Solving the Goldstone boson catastrophe and two-loop Higgs masses in non-supersymmetric models

Johannes Braathen
in collaboration with Mark Goodsell and Florian Staub

based on *arXiv:1609.06977* and *arXiv:1705.xxxxx*

Laboratoire de Physique Théorique et Hautes Énergies

20th Planck Conference
May 25, 2017

The context

Higgs mass m_h as a probe for New Physics

- No direct sign of BSM Physics since Higgs discovery in 2012
- m_h sensitive to New Physics via
  - $\rightarrow$ tree-level value (predicted in some models eg. SUSY)
  - $\rightarrow$ effect of heavy new particles in loops, with large couplings as well (eg. stops)
- Computations performed with **effective potential** and/or **diagrammatic techniques**

State of the art

- **SM**: V_{eff} (relates $m_h^2 \leftrightarrow \lambda$) is known to full 2-loop (*Ford, Jack and Jones '92*) + leading – QCD – 3-loop and 4-loop (*Martin '13, Martin '15*)
- Some results for m_h^2 in specific SUSY theories: **MSSM** (leading – SQCD – 3-loop order); **NMSSM** (2-loop); **Dirac Gaugino models** (leading – SQCD – 2-loop: *J.B., Goodsell, Slavich '16*)
- **Generic theories**: V_{eff} computed to 2-loop (*Martin '01*), 2-loop tadpoles and scalar masses (in gaugeless limit) implemented in `SARAH` (*Goodsell, Nickel, Staub '15*)

The Goldstone Boson Catastrophe

- Beyond one loop, V_{eff} only computed in Landau gauge $\Rightarrow$ Goldstones are treated as actual massless bosons *i.e.* $(m_G^2)^{\text{OS}} = 0$

Remark Having Goldstones appearing in calculation not related to global/gauged symmetry, but to the gauge choice

- By **choice** (for simplicity) V_{eff} is computed with running masses:

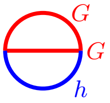
$$(m_G^2)^{\text{run.}} = (m_G^2)^{\text{OS}} - \Pi_G((m_G^2)^{\text{OS}}) = -\Pi_G(0),$$

where Π_G is the Goldstone self-energy

- V_{eff} contains logs of $(m_G^2)^{\text{run.}}$, eg

$$V_{\text{SM}}^{(2)} \supset -3\lambda^2 v^2 I(m_h^2, m_G^2, m_G^2) \underset{m_G^2 \to 0}{=} -6\lambda^2 v^2 A(m_G^2)\frac{A(m_h^2)}{m_h^2}$$

$$\Rightarrow \frac{\partial V_{\text{SM}}^{(2)}}{\partial v} \supset -12\lambda^3 v^3 \log\frac{m_G^2}{Q^2}\frac{A(m_h^2)}{m_h^2}$$



$I(m_h^2, m_G^2, m_G^2)$

with $A(x) = x(\log(x/Q^2) - 1)$.

- Under RG flow, $(m_G^2)^{\text{run.}}$ may
  - $\rightarrow$ become 0 $\Rightarrow$ infrared divergence in V_{eff} and/or its derivatives
  - $\rightarrow$ change sign $\Rightarrow$ imaginary part in V_{eff} and its derivatives

$\equiv$ **Goldstone boson catastrophe**

First approaches to the GBC

(cf. talk by Florian Staub this morning)

By hand

- if $m_G^2 < 0$, drop the imaginary part of V_{eff}
- tune the renormalisation scale Q to ensure $m_G^2 > 0$ (and even m_G^2 not too small)
  $\Rightarrow$ may be impossible to achieve and is completely ad hoc

In automated codes (SARAH)

- For SUSY theories **only**: rely on the gauge-coupling dependent part of $V^{(0)}$
  - $\rightarrow$ minimize full $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}|_{\text{gaugeless}}$
  - $\rightarrow$ compute tree-level masses with $V^{(0)}|_{\text{gaugeless}}$ (= turn off the D-term potential)
  - $\rightarrow$ yields a fake Goldstone mass of order $\mathcal{O}(m_{EW}) \Rightarrow$ no GBC

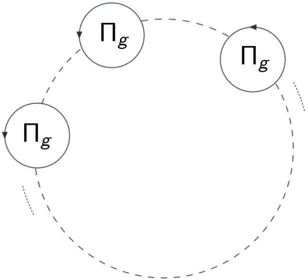
  $\rightarrow$ wrong mass for Goldstones hence wrong contribution to m_h
- Add a regulator mass $m^2_{\text{reg.}} = RQ^2$ for massless particles $\rightarrow$ unwanted new dependence on R, changes the relative size of Goldstone contributions

$+$ both methods spoil gauge invariance, etc.

$\Rightarrow$ especially wrong when the scalars – in particular the pseudo-scalars – give large contributions to the Higgs mass $\longrightarrow$ non-SUSY models

Resummation of the Goldstone contribution

SM: Martin 1406.2355; Ellias-Miro, Espinosa, Konstandin 1406.2652.
MSSM: Kumar, Martin 1605.02059.
Generic th.: JB, Goodsell 1609.06977.



