Intermediate Charge-Breaking Phases in the 2-Higgs-Doublet Model

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Institute for Theoretical Physics

based on [2308.04141]

in collaboration with Mayumi Aoki, Lisa Biermann, Igor P. Ivanov, Margarete Mühlleitner, and Hiroto Shibuya

Scalars 2023

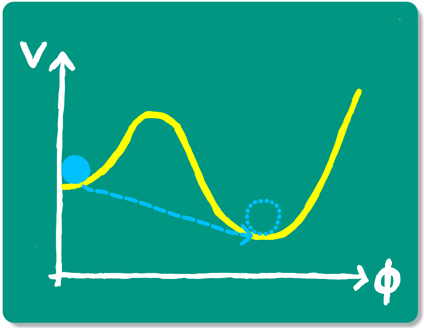
An opportunity to discuss various aspects of scalar particles.

Warsaw, 16/09/2023

Evolution of the Universe around the electroweak epoch?

Excellent testbed for BSM physics with extended scalar sectors

- Exotic intermediate phases such as charge-breaking ones (massive photons, ...)?



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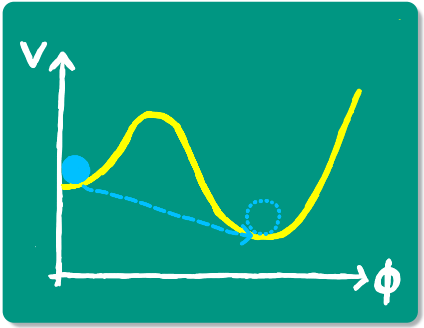
  EW-symmetric (high T) $\longrightarrow$ neutral
  $\longrightarrow$ charge-breaking $\longrightarrow$ neutral ($T = 0$)



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- Multi-step phase transitions? E.g.:

  EW-symmetric (high T) $\longrightarrow$ neutral
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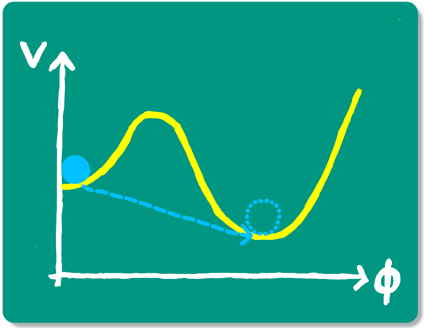
- First-order phase transitions between charge-breaking and neutral phases?



The CP-conserving 2HDM (type I) with softly broken $\mathbb{Z}_2$ symmetry

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c.\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
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with

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and real fields ρ_i, η_i, ζ_i, ψ_i ($i = 1, 2$), and VEVs $\bar{\omega}_j$ ($j = 1, 2, \mathrm{CP}, \mathrm{CB}$)

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- Present-day EW-breaking vacuum at zero temperature $T = 0$ (with $v_j \equiv \bar{\omega}_j|_{T=0}$):

$$v_{\text{CB}} = v_{\text{CP}} = 0 \quad \text{and} \quad v^2 \equiv v_1^2 + v_2^2 = (246.22\ \text{GeV})^2 \quad \text{and} \quad \tan\beta \equiv v_2 / v_1$$

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Phase diagram in (m_{11}^2, m_{22}^2) plane

Start with toy model with $m_{12}^2 = 0$; derive e.g. with geometric methods *[Ivanov '08]*:

Phase diagram in (m^2_{11}, m^2_{22}) plane

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- **Bounded-from-below conditions:**

$$\lambda_{1,2} > 0\,, \quad \sqrt{\lambda_1\lambda_2} + \lambda_3 > 0\,,$$
$$\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - \lambda_5 > 0$$

- **Conditions for a CB vacuum:**

$$\sqrt{\lambda_1\lambda_2} - \lambda_3 > 0\,, \quad \lambda_4 > |\lambda_5|$$

and

$$m^2_{11}\sqrt{\lambda_2} + m^2_{22}\sqrt{\lambda_1} < 0\,,$$
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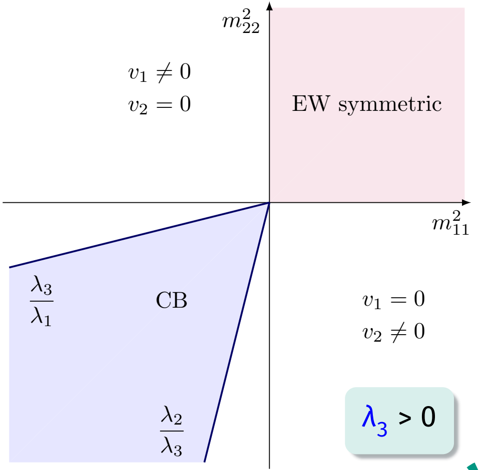

