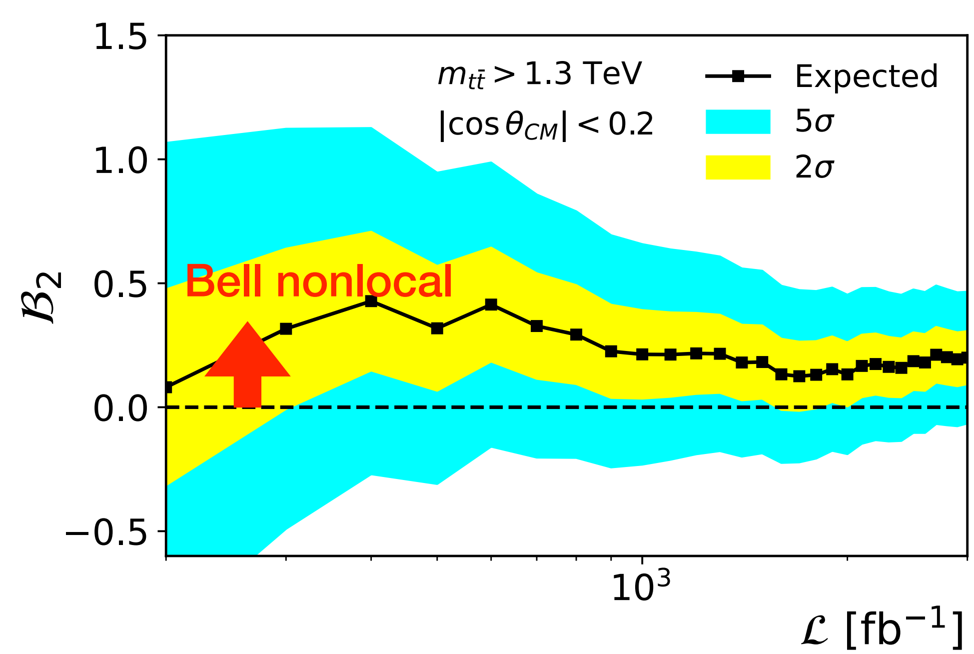
$$\sqrt{2}S_{\mathrm{CHSH}}/2 = 2.20 \pm 0.1 > 2$$ $\Rightarrow$ Bell nonlocality $\sim 1.8\sigma$

MC-sim: semi-leptonic decay, $pp \to t\bar{t} \to (b\ell\nu)(\underbrace{bjj}_{})$

[Dong, Goncalves, Kong, Navarro (2023)]

