Natural Alignment & Quartic Coupling Unification in multi-Higgs Doublet Models

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Based on:
- P.S.B. Dev, AP, JHEP1412 (2014) 024
- AP, PRD93 (2016) 075012
- N. Darvishi, AP, PRD99 (2019) 115014

Outline:

- Brief history of symmetries for natural SM alignment
- SM alignment in the 2HDM and beyond
- Quartic coupling unification in the 2HDM
- Phenomenological implications at the LHC
- Conclusions

• Brief history of symmetries for natural SM alignment

– Flavour unitarity of the CKM mixing matrix
[Gell-Mann, Levy '60; Cabbibo '63; Kobayashi, Maskawa '73]

– GIM mechanism to explain the smallness of the strangeness-changing interaction at the quantum level (requires the existence of the c-quark)
[Glashow, Iliopoulos, Maiani '70]

– Conditions for diagonal neutral currents in Z-boson interactions to quarks
[Paschos '77]

– Natural diagonal neutral currents in Z- **&** multi-Higgs-boson interactions to quarks [Glashow, Weinberg '77]

– Renormalizable models with partial flavour non-conservation at tree level (GIM suppressed). [Branco, Grimus, Lavoura '96]

– Yukawa alignment in the 2HDM broken by RG effects (not enforced by symmetries) [Pich, Tuzon '09]

– Natural SM alignment of New Physics [this talk]

• SM Alignment in the 2HDM and beyond

• 2HDM potential

[T. D. Lee '73;
Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
\mathrm{V} \;=\; & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)\,.
\end{aligned}
$$

• Physical (CP-conserving) spectrum:

CP-even Higgs bosons $\boldsymbol{H}$ and $\boldsymbol{h}$; CP-odd scalar $\boldsymbol{a}$; charged scalars $\boldsymbol{h}^\pm$.

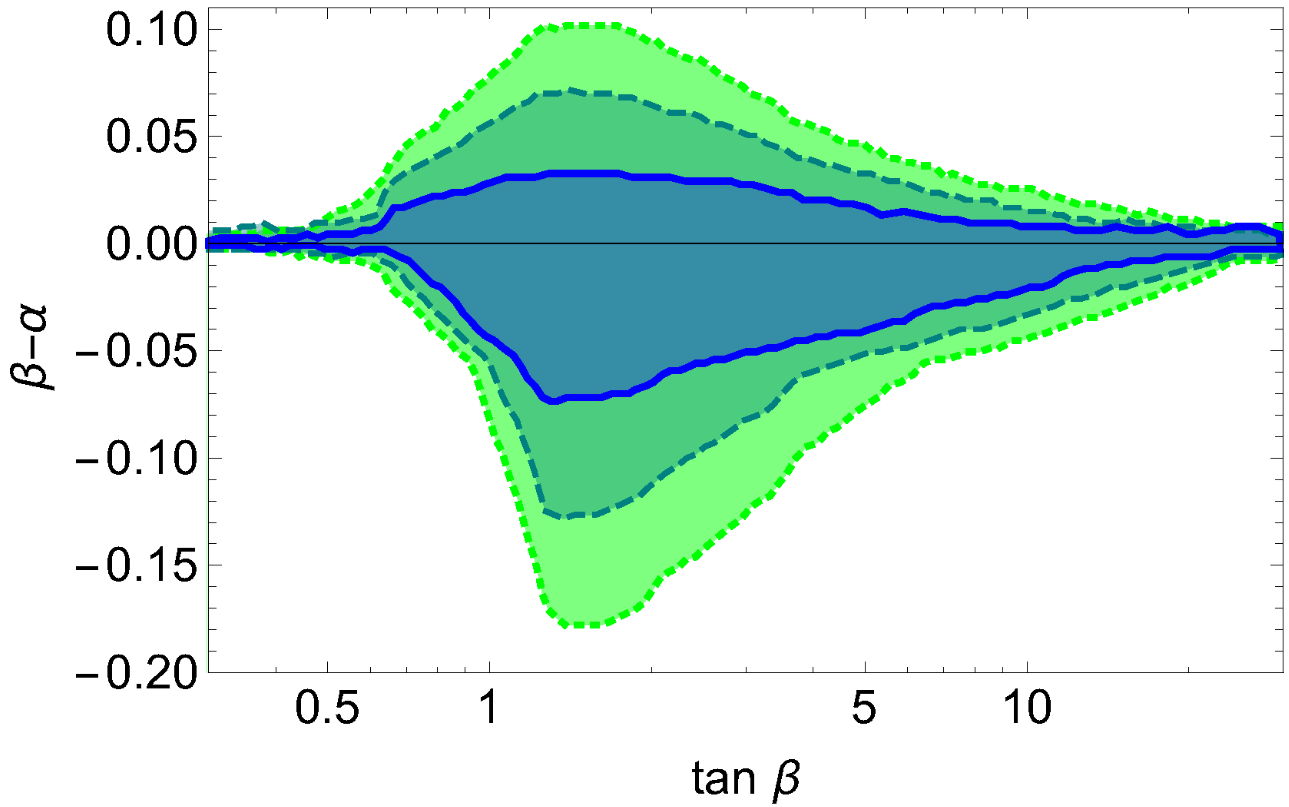
• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} \;=\; \cos(\beta-\alpha)\,, \qquad g_{hVV} \;=\; \sin(\beta-\alpha)\,,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

• Global fit to SM mis-alignment

[e.g. D. Chowdhury, O. Eberhardt, JHEP11 (2015) 052.]



Pheno favoured limit $\beta \to \alpha$: $g_{HVV} = \cos(\beta - \alpha) \;\to\; g_{H_{\text{SM}}VV} = 1.$

• SM Alignment $\beta \to \alpha$:

(i) Decoupling: [Georgi, Nanopoulos '79; Gunion, Haber '03; Ginzburg, Krawczyk '05]

$$M_h^2 \;\simeq\; M_a^2 + \lambda_5 v^2 \;\simeq\; M_{h^\pm}^2 \;\gg\; v_{\rm SM}^2$$

$$M_H^2 \;\simeq\; 2\lambda_{\rm SM} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2}\Big[s_\beta^2\,(2\lambda_2 - \lambda_{345}) - c_\beta^2\,(2\lambda_1 - \lambda_{345}) \;+\; \dots\Big]^2$$

(ii) Fine-tuning: [Krawczyk et al. '99; Carena, Low, Shah, Wagner '14; Dev, AP '14]

$$\lambda_7 t_\beta^4 \;-\; (2\lambda_2 - \lambda_{345}) t_\beta^3 \;+\; 3(\lambda_6 - \lambda_7)\, t_\beta^2 \;+\; (2\lambda_1 - \lambda_{345}) t_\beta - \lambda_6 \;=\; 0$$

(iii) Natural SM alignment (independent of $M_{h^\pm}$ and t_β): [Dev, AP '14]

$$\lambda_1 = \lambda_2 = \frac{\lambda_{345}}{2} \quad (\text{with } \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5), \qquad \lambda_6 = \lambda_7 = 0$$

Symmetries:

- Sp(4): $\lambda_4 = \lambda_5 = 0 \;\to\; \lambda_{345} = \lambda_3$
- SU(2): $\lambda_5 = 0 \;\to\; \lambda_{345} = \lambda_{34} \equiv \lambda_3 + \lambda_4$
- SO(2)$\times\mathcal{CP}$: $\lambda_{3,4,5} \;\neq\; 0$

• Maximally Symmetric Two Higgs Doublet Model

[P.S.B. Dev, AP '14]