[Adapted from arXiv:1406.2652]

- Power counting $\to$ most divergent contribution to V_{eff} at ℓ-loop = ring of $\ell - 1$ Goldstone propagators and $\ell - 1$ insertions of 1PI subdiagrams Π_g involving **only** heavy particles
- Π_g obtained from Π_G, Goldstone self-energy, by removing "soft" Goldstone terms
- Resumming Goldstone rings $\Leftrightarrow$ shifting the Goldstone tree-level mass by Π_g in the 1-loop Goldstone term

$$\hat{V}_{\text{eff}} = V_{\text{eff}} + \frac{1}{16\pi^2}\left[f(m_G^2 + \Pi_g) - \sum_{n=0}^{\ell-1} \frac{(\Pi_g)^n}{n!} \left(\frac{d}{dm_G^2}\right)^n f(m_G^2)\right]$$

$\to$ ℓ-loop resummed V_{eff}, free of leading Goldstone boson catastrophe

A word on the extension of the resummation procedure for generic theories

arXiv:1609.06977

Additional difficulties !

- several Goldstones
- scalar mixing

$\Rightarrow$ Single out the Goldstones (index $G, G', ...$) and express their masses

$$m_G^2 = -\sum_i \frac{1}{v_i} (\tilde{R}_{iG})^2 \left. \frac{\partial (V_{\text{eff}} - V^{(0)})}{\partial \phi_i^0} \right|_{\phi_i^0 = 0} = \mathcal{O}(\text{1-loop})$$

($\tilde{R}_{ij}$: rotation matrices in tree-level minimum of V_{eff})

Issues with the resummation

- (technical) taking derivatives of $\hat{V}_{\text{eff}}$ can be very difficult (involves derivatives of the rotation matrices, etc.) $\rightarrow$ in practice resummation was **only** used to find the **tadpole equations**.
- (conceptual) the choice of "soft" Goldstone terms to remove from Π_G to find Π_g may be ambiguous and it is difficult to justify which terms to keep

Our solution: setting the Goldstone boson on-shell arXiv:1609.06977

Adopt an on-shell scheme for the Goldstone(s)

- Replace $(m_G^2)^{\text{run.}}$ by $(m_G^2)^{\text{OS}}(=0)$ and $\Pi_G(0)$

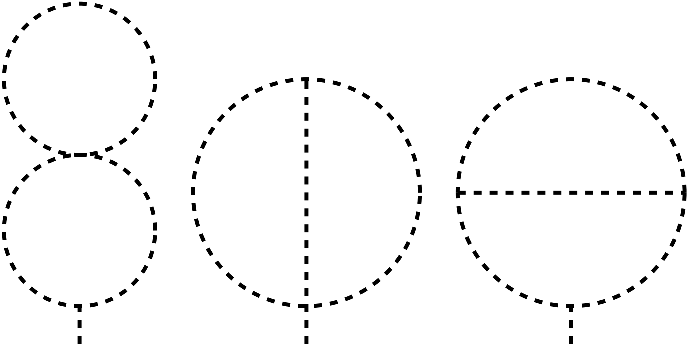
$$\underbrace{(m_G^2)^{\text{run.}}}_{\substack{\text{tree-level mass}\\ \text{in loop-corrected}\\ \text{minimum}}} = \underbrace{(m_G^2)^{\text{OS}}}_{\text{on-shell mass}} - \underbrace{\Pi_G((m_G^2)^{\text{OS}})}_{\text{self-energy}} = -\Pi_G(0)$$

- This can be done **directly** in the tadpole equations or mass diagrams!

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

2-loop tadpole diagrams involving scalars only:

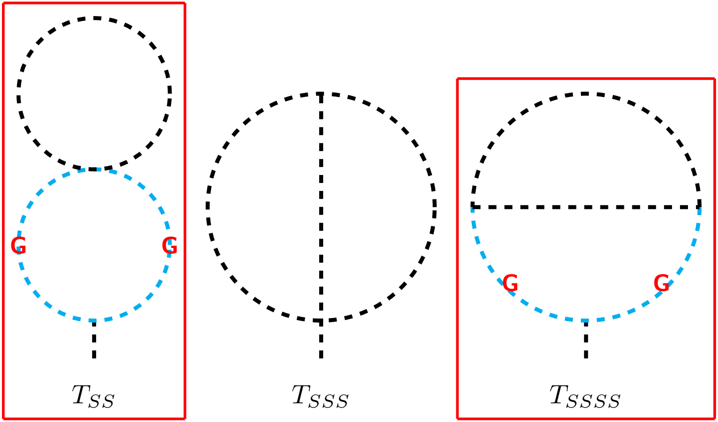
The GBC also appears in diagrams with scalars and fermions or gauge bosons, and is cured with the same procedure → we present the purely scalar case.