boosted top-tagging

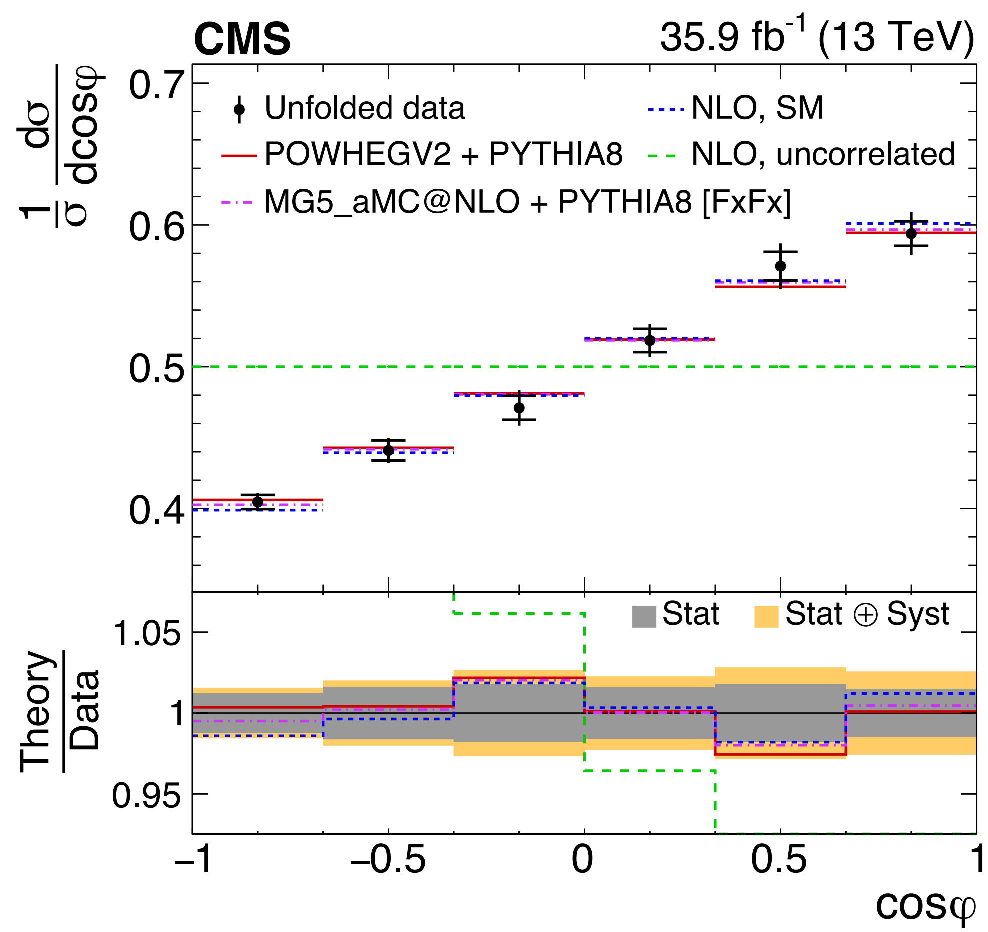




Entanglement in CMS

Recently, D has been measure by CMS in di-leptonic $pp \to t\bar{t}$

[Phys. Rev. D 100, 072002]



$$D \equiv \frac{\mathrm{Tr}[C]}{3} < -\frac{1}{3}$$ entanglement

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1 - D\cos\varphi}{2}$$

CMS result:

$$D = -0.237 \pm 0.011 > -\frac{1}{3}$$ entanglement is not detected

To see the entanglement, selecting certain kinematical regions is crucial.

A dedicated analysis is needed.

$H \to WW^*, ZZ^*$

[Barr (2022)]
[Aguilar-Saavedra ,Bernal, Casas, Moreno (2022)]
[Aguilar-Saavedra (2023)]
[Fabbrichesi, Floreanini, Gabrielli, Marzola (2023)]

- Conceptually less clear since one particle is off-shell.

  $\Rightarrow$ virtual particle with mass shifted: $m_{V*} = f \cdot m_V \quad (0 < f < 1)$

- two qutrits (rather than qubits)

- the final state is pure:

$$\left.\begin{aligned} &|\Psi_{VV*}\rangle \simeq |+-\rangle - \beta|00\rangle + |-+\rangle \\ &\beta = 1 + \frac{m_H^2 - (1+f)^2 m_V^2}{2 f m_V^2} \sim 1 \end{aligned}\right\} \Rightarrow \text{(almost) maximally entangled}$$

CGLMP Qutrit inequality

CGLMP function

[Collins Gisin Linden Massar Popescu (2002)]

$$I_3 \equiv P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$-P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$$

*) $P(A_i = B_j + k)$ is the probability that A_i and B_j are differ by k mod 3

$$I_3 \leq \begin{cases} 2 & \text{Local theories} \\ 1 + \sqrt{11/3} \simeq 2.9149 & \text{Quantum Mechanics} \end{cases}$$

Quantum state tomography

- It is convenient to reconstruct the density matrix from the kinematics, then analysis entanglement and nonlocality

- density matrix is 9 x 9 Hermitian matrix with unit trace. It can be expanded by two sets of Gell-Mann matrices and $8+8+64=80$ real parameters (9^2-1)

$$\rho = \frac{1}{9}(\mathbf{1}\otimes\mathbf{1}) + \frac{1}{3}\sum_{i=1}^{8} a_i(\lambda_i\otimes\mathbf{1}) + \frac{1}{3}\sum_{j=1}^{8} b_j(\mathbf{1}\otimes\lambda_j) + \sum_{i,j=1}^{8} c_{ij}(\lambda_i\otimes\lambda_j)$$

- real parameters a_i, b_j, c_{ij} can be reconstructed from the directions of two charged leptons, $\mathbf{n}_1$ and $\mathbf{n}_2$, using the eight **Wigner P functions**, Φ_i^P

$$a_i = \frac{1}{2}\left\langle \Phi_i^P \mathbf{n_1} \right\rangle_{\rm av} \qquad b_i = \frac{1}{2}\left\langle \Phi_i^P \mathbf{n_2} \right\rangle_{\rm av} \qquad c_{ij} = \frac{1}{4}\left\langle (\Phi_i^P \mathbf{n_1})(\Phi_j^P \mathbf{n_2}) \right\rangle_{\rm av}$$