$\lambda_3 > 0$

Thermal evolution in the high-temperature limit

One-loop thermal corrections in high-T limit ($\lambda_{1,2,3,4,5}$ stay T-independent):

$$m_{11}^2(T) = m_{11}^2 + c_1 T^2$$
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including gauge and Yukawa couplings

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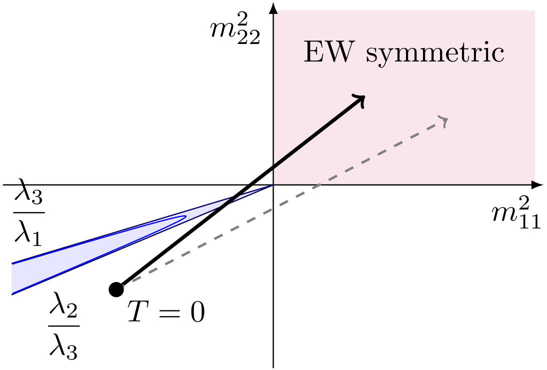

(blue hyperbola: $m_{12}^2 \neq 0$)

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$m_{22}^2(T)$

Follow-up questions

So far already known [Ivanov '08; Ginzburg, Ivanov, Kanishev '09], but:

- Existence of CB phases using full one-loop-corrected effective 2HDM potential (beyond high-T limit)?
- Intermediate CB phases vs. collider constraints?
- Sequences of phase transitions? EW-symmetry restoration at high T?

λ_3

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Electroweak symmetry (non-)restoration in the 2HDM

Is the EW symmetry always restored at $T \to \infty$? $\Rightarrow$ Check for minimum at origin

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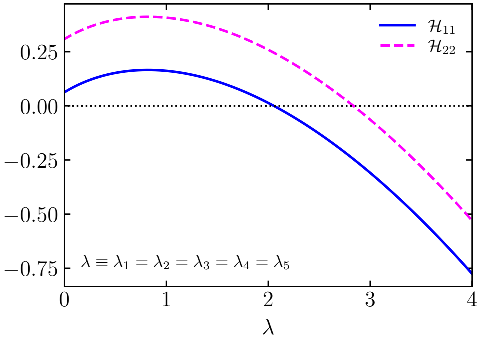

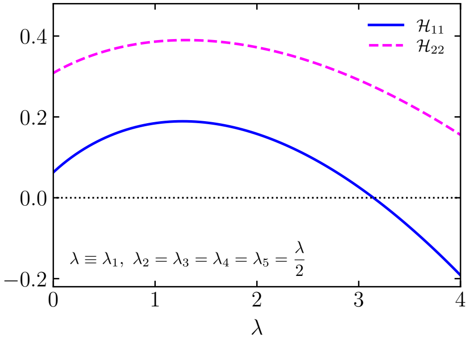

$\Rightarrow$ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

($\mathcal{H}_{ii}$ ~ eigenvalues of the Hessian of leading T^2 term of the full one-loop potential)

Scans of the 2HDM parameter space

- Scan over parameter space and generate *seed points* at $T = 0$
  - Parameter points with a suitable trajectory for $T > 0$ through the CB region
  - VEV $v = 246.22$ GeV and light CP-even Higgs mass $m_h = 125.09$ GeV fixed at $T = 0$

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- Locating the minima and evolving the VEVs for $T > 0$ for full one-loop effective potential including thermal corrections: BSMPT *[Basler, Mühlleitner, Müller '18–'21]*

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- Use ScannerS *[Coimbra et al. '13–'20]* to apply constraints to selected points:

Theoretical constraints:
bounded-from-below, perturbativity, perturbative unitarity *[Akeroyd, Arhrib, Naimi '00]*, absolute stability *[Barroso, Ferreira, Ivanov, Santos '13]*

Experimental constraints:
flavour physics, Higgs searches at colliders, STU-parameters *[Peskin, Takeuchi '92]*

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Two strategies for scans

- *Scan (i)*: intermediate CB phase in high-T approximation **and** in full one-loop treatment: experimental constraints exclude all points!
- *Scan (ii)*: intermediate CB phase in full one-loop treatment, enforce all experimental constraints

bounded-from-below, perturbativity, perturbative unitarity *[Akeroyd, Arhrib, Naimi '00]*, absolute stability *[Barroso, Ferreira, Ivanov, Santos '13]*

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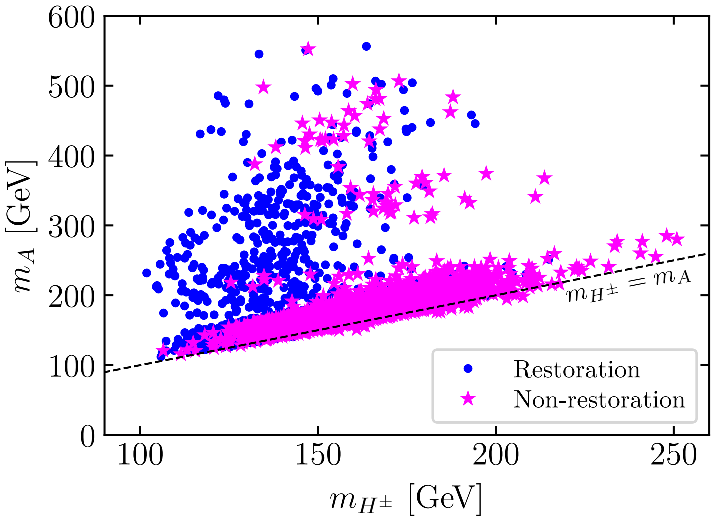
Results of scan *(ii)* (no CB phase required)



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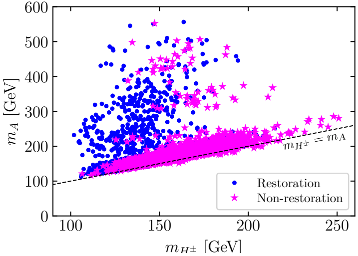

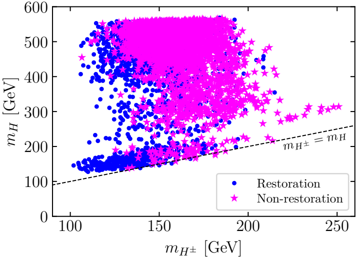

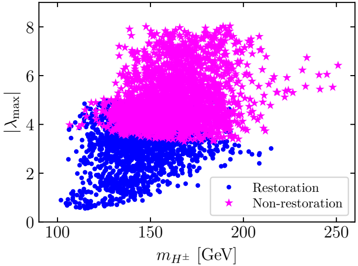

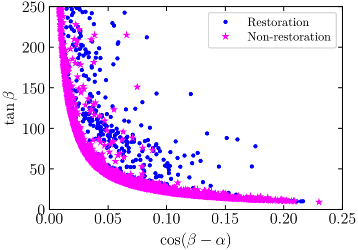

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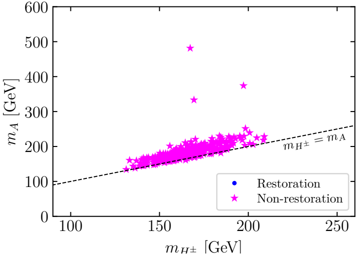

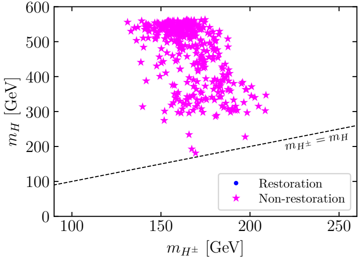

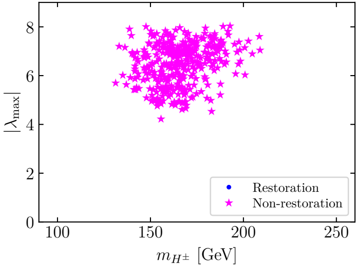

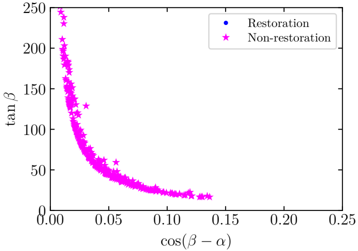

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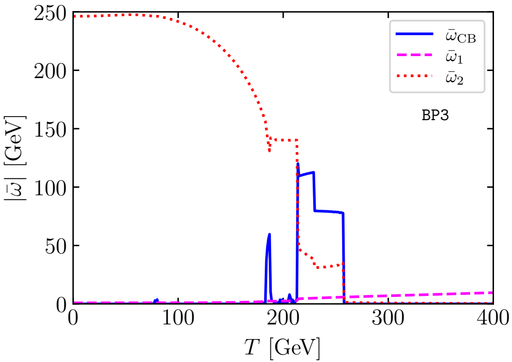
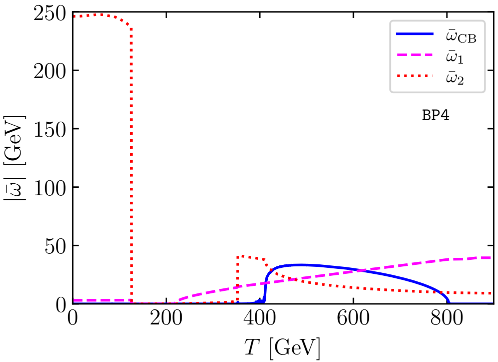
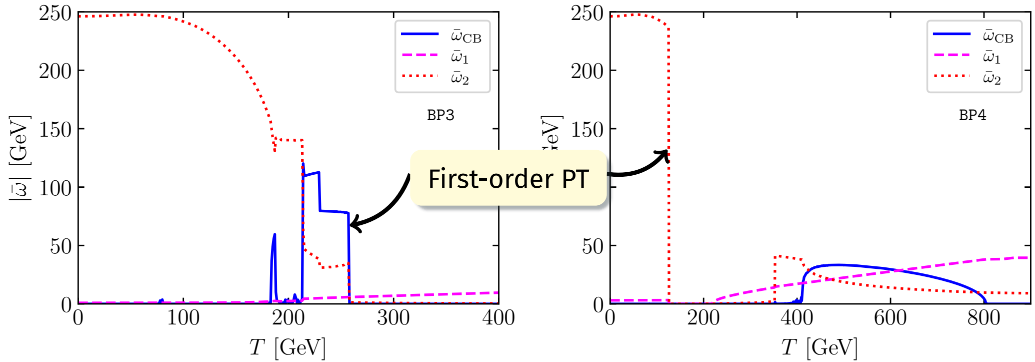
Benchmark points





	m_H (GeV)	m_A (GeV)	$m_{H^\pm}$ (GeV)	$\tan\beta$	$\cos(\beta-\alpha)$	m_{12}^2 (GeV^2)
BP3	342.52	230.02	183.72	286.00	0.009	410.17
BP4	558.56	194.52	168.43	80.84	0.026	3857.90

Benchmark points



	m_H (GeV)	m_A (GeV)	$m_{H^\pm}$ (GeV)	$\tan\beta$	$\cos(\beta-\alpha)$	m_{12}^2 (GeV^2)
BP3	342.52	230.02	183.72	286.00	0.009	410.17
BP4	558.56	194.52	168.43	80.84	0.026	3857.90

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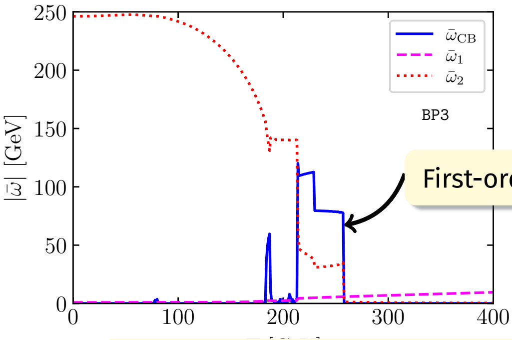

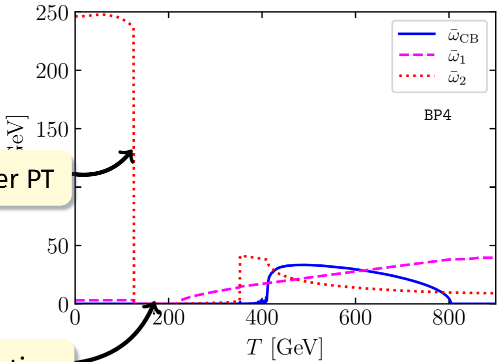

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Summary and conclusions

Phase transitions including intermediate charge-breaking phases in the 2HDM

- Intermediate CB phases can occur in the CP-conserving 2HDM **with full one-loop thermal corrections**
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See [2308.04141] for more details!

