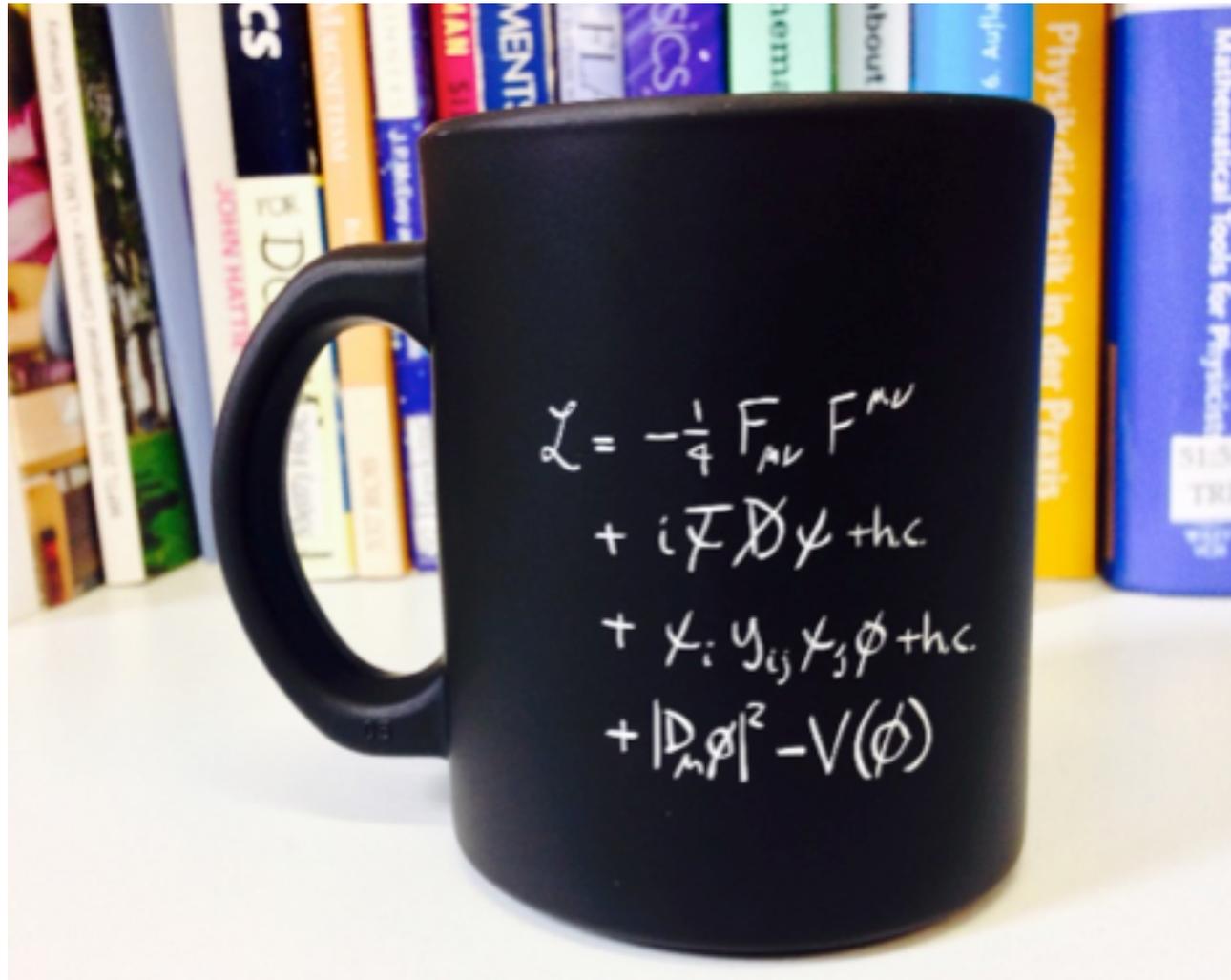


## Hierarchion, a unified explanation of the SM hierarchies and neutrino masses

Rick S Gupta (IPPP Durham)

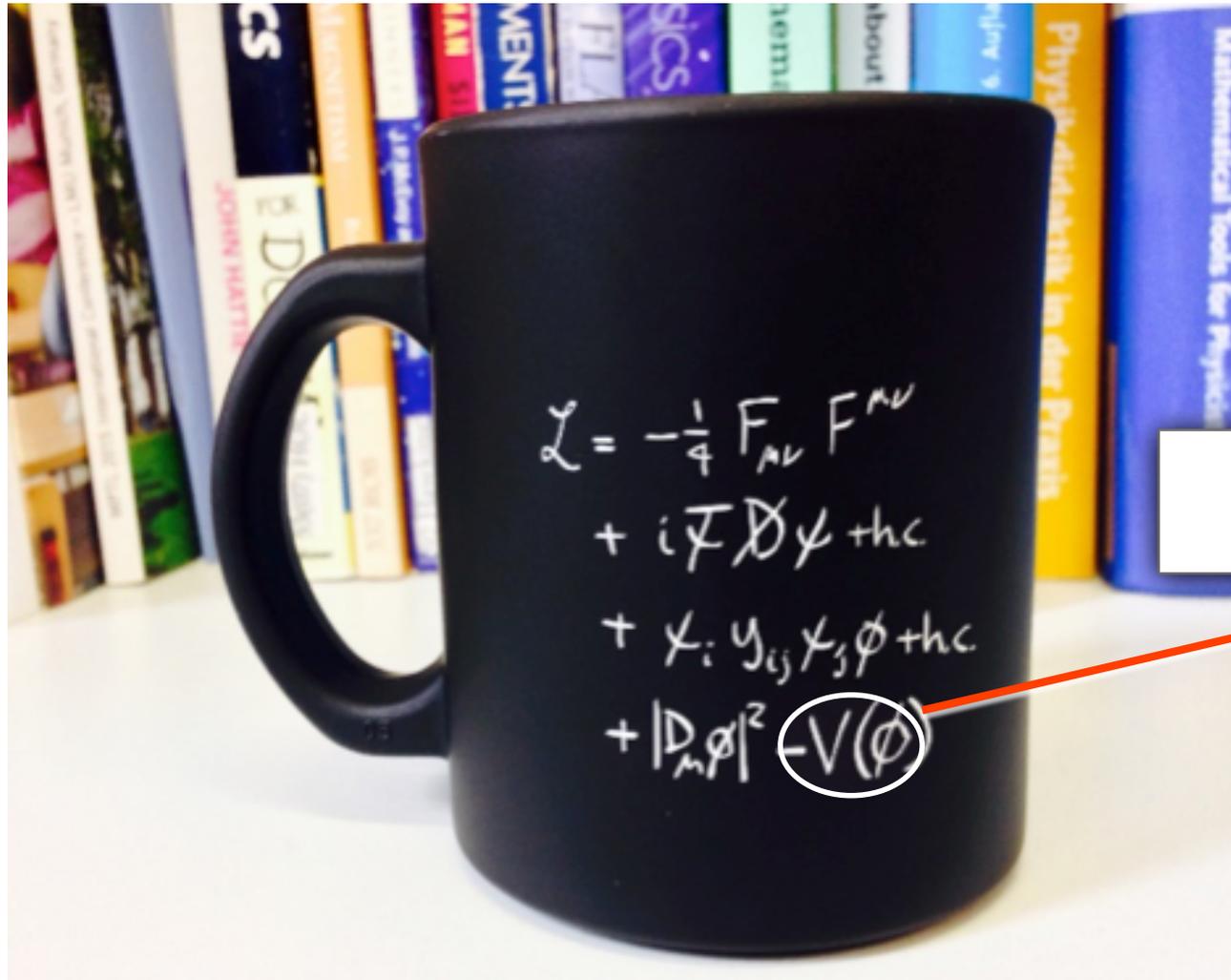
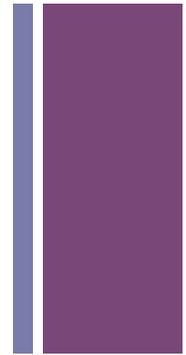
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# SM hierarchies



+

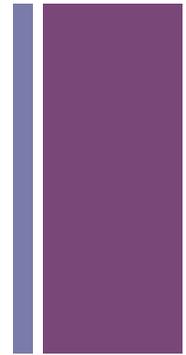
# SM hierarchies



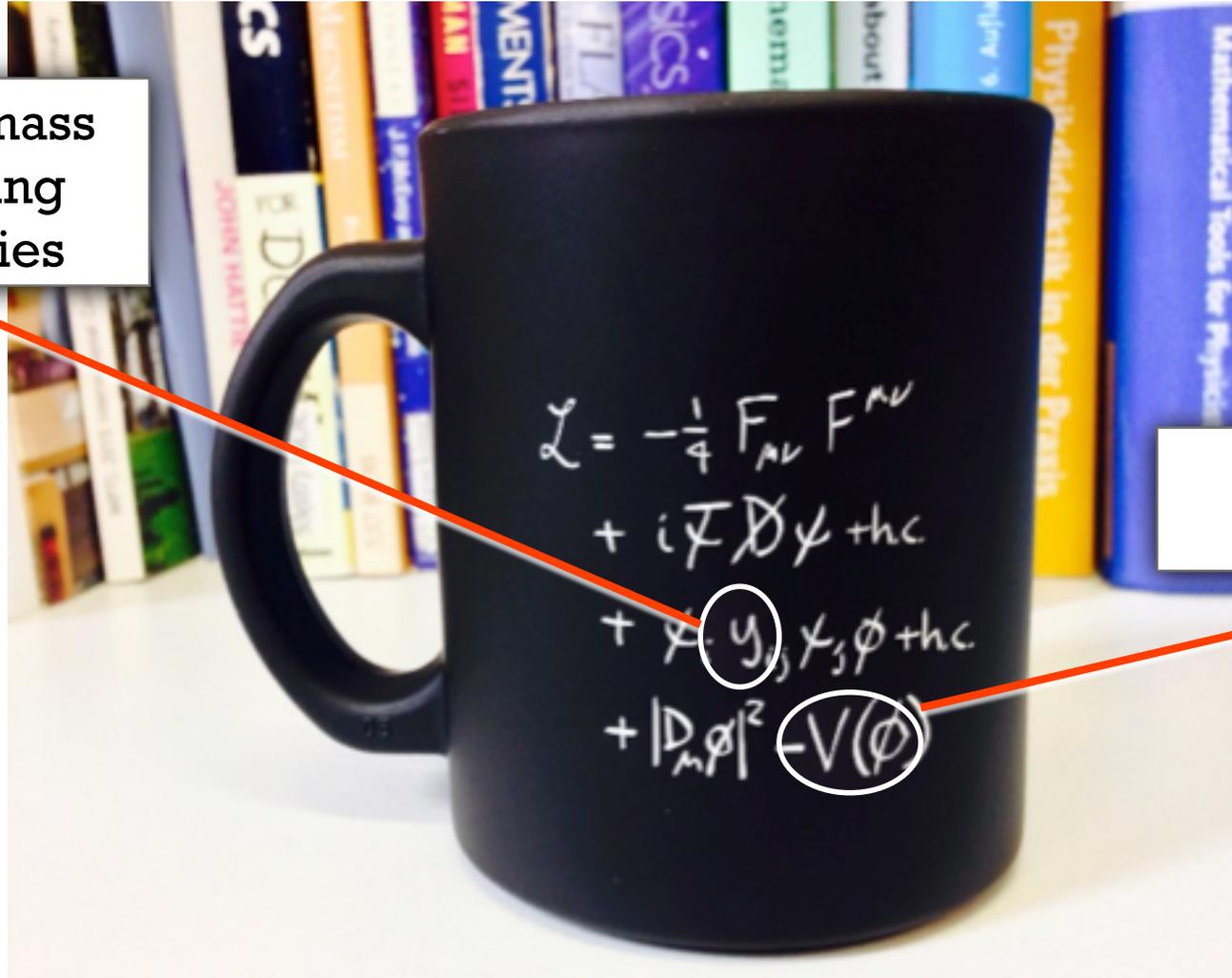
Higgs mass naturalness

+

# SM hierarchies



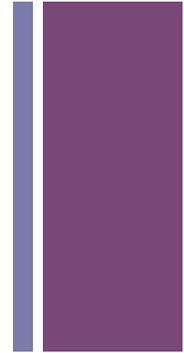
Fermion mass  
and mixing  
hierarchies



Higgs mass  
naturalness

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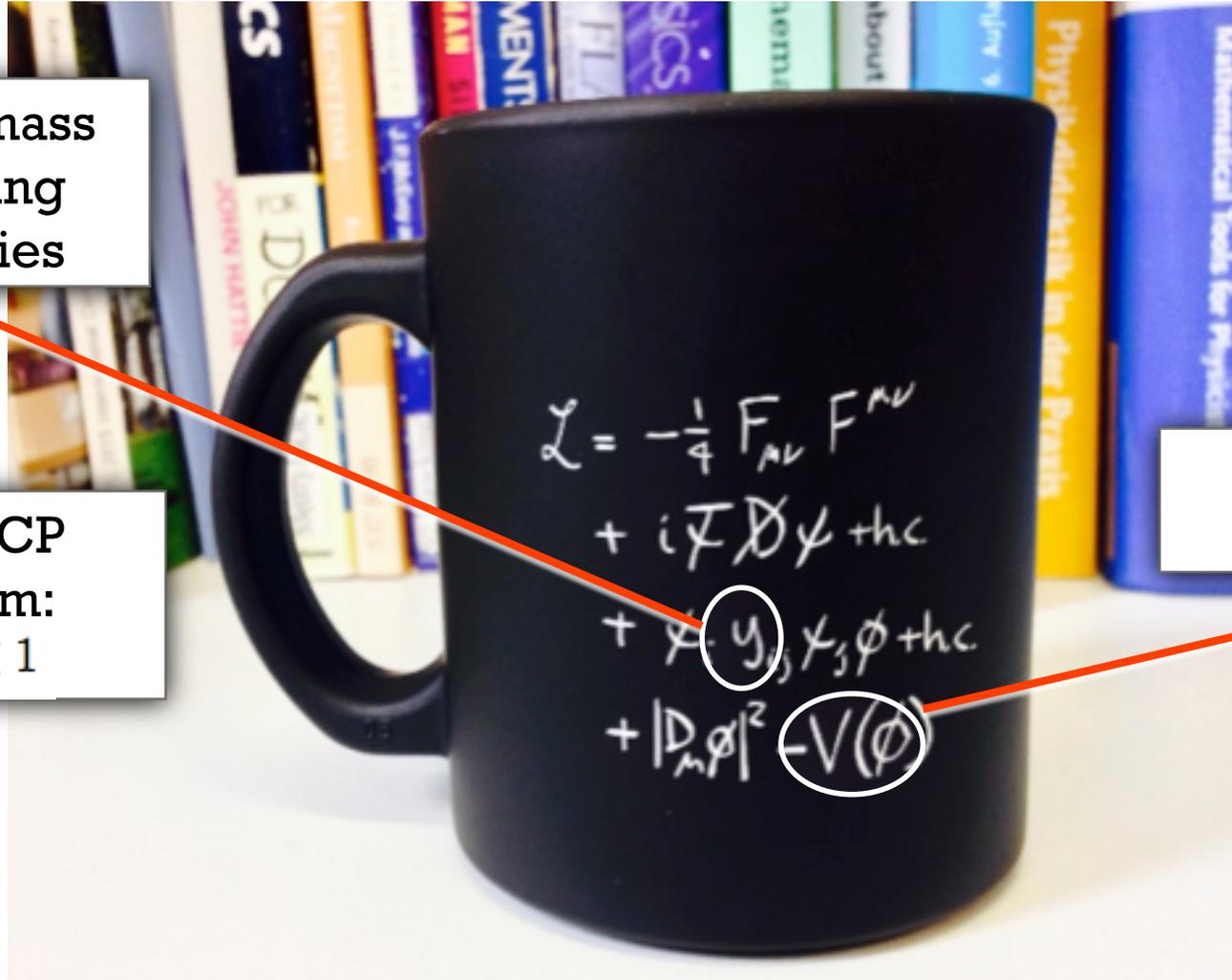
# SM hierarchies



Fermion mass  
and mixing  
hierarchies

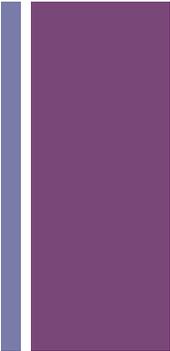
Strong CP  
problem:  
 $\theta_{\text{QCD}} \ll 1$

Higgs mass  
naturalness





# SM hierarchies+Neutrino masses

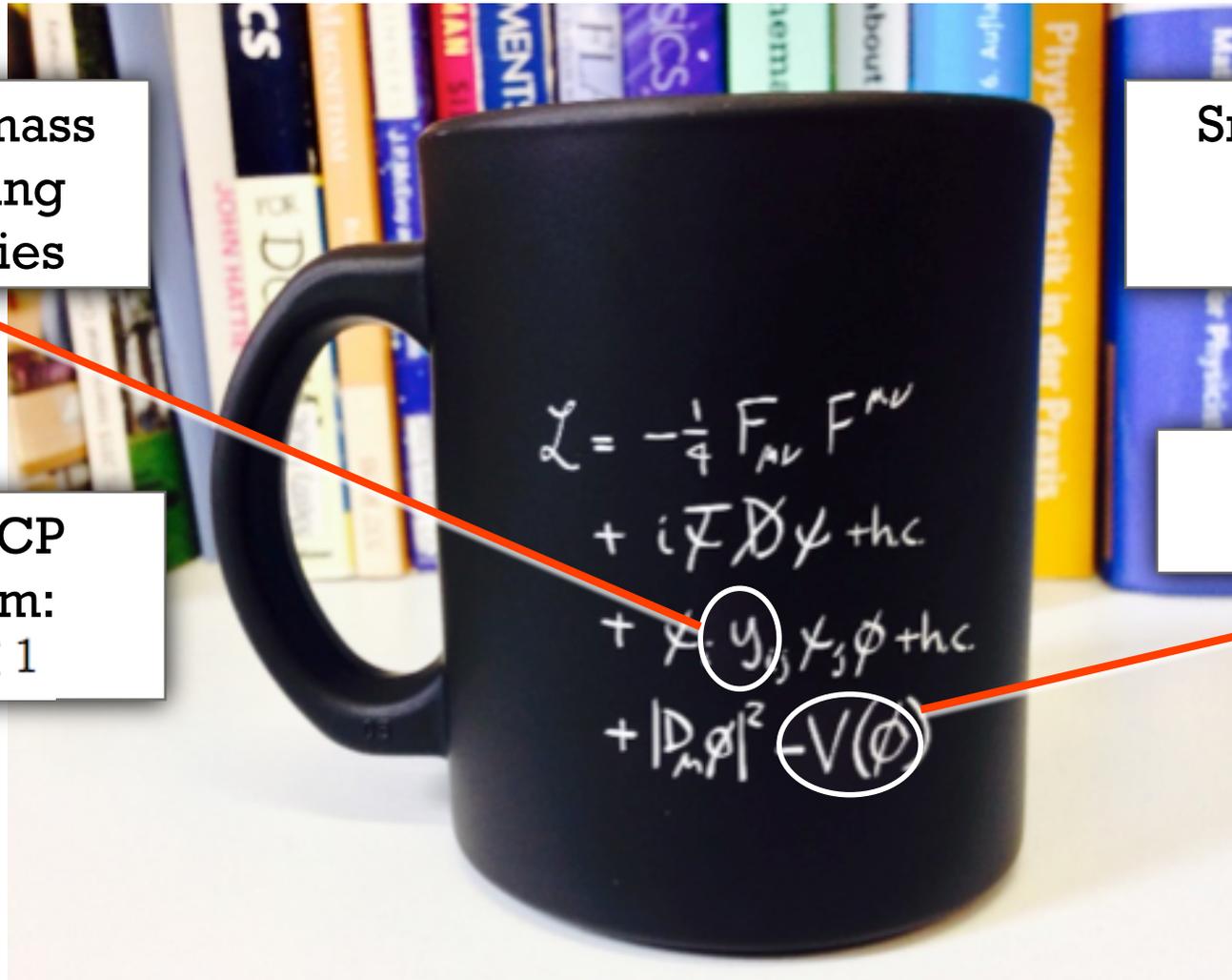


Fermion mass  
and mixing  
hierarchies

Smallness of  
neutrino  
masses

Strong CP  
problem:  
 $\theta_{\text{QCD}} \ll 1$

Higgs mass  
naturalness





# The hierarchion

- We present a model where we solve all the hierarchies in SM plus neutrino masses.
- We find a way to **combine**
  1. The **Relaxion-Clockwork solution** to **Higgs mass hierarchy** problem
  2. The **Nelson Barr Solution** to the **strong CP** problem
  3. The **Froggatt-Nielsen solution** for the **SM flavour puzzle**
  4. The **see-saw mechanism** for **neutrino masses**

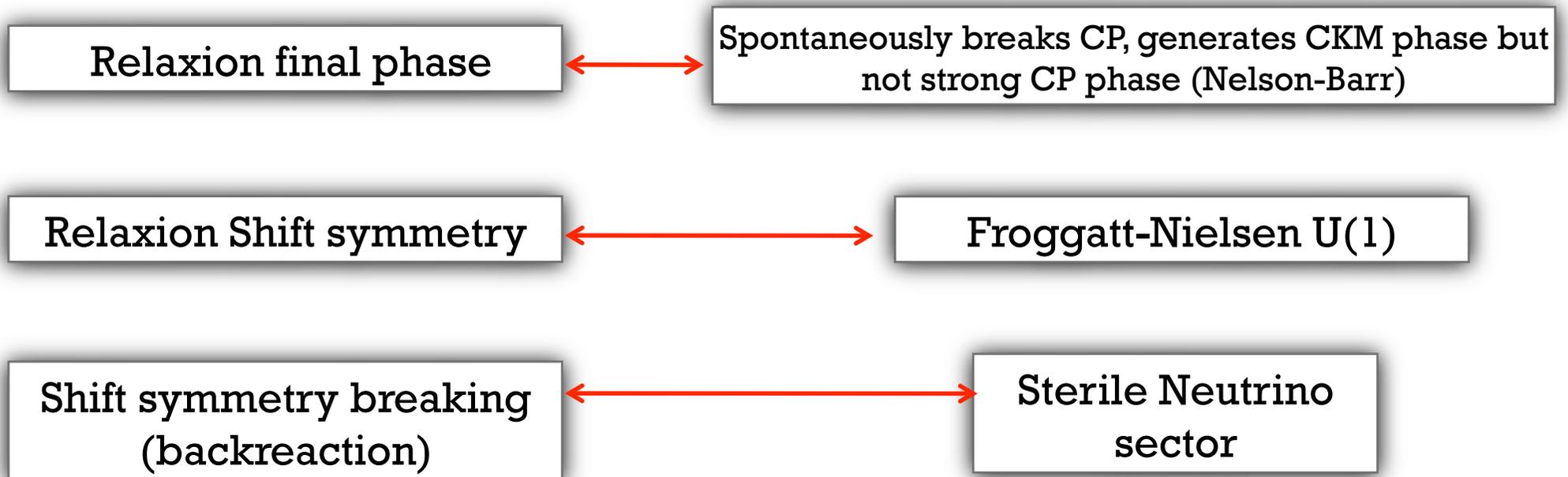
in a **unified framework** with a **single light degree of freedom** that we call the **hierarchion**.

Davidi, RSG, Perez, Redigolo and Shalit (arXiv: 1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv: 1712.XXXXX)

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# The hierarchion: basic picture

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Davidi, RSG, Perez, Redigolo and Shalit (arXiv: 1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv: 1712.XXXXX)



# The hierarchion: basic picture

Relaxion final phase



Spontaneously breaks CP, generates CKM phase but not strong CP phase (Nelson-Barr)

Relaxion Shift

The hierarchion is at the same time the *relaxion*,

the *family* of a global flavor symmetry and the *CKM phase* of NB models.

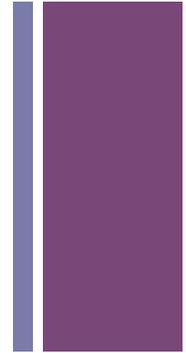
U(1)

Shift symmetry breaking  
(backreaction)

sector



# Relaxions: basic ingredients



1. In relaxion models the value of  $\mu^2$ , the **Higgs mass squared term** in the Higgs potential **changes during the course of inflation.**

$$\mu^2(\phi) = \kappa\Lambda_H^2 - \Lambda_H^2 \cos \frac{\phi}{F}$$

2. It varies with the **classical value** of a **scalar field  $\Phi$** , which **slowly rolls** because of a potential:

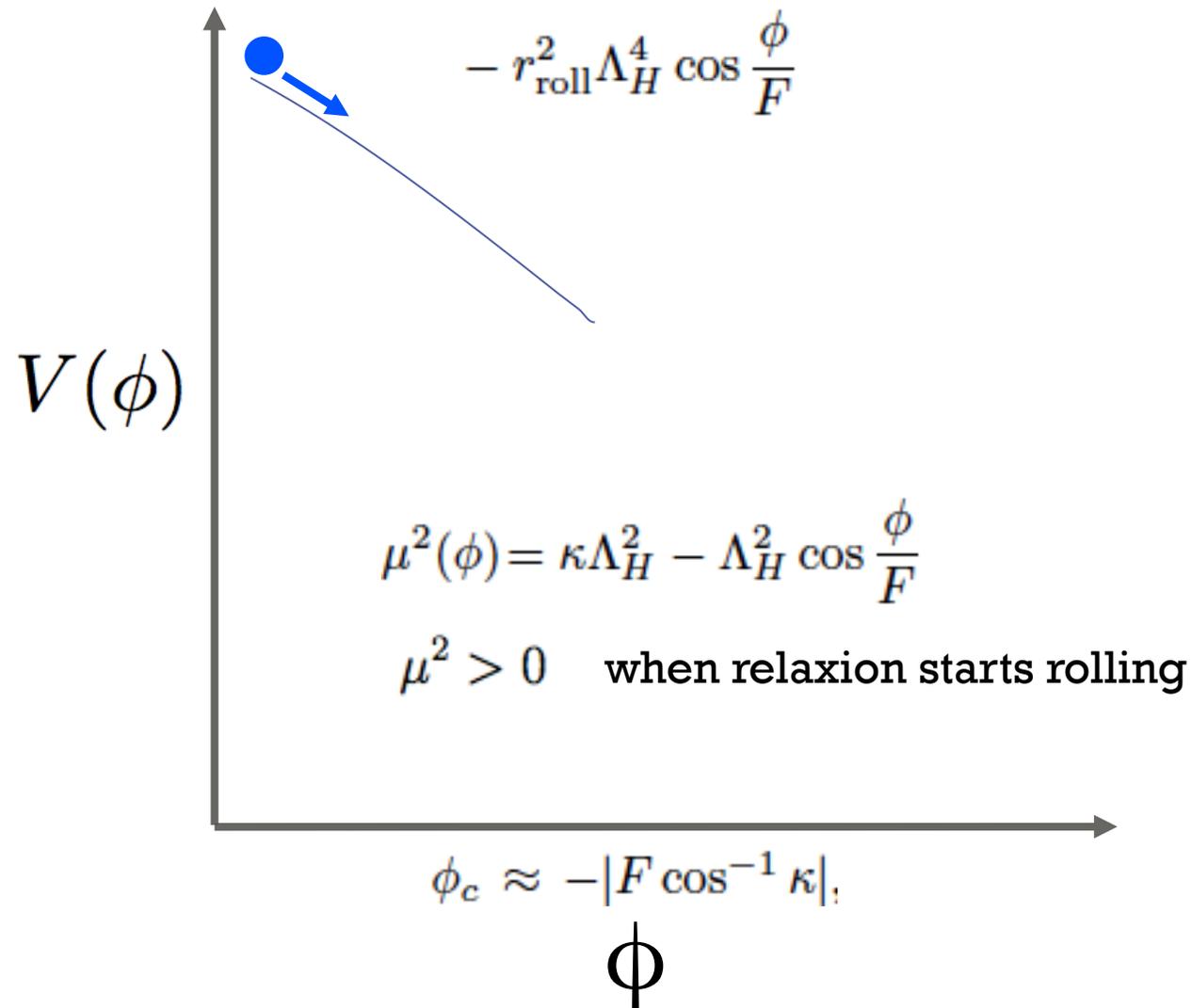
$$-r_{\text{roll}}^2 \Lambda_H^4 \cos \frac{\phi}{F}$$

3. **Backreaction** potential:

$$V_{\text{br}} = -\Lambda_{\text{br}}^4 \cos \frac{\phi}{f}, \quad \Lambda_{\text{br}}^4 \sim M_{\text{br}}^{4-j} (v+h)^j$$

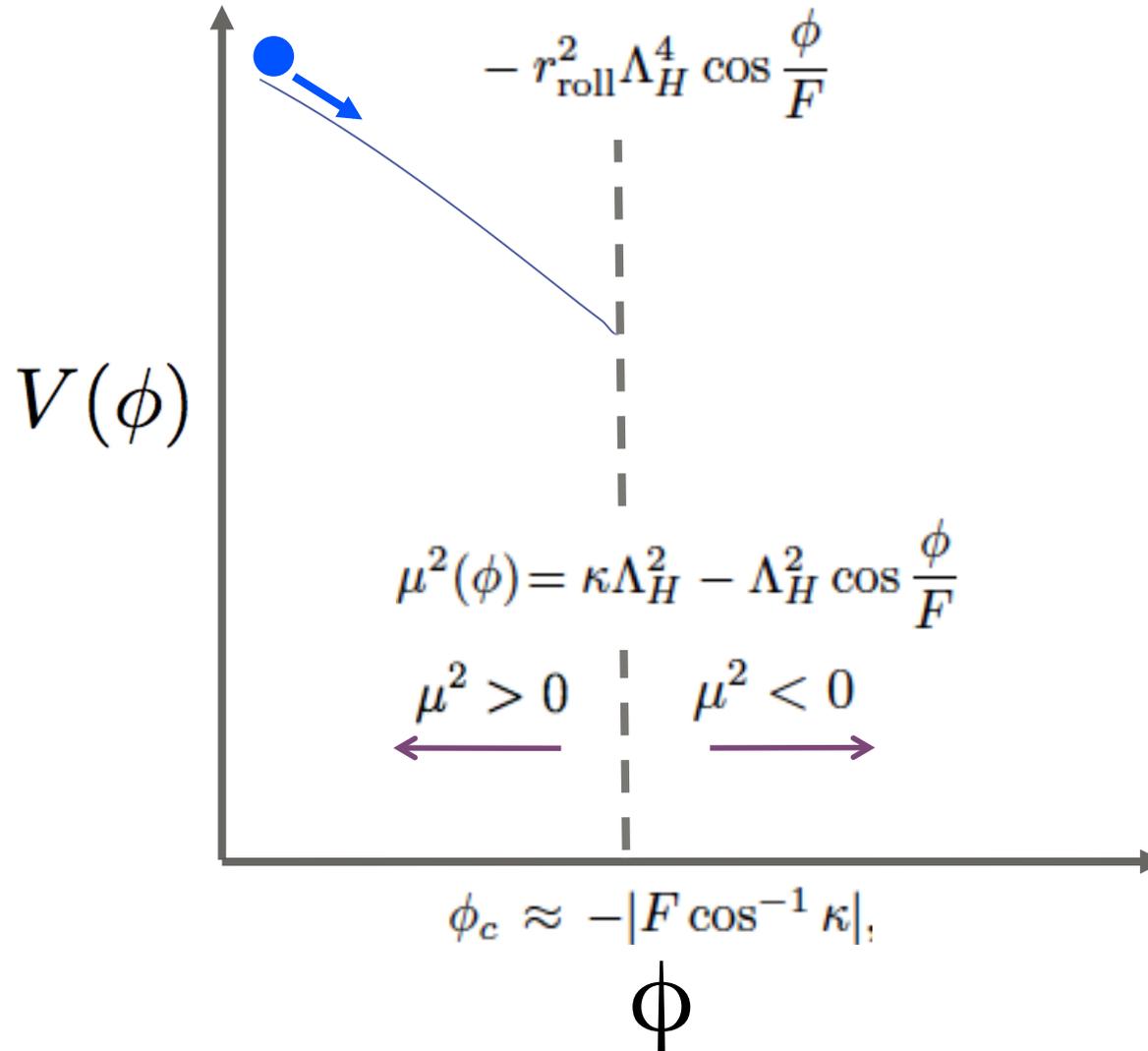
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# The Relaxion Mechanism



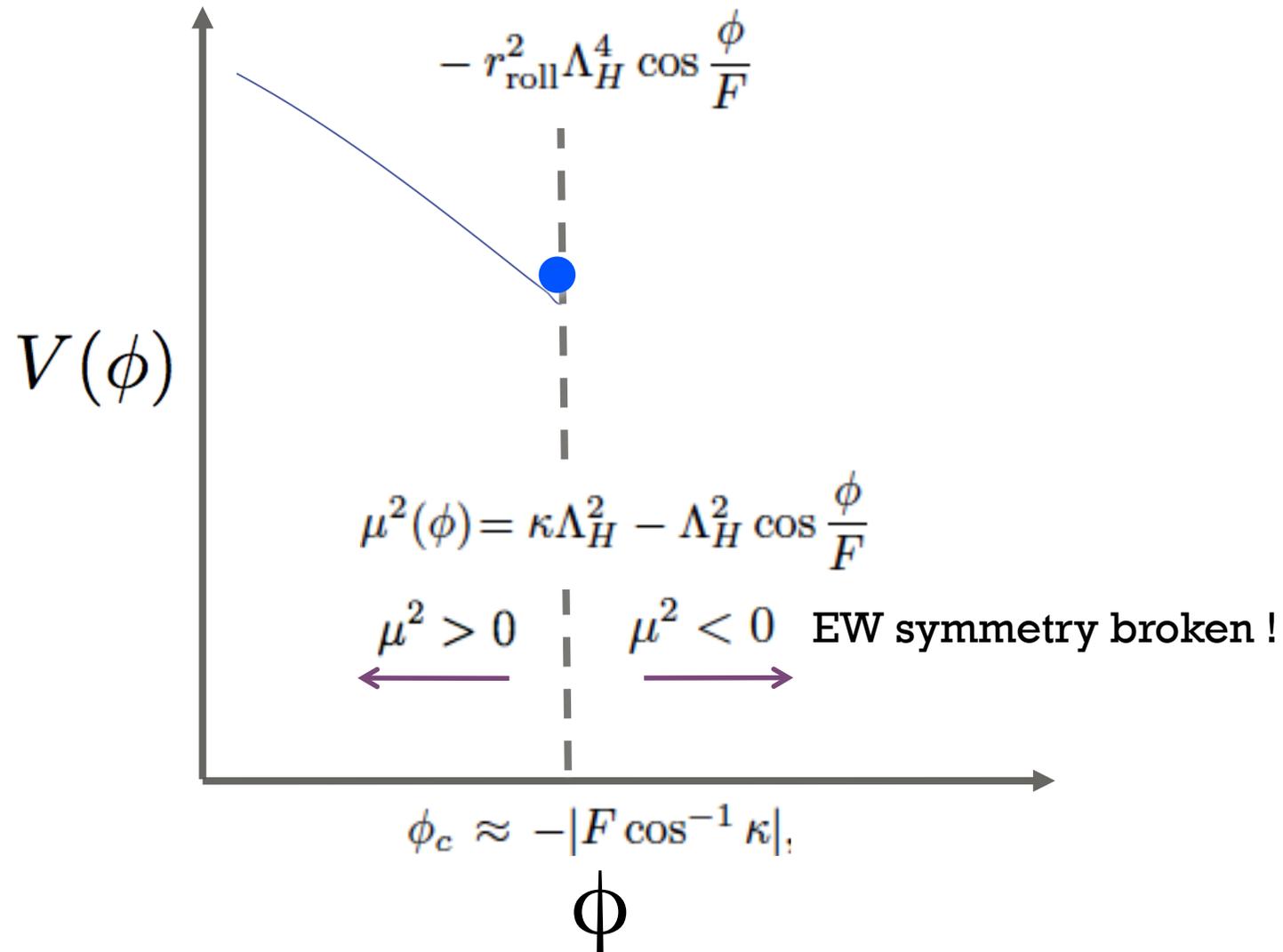
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# The Relaxion Mechanism