T_{SS} T_{SSS} T_{SSSS}

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

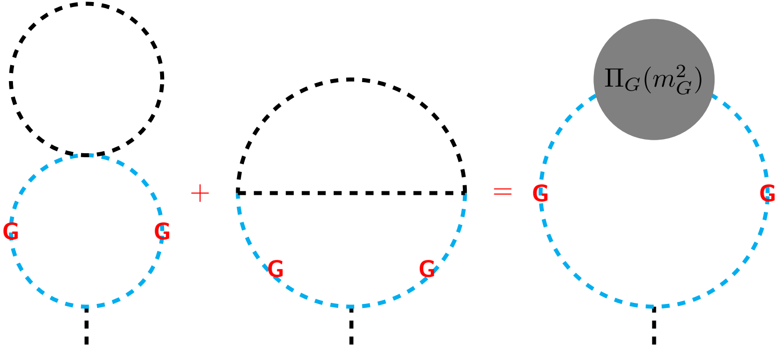
2-loop tadpole diagrams involving scalars only:



Some diagrams of T_{SS} and T_{SSSS} topologies diverge for $m_G^2 \to 0$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

2-loop divergences in tadpole diagrams (involving scalars only) ...



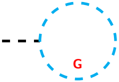
... rewritten as a one-loop diagram with insertion of $\Pi_G(m_G^2)$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

What happens when setting the Goldstone on-shell?

- Contribution of the Goldstone(s) to the 1-loop tadpole:

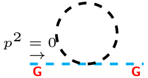
$$T_S \supset$$



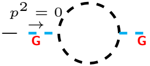
$$\propto A(m_G^2) = m_G^2\Big(\log\frac{m_G^2}{Q^2} - 1\Big)$$

- At 1-loop order the scalar-only diagrams in $\Pi_G(0)$ are

$$(m_G^2)^{\text{run.}} = \underbrace{(m_G^2)^{\text{OS}}}_{=0} -$$



$$-$$



$$+\cdots$$

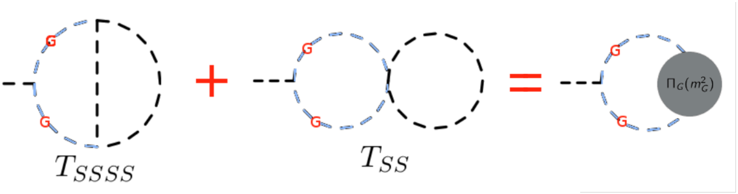
- Shifting m_G^2 by a 1-loop quantity, $\Pi_G(0)$, in the 1-loop tadpole

$\Rightarrow$ **2-loop shift !**

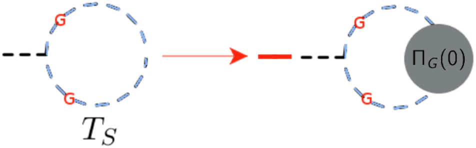
$$A((m_G^2)^{\text{run.}}) = \underbrace{A(0)}_{=0} - \underbrace{\log\frac{m_G^2}{Q^2}}_{\text{1-loop}} \underbrace{\Pi_G(0)}_{\text{1-loop}}$$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

- 2-loop divergent tadpole diagrams



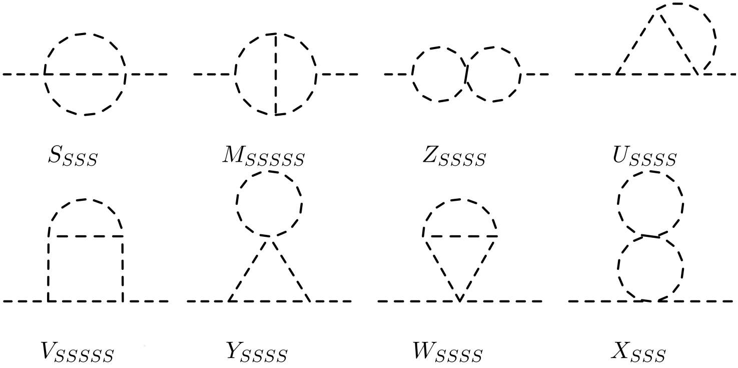
- shifting the Goldstone term in the 1-loop tadpole T_S



$\Rightarrow$ the divergent parts from the diagrams and the shift will cancel out!

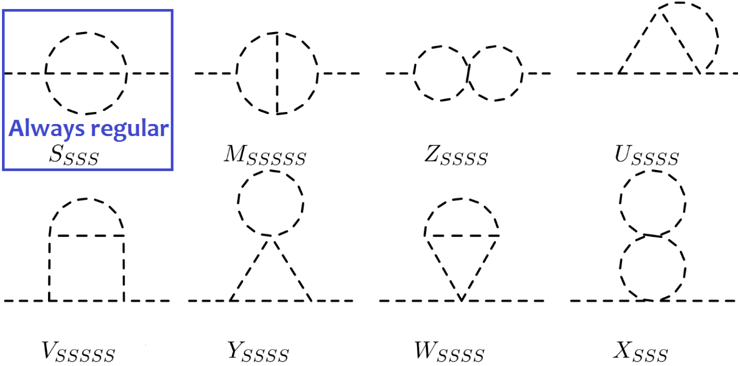
Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  ⇒ true at 1-loop order
  ⇒ at 2-loop, ∃ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



