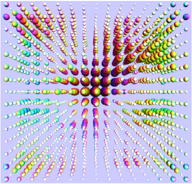
Lattice gauge theory beyond the standard model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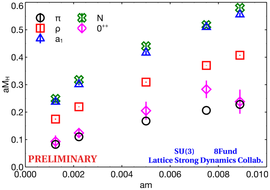

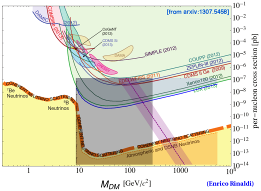

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Planck 2017, 22 May

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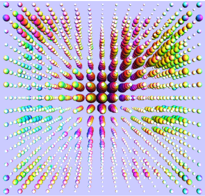
and now for something completely different

Overview

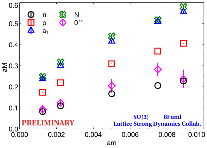
Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems

Lattice calculations are especially important
when QCD-based intuition may be unreliable



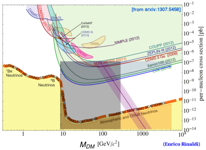
(Image credit: Claudio Rebbi)

- A high-level summary of lattice gauge theory



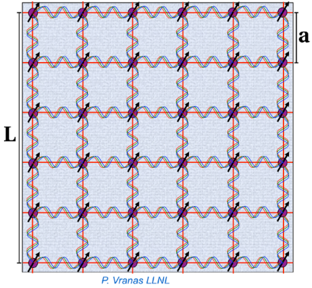
- High-precision non-perturbative QCD (briefly)

- Non-QCD strong dynamics
  - Near-conformal dynamics for composite Higgs
  - Bosonic baryons for composite dark matter



The essence of lattice gauge theory

Lattice discretization is a non-perturbative regularization of QFT



Formulate theory on a finite, discrete euclidean spacetime $\longrightarrow$ **the lattice**

Spacing between lattice sites ("a") introduces UV cutoff scale $1/a$

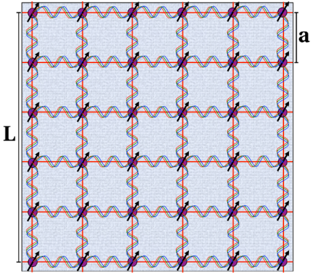
Remove cutoff by taking continuum limit: $a \to 0$ with $L/a \to \infty$

Finite number of degrees of freedom $\left(\sim 10^9\right)$

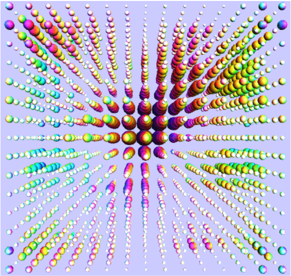
$\longrightarrow$ numerically compute observables via importance sampling

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \qquad \longrightarrow \qquad \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}(\Phi_k)$$

Features of lattice gauge theory



P. Vranas LLNL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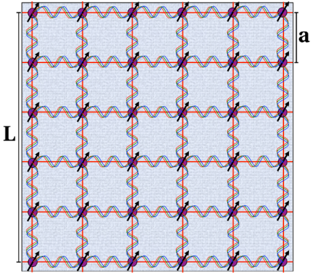
- Fully non-perturbative predictions from first principles (lagrangian)
- Fully gauge invariant—no gauge fixing required
- Applies directly in four dimensions
- Euclidean SO(4) rotations & translations ($\longrightarrow$ Poincaré symmetry) recovered automatically in the $a \to 0$ continuum limit

Limitations of lattice gauge theory



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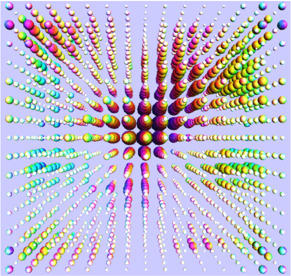
(Image credit: Claudio Rebbi)

- Lattice action discretizes UV-complete lagrangian,
  (usually) including only strong sector
- Super-Poincaré symmetry (usually) not recovered automatically
- Finite volume (usually) needs to contain all correlation lengths
  $\longrightarrow$ unphysically large masses extrapolated to chiral limit via EFT
- Obstructions to real-time dynamics, chiral gauge theories