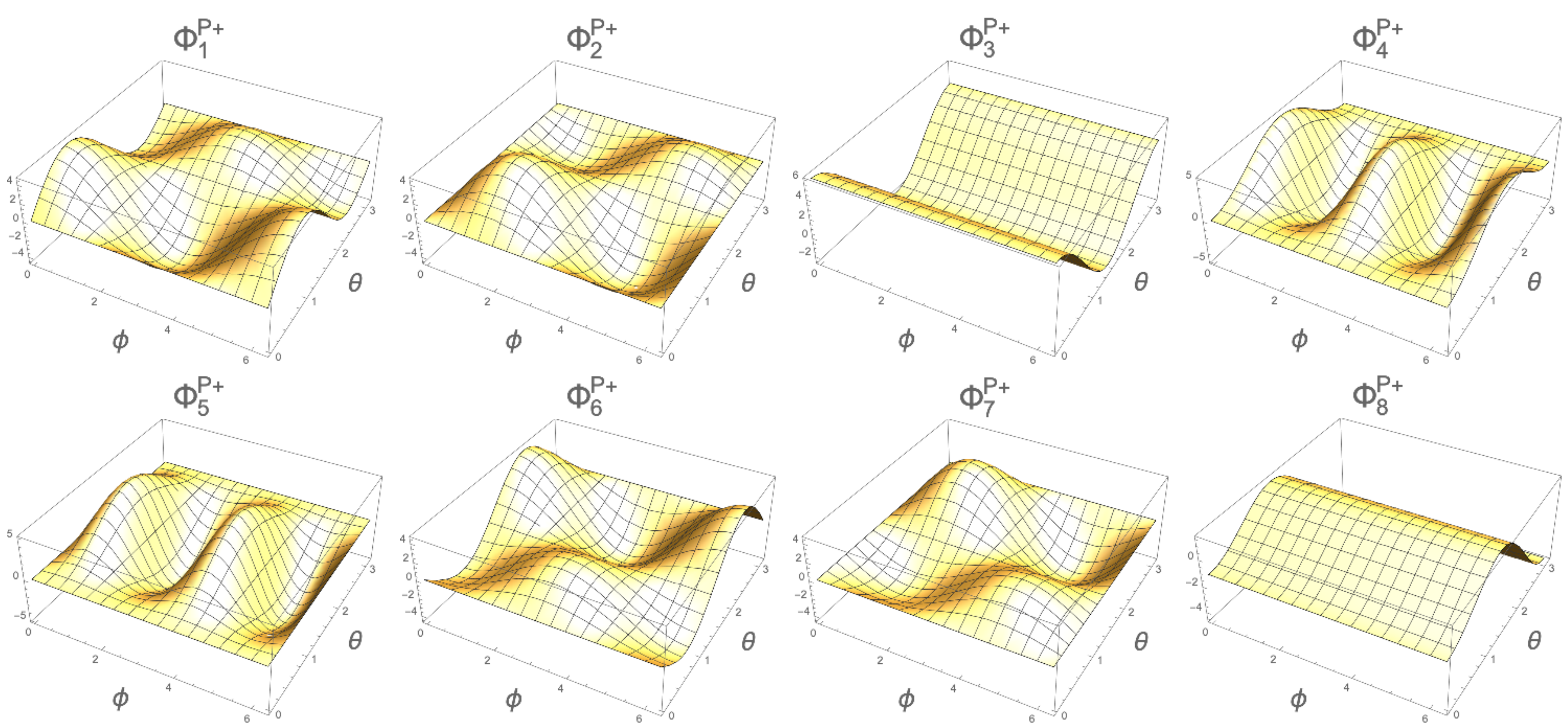
[Ashby-Pickering, Barr, Wierzchucka (2022)]