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# The Relaxion Mechanism





# Backreaction potential

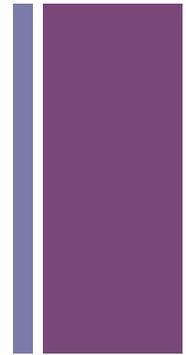
- **Backreaction** term in the potential turns on only when upon EWSB.

$$V_{\text{br}} = -\Lambda_{\text{br}}^4 \cos \frac{\phi}{f}, \quad \Lambda_{\text{br}}^4 \sim M_{\text{br}}^{4-j} (v+h)^j$$

- Example: QCD axion potential

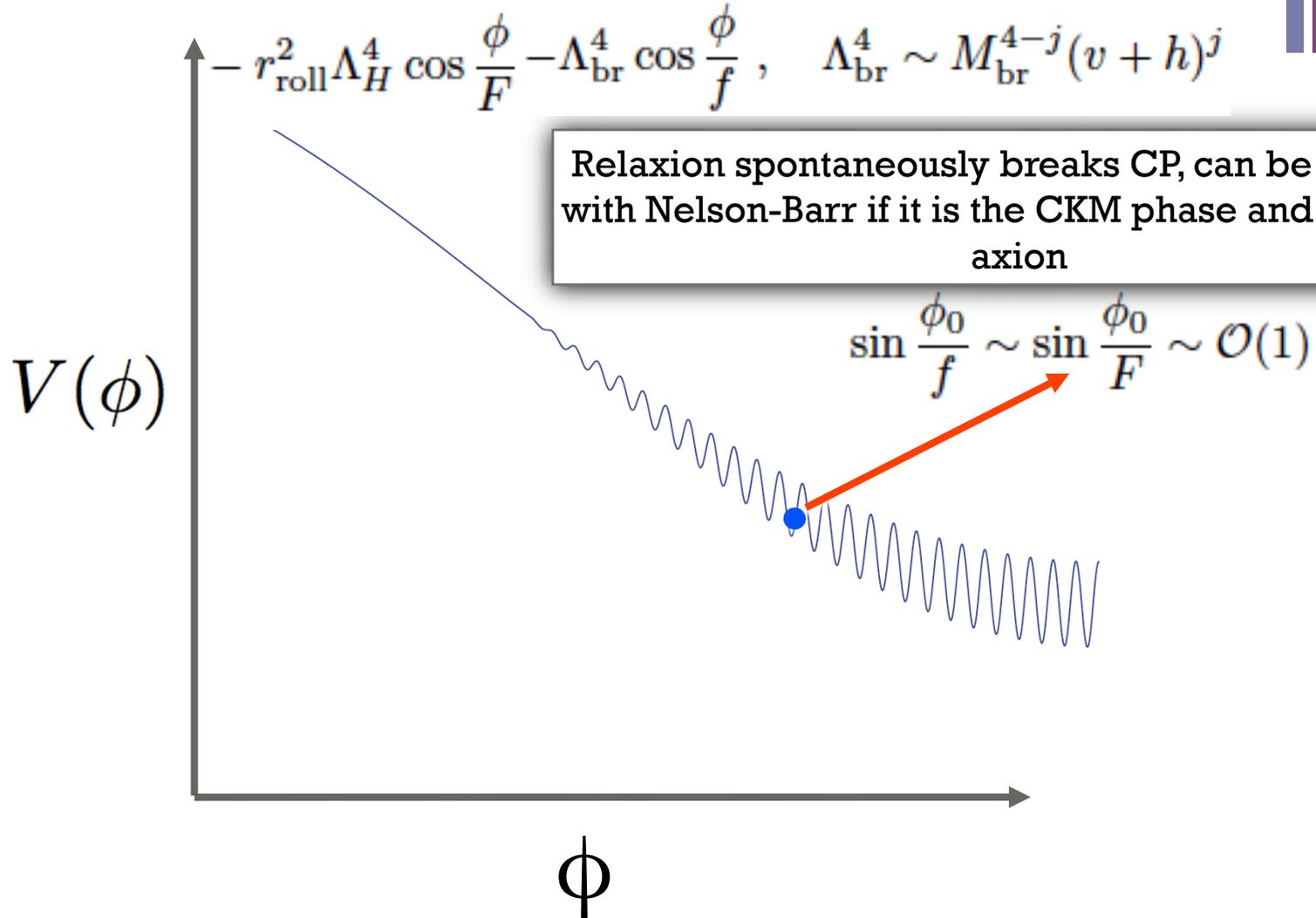
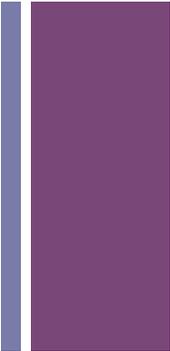
$$\Delta V \sim y_u v f_\pi^3 \cos \left( \frac{\phi}{f_a} \right)$$

- In simplest model relaxion is QCD axion.



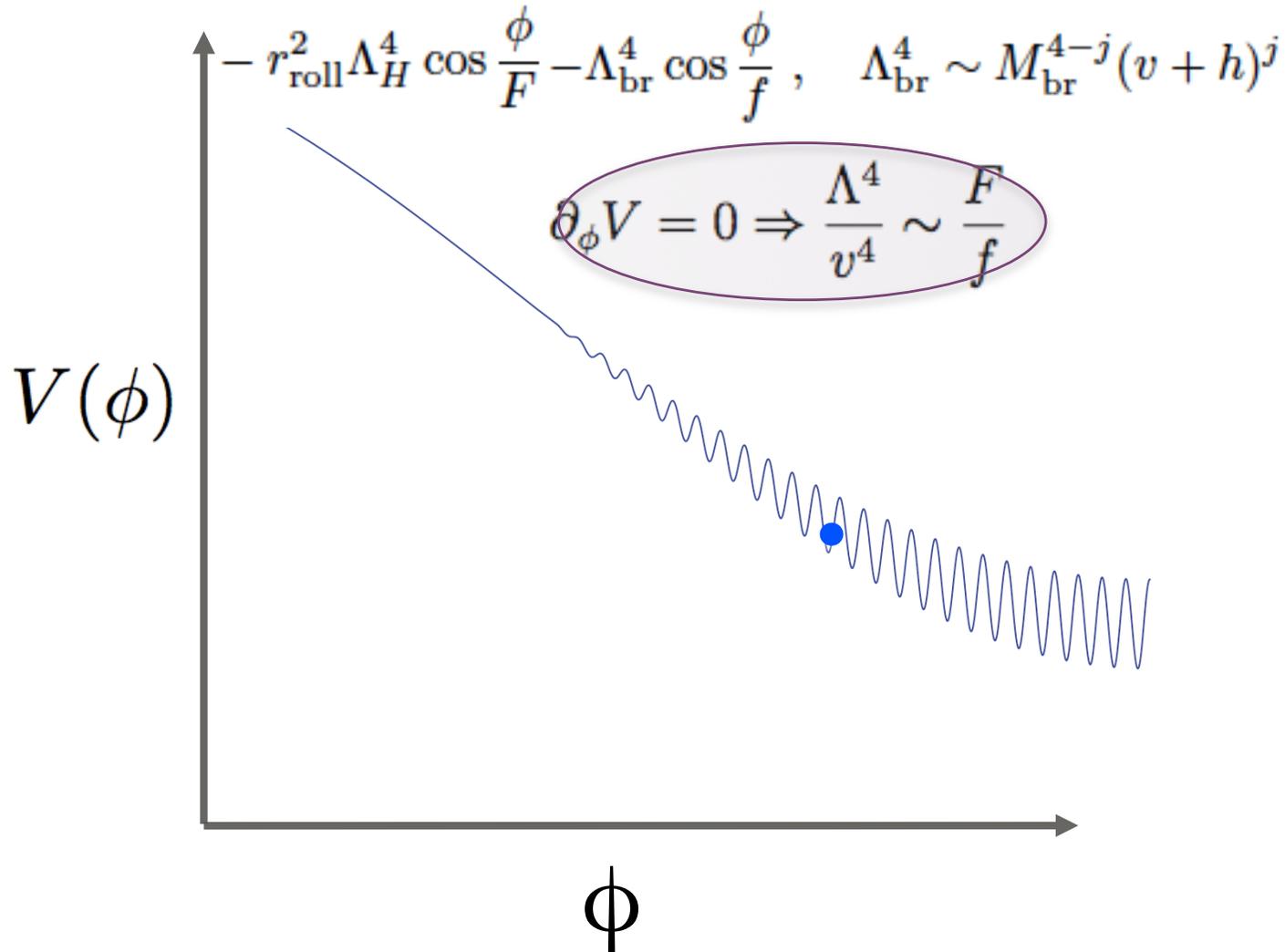
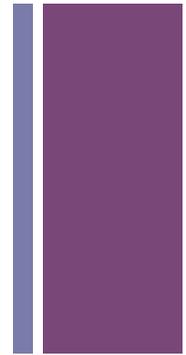