$G_{\boldsymbol{\Phi}} = \mathrm{SU}(2)_L \otimes \mathrm{Sp}(4)/\mathrm{Z}_2 \simeq \mathrm{SU}(2)_L \otimes \mathrm{SO}(5).$

$$\mathrm{V} \;=\; -\,\mu^2\Big(|\Phi_1|^2+|\Phi_2|^2\Big)+\lambda\,\Big(|\Phi_1|^2+|\Phi_2|^2\Big)^2 \;=\; -\frac{\mu^2}{2}\,\boldsymbol{\Phi}^\dagger\,\boldsymbol{\Phi} + \frac{\lambda}{4}\,\big(\boldsymbol{\Phi}^\dagger\,\boldsymbol{\Phi}\big)^2\,,$$

where

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

$$\boldsymbol{\Phi} \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}\,, \qquad \text{with}\;\; U_L \in\; \mathrm{SU}(2)_L:\; \boldsymbol{\Phi} \;\mapsto\; \boldsymbol{\Phi}' \;=\; U_L\,\boldsymbol{\Phi}\,,$$

such that under **global field transformations**,

[AP, Phys. Lett. B706 (2012) 465.]

$$\mathrm{Sp}(4):\quad \boldsymbol{\Phi} \;\mapsto\; \boldsymbol{\Phi}' \;=\; U\,\boldsymbol{\Phi}\,, \quad \text{with}\; U \in \mathrm{U}(4)\;\;\&\;\; U\mathrm{C}U^{\mathsf{T}} = \;\mathrm{C} \equiv i\sigma^2 \otimes \sigma^0$$

$\mathrm{SU}(2)_L$ gauge kinetic terms remain invariant.

Breaking Effects: $-m_{12}^2\,\phi_1^\dagger\phi_2$, $\mathrm{U}(1)_Y$ coupling g', Yukawa couplings $\mathbf{Y}^{u,d}$.

References (*an incomplete list on SM Alignment in the 2HDM*)

- **On the SM Higgs basis (also Decoupling of FCNC Effects):**
  H. Georgi and D. V. Nanopoulos, Phys. Lett. B82 (1979) 95.

- **Alignment via Decoupling:**
  - J. F. Gunion, H. E. Haber, Phys. Rev. D67 (2003) 075019.
  - I. F. Ginzburg, M. Krawczyk, Phys. Rev. D72 (2005) 115013.

- **Alignment via Fine-tuning:**
  - P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, M. Krawczyk, Phys. Lett. B496 (2000) 195.
  - A. Delgado, G. Nardini, M. Quiros, JHEP1307 (2013) 054.
  - M. Carena, I. Low, N. R. Shah, C. E. M. Wagner, JHEP1404 (2014) 015.

- **Natural Alignment without Decoupling and without Fine-tuning:**
  - P.S.B. Dev, AP, JHEP1412 (2014) 024.
  - B. Grzadkowski, O. M. Ogreid, P. Osland, Phys. Rev. D94 (2016) 115002.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226.
- **$\mathbf{Z}_2$ symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.
- **Inert $\mathbf{Z}_2$ symmetry:** N. G. Deshpande, E. Ma, Phys. Rev. D18 (1978) 2574.
- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
- **Custodial SU(2)$_L$-preserving symmetry:**
  P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.
- **Bilinear formalism:**
  M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
  C. C. Nishi, Phys. Rev. D74 (2006) 036003.
- **SU(2)$_L\otimes$U(1)$_Y$-preserving symmetries:** <u>6</u>
  I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
  P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.
- **Hypercustodial SU(2)$_L$-preserving symmetries:** <u>13</u>
  R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.
- **On completeness and uniqueness of classification:**
  AP, Phys. Lett. B706 (2012) 465.

- **Natural Alignment Beyond the 2HDM** [AP '16]

– **nHDM potential** with m **inert scalar doublets:**

$$V_{n\text{HDM}} \;=\; V_{\text{sym}} \;+\; V_{\text{inert}} \;+\; \Delta V_{\text{soft}}\ ,$$

– **<u>3</u> continuous alignment symmetries** in the **field space of the active EWSB sector** $(N_H = n - m)$:

(i) $\text{Sp}(2N_H) \times \mathcal{D}$ (ii) $\text{SU}(N_H) \times \mathcal{D}$ (iii) $\text{SO}(N_H) \times \mathcal{CP} \times \mathcal{D}$,

where $\mathcal{D}$ acts on the inert sector *only*.

– **Symmetry invariants:**

$$\begin{aligned}
\text{(i)} \quad S &= \Phi_1^\dagger \Phi_1 \;+\; \Phi_2^\dagger \Phi_2 \;+\; \ldots \;=\; \tfrac{1}{2}\, \mathbf{\Phi}^\dagger\, \mathbf{\Phi} \\
\text{(ii)} \quad D^a &= \Phi_1^\dagger \sigma^a \Phi_1 \;+\; \Phi_2^\dagger \sigma^a \Phi_2 \;+\; \ldots \\
\text{(iii)} \quad T &= \Phi_1 \Phi_1^{\mathsf{T}} \;+\; \Phi_2 \Phi_2^{\mathsf{T}} \;+\; \ldots
\end{aligned}$$

– **Symmetric part of the scalar potential:**

$$V_{\text{sym}} \;=\; -\mu^2\, S \;+\; \lambda_S\, S^2 \;+\; \lambda_D\, D^a D^a \;+\; \lambda_T\, \text{Tr}\,(T\,T^*)\ .$$

– **Inert part of the scalar potential:**

$$V_{\text{inert}} = \widehat{m}^2_{\hat{a}\hat{b}}\, \widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}} + \lambda_{\hat{a}\hat{b}\hat{c}\hat{d}}\, (\widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}} \widehat{\Phi}_{\hat{d}}) + \lambda_{\hat{a}\hat{b}cd}\, (\widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c \Phi_d)$$
$$+ \lambda_{a\hat{b}\hat{c}d}\, (\Phi^\dagger_a \widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}} \Phi_d) + \Big[\lambda_{a\hat{b}c\hat{d}}\, (\Phi^\dagger_a \widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c \widehat{\Phi}_{\hat{d}}) + \text{H.c.} \Big]$$

$$\mathbf{Z}_2^{\mathbf{I}} : \quad \Phi_a \to \Phi_a \quad (a = 1, 2, \dots, N_H)\,, \qquad \widehat{\Phi}_{\hat{b}} \to -\widehat{\Phi}_{\hat{b}} \quad (\hat{b} = \hat{1}, \hat{2}, \dots, \widehat{m})$$

– **Soft-symmetry Breaking:**

$$\Delta V_{\text{soft}} = m^2_{ab}\, \Phi^\dagger_a \Phi_b$$

– **Minimal Symmetry of Alignment in the Higgs basis:**

$$\mathbf{Z}_2^{\mathbf{EW}} : \quad \Phi'_1 \to \Phi'_1\,, \qquad \Phi'_{a'} \to -\Phi'_{a'} \qquad (a' = 2, 3, \dots, N_H)$$

where m^2_{ab} becomes diagonal.

$\Longrightarrow$ $\boxed{\textbf{Minimal Alignment Symmetry:} \quad \mathbf{Z}_2^{\mathbf{EW}} \times \mathbf{Z}_2^{\mathbf{I}}}$ [AP '16]

• Quartic coupling unification in the MS-2HDM

[Dev, AP '14; N. Darvishi, AP '19]

Symmetry-breaking of $Sp(4)/Z_2 \sim SO(5)$:

- Soft breaking (e.g. through m_{12}^2):

$$M_H^2 \;=\; 2\lambda_2 v^2, \qquad M_h^2 \;=\; M_a^2 \;=\; M_{h^\pm}^2 \;=\; \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}$$