Quantum state tomography

- Wigner functions for $W^\pm \to \ell^\pm \nu$

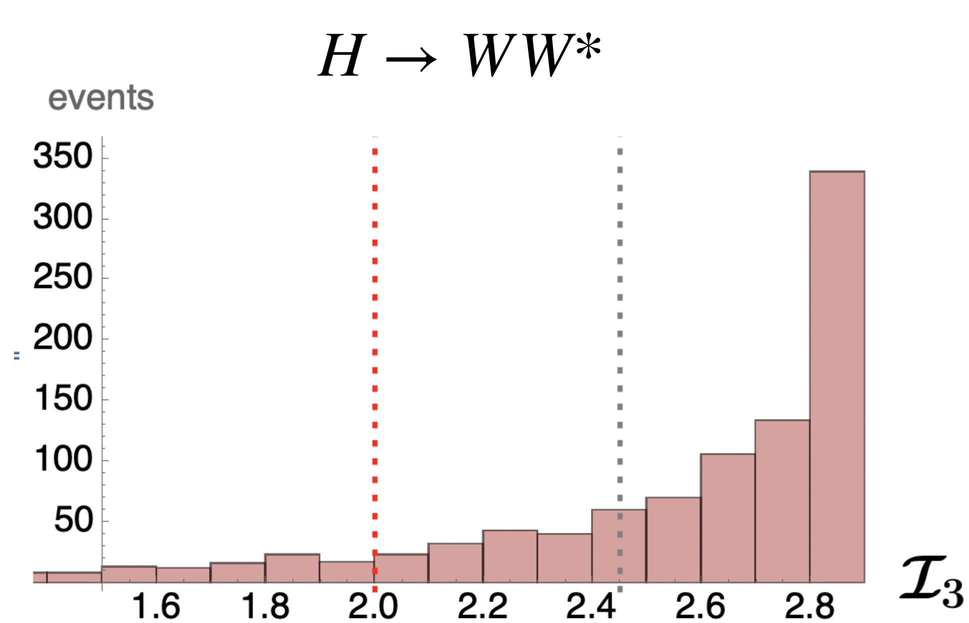
[Ashby-Pickering, Barr, Wierzchucka (2022)]

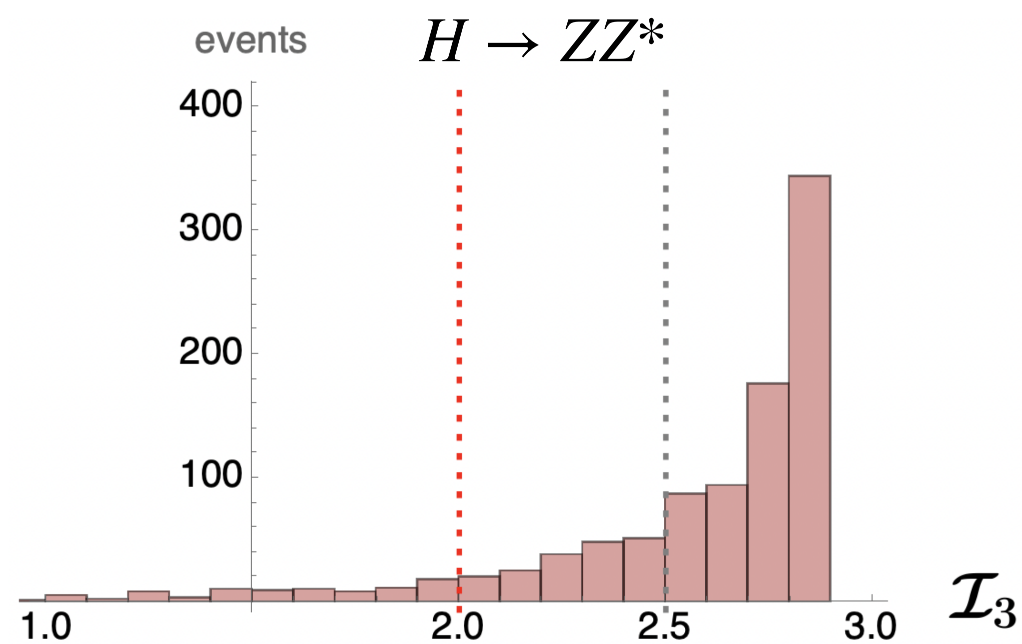
$$\Phi_1^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi$$
$$\Phi_2^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi$$
$$\Phi_3^{P\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5)$$
$$\Phi_4^{P\pm} = 5\sin^2\theta\cos 2\phi$$
$$\Phi_5^{P\pm} = 5\sin^2\theta\sin 2\phi$$
$$\Phi_6^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\cos\phi$$
$$\Phi_7^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\sin\phi$$
$$\Phi_8^{P\pm} = \frac{1}{4\sqrt{3}}(\pm 12\cos\theta - 15\cos 2\theta - 5)$$