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THANK YOU FOR YOUR ATTENTION! ☺

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Backup

$\mathbb{Z}_2$ symmetry

To avoid dangerously large flavour-changing neutral currents at tree level:

- Impose discrete $\mathbb{Z}_2$ symmetry:

$$\Phi_1 \longrightarrow \Phi_1, \quad \Phi_2 \longrightarrow -\Phi_2$$

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	u-type	d-type	leptons
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-Specific	Φ_2	Φ_2	Φ_1
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Types of vacua in the 2HDM

Type of vacuum	$\sqrt{2}\,\langle\Phi_1\rangle$	$\sqrt{2}\,\langle\Phi_2\rangle$
Neutral EW-symmetric	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
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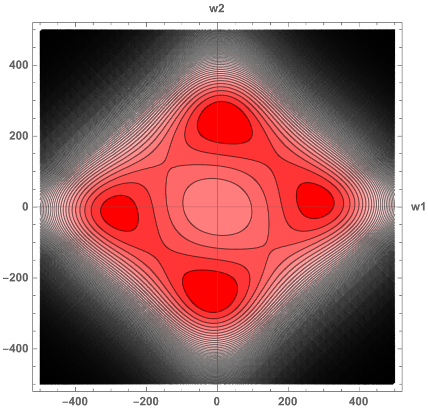

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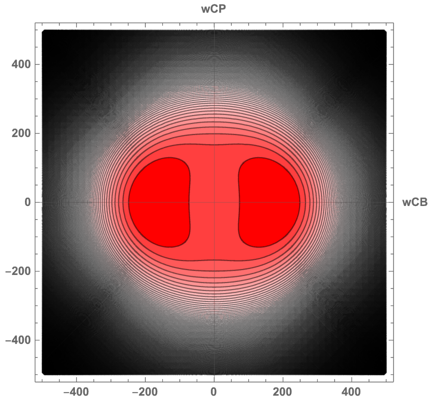

Can we classify the different vacua geometrically?

Tilde basis

Introduce rescaled fields with $k^4 \equiv \sqrt{\lambda_2/\lambda_1}$:

$$\Phi_1 = k\tilde{\Phi}_1, \quad \Phi_2 = k^{-1}\tilde{\Phi}_2 \qquad \Leftrightarrow \qquad \tilde{\Phi}_1 = k^{-1}\Phi_1, \quad \tilde{\Phi}_2 = k\Phi_2$$

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Rescaled terms in V:

$$\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 = \lambda_1 k^4\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right)^2 + \lambda_2 k^{-4}\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^2 = \tilde{\lambda}\left[\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right)^2 + \left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^2\right]$$

$$m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) = \tilde{m}_{11}^2\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right) + \tilde{m}_{22}^2\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)$$

with $\quad \tilde{\lambda} \equiv \sqrt{\lambda_1\lambda_2}, \quad \tilde{m}_{11}^2 \equiv k^2 m_{11}^2, \quad \tilde{m}_{22}^2 \equiv k^{-2} m_{22}^2$

- Other quartic terms and m_{12}^2 term remain unchanged

Potential in the tilde basis

$$V = \tilde{m}_{11}^2 \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 + \tilde{m}_{22}^2 \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - m_{12}^2 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2 + h.c.\right) + \frac{\tilde{\lambda}}{2}\left[\left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_1\right)^2 + \left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_2\right)^2\right]$$
$$+ \lambda_3 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_1\right)\left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_2\right) + \lambda_4 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2\right)\left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_1\right) + \frac{\lambda_5}{2}\left[\left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2\right)^2 + \mathrm{h.c.}\right]$$

Bilinear form

Introduce vector $r^\mu = \left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1 + \tilde{\Phi}_2^\dagger\tilde{\Phi}_2,\ 2\,\mathrm{Re}\,\tilde{\Phi}_1^\dagger\tilde{\Phi}_2,\ 2\,\mathrm{Im}\,\tilde{\Phi}_1^\dagger\tilde{\Phi}_2,\ \tilde{\Phi}_1^\dagger\tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^T$

$\Rightarrow$ **2HDM potential in bilinear form** (following *[Ivanov '06-'08]*):

$$V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$$

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where ($\Lambda_{\mu\nu}$ diagonal in tilde basis!)

$$M_\mu = \left(M_0, M_1, M_2, M_3\right)^T = \left(-\frac{\tilde m_{11}^2 + \tilde m_{22}^2}{2},\ -\mathrm{Re}\,m_{12}^2,\ \mathrm{Im}\,m_{12}^2,\ \frac{\tilde m_{11}^2 - \tilde m_{22}^2}{2}\right)^T$$