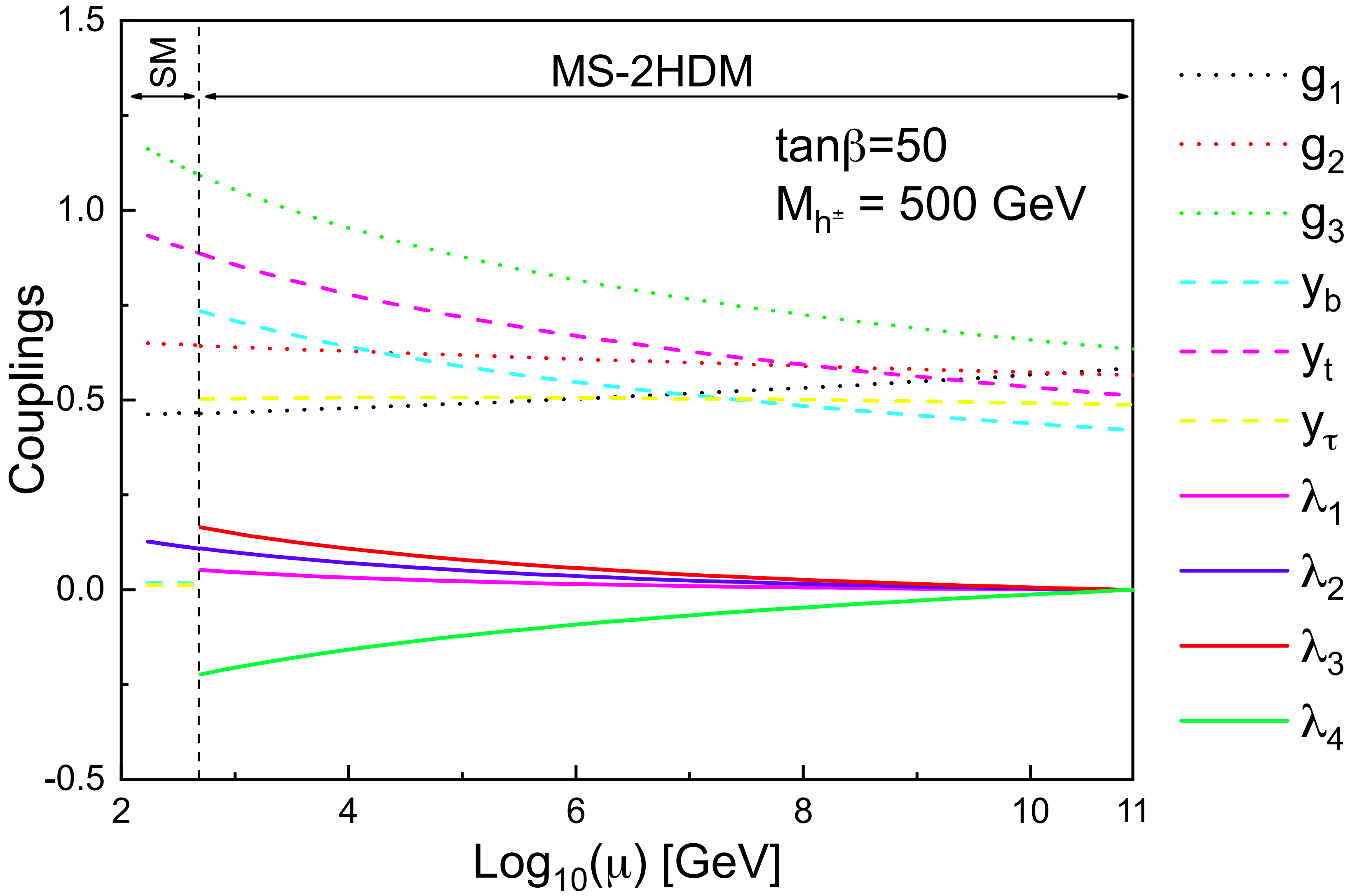
Heavy Higgs spectrum is degenerate at tree level.

- Explicit breaking through RG running (two loops):

$$\begin{aligned} \mathrm{Sp}(4)/\mathrm{Z}_2 \otimes \mathrm{SU}(2)_L \quad & \xrightarrow{g'\neq 0} \quad \mathrm{SU}(2)_{\mathrm{HF}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ & \xrightarrow{\mathbf{Y}^{u,d}} \quad \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ & \xrightarrow[\langle \Phi_{1,2} \rangle]{m_{12}^2} \quad \mathrm{U}(1)_{\mathrm{em}} \end{aligned}$$

• Quartic Coupling Unification (two loops)

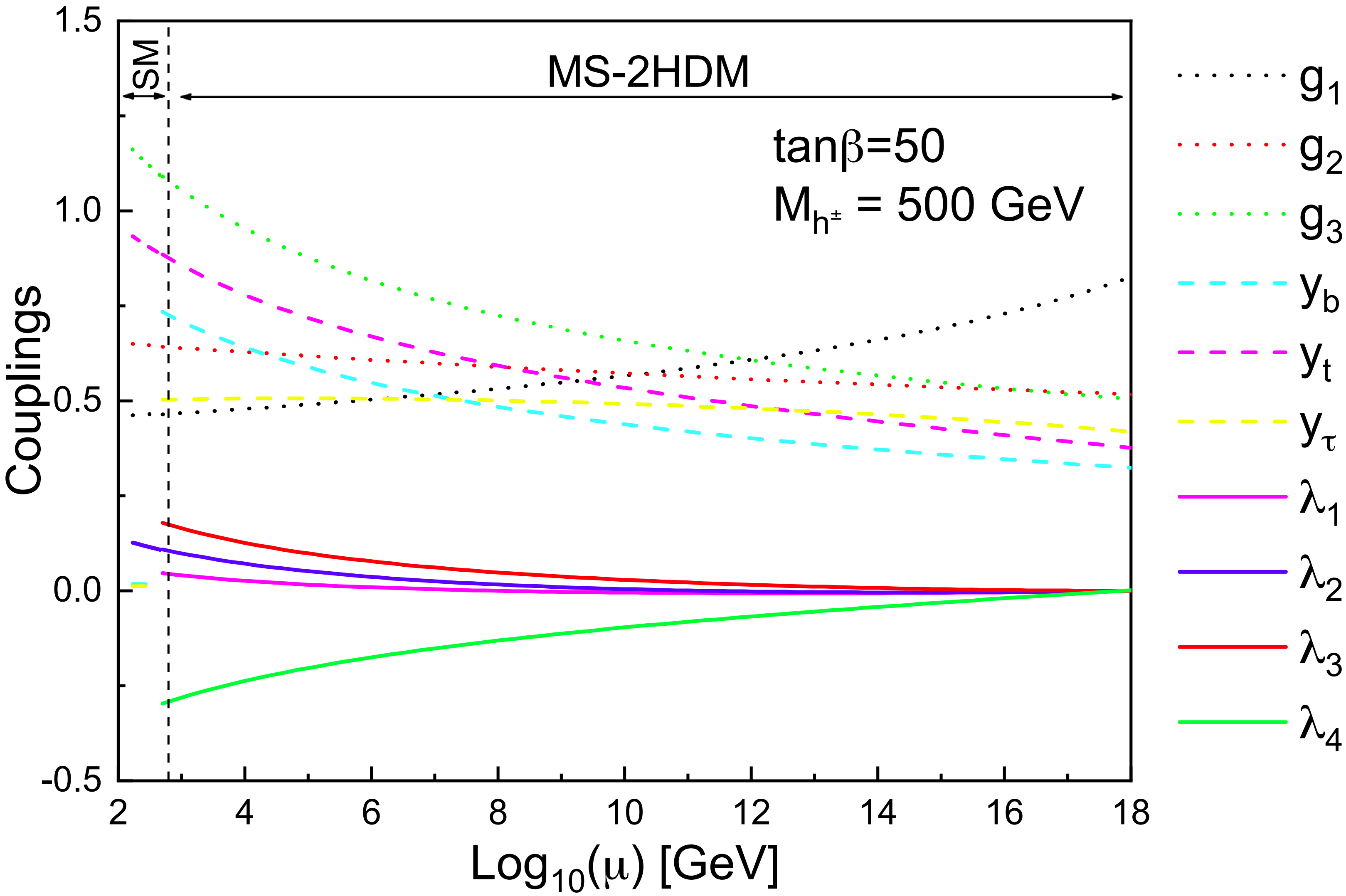
[N. Darvishi, AP '19]