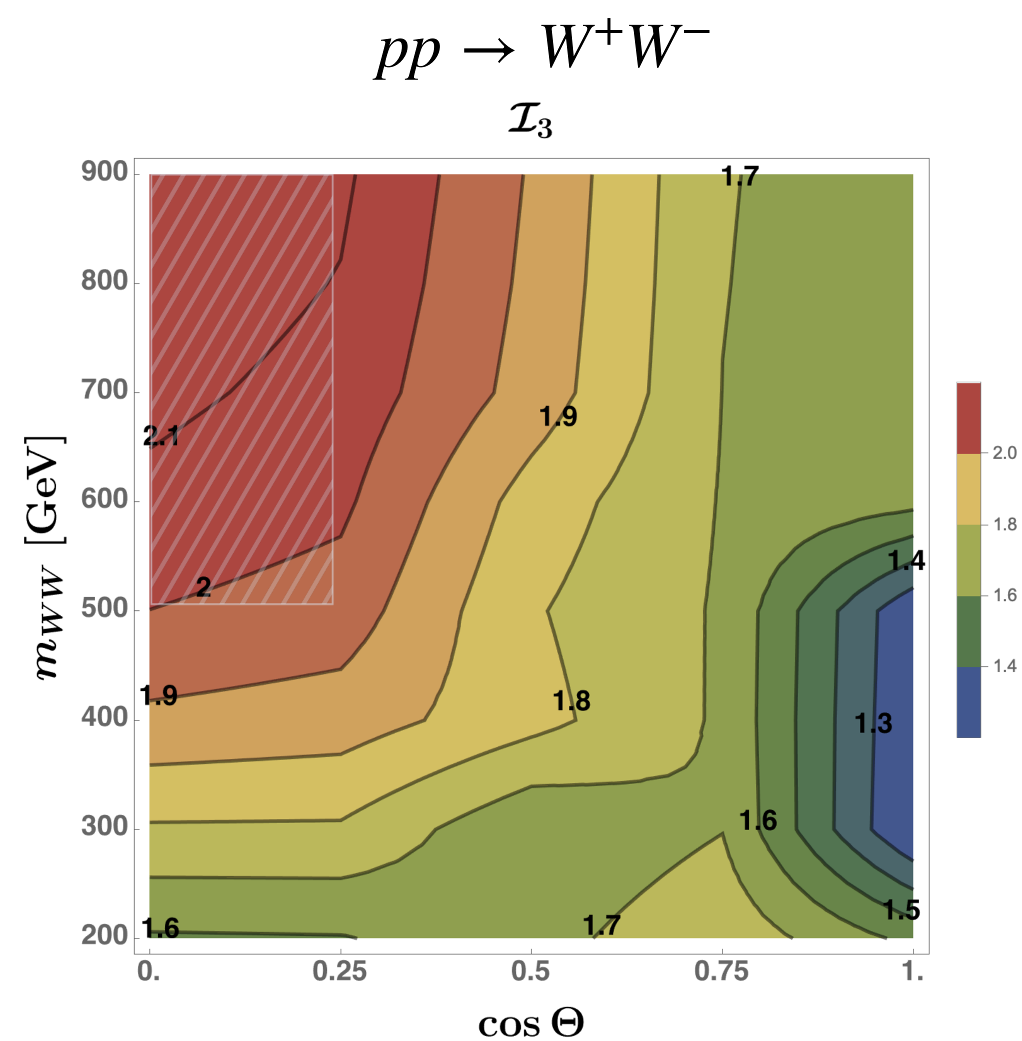


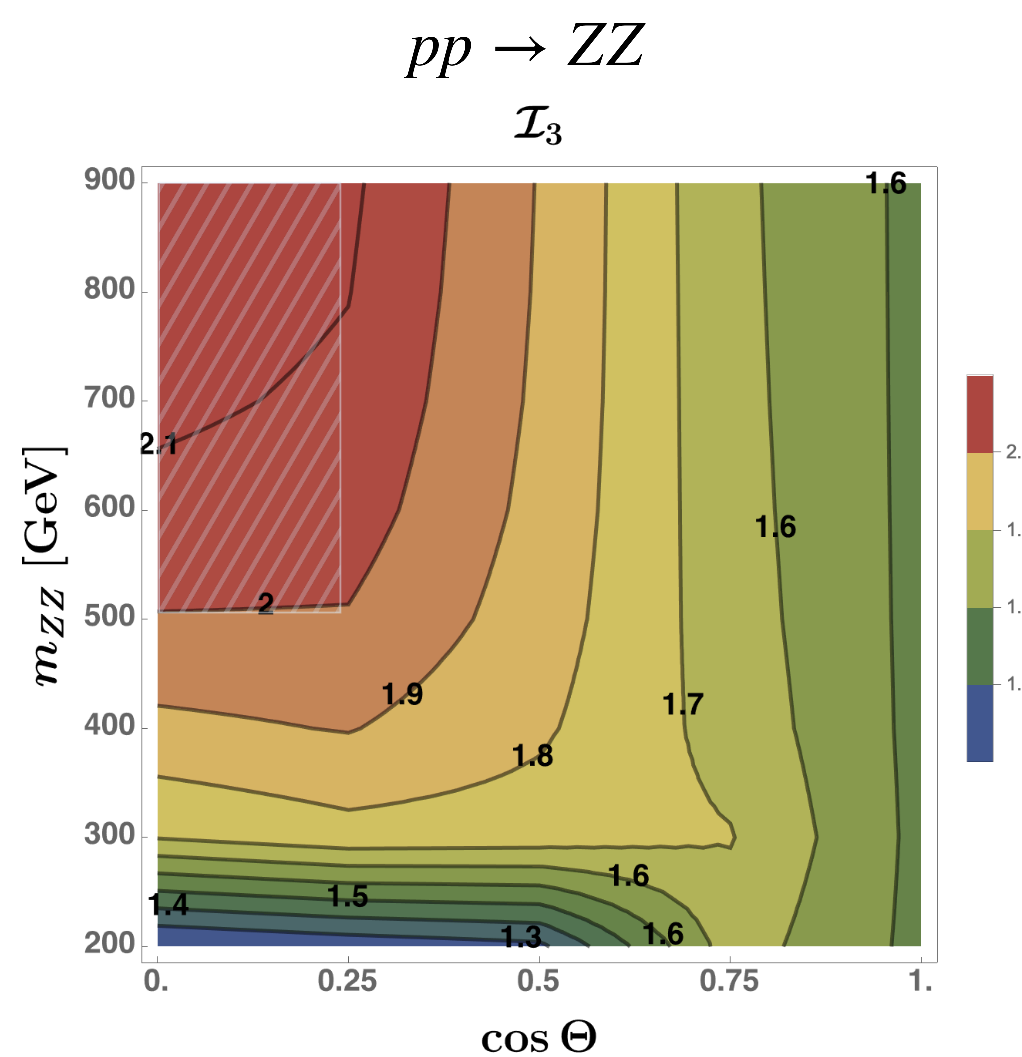
CGLMP function I_3 in optimal measurement axes

[Fabbrichesi, Floreanini, Gabrielli, Marzola (2023)]