Lattice QCD for BSM

High-precision non-perturbative QCD calculations
reduce uncertainties and help resolve potential new physics

- Hadronic matrix elements & form factors for flavor physics
  Sub-percent precision for easiest observables (arXiv:1607.00299)
- Hadronic contributions to $(g-2)_\mu$ (arXiv:1311.2198)
  Targeting $\sim$0.1% precision for vac. pol., $\sim$10% for light-by-light
- m_c, m_b and $\alpha_s(m_Z)$ to $\sim$0.1% for Higgs couplings (arXiv:1404.0319)
- High-temp. topological suscept. for axion DM (arXiv:1606.07494)
- Nucleon electric dipole moment, form factors (arXiv:1701.07792)

Lattice gauge theory beyond QCD

Lattice calculations especially important for non-QCD strong dynamics

These are exploratory investigations of representative systems
to elucidate generic dynamical phenomena & connect with EFT

arXiv:1309.1206 arXiv:1510.05018 arXiv:1701.07782

Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.

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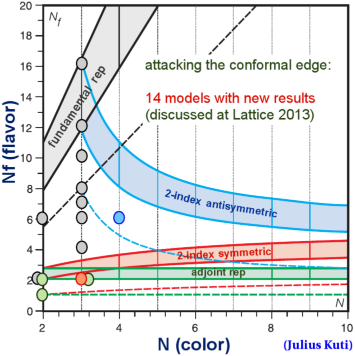
Non-QCD strong dynamics

Two main directions (not mutually exclusive)

- Near-conformal dynamics from many fermionic d.o.f.
  $\longrightarrow$ large number of fundamental fermions or a few in a larger rep
- Different symmetries from different gauge group or reps
  $\longrightarrow$ (pseudo)real reps for cosets SU(n)/Sp(n) or SU(n)/SO(n)



All near-conformal lattice studies so far observe a very light singlet scalar qualitatively different from QCD

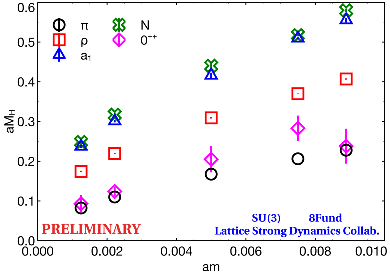
Example: SU(3) with $N_F = 8$ fund.
work by LSD Collaboration
(arXiv:1601.04027, arXiv:1702.00480)

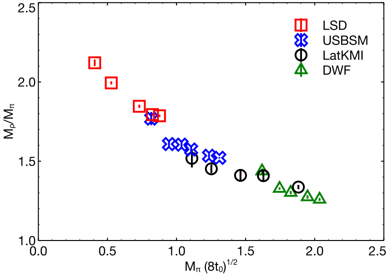
Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	DS
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Colorado	Anna Hasenfratz, Ethan Neil
UC Davis	Joseph Kiskis
Edinburgh	Oliver Witzel
Livermore	Pavlos Vranas
Oregon	Graham Kribs
RBRC	Enrico Rinaldi
Yale	Thomas Appelquist, George Fleming, Andrew Gasbarro

Exploring the range of possible phenomena
in strongly coupled gauge theories

Light scalar in 8-flavor spectrum





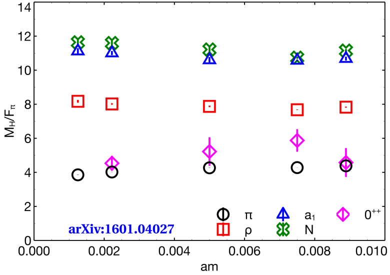
Flavor-singlet scalar degenerate with pseudo-Goldstones down to lightest fermion masses we can fit into $64^3 \times 128$ lattices

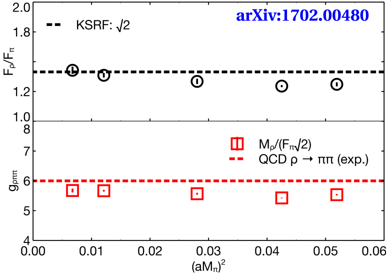
Both M_S and M_P are less than half the vector mass M_V, and the hierarchy is growing as we approach the chiral limit

This is very different from QCD

Controlled chiral extrapolations need EFT that includes scalar...