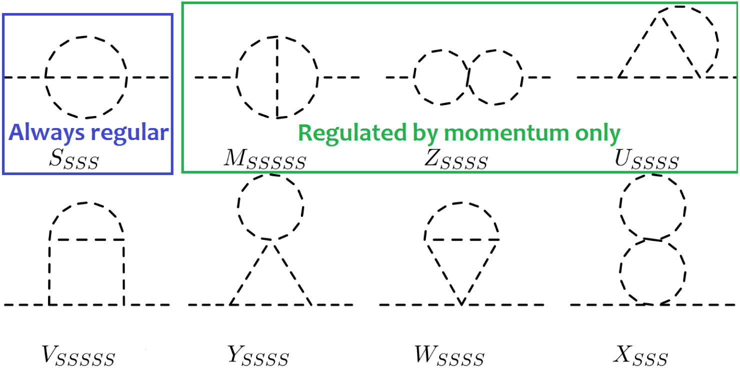
Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  $\Rightarrow$ true at 1-loop order
  $\Rightarrow$ at 2-loop, $\exists$ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



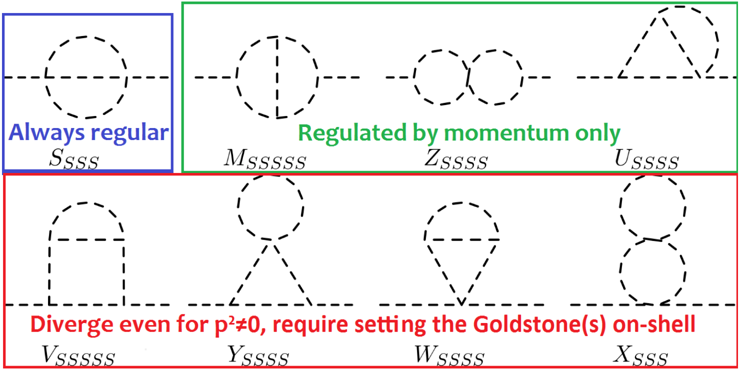
Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  $\Rightarrow$ true at 1-loop order
  $\Rightarrow$ at 2-loop, $\exists$ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



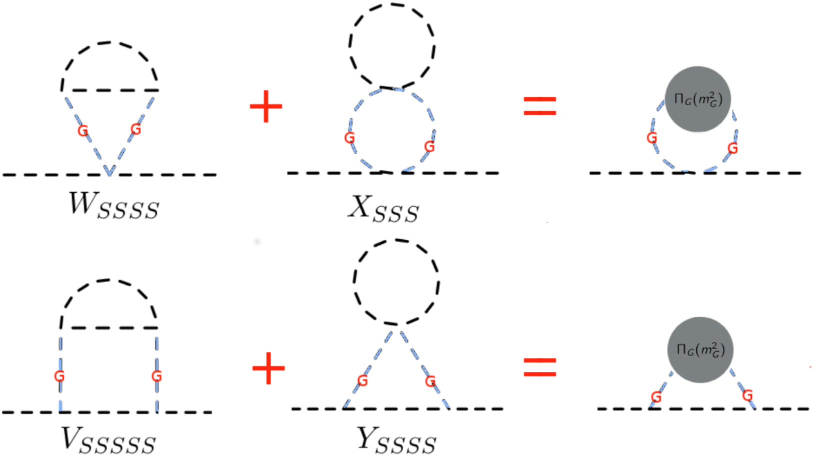
Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  $\Rightarrow$ true at 1-loop order
  $\Rightarrow$ at 2-loop, $\exists$ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



Canceling the IR divergences in the mass diagrams arXiv:1609.06977

Rewrite the divergent two-loop mass diagrams

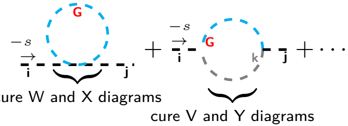


Canceling the IR divergences in the mass diagrams arXiv:1609.06977

Setting the Goldstone(s) on-shell in mass diagrams

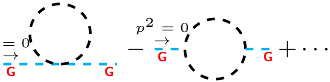
- Goldstone contributions to the 1-loop scalar self-energy

$$\Pi^{(1)}_{ij}(s = \text{-}p^2) =$$



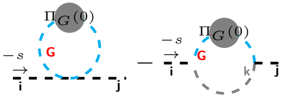
- Again, shifting the Goldstone mass to on-shell scheme gives

$$(m_G^2)^{\text{run.}} = -$$



$\rightarrow$ 2-loop shift to the mass diagrams

$$\delta\Pi^{(1)}_{ij}(s) = -$$



$\longrightarrow$ cancels the divergence in the V, X, Y, W mass diagrams !