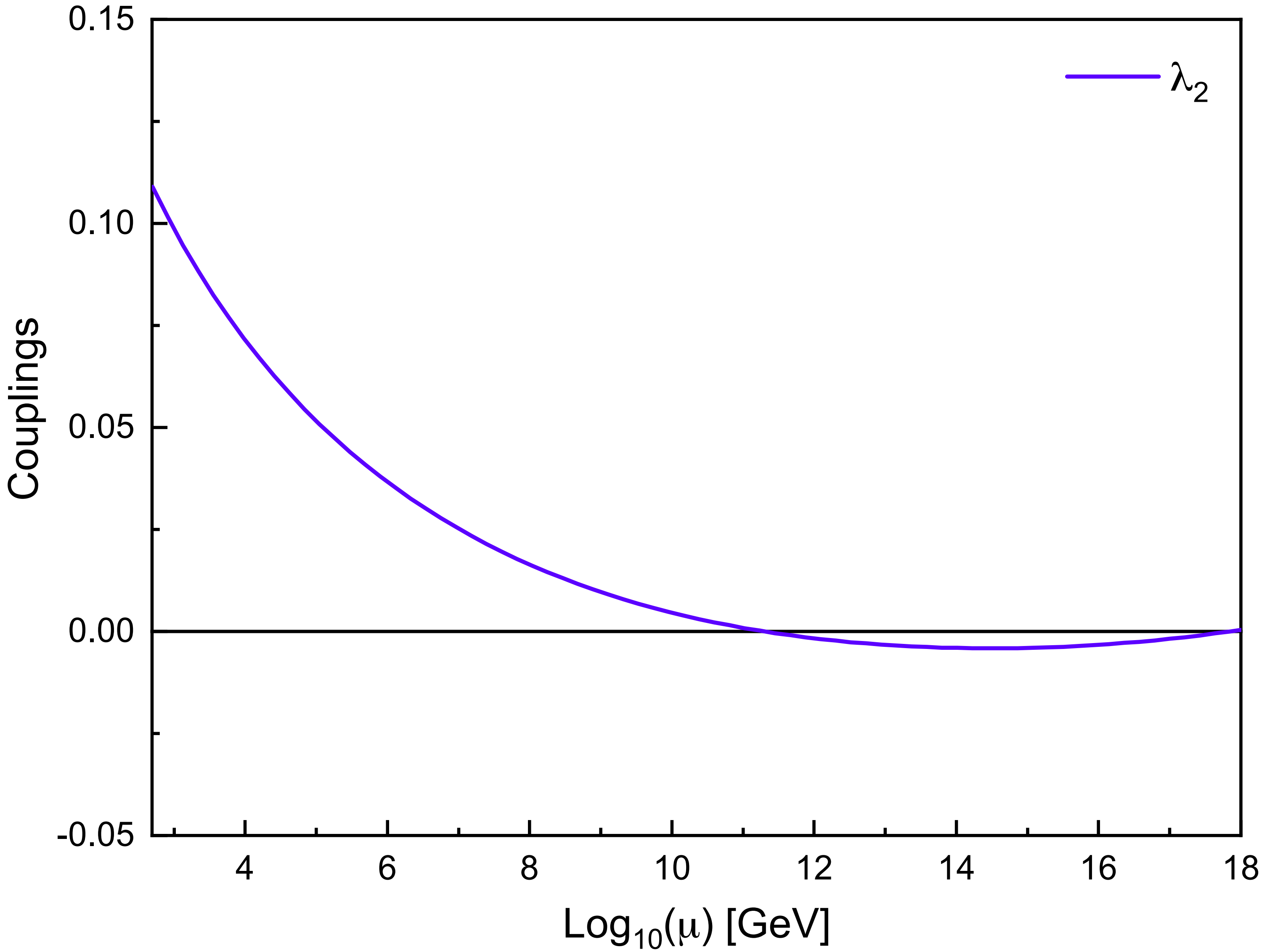
First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order $m_{\rm Pl}$)

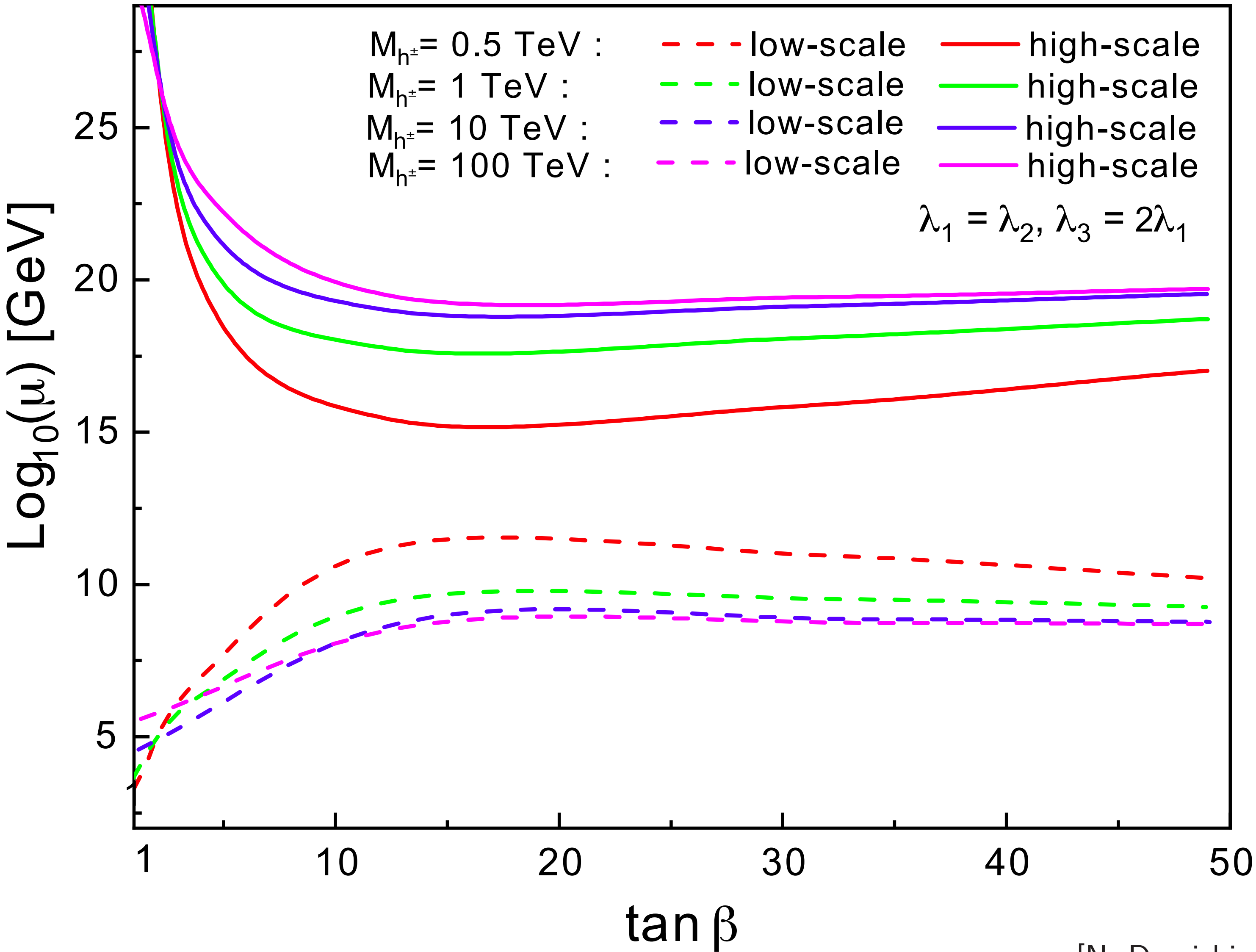
[N. Darvishi, AP '19]



A closer look at the RG evolution of λ_2



Low- and high-scale quartic coupling unification: $\tan\beta$ vs $\mu_X^{(1,2)}$



[N. Darvishi, AP '19]