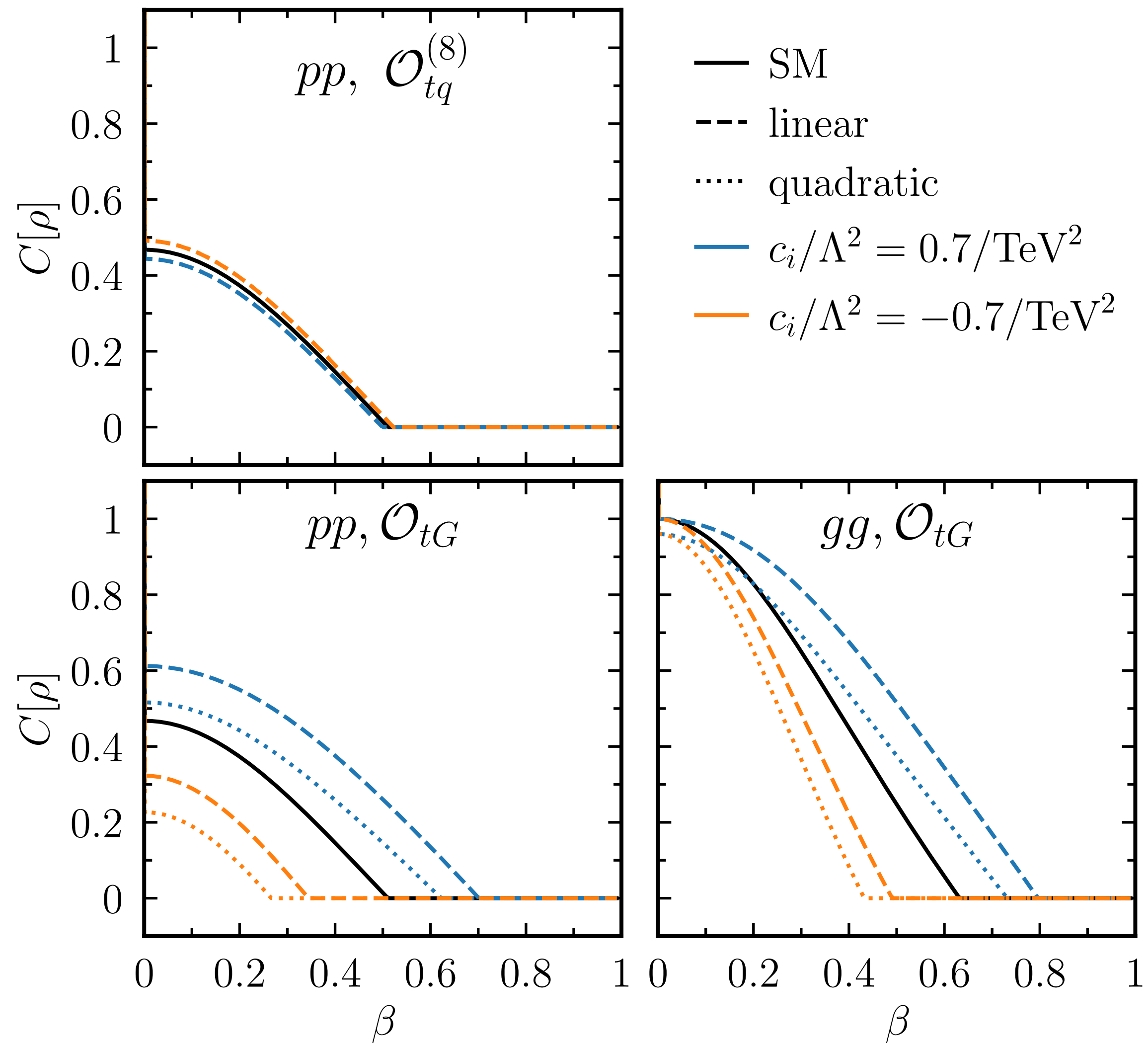




Effect of BSM $pp \to t\bar{t}$

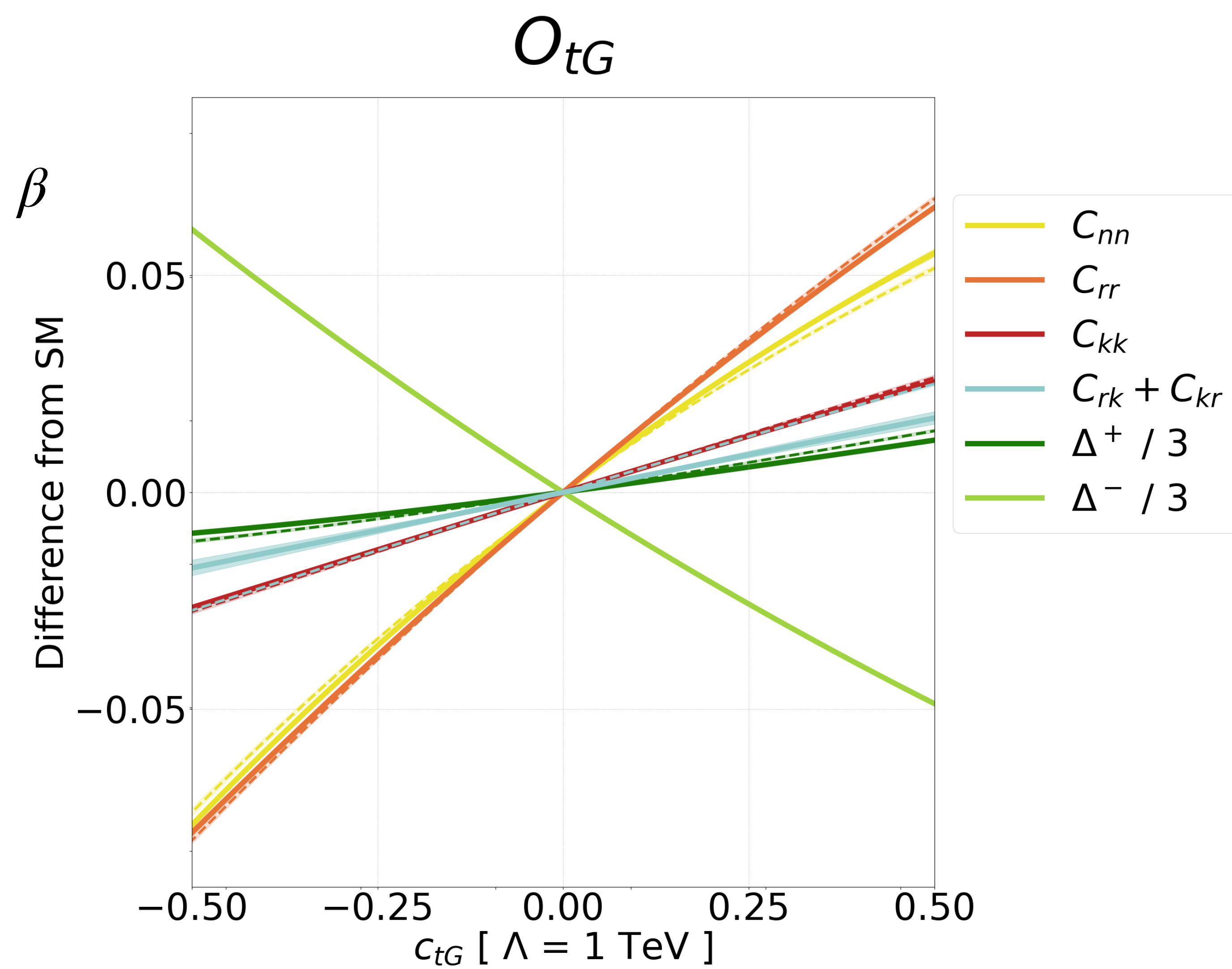
$$\mathcal{O}_{tG} = g_S\, \overline{Q} T_A \tilde{\varphi} \sigma^{\mu\nu} t\, G^A_{\mu\nu} \qquad \mathcal{O}^8_{tq} = \sum_{\mathrm{f}=1}^{2} (\bar{q}_{\mathrm{f}} \gamma_\mu T_A q_{\mathrm{f}})(\bar{t}\gamma^\mu T^A t)$$

[Aoude Madge Maltoni Mantani (2022)]



β: top velocity in the $t\bar{t}$ rest frame

[Severi Vryonidou (2023)]



$$\Delta^{\pm} = \pm(C_{kk} + C_{rr}) - C_{nn} - 1$$

Effect of BSM $H \to \tau^+\tau^-$

[Altakach, Lamba, Maltoni, Mawatari, Sakurai (2023)]

- Under CP, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$
- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \ \geq 0$$

- From MC-sim (assuming the SM, i.e. $\delta = 0$), we find

$$A = 0.112 \pm 0.085 \qquad (\mathrm{FCC_{ee}})$$

- CP-violating Yukawa interaction gives

$$\mathscr{L}_{\rm int} \propto H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\psi_\tau \Rightarrow C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow A(\delta) = 4\sin^2 2\delta$$

$$\Rightarrow \boxed{|\delta| < 6.4^o \quad (\mathrm{FCC_{ee}})}$$

Summary

- Quantum property measurements (entanglement and Bell inequality) at LHC and future lepton colliders have recently been studied.
- Such tests are important to look for beyond QM at high energies but it is also sensitive to BSM in the QFT framework.
- Promising processes: $pp \to t\bar{t}, VV, \quad H \to VV^*, \tau^+\tau^-$
- Experiments by ATLAS and CMS a bit behind the theoretical studies

Future prospects

- Feasibility studies for $pp \to VV,\, H \to VV^*$ including background
- $pp \to H(Z) \to \tau^+\tau^-$ at the LHC, tau decay modes other than $\tau \to \pi\nu$
- Entanglement in 3 parties? $pp \to t\bar{t}W$
- Testing not only the quantum state but also quantum process (CPTP maps).

See [Eckstein, Horodecki 2103.12000] for details

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen