Classifying Accidental Continuous Symmetries in nHDMs

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Outline

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Multi-Higgs Doublet Models (nHDMs)

- The nHDMs contain n doublet fields, ϕ_n $(n \geq 2)$ with identical quantum numbers.
- General scalar potential with n doublets [Botella, Silva, '95]

$$V = \sum_{i,j=1}^{n} m_{ij}^2 (\Phi_i^\dagger \Phi_j) + \sum_{i,j\,k,l=1}^{n} \lambda_{ijkl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$$

- The nHDMs potential contain n^2 physical mass terms along with $n^2(n^2+1)/2$ physical couplings.

Construction of nHDMs in the bilinear formalism

- In the bilinear scalar field space, fields are $4n$-dimensional complex multiplets, realizing $\mathrm{GL}(4n,\mathbb{C})$

$$\mathbf{\Phi_2} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}_{\mathbf{8-Dim}}, \quad \mathbf{\Phi_3} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \\ i\sigma^2\phi_3^* \end{pmatrix}_{\mathbf{12-Dim}}, \quad \cdots, \mathbf{\Phi_n} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ . \\ . \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \\ i\sigma^2\phi_3^* \\ . \\ . \end{pmatrix}_{\mathbf{4n-Dim}}$$

- The $\mathbf{\Phi_n}$-multiplet is $SU(2)_L$-covariant.
- The charge conjugation operator: $\quad \mathbf{\Phi}_n = C\mathbf{\Phi}_n^*, \quad C = \sigma^2 \otimes \mathbf{1}_n \otimes \sigma^2$.
- $U(1)_Y$-violating parts do not appear in the potential.

Construction of the nHDMs in the bilinear formalism

Realizing $\mathrm{GL}(4n,\mathbb{C}) \quad \Longrightarrow \quad$ *null* $n(2n-1)$-vector $R_n^{\tilde{A}} = \mathbf{\Phi}_n^\dagger \Sigma_n^{\tilde{A}} \mathbf{\Phi}_n$

$$\Sigma_n^{\tilde{A}} = \left(\sigma^0 \otimes t_S^a \otimes \sigma^0, \quad \sigma^i \otimes t_A^b \otimes \sigma^0\right), \quad \tilde{A} = 0, 1, 2, \cdots, n(2n-1)$$

$$R_n^{\tilde{A}} = \begin{pmatrix} \left.\mathbf{\Phi_n^\dagger}\left(\sigma^{\mathbf{0}} \otimes \mathbf{1_n} \otimes \sigma^{\mathbf{0}}\right)\mathbf{\Phi_n}\right\} \mathbf{1} \\ \left.\begin{matrix}\mathbf{\Phi_n^\dagger}\left(\sigma^{\mathbf{0}} \otimes \mathbf{t_S^a} \otimes \sigma^{\mathbf{0}}\right)\mathbf{\Phi_n} \\ \mathbf{\Phi_n^\dagger}\left(\sigma^{\mathbf{3}} \otimes \mathbf{t_A^b} \otimes \sigma^{\mathbf{0}}\right)\mathbf{\Phi_n}\end{matrix}\right\} \mathbf{n^2-1} \\ \left.\mathbf{\Phi_n^\dagger}\left(\sigma^{\mathbf{1,2}} \otimes \mathbf{t_A^b} \otimes \sigma^{\mathbf{0}}\right)\mathbf{\Phi_n}\right\} \mathbf{n(n-1)} \end{pmatrix}_{n(2n-1)\text{-Dim}}, \qquad \mathbf{t_{S,A}} \in \mathrm{SU(n)}$$

The $R_n^{\tilde{A}}$-vector for 2HDM and 3HDM

$$R_2^{\tilde{A}} = \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ -i[\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1] \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \\ -[\phi_1^T i\sigma^2\phi_2 - \phi_2^\dagger i\sigma^2\phi_1^*] \\ i[\phi_1^T i\sigma^2\phi_2 + \phi_2^\dagger i\sigma^2\phi_1^*] \end{pmatrix}_{\text{6-Dim}}$$