Automated two-loop mass computations free of the Goldstone boson catastrophe

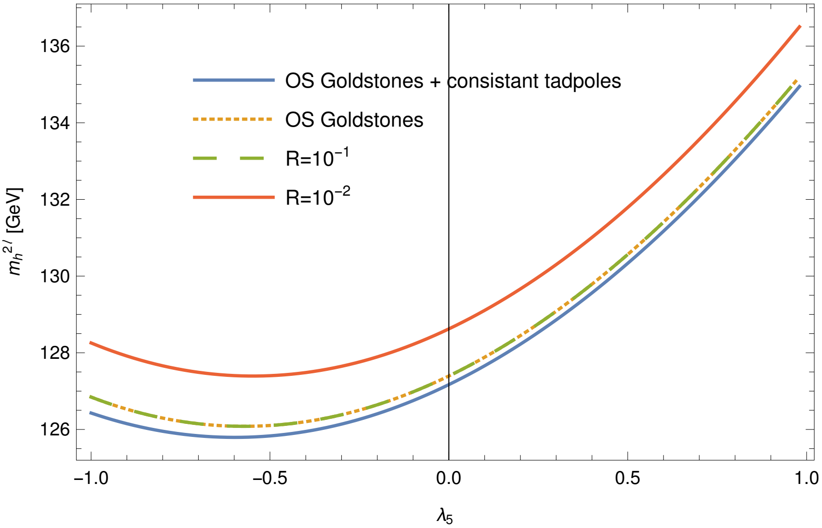
- On-shell Goldstones ⇒ regularised loop functions, free of GBC
- Implemented in new routines in `SARAH`/`SPheno` spectrum generator (`SARAH` = *Mathematica package, creates* `SPheno` *code for model to study*, cf. Florian Staub's talk this morning and arXiv:0806.0538, arXiv:1309.7223, arXiv:1503.04200)
- In particular useful for study of Higgs masses in non-SUSY theories where pseudo-scalar contributions are **large**.

In the following: a few checks and examples of results for $m_h^{2\ell}$ in non-SUSY models → here 2HDM

based on 1705.xxxxx (to appear soon)

Two-loop Higgs masses in the 2HDM

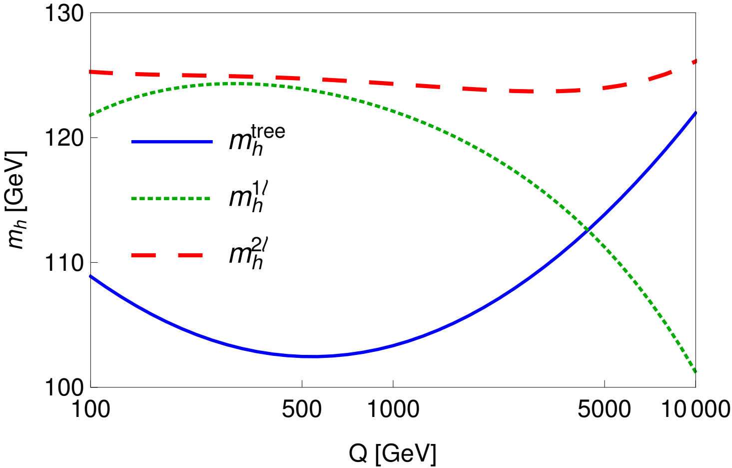
No more GBC!



Smaller value of $R \to$ effect of several GeV on m_h...

Two-loop Higgs masses in the 2HDM

Improved renormalisation scale dependence



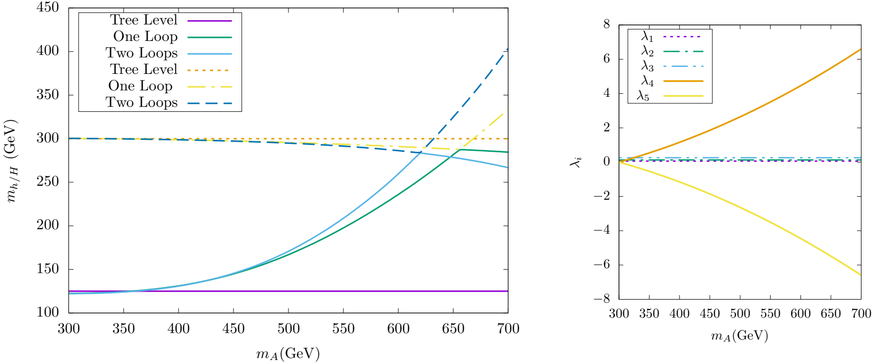
with $\tan\beta = 50$, $\lambda_1 = \lambda_2 = 0.0917$, $\lambda_3 = 0.3334$, $\lambda_4 = 0.8$, $\lambda_5 = -0.95$, $m_{12}^2 = -50\,000$ GeV^2 at scale $Q = 160$ GeV.