• Misalignment in the MS-2HDM

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_{\mathcal{S}}^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} \xrightarrow[\textbf{approx.}]{\textbf{seesaw}} \quad M_H^2 \simeq \widehat{A} - \frac{\widehat{C}^2}{\widehat{B}} \quad \& \quad M_h^2 \simeq \widehat{B} \gg \widehat{A},\, \widehat{C}$$

Light-to-heavy scalar mixing:

$$\theta_{\mathcal{S}} \equiv \frac{\widehat{C}}{\widehat{B}} = \frac{v^2 s_\beta c_\beta \left[s_\beta^2\,(2\lambda_2 - \lambda_{34}) - c_\beta^2\,(2\lambda_1 - \lambda_{34}) \right]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2\,(\lambda_1 + \lambda_2 - \lambda_{34})} \ll 1$$

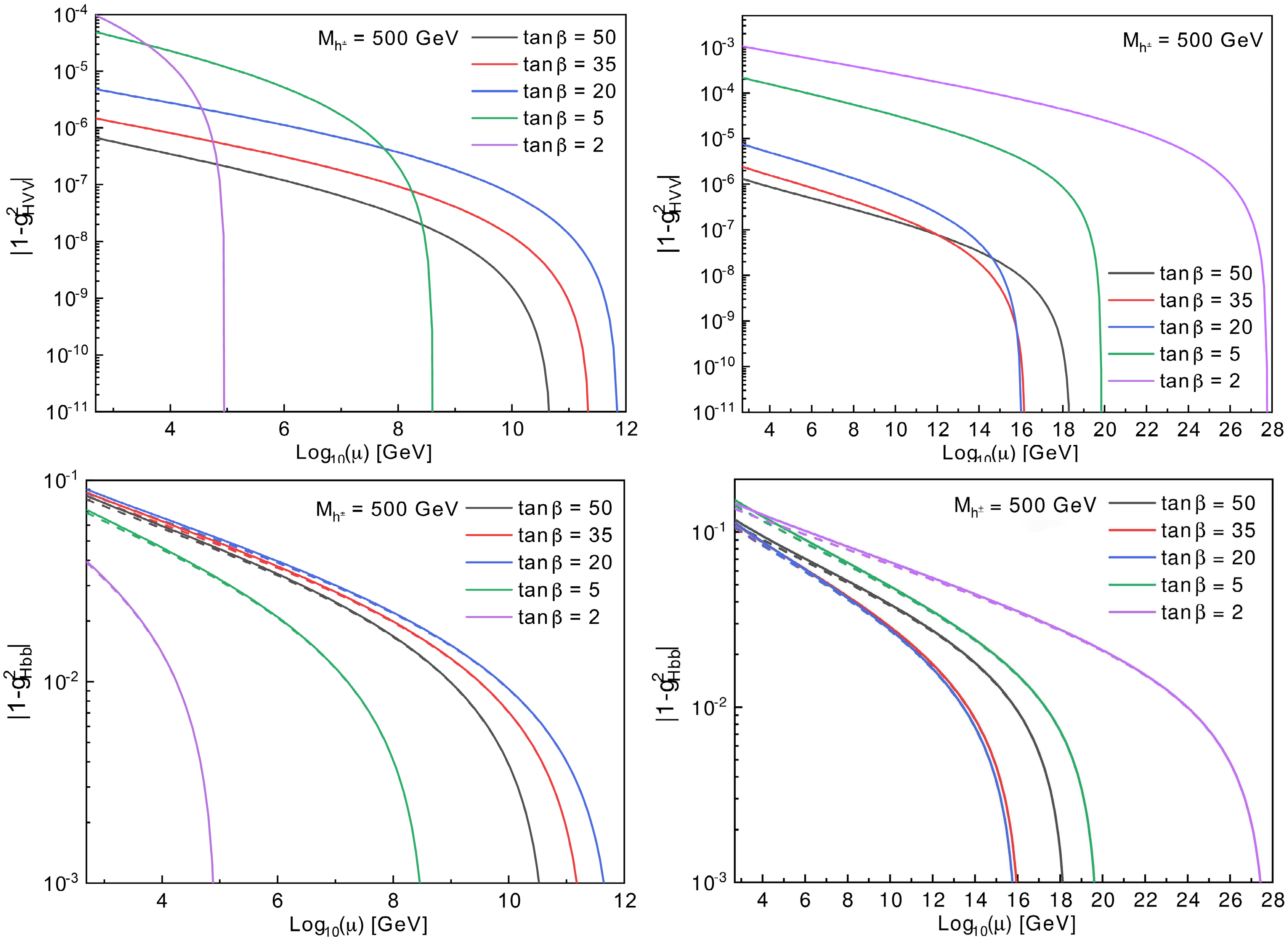
Higgs couplings to $V = W, Z$:

$$g_{HVV} \simeq 1 - \frac{1}{2}\,\theta_{\mathcal{S}}^2\,, \qquad g_{hVV} \simeq -\,\theta_{\mathcal{S}}$$

Higgs couplings to quarks:

$$g_{Huu} \simeq 1 + t_\beta^{-1}\,\theta_{\mathcal{S}}\,, \qquad g_{Hdd} \simeq 1 - t_\beta\,\theta_{\mathcal{S}}\,,$$
$$g_{huu} \simeq -\theta_{\mathcal{S}} + t_\beta^{-1}\,, \qquad g_{hdd} \simeq -\theta_{\mathcal{S}} - t_\beta\,.$$

Predictions for Higgs-boson couplings to $V = W, Z$ and b-quarks



Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h^\pm} = 500\,\mathrm{GeV}$.

[N. Darvishi, AP '19]

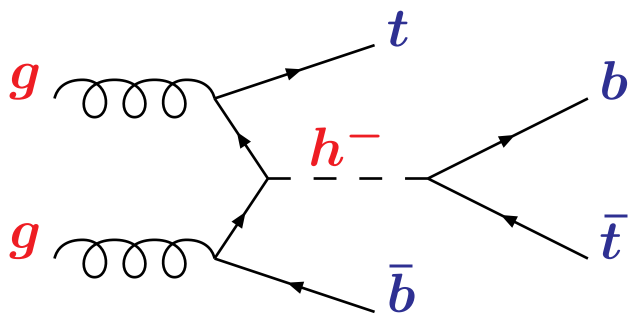
Couplings	ATLAS	CMS	$\tan\beta = 2$	$\tan\beta = 20$	$\tan\beta = 50$
$\lvert g^{\text{low-scale}}_{HZZ}\rvert$	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$\lvert g^{\text{high-scale}}_{HZZ}\rvert$			0.9981	0.9999	0.9999
$\lvert g^{\text{low-scale}}_{Htt}\rvert$	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$\lvert g^{\text{high-scale}}_{Htt}\rvert$			1.0987	1.0003	1.0001
$\lvert g^{\text{low-scale}}_{Hbb}\rvert$	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$\lvert g^{\text{high-scale}}_{Hbb}\rvert$			0.8810	0.9449	0.9427

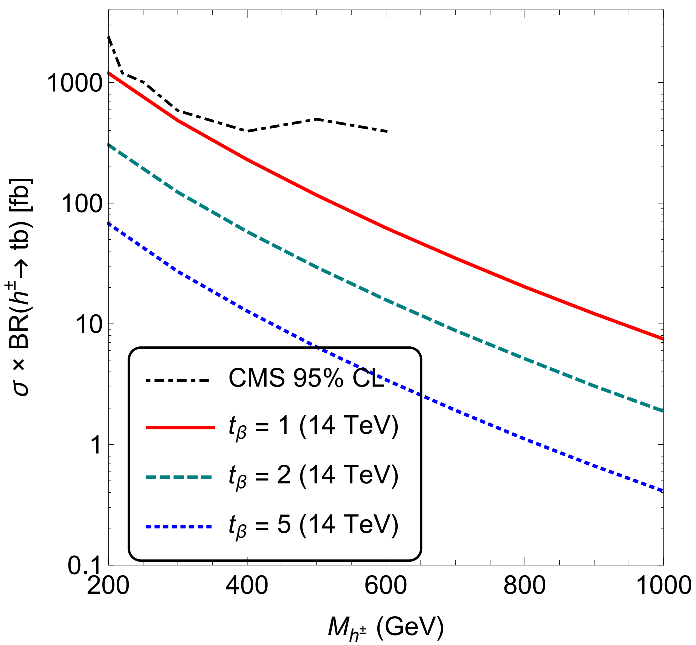
$\longrightarrow$ Misalignment predictions consistent with experiment