Another generic feature: broad vector resonance





Without EFT, roughly constant ratio $M_V/F_P \simeq 8 \rightsquigarrow M_V \simeq 2$ TeV$/\sin\theta$

[NB: expect $M_P/F_P \to 0$ in chiral limit!]

We measure $F_V \approx F_P\sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P\sqrt{2})$

gives vector width $\Gamma_V = \dfrac{g_{VPP}^2 M_V}{48\pi} \simeq 450$ GeV — hard to see at LHC

Work in progress: Constraining EFT

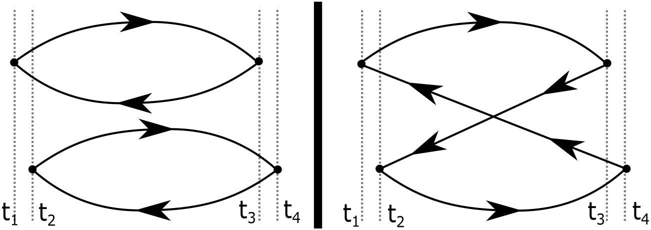
There are many candidate EFTs that include PNGBs + light scalar

(linear σ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaeble–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

We are now computing $2 \to 2$ elastic scattering of PNGBs & scalar, as well as scalar form factor of PNGB

Subsequent step: Analog of πK scattering in mass-split system



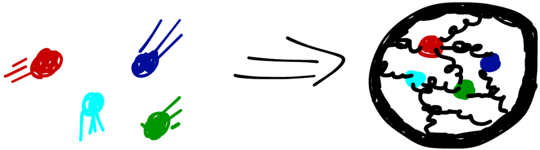
Composite dark matter

Many possibilities: (arXiv:1604.04627)

dark baryon, dark nuclei, dark pion, dark quarkonium, dark glueball. . .

Example: Stealth Dark Matter (arXiv:1503.04203, arXiv:1503.04205)

- Deconfined charged fermions produce relic densnity
- Confined SM-singlet dark baryon detectable via form factors



For QCD-like SU(3) model, direct detection constrains $M_{DM} \gtrsim 20$ TeV due to leading magnetic moment interaction (arXiv:1301.1693)

A lower bound for stealth dark matter

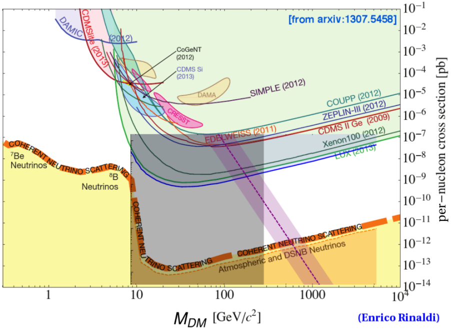
SU(4) bosonic baryons forbid both leading magnetic moment
and sub-leading charge radius interactions in non-rel. EFT

The polarizability is unavoidable — compute it on the lattice
to place a lower bound on the direct detection rate

Nuclear cross section $\propto Z^4/A^2$,
these results specific to Xenon

Uncertainties dominated
by nuclear matrix element

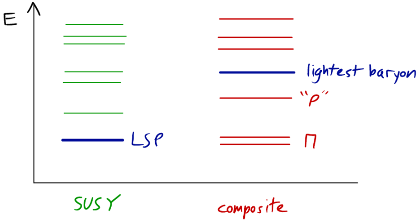
Shaded region is complementary
constraint from particle colliders

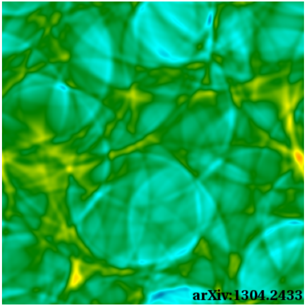


Future plans: Colliders and gravitational waves

Other composite dark-sector states
can be discovered at colliders

Additional lattice input can help
predict production and decays





Confinement transition in early universe
may produce gravitational waves

First-order transition $\longrightarrow$ colliding bubbles

Lattice calculations needed
to predict properties of transition

Outlook: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems and BSM physics

- High-precision non-perturbative QCD helps resolve new physics
- Exploring generic features of representative systems beyond QCD
  - Near-conformal dynamics with connections to composite Higgs
  - SU(4) bosonic baryons make composite dark matter stealthier

Thank you!