Two-loop Higgs masses in the 2HDM

The danger of using masses as inputs

Studies of 2HDM usually take tree-level Higgs masses as inputs instead of couplings from scalar potential, eg here inputs are

$$m_h, m_H, m_{H^\pm}, m_A, m_{12}^2, \tan\alpha, \tan\beta$$



Huge loop corrections → non physical parameter point (too large couplings)

Our results and outlooks

- **Analytic results** for generic theories (scalars, fermions, gauge bosons), *avoiding the Goldstone boson catastrophe*
  - → full **two-loop tadpole equations**
  - → **two-loop mass diagrams** for neutral scalars in *gaugeless limit*, in a *generalised effective potential approach* ($i.e.$ neglect terms of order $\mathcal{O}(s)$ and higher)

- **Numerical implementation** in `SARAH`
  (*illustrated in 1705.xxxxx, soon made public*)
  - → no more numerical instability associated with the GBC
  - → **automated Higgs mass calculations in both SUSY and non-SUSY models**

- Further work on the GBC
  - extend the solution of GBC to higher loop order
    - → on-shell method still working?
    - → how to formalise/prove the resummation prescription? ($i.e.$ how to find Π_g)
  - extend mass-diagram calculations to quartic order in the gauge couplings (go beyond the gaugeless limit)
  - investigate further the link between resummation and on-shell method
  - use similar techniques to address other IR divergences ?

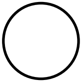
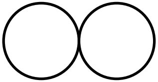
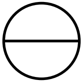
Thank you for your attention !

Backup slides

The effective potential

$$V_{\text{eff}} = V^{(0)} + \text{quantum corrections}$$

- Potential for scalars, including quantum corrections = 1PI vacuum graphs computed loop by loop

1-loop  ; 2-loop  +  ; etc.

- Expressed as a function of **running tree-level masses** of particles, in some **minimal substraction scheme** ($\overline{\mathrm{MS}}$, $\overline{\mathrm{DR}}'$, etc.)
- First derivative of V_{eff}: **tadpole equation** ($\leftrightarrow$ minimum condition), relates vev and mass-squared parameters
- Second derivative: same as self-energy diagrams, but with **zero external momentum** $\rightarrow$ **approximate scalar masses**

Illustration: the abelian Goldstone model

- 1 complex scalar $\phi = \frac{1}{\sqrt{2}}(v + h + iG)$, no gauge group and only a potential

$$V^{(0)} = \mu^2|\phi|^2 + \lambda|\phi|^4$$

v: true vev, to all orders in perturbation theory (PT)

- SM: G^+, G^0 Goldstones do not mix, and can be treated separetely
$\rightarrow$ this model captures the behaviour of the GBC in the SM
- V_{eff} at 2-loop order:

$$V_{\text{eff}} = V^{(0)} + \underbrace{\frac{1}{16\pi^2}\Big[f(m_h^2) + f(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{1}{(16^2)^2}\Bigg[\lambda\Big(\frac{3}{4}A(m_G^2)^2 + \frac{1}{2}A(m_G^2)A(m_h^2)\Big) - \lambda^2 v^2 I(m_h^2, m_G^2, m_G^2) + \overbrace{\cdots}^{\text{no Goldstone}}\Bigg]}_{\text{2-loop}} + \mathcal{O}(\text{3-loop}$$

where $f(x) = \frac{x^2}{4}(\log x/Q^2 - 3/2)$, $A(x) = x(\log x/Q^2 - 1)$ and $I \propto$ 

- Tree-level masses: $m_h^2 = \mu^2 + 3\lambda v^2$, $m_G^2 = \mu^2 + \lambda v^2$

Illustration: the abelian Goldstone model

Tree-level tadpole

$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

Illustration: the abelian Goldstone model

Tree-level tadpole equation

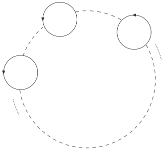
$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole equation

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\Big[3A(m_h^2) + A(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\overbrace{\log\frac{m_G^2}{Q^2}}^{\textbf{GBC!}}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2}A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2\to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

More details on the resummation of Goldstone contributions

$R_\ell \equiv$ 

$$\propto \int \frac{d^d k}{i(2\pi)^d} \left(\frac{\Pi_g}{k^2 - m_G^2} \right)^{\ell - 1}$$

$$\propto \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} \int \frac{d^d k}{i(2\pi)^d} \log(k^2 - m_G^2)$$

$$= \frac{1}{16\pi^2} \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} f(m_G^2)$$

so $\displaystyle\sum_\ell R_\ell = \frac{1}{16\pi^2} f(m_G^2 + \Pi_g)$

where $f(x) = \frac{x^2}{4}(\overline{\log}\, x - \frac{3}{2})$

Extending the resummation to generic theories arXiv:1609.06977

Generic theories: J.B., Goodsell arXiv:1609.06977

Real scalar fields $\varphi_i^0 = v_i + \phi_i^0$, where v_i are the vevs **to all order in PT**

$$V^{(0)}(\{\varphi_i^0\}) = V^{(0)}(v_i) + \frac{1}{2} m_{0,ij}^2 \phi_i^0 \phi_j^0 + \frac{1}{6} \hat{\lambda}_0^{ijk} \phi_i^0 \phi_j^0 \phi_k^0 + \frac{1}{24} \hat{\lambda}_0^{ijkl} \phi_i^0 \phi_j^0 \phi_k^0 \phi_l^0$$