• Phenomenological implications at the LHC

Discovery channels for aligned Higgs doublets:

• $gg \to t\bar{b}h^- \to t\bar{b}\bar{t}b$ [Dev, AP '14]





$p_T^\ell > 20$ GeV,

$|\eta^\ell| < 2.5$,

$\Delta R^{\ell\ell} > 0.4$,

$M_{\ell\ell} > 12$ GeV,

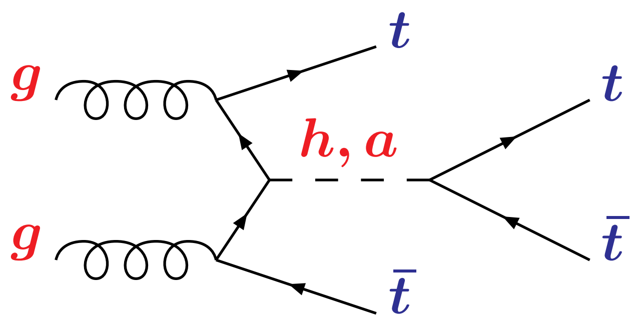
$|M_{\ell\ell} - M_Z| > 10$ GeV,

$p_T^j > 30$ GeV,

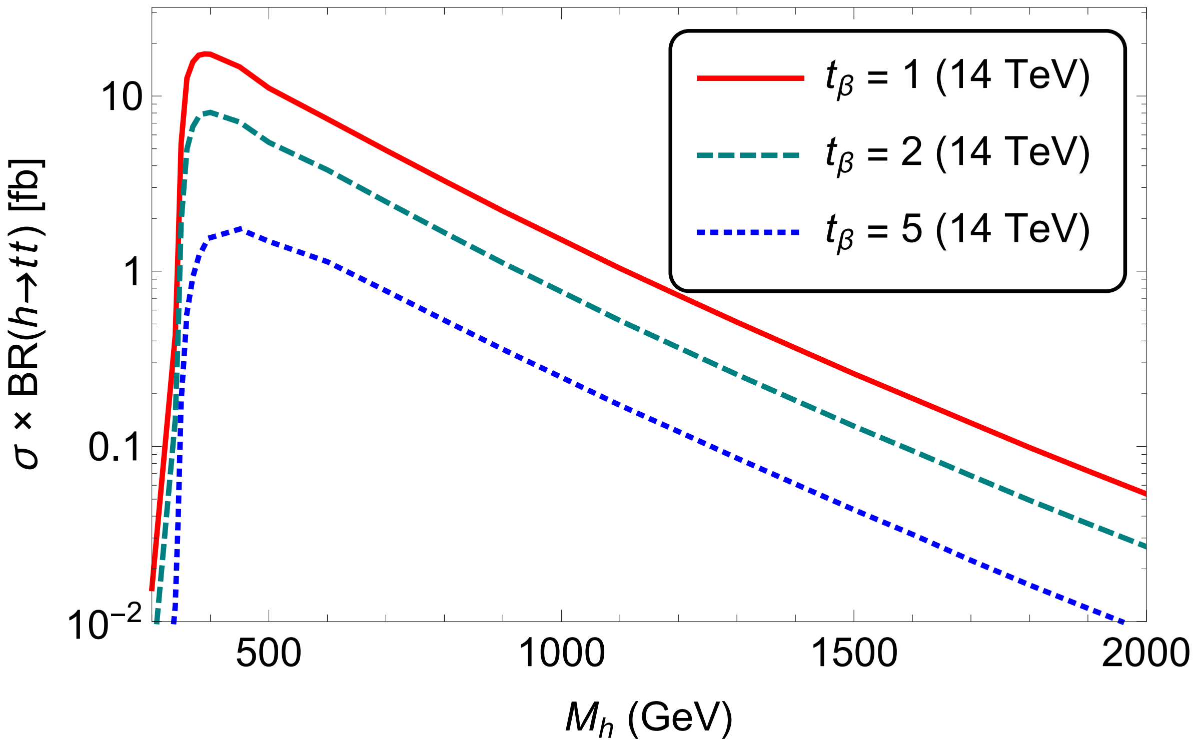
$|\eta^j| < 2.4$,

$\not{E}_T > 40$ GeV.

• $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$

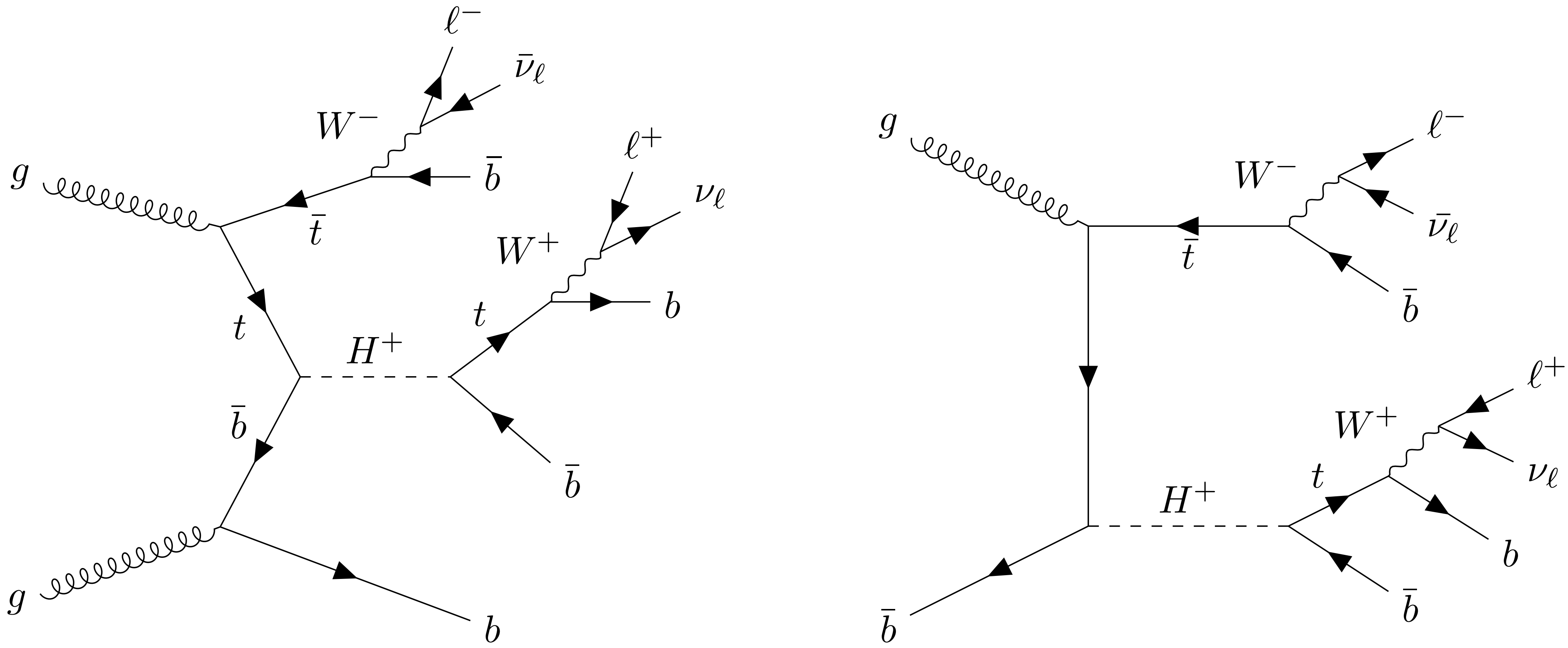


[Dev, AP '14]