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Thank you!

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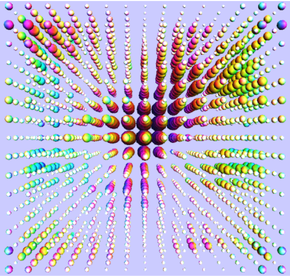
Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations Φ_k with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_k]}$



(Image credit: Claudio Rebbi)

HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

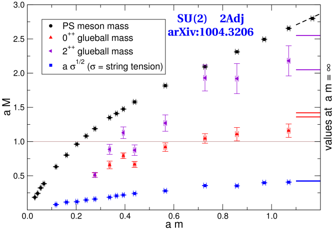
Fermions $\longrightarrow$ extensive action computation, so best to update entire system at once

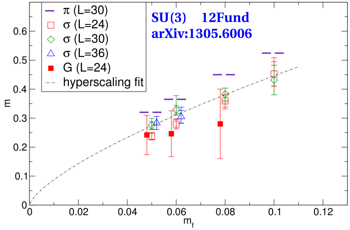
Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension ("MD time" τ) and stochastic canonical momenta for all field variables
2. Run inexact MD evolution along a trajectory in τ to generate new four-dimensional field configuration
3. Apply MRT accept/reject test to MD discretization error

Backup: Light scalars beyond QCD

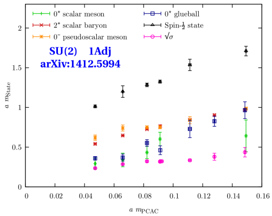
Not so shocking in mass-deformed IR-conformal theories

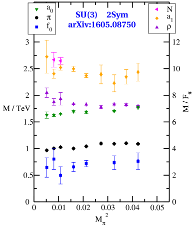


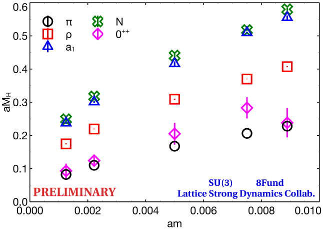


More surprising in systems apparently exhibiting

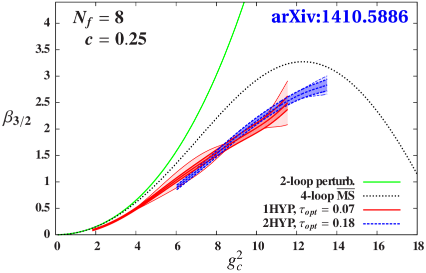
spontaneous chiral symmetry breaking

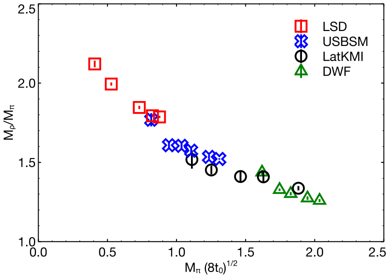






Backup: 8-flavor SU(3) infrared dynamics





- β function is monotonic up to fairly strong couplings
  No sign of approach towards conformal IR fixed point $[\beta(g_\star^2) = 0]$