$$R_3^{\tilde{A}} = \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ \phi_1^\dagger\phi_3 + \phi_3^\dagger\phi_1 \\ \phi_2^\dagger\phi_3 + \phi_3^\dagger\phi_2 \\ -i[\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1] \\ -i[\phi_1^\dagger\phi_3 - \phi_3^\dagger\phi_1] \\ -i[\phi_2^\dagger\phi_3 - \phi_3^\dagger\phi_2] \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \\ \frac{1}{\sqrt{3}}[\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 - 2\phi_3^\dagger\phi_3] \\ -[\phi_1^T i\sigma^2\phi_2 - \phi_2^\dagger i\sigma^2\phi_1^*] \\ -[\phi_1^T i\sigma^2\phi_3 - \phi_3^\dagger i\sigma^2\phi_1^*] \\ -[\phi_2^T i\sigma^2\phi_3 - \phi_3^\dagger i\sigma^2\phi_2^*] \\ i[\phi_1^T i\sigma^2\phi_2 + \phi_2^\dagger i\sigma^2\phi_1^*] \\ i[\phi_1^T i\sigma^2\phi_3 + \phi_3^\dagger i\sigma^2\phi_1^*] \\ i[\phi_2^T i\sigma^2\phi_3 + \phi_3^\dagger i\sigma^2\phi_2^*] \end{pmatrix}_{\text{15-Dim}}$$

The nHDM potential and the maximal symmetry

- The nHDM potential base on the *null* $n(2n-1)$-vector $R_n^{\tilde{A}}$

$$V = -\frac{1}{2} M_{\tilde{A}} R^{\tilde{A}} + \frac{1}{4} L_{\tilde{A}\tilde{A}'} R^{\tilde{A}} R^{\tilde{A}'}$$

[Maniatis, '06; Nishi, '06; Ivanov, '06; Battye,Pilaftsis'11]

- The gauge-kinetic term:

$$T = \frac{1}{2}(D_\mu \mathbf{\Phi}_n)^\dagger (D^\mu \mathbf{\Phi}_n), \quad D_\mu = \sigma^0 \otimes \mathbf{1}_n \otimes (\sigma^0 \partial^0_\mu + i g_w W^i_\mu \sigma^i/2)$$

- To leave the $\mathrm{SU}(2)_L$ gauge-kinetic term canonical:

$$\mathbf{GL(4n,\mathbb{C})} \xrightarrow{\text{reduces to}} \mathbf{Sp(2n)/Z_2} \;\; \textit{Symplectic group}$$

The maximal symmetry group: $\mathbf{Sp(2n)/Z_2 \otimes SU(2)_L}$

Co-adjoint reps of $\mathrm{Sp}(2n)$

- The reps of $\mathrm{Sp}(2n)/Z_2$ group in the $\mathbf{\Phi}_n$-basis:

$$K_n^{\tilde{B}} = \left(\sigma^0 \otimes t_A^b \otimes \sigma^0, \quad \sigma^i \otimes t_S^a \otimes \sigma^0\right)$$

- The Lie commutation structure between Σ_n^A and $K_n^{\tilde{B}}$ generators:

$$[K_n^{\tilde{B}}, \Sigma_n^A] = 2i f_n^{\tilde{B}AA'} \Sigma_n^{A'}$$

- The infinitesimal $\mathrm{Sp}(2n)/Z_2$ transformations of the $R_n^{\tilde{A}}$ vector:

[Pilaftsis, '11]

$$\delta R_n^A = i\theta^{\tilde{B}} \mathbf{\Phi}_n^\dagger [\Sigma^A{}_n, K_n^{\tilde{B}}] \mathbf{\Phi}_n = 2\theta^{\tilde{B}} f_n^{\tilde{B}AA'} R_n^{A'}$$

- The **Co-adjoint reps** of $\mathrm{Sp}(2n)/Z_2$:

$$T^{\tilde{B}}_{n_{AB}} = -\frac{1}{2} i f_n^{\tilde{B}AA'} = \frac{1}{2} Tr([\Sigma_n^A, K_n^{\tilde{B}}] \Sigma_n^{A'})$$

Continuous symmetries as subgroups of $\mathrm{Sp}(2n)$

- The symplectic subgroup:

$$\mathrm{Sp}(2n) \supset \mathrm{Sp}(2p) \otimes \mathrm{Sp}(2q) \qquad \text{with } p + q = n$$

- The $\mathrm{SU}(n)$ maximal subgroups:

$$\supset \mathrm{SU}(n) \otimes \mathrm{U}(1)$$

- $\mathrm{SU}(n)$ and $\mathrm{SO}(n)$ symmetry subgroups:

$$\begin{aligned} \mathrm{SU}(n) &\supset \mathrm{SU}(p) \otimes \mathrm{SU}(q) \otimes \mathrm{U}(1) && \text{with } p + q = n \\ &\supset \mathrm{Sp}(n) \quad (\text{for even } n) \\ &\supset \mathrm{SO}(n) \\ \mathrm{SO}(n) &\supset \mathrm{SO}(p) \otimes \mathrm{SO}(q) && \text{with } p + q = n \end{aligned}$$

Building the potential with Prime invariants

- Maximal block:

$$\begin{cases} \mathrm{Sp}(2n)/Z_2: & S_n = \Phi^+\Phi & \text{with } \Phi = \begin{pmatrix} \Phi \\ i\sigma^2\Phi^* \end{pmatrix} \\ \mathrm{SU}(n)\times U(1): & D_n^a = \Phi^\dagger\sigma^a\Phi & \text{with } \Phi = (\phi_1, \phi_2, \cdots, \phi_n)^T \\ \mathrm{SO}(n): & T_n = \Phi\Phi^T & \text{with } \Phi = (\phi_1, \phi_2, \cdots, \phi_n)^T \end{cases}$$

- Minimal block:

$$\begin{cases} \mathrm{Sp}(2)/Z_2: & \begin{cases} S_{(11)} = \phi_1^+\phi_1 & \text{for } \begin{pmatrix} \phi_1 \\ \tilde{\phi}_1 \end{pmatrix} \\ S'_{(12)} = \phi_1^+\phi_1 + \phi_2^+\phi_2 = S'_{(21)} = S & \text{for } \begin{pmatrix} \phi_1 \\ \tilde{\phi}_2 \end{pmatrix} \end{cases} \\ \mathrm{SU}(2)\times\mathrm{U}(1): & \begin{cases} D^a_{(12)} = \phi_1^+\sigma^a\phi_1 + \phi_2^+\sigma^a\phi_2 = D^a_{(21)} & \text{for } \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \\ D'^a{}_{(12)} = \phi_1^+\sigma^a\phi_1 + \tilde{\phi}_2\sigma^a\tilde{\phi}_2 = -D'^a{}_{(21)} & \text{for } \begin{pmatrix} \phi_1 \\ \tilde{\phi}_2 \end{pmatrix} \end{cases} \\ \mathrm{SO}(2): & \begin{cases} T_{(12)} = \phi_1\phi_1^T + \phi_2\phi_2^T = T_{(21)} & \text{for } \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \end{cases} \end{cases}$$

Symmetric part of the scalar potential: $V = -\mu^2 S + \lambda_s S^2 + \lambda_D D^a D^a + \lambda_T Tr(TT^*)$

Two Higgs Doublet Model

The maximal symmetry of 2HDM: $\mathbf{Sp(4)/Z_2)} \otimes \mathbf{SU(2)_L}$

- All continuous symmetries in 2HDM:

  (i) $\mathrm{Sp}(4)/Z_2 \cong \mathrm{SO}(5)$,

  (j) $\mathrm{Sp}(2)/Z_2 \otimes \mathrm{Sp}(2)/Z_2 \cong \mathrm{SO}(4)$,

  (k) $\mathrm{SO}(3)$,

  (l) $\mathrm{O}(3) \otimes \mathrm{O}(2)$,

  (m) $\mathrm{O}(2) \otimes \mathrm{O}(3)$,

  (n) $\mathrm{O}(2) \otimes \mathrm{O}(2)$,

  (o) $\mathrm{O}(2)$

- The 2HDM $\mathrm{Sp}(4)/Z_2$-invariant potential (MS-2HDM):

$$V_{\text{MS-2HDM}} = -\mu^2 \left(|\Phi_1|^2 + |\Phi_2|^2\right) + \lambda \left(|\Phi_1|^2 + |\Phi_2|^2\right)^2$$

with the parameters:

$$\mu_1^2 = \mu_2^2 = \mu^2, \quad m_{12}^2 = 0, \quad 2\lambda_2 = 2\lambda_1 = \lambda_3 = 2\lambda, \quad \lambda_4 = \mathrm{Re}(\lambda_5) = \lambda_6 = \lambda_7 = 0$$

The list of accidental symmetries of 2HDM potential

No.	Symmetry	Non-zero parameter space of 2HDM
1	CP1	μ_1^2, μ_2^2, Re(m_{12}^2), λ_1, λ_2, λ_3, λ_4, Re(λ_5), Re(λ_6) =Re(λ_7)
2	Z_2	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4, Re(λ_5)
3	CP2	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, Re(λ_5)
4	U(1)$_{\mathrm{PQ}}$	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4
5	CP1$\otimes$SO(2)	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, $\lambda_5 = 2\lambda_1 - \lambda_{34}$
6	SU(2)$_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = 2\lambda_1 - \lambda_3$
7	Sp(2)$_{\mathrm{HF}}$	μ_1^2, μ_2^2, Re(m_{12}^2), λ_1, λ_2, λ_3, $\lambda_4 =$ Re(λ_5), Re(λ_6)=Re(λ_7)
8	(CP1 $\rtimes S_2$)$\otimes$Sp(2)$_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 =$ Re(λ_5), Re(λ_6)=Re(λ_7)
9	$(S_2 \rtimes Z_2)$$\otimes$Sp(2)$_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, Re(λ_5)= $\pm\lambda_4$
10	U(1)$_{\mathrm{PQ}}$ $\otimes$Sp(2)$_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, λ_4
11	Sp(2)$\otimes$Sp(2)	μ_1^2, μ_2^2, λ_1, λ_2, λ_3
12	$S_2$$\otimes$Sp(2)$\otimes$Sp(2)	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3
13	Sp(4)$/Z_2$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$

[Pilaftsis, '11]

Three Higgs Doublet Model

The maximal symmetry of 3HDM: $\mathbf{Sp(6)/Z_2 \otimes SU(2)_L}$

- In addition to 2HDM continuous symmetries, there are:

$$(p)\,\mathrm{Sp}(6)/Z_2, \qquad (q)\,\mathrm{Sp}(4)/Z_2 \otimes \mathrm{Sp}(2),$$
$$(r)\,\mathrm{Sp}(2)/Z_2 \otimes \mathrm{Sp}(2) \otimes \mathrm{Sp}(2), \qquad (s)\,\mathrm{SU}(3) \otimes \mathrm{U}(1)$$

- The 3HDM $\mathrm{Sp}(6)/Z_2$-invariant potential (MS-3HDM):

$$V_{\text{MS-3HDM}} = -\mu^2 \left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right) + \lambda \left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right)^2$$

with these relations between non zero parameters:

$$\mu_1 = \mu_2 = \mu_3 = \mu, \quad \lambda_{11} = \lambda_{22} = \lambda_{33} = 2\lambda_{1122} = 2\lambda_{1133} = 2\lambda_{2233} = 2\lambda$$

The preliminary list of accidental symmetries of 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
1	CP1	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2),Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, Re(λ_{1212}), Re(λ_{1313}), Re(λ_{2323}), Re(λ_{1213}) = Re(λ_{2113}), Re(λ_{1223}) = Re(λ_{2123}),Re(λ_{1323}) = Re(λ_{1332}), Re(λ_{1112}) = Re(λ_{2212}) = Re(λ_{3312}), Re(λ_{1113}) = Re(λ_{2213}) = Re(λ_{3313}),Re(λ_{1123}) = Re(λ_{2223}) = Re(λ_{3323})
2	Z_2	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {m_{13}^2, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1232}, λ_{1113}, λ_{2213}, λ_{3313} and h.c.}
2’	Z_2'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {m_{23}^2, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1213}, λ_{1123}, λ_{2223}, λ_{3323} and h.c.}
3	$Z_2 \otimes Z_2'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {λ_{1212}, λ_{1313}, λ_{2323} and h.c.}
4	Z_3	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {λ_{1213}, λ_{1323}, λ_{2123} and h.c.}
5	Z_4	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {λ_{1212}, λ_{1323} and h.c.}
6	$\mathrm{U}(1)_{\mathrm{PQ}}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {λ_{1323} and h.c.}
6’	$\mathrm{U}(1)'_{\mathrm{PQ}}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, {m_{23}^2, λ_{2323}, λ_{1123}, λ_{2223}, λ_{3323} and h.c.}
7	$\mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)'_{\mathrm{PQ}}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}

The preliminary list of accidental symmetries of 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
8	$Z_2 \otimes \mathrm{U}(1)'_{\mathrm{PQ}}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, { λ_{2323} and h.c.}
9	CP2	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, λ_{1212}, $\lambda_{1313} = \lambda_{2323}$, {$\lambda_{1112} = -\lambda_{2212}$ and h.c.}
10	$\mathrm{CP1} \otimes \mathrm{SO}(2)$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1212} = 2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, $\lambda_{1313} = \lambda_{2323}$,
11	D_3	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, { $\lambda_{2131} = -\lambda_{1232}$, λ_{1323} and h.c.}
12	D_4	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, λ_{1212}, $\lambda_{1331} = \lambda_{2332} = \lambda_{3231}$
13	$\mathrm{SU}(2)_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, $\lambda_{1122} = 2\lambda_{11} - \lambda_{1221}$, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\lambda_{1331} = \lambda_{2332}$
14	$\mathrm{Sp}(2)_{\mathrm{HF}}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Re}(m_{13}^2)$,$\mathrm{Re}(m_{23}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=$\mathrm{Re}(\lambda_{1313})$, λ_{2332}=$\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{2113})$, $\mathrm{Re}(\lambda_{1223}) = \mathrm{Re}(\lambda_{2123})$,$\mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1332})$, $\mathrm{Re}(\lambda_{1112}) = \mathrm{Re}(\lambda_{2212}) = \mathrm{Re}(\lambda_{3312})$, $\mathrm{Re}(\lambda_{1113}) = \mathrm{Re}(\lambda_{2213}) = \mathrm{Re}(\lambda_{3313})$,$\mathrm{Re}(\lambda_{1123}) = \mathrm{Re}(\lambda_{2223}) = \mathrm{Re}(\lambda_{3323})$

The preliminary list of accidental symmetries of 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
15	$(\mathrm{CP1} \rtimes S_2)\otimes \mathrm{Sp}(2)_{\mathrm{HF}}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Re}(m_{13}^2)=\mathrm{Re}(m_{23}^2)$, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\mathrm{Re}(\lambda_{1212})$ $\lambda_{1331}=\lambda_{2332}=\mathrm{Re}(\lambda_{1313})=\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1323})=\mathrm{Re}(\lambda_{1332})$, $\mathrm{Re}(\lambda_{1223})=\mathrm{Re}(\lambda_{2123})=\mathrm{Re}(\lambda_{1213})=\mathrm{Re}(\lambda_{2113})$, $\mathrm{Re}(\lambda_{1112})=\mathrm{Re}(\lambda_{2212})=\mathrm{Re}(\lambda_{3312})$, $\mathrm{Re}(\lambda_{1113})=\mathrm{Re}(\lambda_{2213})=\mathrm{Re}(\lambda_{3313})=\mathrm{Re}(\lambda_{1123})=\mathrm{Re}(\lambda_{2223})=\mathrm{Re}(\lambda_{3323})$
16	$(S_2\rtimes Z_2)\otimes \mathrm{Sp}(2)_{\mathrm{HF}}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\mathrm{Re}(\lambda_{1212})$, $\lambda_{1331}=\lambda_{2332}=\mathrm{Re}(\lambda_{1313})=\mathrm{Re}(\lambda_{2323})$
17	A_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{1212}=\lambda_{1313}=\lambda_{2323}$ and h.c. $\}$
18	S_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\lambda_{1212}=\lambda_{1313}=\lambda_{2323}$
19	$\Delta(54)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$ $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}=\lambda_{11}-2\lambda_{1122}$, $\{\lambda_{1213}=\lambda_{2123}=\lambda_{3231}$ and h.c.$\}$
20	$\Sigma(36)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\lambda_{1213}=\lambda_{1323}=\lambda_{1232}$

The preliminary list of accidental symmetries of 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
21	$\mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{Sp}(2)_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233},\ \lambda_{1221}, \lambda_{1331} = \lambda_{2332}$
22	$\mathrm{Sp}(2)\otimes\mathrm{Sp}(2)\otimes\mathrm{Sp}(2)$	$\mu_1^2,\ \mu_2^2,\ \mu_3^2,\ \lambda_{11},\ \lambda_{22},\ \lambda_{33},\ \lambda_{1122},\ \lambda_{1133},\ \lambda_{2233}$
23	$\mathrm{Sp}(4)/Z_2$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233}$
24	$\mathrm{CP1}\times\mathrm{SO}(3)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \lambda_{1212} = \lambda_{1313} = \lambda_{2323} = 2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$
25	$(\mathrm{CP1} \rtimes S_4)\otimes\mathrm{Sp}(2)_{\mathrm{HF}}$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}=\mathrm{Re}(\lambda_{1313})=\mathrm{Re}(\lambda_{2323})=\mathrm{Re}(\lambda_{1212})$
26	$\mathrm{SU}(3)\times\mathrm{U}(1)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332} = 2\lambda_{11} - \lambda_{1122}$
27	$\mathrm{Sp}(6)/Z_2$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33} = 2\lambda_{1122} = 2\lambda_{1133} = 2\lambda_{2233}$

[Ivanov, Vdeovin, '12; Keus et al, '13; Ivanov, Varzielas, '19; $\cdots$]→ Discrete symmetries in 3HDM

[Pilaftsis, '16]→ Continuous symmetries in nHDM

[ND, Pilaftsis, '19 in preparation]→ The complete list of symmetries

Phenomenological implications MS-nHDM

Maximal symmetry $\mathrm{Sp}(2n)/Z_2 \to$ Natural alignment

- Natural SM alignment: alignment limit without decoupling and also without fine-tuning among the quartic couplings.

[See A. Pilaftsis's talk]

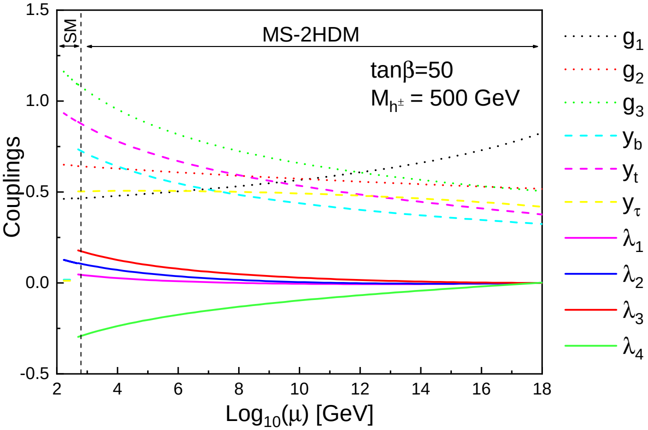
- For example: In MS-2HDM considering soft-term and two loops RGEs effects:

$$\begin{aligned} \mathrm{Sp}(4)/Z_2 \otimes \mathrm{SU}(2)_L \quad & \xrightarrow{g' \neq 0} \mathrm{O}(3) \otimes \mathrm{O}(2) \otimes \mathrm{SU}(2)_L \sim \mathrm{O}(3) \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ & \xrightarrow{\text{Yukawa}} \mathrm{O}(2) \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \sim \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ & \xrightarrow{\langle \Phi_{1,2} \rangle \neq 0} \mathrm{U}(1)_{\mathrm{em}} \end{aligned}$$

[Dev, Pilaftsis, '14; ND, Pilaftsis, '19]

Phenomenological implications of MS-nHDM

- The $Sp(2n)/Z_2$ symmetry is realized at high scale $\mu_{X_2} = 10^{18}$ GeV
- At the threshold scale SM is realized
- Misalignment predictions are consistent with experimental constraints



[See A. Pilaftsis's talk]

Summary and conclusions

- Multi-Higgs Doublet Models (nHDMs) have large number of accidental symmetries as subgroup of Sp($2n$).
- The maximal symmetry of nHDM provides the natural SM alignment with quartic coupling unification.
- The preliminary listing of accidental symmetries as subgroups of Sp($2n$) in nHDMs potentials presented.
- Prime invariants to construct the scalar sector of nHDM potentials introduced.
- We have recovered the maximum of 13 accidental symmetries for the 2HDM potential and identified 27 accidental symmetries for the 3HDM potential along with their allowed parameter spaces.

Thank you!