Realistic simulation analysis with a reconstruction BDT

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]

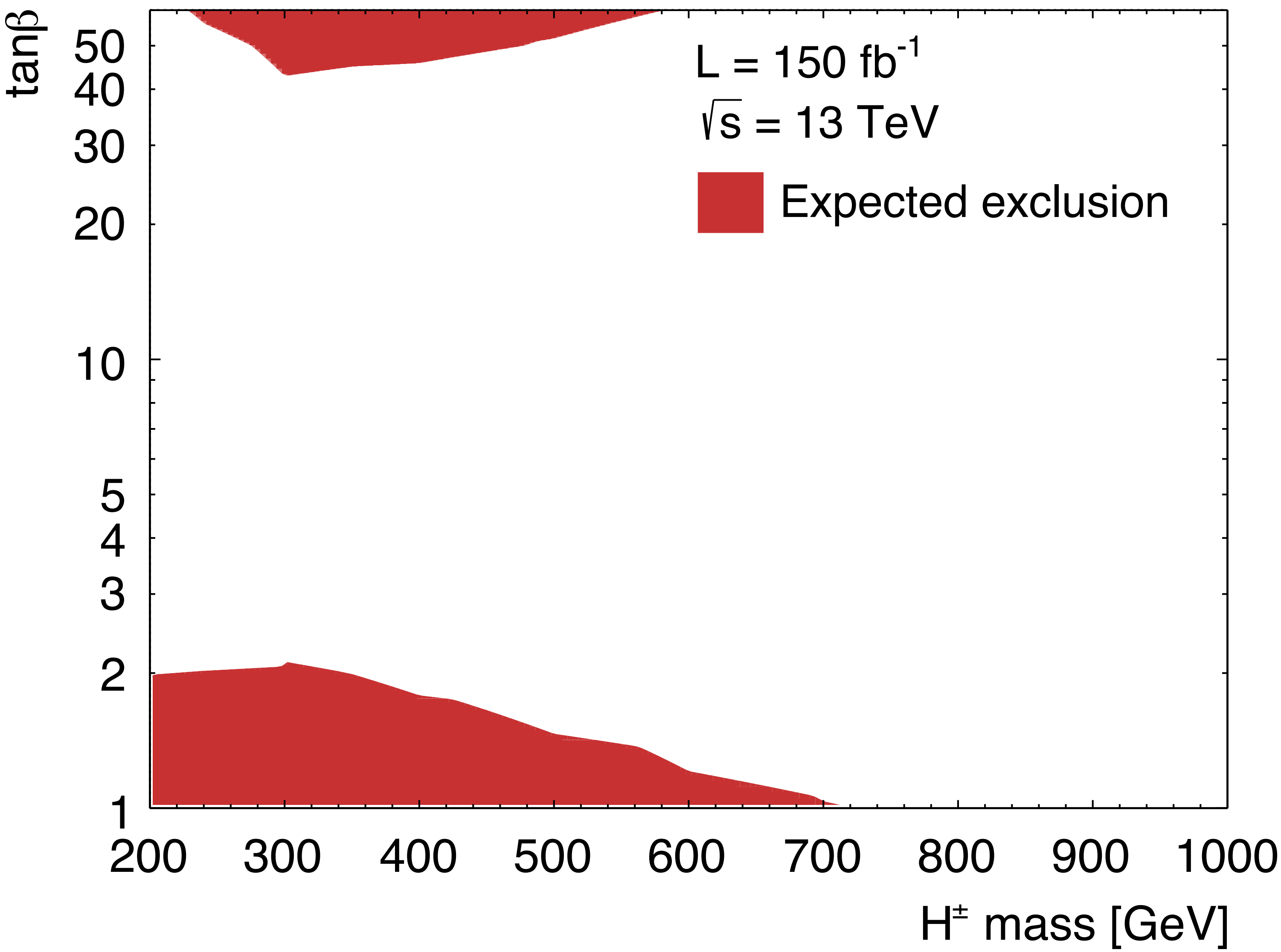


Reconstruction BDT trained on 57 observables:

- $\Delta R(b_i, l^a)$, $\Delta\eta(b_i, l^a)$, $\Delta\phi(b_i, l^a)$, $p_T^{b_i+l^a}$, $m(b_i, l^a)$, where $i = tH, t$ and $a = +, -$
- $\left|m(l^+, b_{tH}) - m(l^-, b_t)\right|$ and $\left|m(l^-, b_{tH}) - m(l^+, b_t)\right|$
- $p_T^{b_j}$, where $j = tH, H, t$
- $\Delta R(b_{tH}, b_k)$, $\Delta\eta(b_{tH}, b_k)$, $\Delta\phi(b_{tH}, b_k)$, $p_T^{b_{tH}+b_k}$, $m(b_{tH}, b_k)$, where $k = H, t$
- $\Delta R(t_{H^a}, b_H)$, $\Delta\eta(t_{H^a}, b_H)$, $\Delta\phi(t_{H^a}, b_H)$, $p_T^{t_{H^a}, b_H}$, $m(t_{H^a}, b_H)$, where $a = +, -$
- $\Delta R(t_{H^a}, t_c)$, $\Delta\eta(t_{H^a}, t_c)$, $\Delta\phi(t_{H^a}, t_c)$, where $(H^a, t_c) = (H^+, \bar{t})$ or (H^-, t)
- $m(H^a) - m(b_H)$, where $a = +, -$
- $m(H^+) - m(\bar{t})$ and $m(H^-) - m(t)$
- $p_T^{H^\pm + t_{\text{other}}}$
- $m(H^\pm, t_{\text{other}})$

Results

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]



• Conclusions

- Symmetries for natural alignment *without* decoupling in multi-HDMs:

  (i) $\mathrm{Sp}(2N_H)$ (ii) $\mathrm{SU}(N_H)$ (iii) $\mathrm{SO}(N_H) \times \mathcal{CP}$

  $N_H > 1$: number of EWSB Higgs doublets

- Soft breaking $\longrightarrow$ minimal alignment symmetry: $\mathrm{Z}_2^{\mathrm{EW}} \times \mathrm{Z}_2^{\mathrm{I}}$
  $\rightarrow$ Naturally aligned heavy Higgs sector is $\mathrm{Z}_2^{\mathrm{EW}}$ odd.

- Quartic coupling unification for maximally symmetric nHDMs:

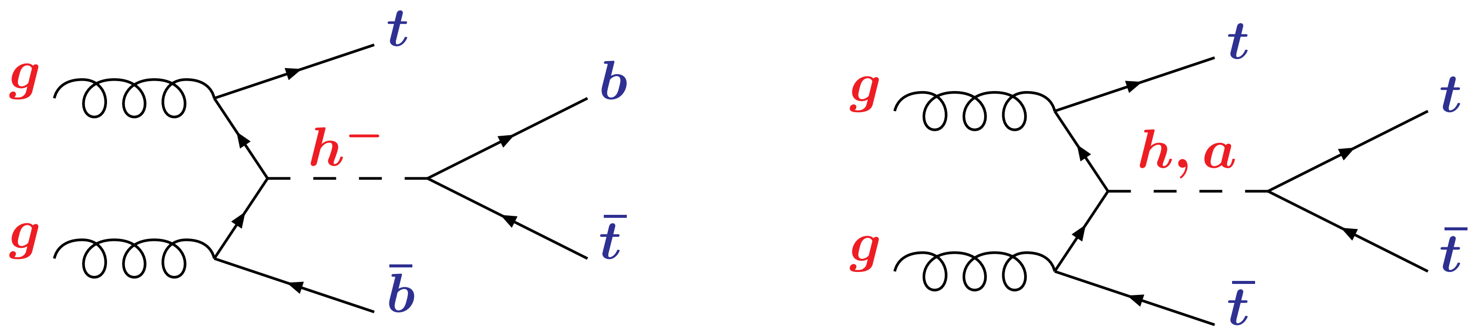
  $$G_\Phi = \mathrm{SU}(2)_L \otimes \mathrm{Sp}(2n)/\mathrm{Z}_2 \qquad \text{(here } n = 2\text{)}.$$

  INPUT: $M_{h^\pm}$ & $\tan\beta \rightarrow \mu_X^{(1)} \sim 10^{11}$ GeV & $\mu_X^{(2)} \sim 10^{19}$ GeV.