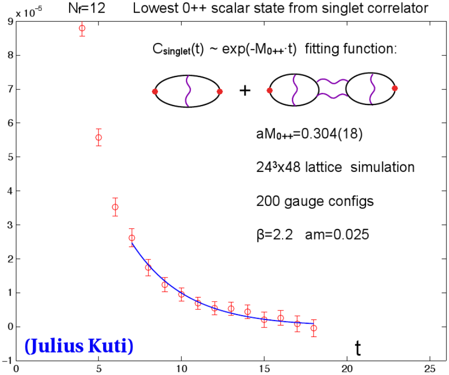
- Ratio M_V/M_P increases monotonically as masses decrease
  as expected for spontaneous chiral symmetry breaking (SχSB)
  Mass-deformed conformal hyperscaling predicts constant ratio

Want to strengthen conclusion by matching to low-energy EFT,
but must go beyond QCD-like χPT to include light scalar…

Backup: Technical challenge for scalar on lattice

Only the new strong sector is included in the lattice calculation
$\Longrightarrow$ The flavor-singlet scalar mixes with the vacuum

Leads to noisy data and relatively large uncertainties

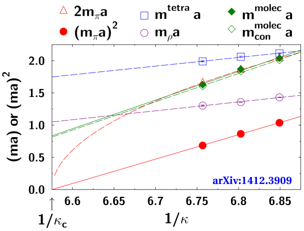


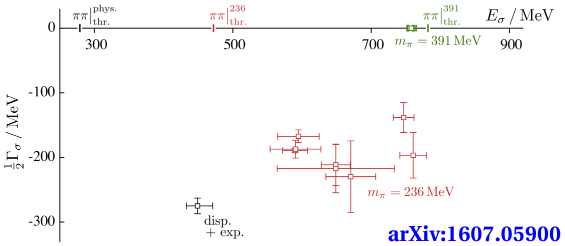
Fermion propagator computation is relatively expensive

"Disconnected diagrams" formally need propagators at all L^4 sites

In practice estimate stochastically to control computational costs

Backup: Isosinglet scalar in QCD spectrum





In lattice QCD, the isosinglet scalar is much heavier than the pion

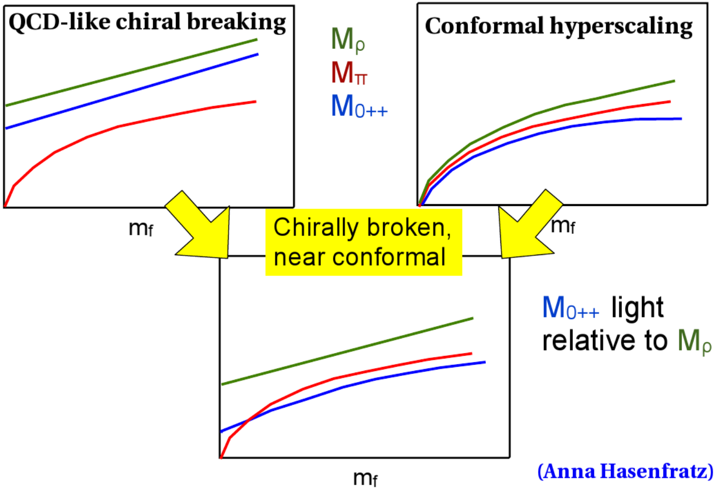
Generally $M_S \gtrsim 2M_P$, and for heavy quarks $M_S > M_V$

For a large range of quark masses m

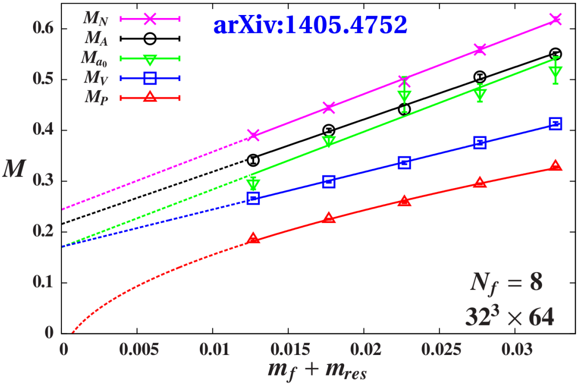
it mixes significantly with two-pion scattering states

Backup: Qualitative picture of light scalar

Light scalar likely related to near-conformal dynamics
(unconfirmed interpretation as PNGB of approx. scale symmetry)