# The Relaxion Mechanism



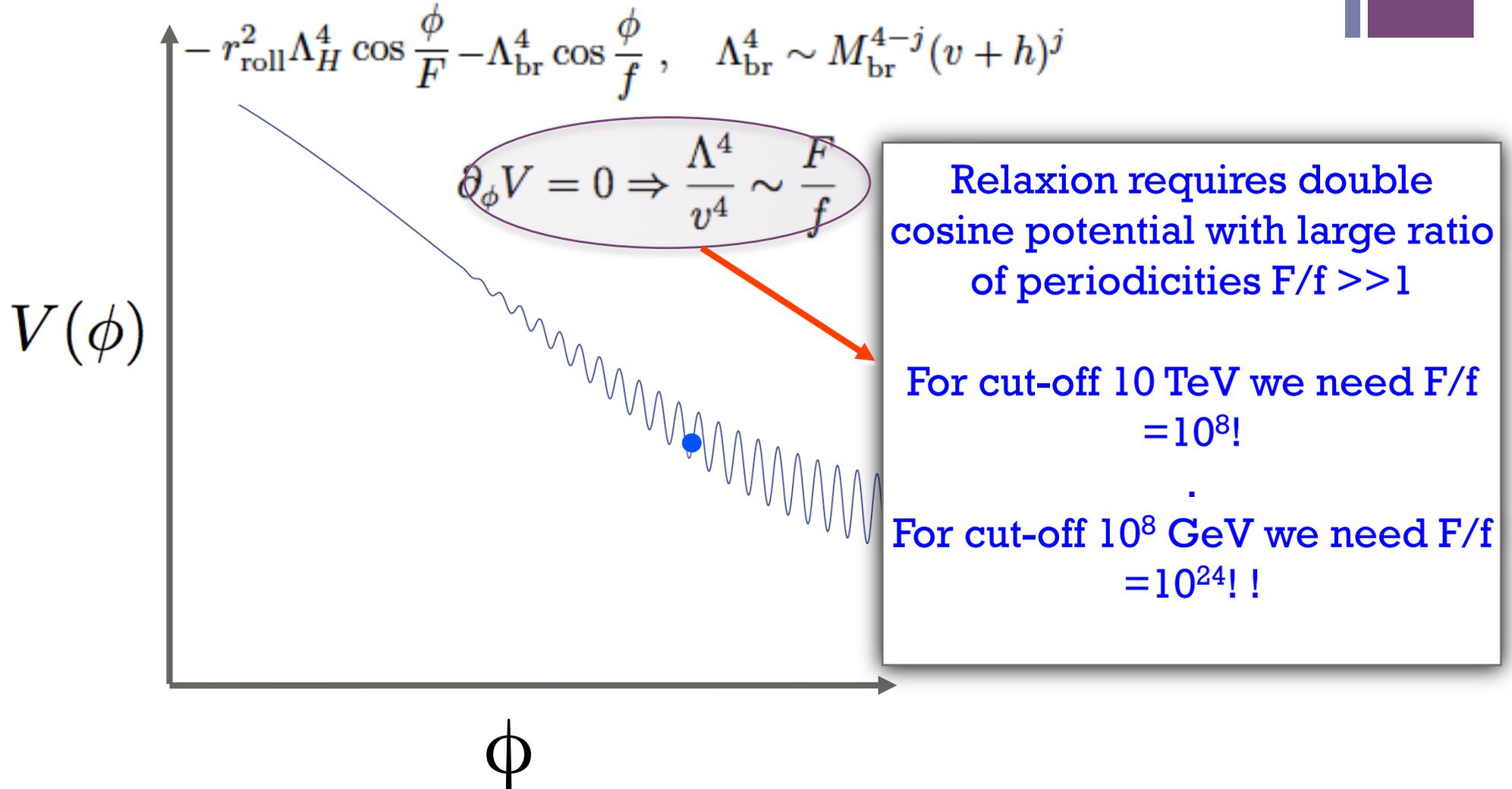
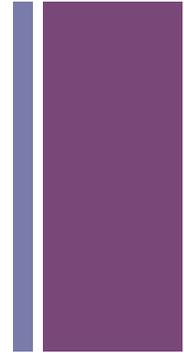
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# The Relaxion Mechanism



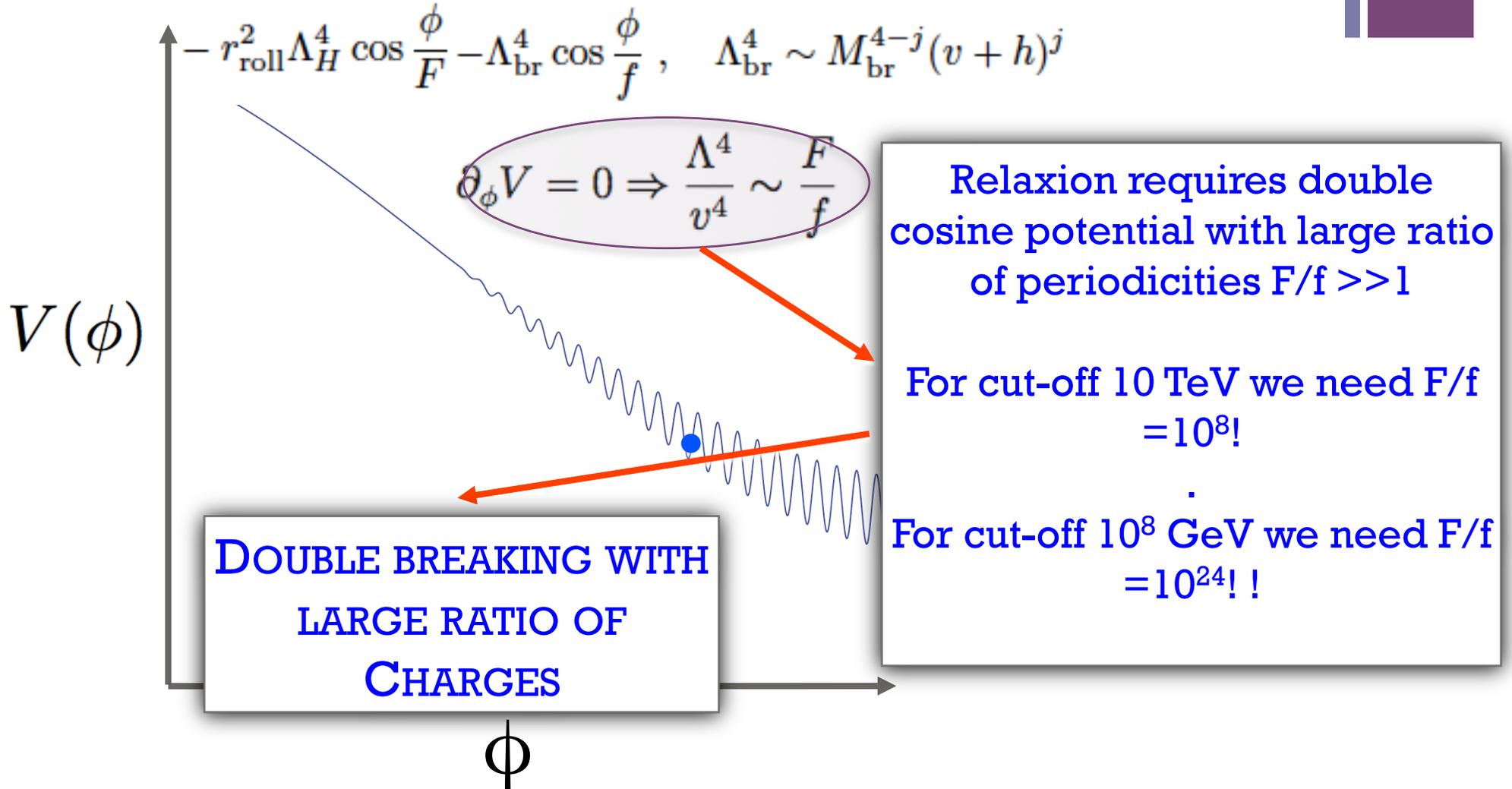
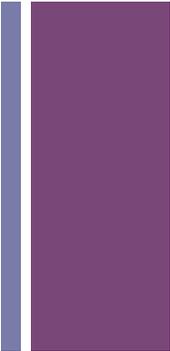
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# The Relaxion Mechanism



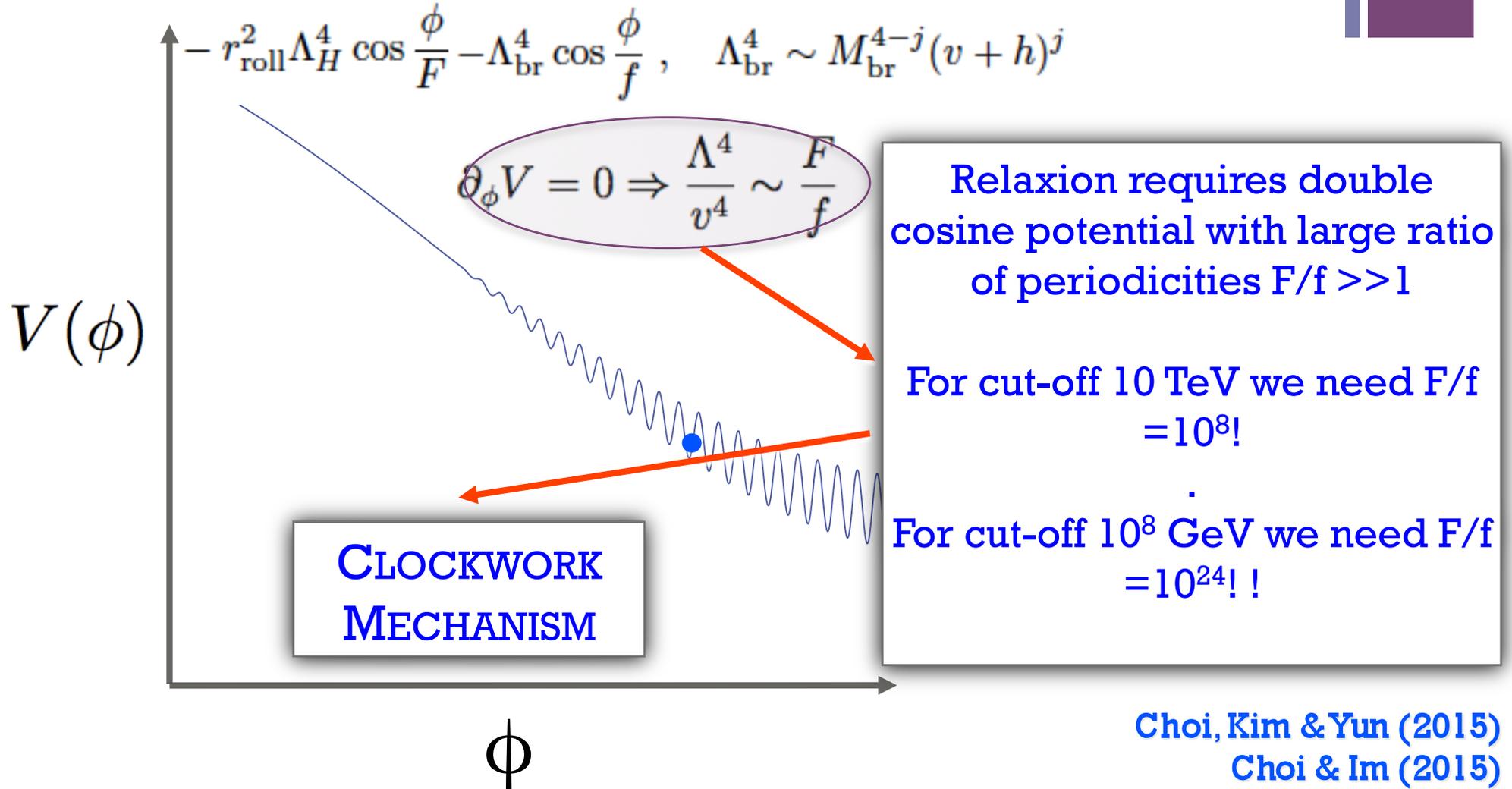
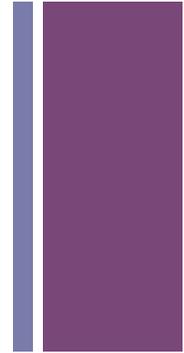
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# The Relaxion Mechanism



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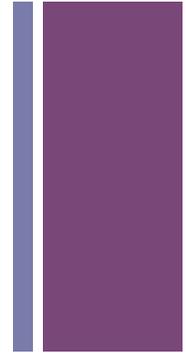
# The Relaxion Mechanism



Choi, Kim & Yun (2015)  
Choi & Im (2015)  
Kaplan & Rattazzi (2015)



# Choi-Kim-Yun alignment/ Clockwork Mechanism



- Multiple axions. Potential for linear fields:

$$V(\phi) = \sum_{j=0}^N \left( -m^2 \phi_j^\dagger \phi_j + \frac{\lambda}{4} |\phi_j^\dagger \phi_j|^2 \right) + \sum_{j=0}^{N-1} \left( \epsilon \phi_j^\dagger \phi_{j+1}^3 + h.c. \right)$$