$m_{0,ij}^2$ solution of the tree-level tadpole equation

To work in minimum of loop-corrected $V_{\text{eff}} \to$ new couplings m_{ij}^2

$\Downarrow$

Diagonalise to work with mass eigenstates in both bases

$$(\phi_i^0, m_{0,ij}^2) \overset{\phi_i^0 = \tilde{R}_{ij} \tilde{\phi}_j}{\longrightarrow} (\tilde{\phi}_i, \tilde{m}_i) \text{ (no loop corrections)}$$

$$(\phi_i^0, m_{ij}^2) \overset{\phi_i^0 = R_{ij} \phi_j}{\longrightarrow} (\phi_i, m_i) \text{ (with loop corrections)}$$

$\Downarrow$

Single out the Goldstone boson(s), index $G, G', \ldots$ and its/their mass(es)

$$m_G^2 = -\sum_i \frac{1}{v_i} (\tilde{R}_{iG})^2 \left. \frac{\partial (V_{\text{eff}} - V^{(0)})}{\partial \phi_i^0} \right|_{\phi_i^0 = 0} = \mathcal{O}(\text{1-loop})$$

Consistent solution of the tadpole equations arXiv:1609.06977

- Schematically, tadpole equations are of the form

$$m^2 = m_0^2 - \frac{1}{v}\frac{\partial \Delta V(m^2)}{\partial v}, \quad \text{with} \begin{cases} & \text{Tree-level masses} \\ m^2 & \text{in } \textbf{loop-corrected} \text{ minimum} \\ m_0^2 & \text{in } \textbf{tree-level} \text{ minimum} \end{cases}$$

  and need to be solved iteratively $\Rightarrow$ time-consuming!

- Expressing loop corrections as functions of m_0^2, and similarly tree-level couplings, makes solving the tadpole equation much easier.
- Generalise the procedure used for Goldstone bosons, and define mass shifts $\Delta_{ij} = -\tilde{R}_{ki}\tilde{R}_{kj}\ \frac{1}{v_k}\frac{\partial \Delta V}{\partial \phi_k^0}\Big|_{\phi^0=0}$

$$(m_{ij}^2, m_{0,ij}^2) \xrightarrow{diagonalise} m_i^2 = \bar{m}_i^2 + \Delta_{ii}$$

- One-loop terms with $\bar{m}_i^2 \Rightarrow$ two-loop shift! $\searrow$

$$\boxed{m^2 = m_0^2 - \frac{1}{v}\frac{\partial \Delta V(\bar{m}^2)}{\partial v} - \delta\left(\frac{\partial \Delta V(\bar{m}^2)}{\partial v}\right)}$$

More details about the calculations for the scalar-only tadpole

Divergent terms

- From T_{SS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \sum_{l\neq G} \lambda^{GGll} \lambda^{GGp}\, \overline{\log}\, m_G^2 A(m_l^2)$$

- From T_{SSSS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \lambda^{pGG} \lambda^{Gkl} \lambda^{Gkl}\, \overline{\log}\, m_G^2 P_{SS}(m_k^2, m_l^2)$$

Setting the Goldstone mass on-shell

$$\Pi^{(1),S}_{GG}\left(p^2\right) = \frac{1}{2}\lambda^{GGjj} A(m_j^2) - \frac{1}{2}(\lambda^{Gjk})^2 B(p^2, m_j^2, m_k^2)$$

- Hence a 2-loop shift:

$$\frac{\partial V_S^{(2)}}{\partial \phi_r^0}((m_G^2)^{\mathrm{OS}}) = \left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{m_G^2 \to (m_G^2)^{\mathrm{OS}}} - \frac{1}{4} R_{rp} \lambda^{GGp}\, \overline{\log}(m_G^2)^{\mathrm{OS}} \Big(\lambda^{GGjj} A(m_j^2) - (\lambda^{Gjk})^2 B(0, r$$

The full 2-loop tadpole equation free of GBC

$$\frac{\partial \hat{V}^{(2)}}{\partial \phi_r^0}\bigg|_{\varphi=v} = R_{rp}\bigg[\overline{T}^p_{SS} + \overline{T}^p_{SSS} + \overline{T}^p_{SSSS} + \overline{T}^p_{SSFF} + \overline{T}^p_{FFFS}$$
$$+ \overline{T}^p_{SSV} + \overline{T}^p_{VS} + \overline{T}^p_{VVS} + \overline{T}^p_{FFV} + \overline{T}^p_{\overline{FF}V} + \overline{T}^p_{\text{gauge}}\bigg].$$