Backup: Non-singlet scalar for $N_F = 8$



In earlier work with domain wall fermions at heavier fermion masses the non-singlet scalar is heavier than the vector, $M_{a_0} \gtrsim M_V$

Staggered analyses in progress, but more complicated

Backup: $2 \to 2$ elastic scattering on the lattice

Measure both E_{PP} and $M_P \longrightarrow k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

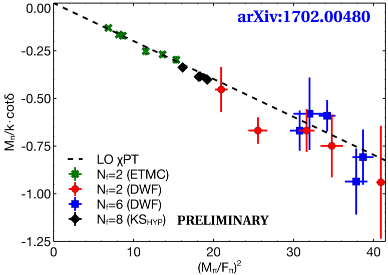
s-wave scattering phase shift: $\cot\delta_0(k) = \frac{1}{\pi k L}\, S\left(\frac{k^2 L^2}{4\pi}\right)$

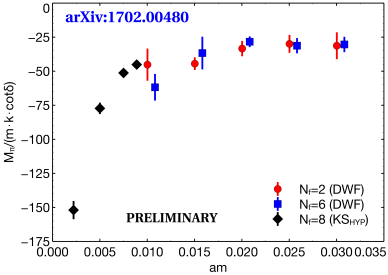
with regularized ζ function $S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$

Effective range expansion:

$$k \cot\delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left(\frac{k^2}{M_P^2}\right) + \mathcal{O}\left(\frac{k^4}{M_P^4}\right)$$

Backup: Initial $2 \to 2$ elastic scattering results





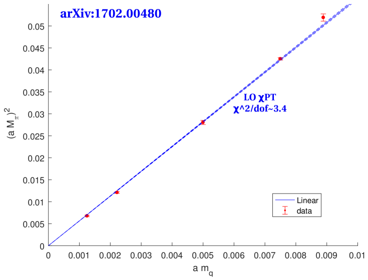
First looking at analog of QCD $\pi\pi$ scattering in $I = 2$ channel
(simplest case with no fermion-line-disconnected diagrams)

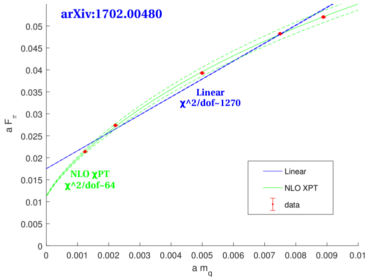
Simplest observable is scattering length $a_{PP} \approx 1/(k \cot \delta)$

$M_P a_{PP}$ vs. M_P^2/F_P^2 curiously close to leading-order χPT prediction

Dividing by fermion mass m reveals expected tension with χPT
which predicts $M_P a_{PP}/m = \text{const.}$ at LO and involves 8 LECs at NLO

Backup: 8f chiral perturbation theory (χPT) fits





In addition to omitting the light scalar

χPT also suffers from large expansion parameter

$$5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889$$

Big ($\sim$50σ) shift in F from linear extrapolation vs. NLO χPT

Fit quality is not good, especially for NLO joint fit with χ^2/d.o.f. $> 10^4$

Backup: NLO chiral perturbation theory formulas

$$M_P^2 = 2Bm\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{128\pi^2\left(2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F}\right) + \frac{\log\left(2Bm/\mu^2\right)}{N_F^2}\right\}\right]$$

$$F_P = F\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{64\pi^2\left(L_4^r + \frac{L_5^r}{N_F}\right) - \frac{1}{2}\log(2Bm/\mu^2)\right\}\right]$$

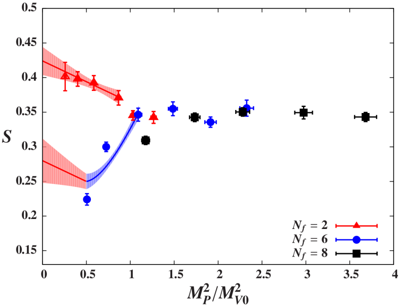
$$\begin{aligned} M_P a_{PP} = \frac{-2Bm}{16\pi F^2}\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{-256\pi^2\left(\left[1 - \frac{2}{N_F}\right](L_4^r - L_6^r)\right.\right.\right. \\ \left. + \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F}\right) - 2\frac{N_F - 1}{N_F^3} \\ \left.\left. + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3}\log\left(2Bm/\mu^2\right)\right\}\right] \end{aligned}$$