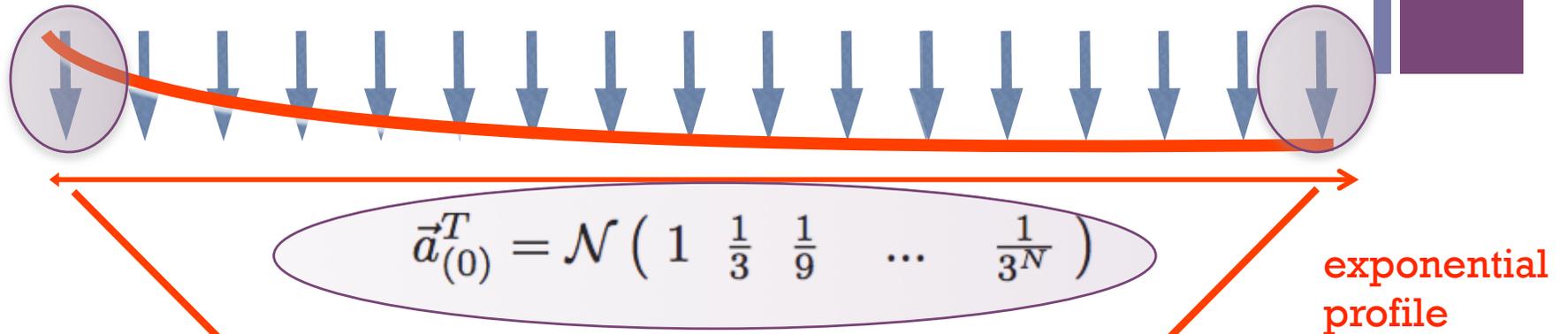
- Symmetry: the fields  $\phi_j$ ,  $j = 0, 1, 2, \dots, N$ , have charges  $Q = 1, 1/3, 1/9, \dots, 1/3^N$

Choi, Kim & Yun (2015)  
Choi & Im (2015)  
Kaplan & Rattazzi(2015)



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# Choi-Kim-Yun alignment/ Clockwork Mechanism



- Double breaking again on 1st and last site:

$$\frac{\pi_0}{32\pi^2 f} G_0 \tilde{G}_0 + \frac{\pi_N}{32\pi^2 f} G_N \tilde{G}_N$$

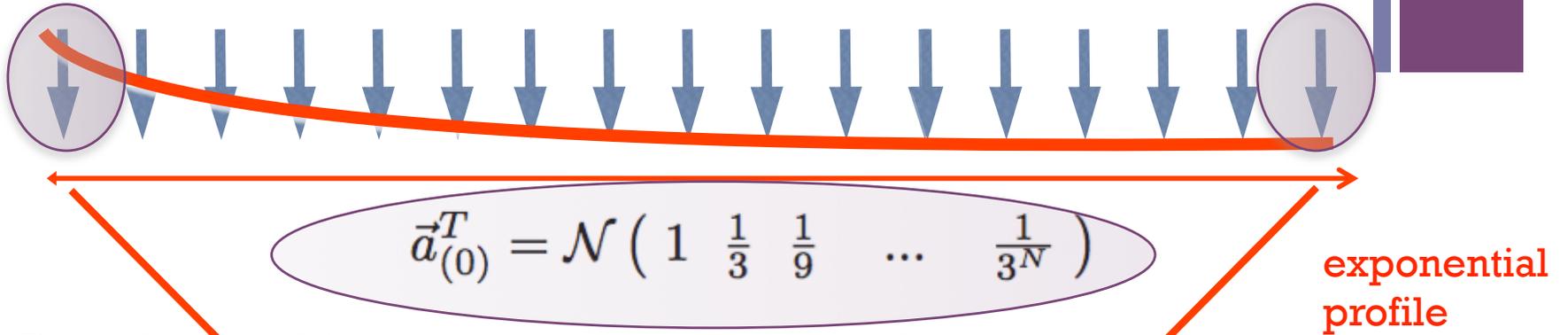
$$\Lambda_N \cos \frac{\pi_N}{f} + \Lambda_0 \cos \frac{\pi_0}{f}$$

$$\Lambda_N \cos \frac{\pi}{3^N f_a} + \Lambda_0 \cos \frac{\pi}{f_a}$$

Choi, Kim & Yun (2014)  
Choi & Im (2015)  
Kaplan & Rattazzi (2015)

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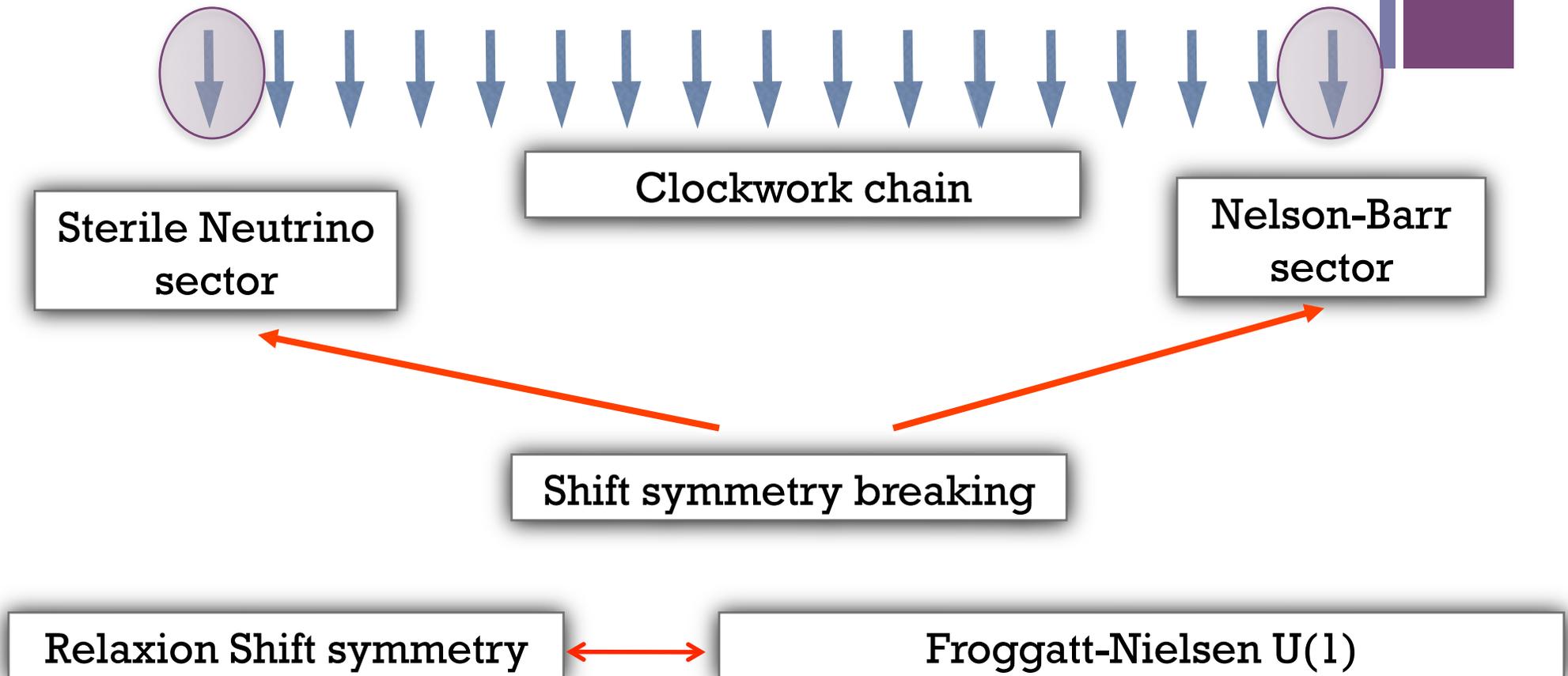
$$\Lambda_N \cos \frac{\pi_N}{f} + \Lambda_0 \cos \frac{\pi_0}{f}$$

$$\Lambda_N \cos \frac{\pi}{3^N f_a} + \Lambda_0 \cos \frac{\pi}{f_a}$$

Relaxion requires double cosine potential with large ratio of periodicities  $F/f \gg 1$

Choi, Kim & Yun (2014)  
Choi & Im (2015)  
Kaplan & Rattazzi (2015)

# + The hierarchion: basic picture

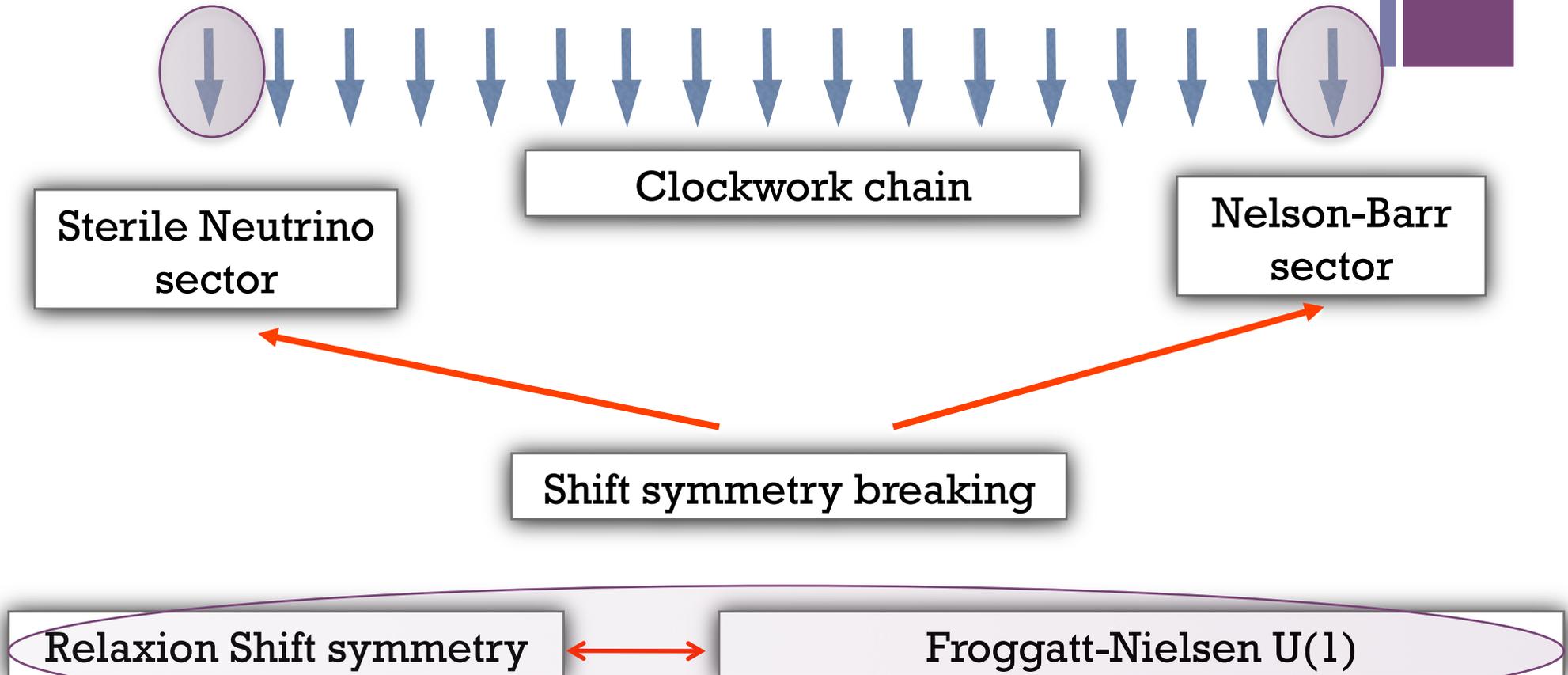


Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1712.XXXXX)

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# The hierarchion: basic picture

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Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1712.XXXXX)



# The Froggatt-Nielsen relaxion

- The relaxion is the pseudo-goldstone boson of a **global symmetry**
- What if we **identify this global symmetry with a flavour symmetry** that explains the fermion mass hierarchies and mixings.
- For example in the Froggatt Nielsen model we can explain the lightness of the electron as follows. The electron Yukawa interaction is:

$$\left(\frac{\Phi}{\Lambda}\right)^{10} L H e^c$$

There is a U(1) symmetry under which  $Q_{e^c} = 10$ ,  $Q_L = 0$ ,  $Q_\Phi = -1$