- Two-loop RG effects give rise to *definite* misalignment predictions for *all* H-couplings to SM particles in terms of $M_{h^\pm}$ & $\tan\beta$.

- **Probing new aligned Higgs doublets via the production channels:**

  (a) $gg \to t\bar{b}h^- \to t\bar{b}\bar{t}b$ **(b)** $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$



More experimental analyses needed

Back-Up Slides

• Symmetries of the 2HDM Potential

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

Introduce the $\mathrm{SU}(2)_L$-covariant 8D complex field multiplet

$$\mathbf{\Phi} \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix} , \quad \text{with} \;\; U_L \in \;\mathrm{SU}(2)_L : \;\mathbf{\Phi} \;\mapsto\; \mathbf{\Phi}' \;=\; U_L\,\mathbf{\Phi}\;.$$

$\mathbf{\Phi}$ satisfies the **Majorana constraint**

$$\mathbf{\Phi} \;=\; \mathrm{C}\,\mathbf{\Phi}^*\;,$$

where C is the **charge conjugation** 8D matrix

$$\mathrm{C} \;=\; \sigma^2 \otimes \sigma^0 \otimes \sigma^2 \;=\; \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\,\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2)\;.$$

• The SO(1,5) Bilinear Formalism

Introduce the *null* 6-Vector

$$\mathrm{R}^{\mathrm{A}} = \mathbf{\Phi}^{\dagger}\Sigma^{\mathsf{A}}\mathbf{\Phi} = \begin{pmatrix} \phi_1^{\dagger}\phi_1 + \phi_2^{\dagger}\phi_2 \\ \phi_1^{\dagger}\phi_2 + \phi_2^{\dagger}\phi_1 \\ -i\left[\phi_1^{\dagger}\phi_2 - \phi_2^{\dagger}\phi_1\right] \\ \phi_1^{\dagger}\phi_1 - \phi_2^{\dagger}\phi_2 \\ \phi_1^{\mathsf{T}} i\sigma^2\phi_2 - \phi_2^{\dagger} i\sigma^2\phi_1^{*} \\ -i\left[\phi_1^{\mathsf{T}} i\sigma^2\phi_2 + \phi_2^{\dagger} i\sigma^2\phi_1^{*}\right] \end{pmatrix},$$

with $\mathsf{A} = \mu,\ 4,\ 5$ and

$$\Sigma^{\mu} = \frac{1}{2}\begin{pmatrix} \sigma^{\mu} & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^{\mu})^{\mathsf{T}} \end{pmatrix} \otimes \sigma^0\,,$$

$$\Sigma^{4} = \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,, \qquad \Sigma^{5} = \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,.$$

• The 2HDM Potential in the SO(1,5) Formalism

$$\mathrm{V} \;=\; -\,\frac{1}{2}\,\mathrm{M_A}\,\mathrm{R^A} \;+\; \frac{1}{4}\,\mathrm{L_{AB}}\,\mathrm{R^A}\,\mathrm{R^B}\;,$$

with

$$\mathrm{M}_{\mathsf{A}} \;=\; \left(\,\mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2\,,\quad 0\,,\quad 0\,\right)\;,$$

$$\mathrm{L}_{\mathsf{AB}} \;=\; \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) & 0 & 0 \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\,.$$

• The 2HDM Potential in the SO(1,5) Formalism

$$\mathrm{V} \;=\; -\,\frac{1}{2}\,\mathrm{M_A\,R^A} \;+\; \frac{1}{4}\,\mathrm{L_{AB}\,R^A\,R^B}\;,$$

with

$$\mathrm{M}_{\mathsf{A}} \;=\; \Big(\,\mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2\,,\quad 0\,,\quad 0\,\Big)\;,$$

$$\mathrm{L}_{\mathsf{AB}} \;=\; \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) & 0 & 0 \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

• Unitary Field Transformations:

[AP, Phys. Lett. B706 (2012) 465.]

$$\mathsf{Sp}(4):\qquad \mathbf{\Phi}\;\mapsto\;\mathbf{\Phi}'\;=\;U\,\mathbf{\Phi}\,,\qquad \text{with}\quad U\in\mathsf{U}(4)\quad \underline{\text{and}}\quad UCU^{\mathsf{T}}=\mathsf{C}$$

$$\mathsf{SO}(5):\qquad \mathsf{R}^{\mathsf{I}}\;\mapsto\;\mathsf{R}'^{\mathsf{I}}\;=\;\mathsf{O}^{\mathsf{I}}{}_{\mathsf{J}}\,\mathsf{R}^{\mathsf{J}}\,,\qquad \text{with}\quad \mathsf{O}\in\mathsf{SO}(5)\;\subset\;\mathsf{SO}(1,5)$$

$$\Longrightarrow\qquad \mathsf{SO}(5)\;\sim\;\mathsf{Sp}(4)/\mathbf{Z}_2$$

• Symmetries of the U(1) $_Y$-Invariant 2HDM Potential

SO(5)-diagonally reduced basis: Im $\lambda_5 = 0$ and $\lambda_6 = \lambda_7$.

The 2HDM potential exhibits a **total** of **13** accidental symmetries:

Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	Re λ_5	$\lambda_6 = \lambda_7$
$(\mathrm{Z}_2)^2 \times \mathrm{SO}(2)$	–	–	0	–	–	–	–	–	0
$\mathrm{O}(2) \times \mathrm{O}(2)$	–	–	0	–	–	–	–	0	0
✓ $\mathbf{O(3) \times O(2)}$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0
$\mathrm{Z}_2 \times \mathrm{O}(2)$	–	–	Real	–	–	–	–	–	Real
$(\mathrm{Z}_2)^3 \times \mathrm{O}(2)$	–	μ_1^2	0	–	λ_1	–	–	–	0
✓ $\mathbf{Z_2 \times [O(2)]^2}$	–	μ_1^2	0	–	λ_1	–	–	$2\lambda_1 - \lambda_{34}$	0
✓ **SO(5)**	–	μ_1^2	0	–	λ_1	$2\lambda_1$	0	0	0
$\mathrm{Z}_2 \times \mathrm{O}(4)$	–	μ_1^2	0	–	λ_1	–	0	0	0
$\mathrm{SO}(4)$	–	–	0	–	–	–	0	0	0
$\mathrm{O}(2) \times \mathrm{O}(3)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	–	0	0
$(\mathrm{Z}_2)^2 \times \mathrm{SO}(3)$	–	μ_1^2	0	–	λ_1	–	–	$\pm\lambda_4$	0
$\mathrm{Z}_2 \times \mathrm{O}(3)$	–	μ_1^2	Real	–	λ_1	–	–	λ_4	Real
$\mathrm{SO}(3)$	–	–	Real	–	–	–	–	λ_4	Real

✓: Natural SM Alignment $\mapsto$

[Dev, AP, JHEP1412 (2014) 024.]

• Symmetry Breaking Scenarios and pseudo-Goldstone Bosons

[AP, Phys. Lett. B706 (2012) 465.]

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo-Goldstone Bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	$T^3,\ T^0$	–	T^3	1 (a)
✓ 5	$Z_2 \times [O(2)]^2$	$T^2,\ T^0$	D_{CP1}	T^2	1 (h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3},\ T^0$	–	$T^{1,2}$	2 $(h,\ a)$
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3,\ T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a,\ h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a,\ h^\pm)$
✓ 13	SO(5)	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h,\ a,\ h^\pm)$

✓: Natural SM Alignment $\mapsto$ [Dev, AP, JHEP1412 (2014) 024.]