Backup: The S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

Non-zero masses and chiral extrapolation needed
to avoid sensitivity to finite lattice volume



$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Moving away from QCD with larger N_F
produces significant reductions

Extrapolation to correct zero-mass limit
becomes more challenging

Backup: Vacuum polarization is just current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

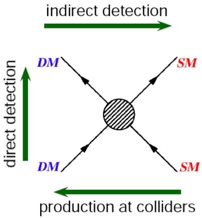
$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Backup: Thermal freeze-out for relic density



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM

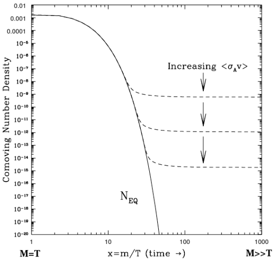
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM

Rapid depletion of Ω_{DM}

Hubble expansion $\longrightarrow$ dilution

leads to freeze-out



Requires coupling between ordinary matter and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\text{ GeV}} \sim 200\alpha$

With strong $\alpha \sim 16$, 'natural' mass scale is $M_{DM} \sim 300$ TeV

At smaller masses $M_{DM} \gtrsim 1$ TeV

thermal relic could be just part of total relic density

Backup: Two roads to natural asymmetric dark matter

Basic idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

- $n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV
  High-dimensional interactions relate baryon# and DM# violation

- $M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right]$
  Sphaleron transitions above $T_s \sim 200$ GeV distribute asymmetries

Both require coupling between ordinary matter and dark matter

Backup: Composite dark matter interactions

Photon exchange via electromagnetic form factors

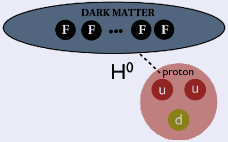
Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow \left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow \left(\overline{\psi}\psi\right) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow \left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs exchange via scalar form factors

Effective Higgs interaction of composite DM
needed for correct Big Bang nucleosynthesis

Higgs couples through $\left\langle B \left| m_\psi \overline{\psi}\psi \right| B \right\rangle$ (σ terms)

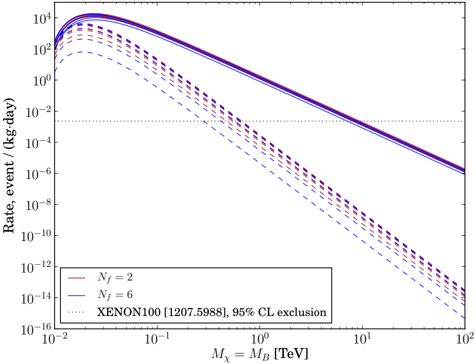


All form factors arise non-perturbatively $\Longrightarrow$ lattice calculations

Backup: SU(3) direct detection constraints

Solid lines are predictions for total number of events XENON100 would observe for SU(3) model with dark baryon mass M_B

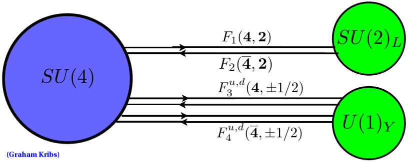
Dashed lines are subleading charge radius contribution suppressed $\sim 1/M_B^2$ relative to magnetic moment contribution



XENON100 results (arXiv:1207.5988) exclude $M_B \lesssim 10$ TeV

SU(N) with even $N \geq 4$ forbids mag. moment...