Notations: see 1609.06977, 1503.03098

The full 2-loop tadpole equation free of GBC

The all-scalar diagrams are

$$\begin{aligned}
\overline{T}^p_{SS} =& \frac{1}{4}\sum_{j,k,l\neq G}\lambda^{jkll}\lambda^{jkp}P_{SS}(m_j^2,m_k^2)A(m_l^2)\\
&+\frac{1}{2}\sum_{k,l\neq G}\lambda^{Gkll}\lambda^{Gkp}P_{SS}(0,m_k^2)A(m_l^2),\\
\overline{T}^p_{SSS} =& \frac{1}{6}\lambda^{pjkl}\lambda^{jkl}f_{SSS}(m_j^2,m_k^2,m_l^2)\Big|_{m_G^2\to 0},\\
\overline{T}^p_{SSSS} =& \frac{1}{4}\sum_{(j,j')\neq(G,G')}\lambda^{pjj'}\lambda^{jkl}\lambda^{j'kl}U_0(m_j^2,m_{j'}^2,m_k^2,m_l^2)\\
&+\frac{1}{4}\sum_{(k,l)\neq(G,G')}\lambda^{pGG'}\lambda^{Gkl}\lambda^{G'kl}R_{SS}(m_k^2,m_l^2),
\end{aligned}$$

where by $(j,j')\neq(G,G')$ we mean that j,j' are not both Goldstone indices.

The full 2-loop tadpole equation free of GBC

The fermion-scalar diagrams are

$$
\begin{aligned}
\overline{T}^p_{SSFF} = & \sum_{(k,l)\neq(G,G')} \bigg\{ \frac{1}{2} y^{IJk} y_{IJl} \lambda^{klp} f^{(0,0,1)}_{FFS}(m_I^2, m_J^2; m_k^2, m_l^2) \\
& - \mathrm{Re}\left[y^{IJk} y^{I'J'k} M^*_{II'} M^*_{JJ'} \right] \lambda^{klp} U_0(m_k^2, m_l^2, m_I^2, m_J^2) \bigg\} \\
& + \frac{1}{2} \lambda^{GG'p} y^{IJG} y_{IJG'} \left(-I(m_I^2, m_J^2, 0) - (m_I^2 + m_J^2) R_{SS}(m_I^2, m_J^2) \right) \\
& - \lambda^{GG'p} \mathrm{Re}\left[y^{IJG} y^{I'J'G'} M^*_{II'} M^*_{JJ'} \right] R_{SS}(m_I^2, m_J^2), \\
\overline{T}^p_{FFFS} = & T^p_{FFFS}\Big|_{m_G^2 \to 0},
\end{aligned}
$$

The full 2-loop tadpole equation free of GBC

The gauge boson-scalar tadpoles are

$$\begin{aligned}
\overline{T}^{p}_{SSV} =& T^{p}_{SSV}\Big|_{m_G^2 \to 0}, \\
\overline{T}^{p}_{VS} =& \frac{1}{4} g^{abii} g^{abp} f^{(1,0)}_{VS}(m_a^2, m_b^2; m_i^2)\Big|_{m_G^2 \to 0} \\
&+ \sum_{(i,k) \neq (G,G')} \frac{1}{4} g^{aaik} \lambda^{ikp} f^{(0,1)}_{VS}(m_a^2; m_i^2, m_k^2), \\
\overline{T}^{p}_{VVS} =& \frac{1}{2} g^{abi} g^{cbi} g^{acp} f^{(1,0,0)}_{VVS}(m_a^2, m_c^2; m_b^2, m_i^2)\Big|_{m_G^2 \to 0} \\
&+ \sum_{(i,j) \neq (G,G')} \frac{1}{4} g^{abi} g^{abj} \lambda^{ijp} f^{(0,0,1)}_{VVS}(m_a^2, m_b^2; m_i^2, m_j^2) \\
&- \frac{1}{4} g^{abG} g^{abG'} \lambda^{GG'p} R_{VV}(m_a^2, m_b^2).
\end{aligned}$$

The full 2-loop tadpole equation free of GBC

The gauge boson-fermion and gauge diagrams are not affected by the Goldstone boson catastrophe

$$\begin{aligned}
\overline{T}^p_{FFV} =& 2g^{aJ}_I \overline{g}^K_{bJ} \mathsf{Re}[M_{KI'} y^{I'Ip}] f^{(1,0,0)}_{FFV}(m_I^2, m_K^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_I \overline{g}^I_{bJ} g^{abp} f^{(0,0,1)}_{FFV}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^p_{\overline{FF}V} =& g^{aJ}_I g^{aJ'}_{I'} \mathsf{Re}[y^{II'p} M^*_{JJ'}] \Big[f_{\overline{FF}V}(m_I^2, m_J^2, m_a^2) + M_I^2 f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \Big] \\
&+ g^{aJ}_I g^{aJ'}_{I'} \mathsf{Re}[M^{IK'} M^{KI'} M^*_{JJ'} y_{KK'p}] f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_I g^{bJ'}_{I'} g^{abp} M^{II'} M^*_{JJ'} f^{(0,0,1)}_{\overline{FF}V}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^p_{\text{gauge}} =& \frac{1}{4} g^{abc} g^{dbc} g^{adp} f^{(1,0,0)}_{\text{gauge}}(m_a^2, m_d^2; m_b^2, m_c^2).
\end{aligned}$$