- Finally take:  $\frac{\langle \Phi \rangle}{\Lambda} = 0.17 \Rightarrow m_e = \mathcal{O}(MeV)$



# The Froggatt-Nielsen relaxion

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# The Froggatt-Nielsen relaxion

- The SM Yukawas are written as:

$$\mathcal{L}_L = y_{ij}^n \left[ \frac{\hat{\Phi}_0}{\Lambda_n} \right]^{|n_{ij}|} L_i \tilde{H} N_j^c + y_{ij}^c \left[ \frac{\hat{\Phi}_0}{\Lambda_c} \right]^{|c_{ij}|} L_i H e_j^c$$

$$\mathcal{L}_Q = y_{ij}^u \left[ \frac{\hat{\Phi}_m}{\Lambda_u} \right]^{|u_{ij}|} Q_i \tilde{H} u_j^c + y_{ij}^d \left[ \frac{\hat{\Phi}_m}{\Lambda_d} \right]^{|d_{ij}|} Q_i H d_j^c$$

$$\Phi_0 \sim \frac{(f + \rho)}{\sqrt{2}} e^{i\phi/f}$$

+

# The Froggatt-Nielsen relaxion

- The SM Yukawas are written as:

Sterile  
neutrinos

$$\mathcal{L}_L = y_{ij}^n \left[ \frac{\hat{\Phi}_0}{\Lambda_n} \right]^{|n_{ij}|} L_i \tilde{H} N_j^c + y_{ij}^c \left[ \frac{\hat{\Phi}_0}{\Lambda_c} \right]^{|c_{ij}|} L_i H e_j^c$$

$$\mathcal{L}_Q = y_{ij}^u \left[ \frac{\hat{\Phi}_m}{\Lambda_u} \right]^{|u_{ij}|} Q_i \tilde{H} u_j^c + y_{ij}^d \left[ \frac{\hat{\Phi}_m}{\Lambda_d} \right]^{|d_{ij}|} Q_i H d_j^c$$

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Relaxion



# The Froggatt-Nielsen relaxion

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Sterile  
neutrinos

$$\mathcal{L}_Q = y_{ij}^u \left[ \frac{\hat{\Phi}_m}{\Lambda_u} \right]^{|u_{ij}|} Q_i \tilde{H} u_j^c + y_{ij}^d \left[ \frac{\hat{\Phi}_m}{\Lambda_d} \right]^{|d_{ij}|} Q_i H d_j^c$$

$$\Phi_0 \sim \frac{(f + \rho)}{\sqrt{2}} e^{i\phi/f}$$

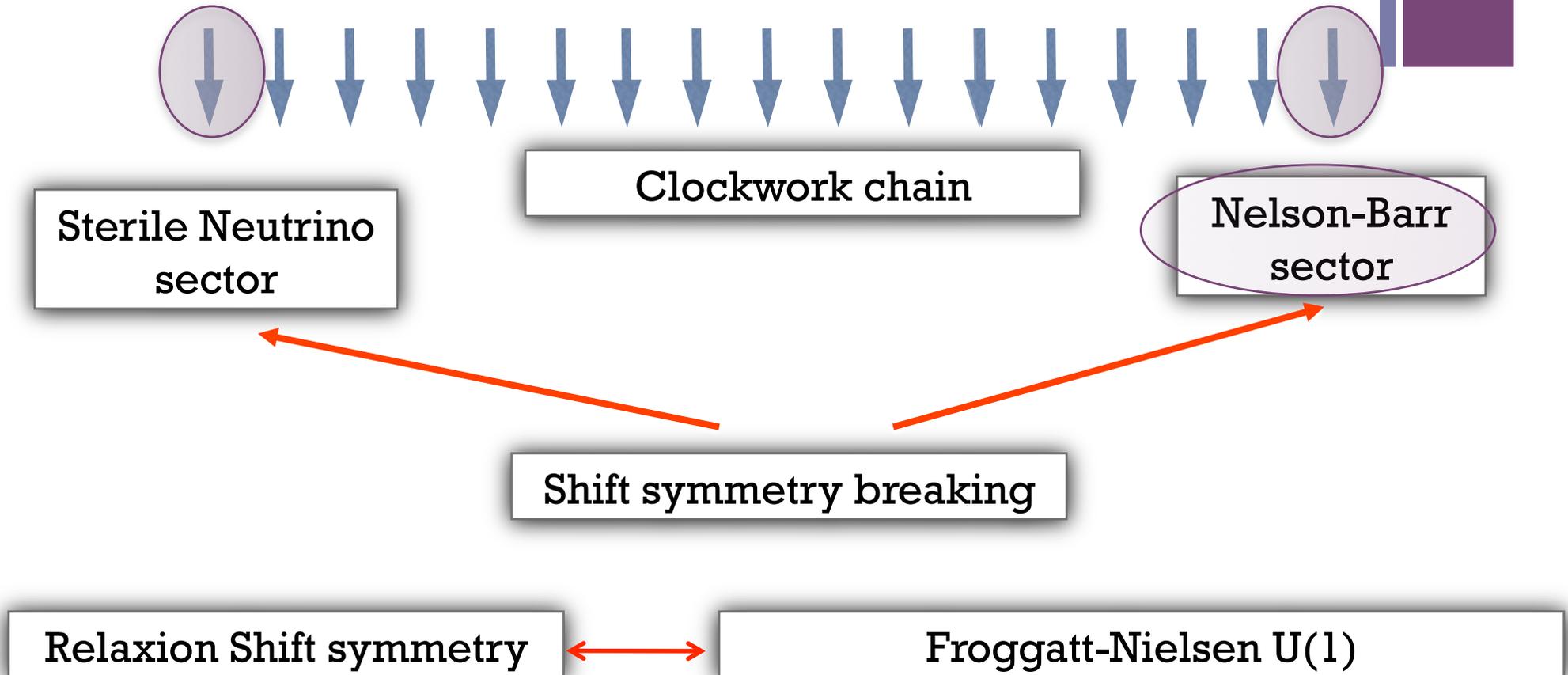
Relaxion

- So far this is an **exact U(1)** and **the relaxion is thus a massless goldstone Boson**. To give the relaxion the required potential we need to **break the U(1) twice**. For this we will use the **seesaw mechanism at the 1<sup>st</sup> site** and **Nelson Barr mechanism at last site..**

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# The hierarchion: basic picture

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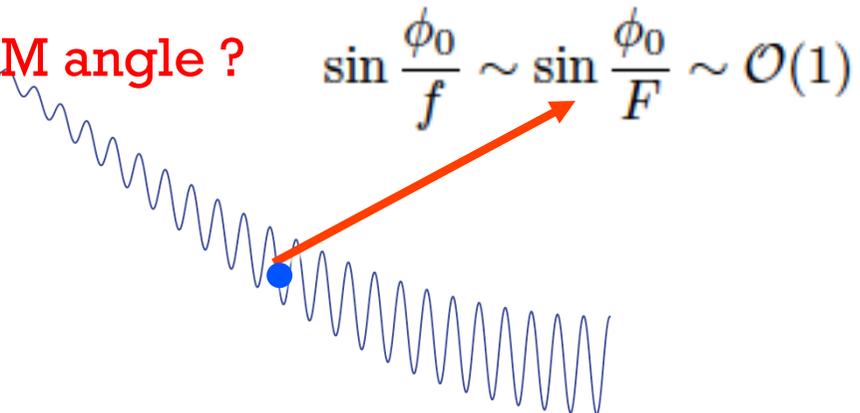


Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1712.XXXXX)



# Nelson-Barr mechanism

- CP is a **good symmetry of UV**.
- **CP broken spontaneously** by a pseudoscalar whose VEV generates CKM phase but not strong CP phase.
- Once CKM phase is generated RG running generates but only at 7 loop level!
- **Relaxion breaks CP spontaneously!**
- Can the **relaxion VEV** be the **CKM angle** ?  $\sin \frac{\phi_0}{f} \sim \sin \frac{\phi_0}{F} \sim \mathcal{O}(1)$



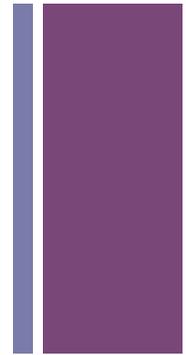


# Nelson-Barr relaxion

- Add a **new vectorlike quark**:

$$\mathcal{L}_Q = y_{ij}^u \left[ \frac{\hat{\Phi}_m}{\Lambda_u} \right]^{|u_{ij}|} Q_i \tilde{H} u_j^c + y_{ij}^d \left[ \frac{\hat{\Phi}_m}{\Lambda_d} \right]^{|d_{ij}|} Q_i H d_j^c + \mu \psi \psi^c$$

- A  $Z_2$  symmetry under which only vectorlike pair charged forbids their couplings of SM quarks.



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# Nelson-Barr relaxion

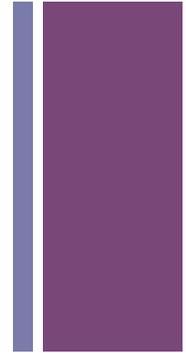
- Break  $Z_2$  and  $U(1)_N$  (thus clockwork  $U(1)$ )

$$\mathcal{L}_\psi^{\text{roll}} = \left[ y_i^\psi \Phi_N + \tilde{y}_i^\psi \Phi_N^* \right] \psi d_i^c$$

- $U(1)_N$  breaking necessary for **presence of physical phase** (else all terms involve  $\partial_\mu \pi_N$  and vanish if  $\pi_N$  put to VEV) as well as **generating rolling potential**.



# Nelson-Barr relaxion



- No strong CP phase!

$$M^d = \begin{pmatrix} (\mu)_{1 \times 1} & (B)_{1 \times 3} \\ (0)_{3 \times 1} & (vY^d)_{3 \times 3} \end{pmatrix} \quad B_i = \frac{f}{\sqrt{2}} \left( y_i^\psi e^{i\theta_N} + \tilde{y}_i^\psi e^{-i\theta_N} \right)$$

$$\bar{\theta}_{\text{QCD}} = \text{Arg}(\det(M^u)) + \text{Arg}(\mu \cdot \det(vY^d)) = 0$$

- This assumes:

$$\text{Arg}(\mu \cdot \det(vY^d)) = 0$$

- Another **crucial requirement** we will discuss soon.

# + Nelson-Barr relaxion

- CKM phase present in effective 3x3 SM quark matrix once VL quark integrated out:

$$\left[ M_{\text{eff}}^d M_{\text{eff}}^{d\dagger} \right]_{ij} \sim v^2 Y_{ik}^d Y_{jk}^d - \frac{v^2 Y_{ik}^d B_k^* B_l Y_{jl}^d}{\mu^2 + B_n B_n^*}$$

Contains CKM phase

$$B_i = \frac{f}{\sqrt{2}} \left( y_i^\psi e^{i\theta_N} + \tilde{y}_i^\psi e^{-i\theta_N} \right)$$

# + Nelson-Barr relaxion

- $U(1)_N$  breaking generates rolling potential:

$$\mathcal{L}_\psi^{\text{roll}} = \left[ y_i^\psi \Phi_N + \tilde{y}_i^\psi \Phi_N^* \right] \psi d_i^c$$

Radiatively

$$V_{\text{roll}} = \mu^2(\phi) |H|^2 + \lambda_H |H|^4 - r_{\text{roll}}^2 \Lambda_H^4 \cos \frac{\phi}{F},$$

$$\mu^2(\phi) = \kappa \Lambda_H^2 - \Lambda_H^2 \cos \frac{\phi}{F},$$

$$\Lambda^2 \sim \frac{y_i^\psi \tilde{y}_j^\psi ((Y^d)^T Y^d)_{ij}}{16\pi^2} f^2$$

# + Radiative threshold contributions to strong CP phase

- **Radiative corrections** to  $\theta_{QCD}$  **vanish** in our model in the limit:

$$y_i^\psi \sim \tilde{y}_i^\psi \sim y_\psi \rightarrow 0$$

- All radiative contributions to  $\theta_{QCD}$  can be systematically evaluated in powers of using symmetry arguments only (spurion analysis). This gives:

$$y_\psi \lesssim 10^{-3}$$

- CKM phase still  $O(1)$ .