Backup: Stealth dark matter model details



Field	$SU(N_D)$	$(SU(2)_L,\ Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\sim m_V\,(F_1F_2 + F_3F_4) + y\,\left(F_1 \cdot HF_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Both vector-like masses m_V and Higgs couplings y are **required**

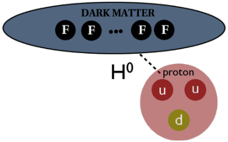
- Higgs couplings ensure rapid meson decay in early universe
- Vector-like masses avoid bounds
  on direct detection via Higgs exchange

Backup: Effective Higgs interaction

With $M_H = 125$ GeV, Higgs exchange may dominate

spin-independent direct detection cross section

$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} \; y_\psi \langle B | \overline{\psi}\psi | B \rangle \;\; y_q \langle N | \overline{q}q | N \rangle \right|^2$$



For quarks $y_q = \dfrac{m_q}{v} \Longrightarrow y_q \langle N | \overline{q}q | N \rangle \propto \dfrac{M_N}{v} \dfrac{\langle N | m_q \overline{q}q | N \rangle}{M_N}$

For dark constituent fermions ψ

there is an additional model parameter, $y_q = \alpha \dfrac{m_\psi}{v}$

In both cases the scalar form factor is most easily determined

using the Feynman–Hellmann theorem $\dfrac{\langle B | m_\psi \overline{\psi}\psi | B \rangle}{M_B} = \dfrac{m_\psi}{M_B} \dfrac{\partial M_B}{\partial m_\psi}$

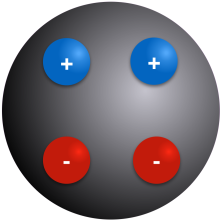
Backup: Stealth dark matter EM form factors

Lightest SU(4) composite dark baryon

Scalar particle $\longrightarrow$ no magnetic moment

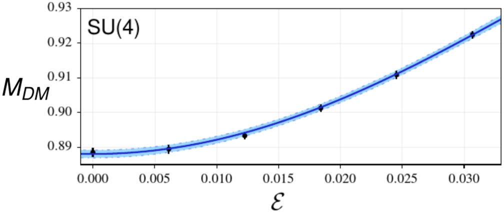
+/- charge symmetry $\longrightarrow$ no charge radius

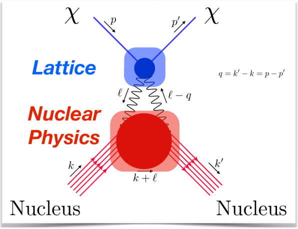
Higgs exchange can be negligibly small



Polarizability places lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



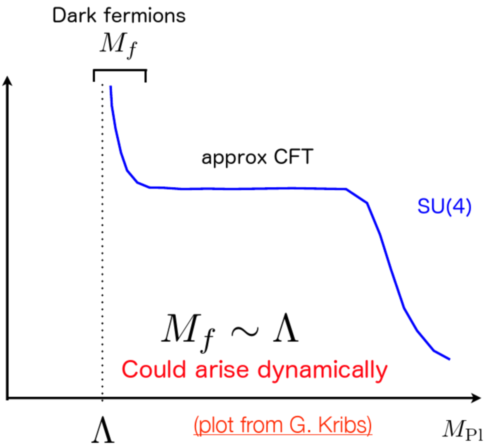


Backup: Stealth dark matter mass scales

Lattice calculations have focused on $m_\psi \simeq \Lambda_D$,
the regime where analytic estimates are least reliable

This mass scale has some theoretical motivation

In addition, collider constraints tighten as mass decreases



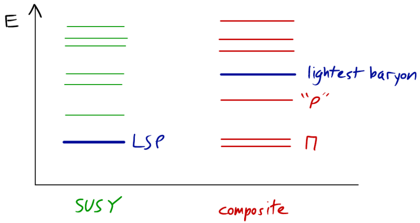
(plot from G. Kribs)

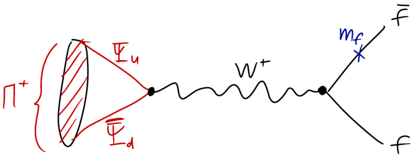
Backup: Stealth dark matter collider detection

Spectrum significantly different from MSSM-inspired models

Very little missing E_T at colliders

Main constraints from much lighter **charged** "Π" states





Rapid Π decays with $\Gamma \propto m_f^2$

Best current constraints recast stau searches at LEP

LHC can also search for $t\bar{b} + \bar{t}b$ from $\Pi^+\Pi^-$ Drell–Yan production