# Anomaly free condition

- We **need**:  $\bar{\theta}_{\text{QCD}} = \text{Arg}(\det(M^u)) + \text{Arg}(\mu \cdot \det(vY^d)) = 0$

$$\text{Arg}(\mu \cdot \det(vY^d)) = 0$$

- If this is non zero it can be **rotated to give** :

$$\frac{n\hat{\theta}}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad n_{\text{QCD}} = \sum_{i=1}^3 (2 [Q_i] + [u_i^c] + [d_i^c])$$

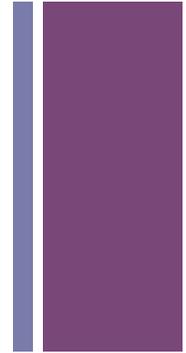
- To have  $\text{Arg}(\mu \cdot \det(vY^d)) = 0$  we need,

$$n_{\text{QCD}} = 0$$

i.e. an **anomaly free Froggatt Nielsen U(1)**.



# Charge assignment



- We find the **anomaly free Froggatt-Nielsen charge assignment**:

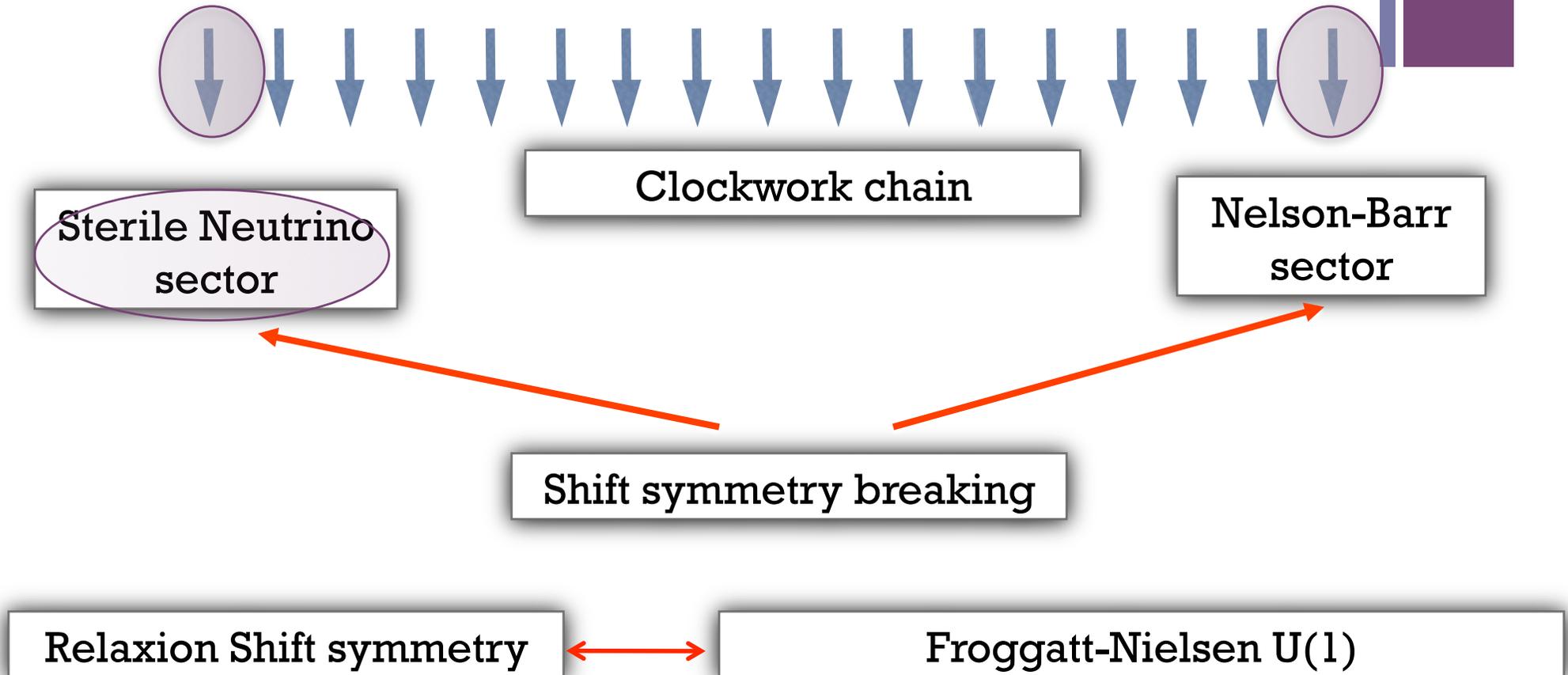
$$\begin{pmatrix} [Q_1] & [Q_2] & [Q_3] \\ [u_1^c] & [u_2^c] & [u_3^c] \\ [d_1^c] & [d_2^c] & [d_3^c] \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ -10 & -6 & 0 \\ 2 & 2 & 2 \end{pmatrix}$$

- Can **explain quark masses and mixings with single 5% tuning** for the Wilson coefficient related to the 12 element of down mass matrix.

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# The hierarchion: basic picture

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Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1712.XXXXX)

## + Breaking the U(1) and neutrino masses (simplified version)

- The **U(1) is broken by** a Majorana mass term for the **sterile neutrinos**

$$\frac{1}{2}M_{ij}N_i^c N_j^c$$

- This generates **neutrino masses**,

$$m_\nu \sim \frac{(Y_N v)^2}{M}$$

- As well as the **backreaction potential**

$$V_{br} \sim (Y_N v)^2 M^2 \cos \frac{\phi}{f} \sim m_\nu M^3 \cos \frac{\phi}{f}$$

# + Breaking the U(1) and neutrino masses (simplified version)

- The **U(1) is broken by** a Majorana mass term for the **sterile neutrinos**

$$\frac{1}{2} M_{ij} N_i^c N_j^c$$

- This generates **neutrino masses**,  $\sim 10^{-6}$  **Size explained by**

No electroweak charged states  
as in other relaxion  
backreaction models.  
Even more elusive at LHC!

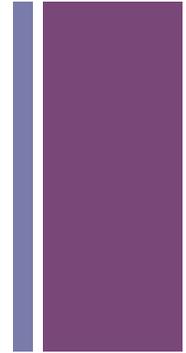
**gatt-Nielsen**

- As well as the **backreaction potential**

$$V_{br} \sim (Y_N v)^2 M^2 \cos \frac{\phi}{f} \sim m_\nu M^3 \cos \frac{\phi}{f}$$



# Phenomenology



- Flavor violating decays:

$$\Gamma(\mu \rightarrow e \phi) \approx \frac{m_e^2 m_\mu}{16\pi f^2} \longrightarrow f \gtrsim 2.8 \cdot 10^7 \text{ GeV}$$

Orders of magnitude improvement possible at MEG

$$\Gamma(K^+ \rightarrow \pi^+ \phi) \approx \frac{m_K}{64\pi} B_s^2 \left[ 1 - \frac{m_\pi^2}{m_K^2} \right] \frac{m_s m_d}{32m f^2} \longrightarrow f \gtrsim 8 \cdot 10^{10} \cdot (1/3)^m \text{ GeV}$$

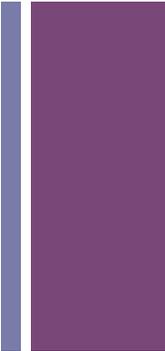
Orders of magnitude improvement possible at NA62, KOTO.

- Also usual constraints from flavor diagonal coupling to electrons, anomaly induced coupling to photons (eg. star cooling bounds).

$$f \gtrsim 6 \cdot 10^8 \text{ GeV}$$



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Bounds from flavor violation in quark sector can be evaded by coupling quarks to intermediate clockwork site, giving them a larger effective  $f$ .

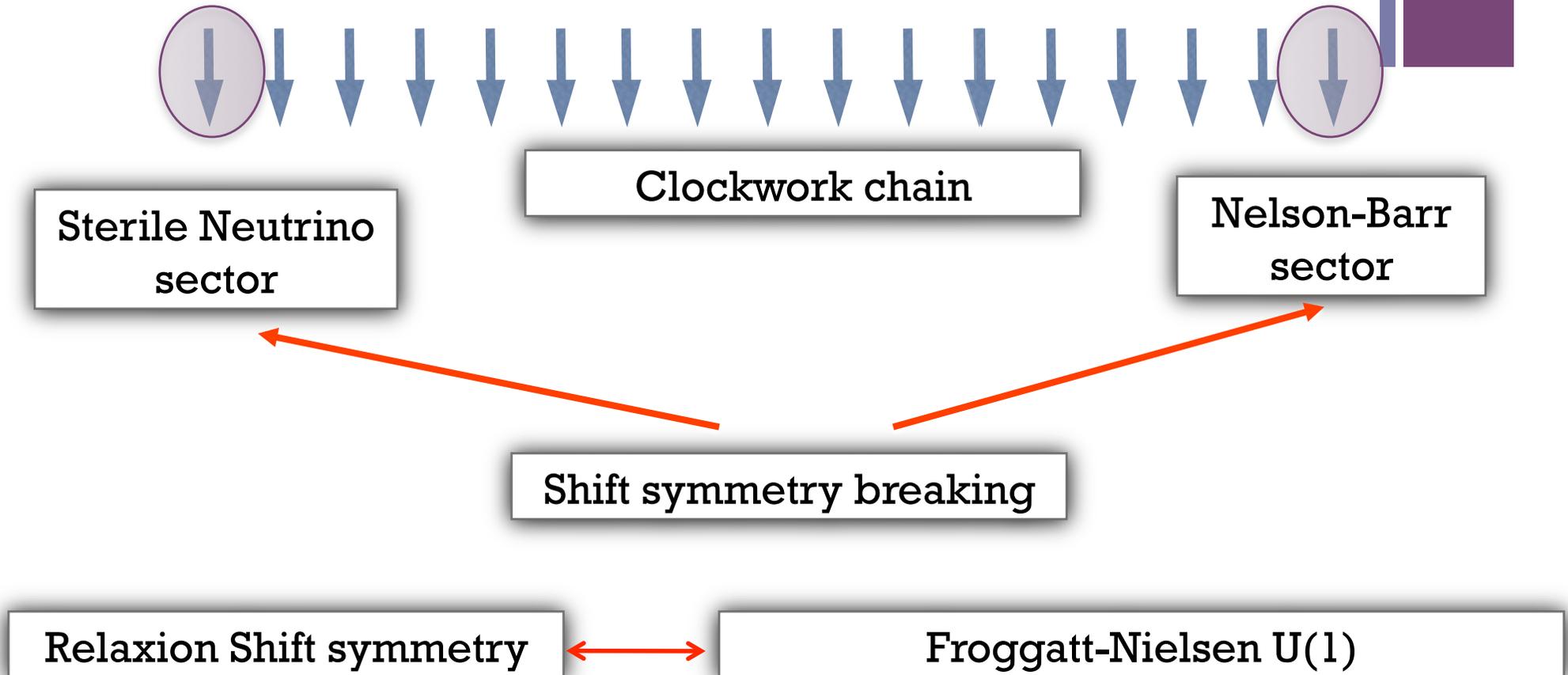
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# The hierarchion: basic picture

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Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1711.00858)  
Davidi, RSG, Perez, Redigolo and Shalit (arXiv:1712.XXXXX)



# Conclusions

- We present a **unified relaxion description** for the **electroweak, strong CP and flavor hierarchies** of the Standard Model, including neutrino masses.
- Single light degree of freedom: **the hierarchion.**
- The ***hierarchion*** is at the same time the ***relaxion***, the ***familon*** of a global flavor symmetry and the ***CKM phase*** of Nelson Barr models.
- **No electroweak states accessible at LHC.**
- **Main Signature: Flavor violating meson and lepton decays.**