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- 'Bounded-from-below' (BFB): $\Lambda_0 > 0\,,\quad \Lambda_0 > \Lambda_i \quad$ for $i = 1, 2, 3 \quad$ (Λ_i can be < 0)

Physical configurations

It follows from the definition of r^μ (Schwarz inequality):

$$r_0 \geq 0, \quad r_\mu r^\mu = r_0^2 - \sum_i r_i^2 \geq 0$$

$\Rightarrow$ Physically realisable configurations inside/on the future lightcone

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Evaluation of minima: (ii) neutral EW-breaking vacuum

Add constraint to potential (*to* surface of cone, $r_\mu r^\mu = r_0^2 - \sum_i r_i^2 = 0$):

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- Plugging solution for $r^\mu = r^\mu(\zeta)$ into constraint $r_0^2 - \sum_i r_i^2 = 0$:

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- Equation has up to 6 solutions for ζ/extrema $\Rightarrow$ extract minimum numerically

Evaluation of minima: (iii) charge-breaking vacuum

From $\frac{dV}{dr^\mu} \stackrel{!}{=} 0$, it follows:

$$\Lambda_0 r_0 = M_0\,, \quad \Lambda_i r_i = M_i$$

(minimum: $\Lambda_i < 0$)

Evaluation of minima: (iii) charge-breaking vacuum

From $\frac{dV}{dr^\mu} \stackrel{!}{=} 0$, it follows: $\Lambda_0 r_0 = M_0\,, \quad \Lambda_i r_i = M_i$ (minimum: $\Lambda_i < 0$)

- Plugging solution into inequalities $r_0 > 0$ and $r_0^2 - \sum_i r_i^2 > 0$:

$$M_0 > 0 \qquad \text{and} \qquad \frac{M_0^2}{\Lambda_0^2} > \sum_i \frac{M_i^2}{\Lambda_i^2}$$

or, slightly rewritten,

$$\tilde{m}_{11}^2 + \tilde{m}_{22}^2 < 0 \qquad \text{and} \qquad \frac{\mu_1^2}{a_1^2} + \frac{\mu_2^2}{a_2^2} + \frac{\mu_3^2}{a_3^2} < 1$$

with $\mu_i^2 = \frac{M_i^2}{M_0^2}, \quad a_i^2 = \frac{\Lambda_i^2}{\Lambda_0^2}$

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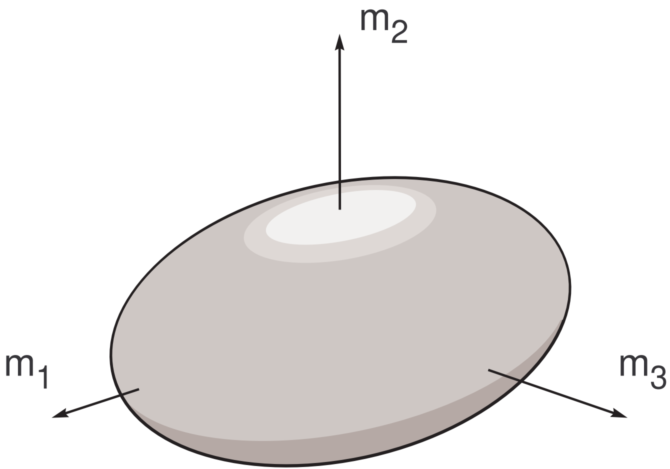
$\Rightarrow$ Points that lie inside an ellipsoid with semi-axes $a_{1,2,3}$

Evaluation of minima: (iii) charge-breaking vacuum: ellipsoid



[Ivanov '08]

Toy model: $\mathbb{Z}_2$-symmetric 2HDM with $m_{12}^2 = 0$

For a simpler graphical representation, discuss **toy model: $\mathbb{Z}_2$-symmetric 2HDM**

$$V_{\text{toy}} = V|_{m_{12}^2=0}$$

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Constraints

- Bounded-from-below:

$$\lambda_{1,2} > 0\,, \quad \sqrt{\lambda_1\lambda_2} + \lambda_3 > 0\,, \quad \sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - \lambda_5 > 0$$

- CB minimum:

$$\sqrt{\lambda_1\lambda_2} - \lambda_3 > 0\,, \quad \lambda_4 > |\lambda_5|$$

and

$$m_{11}^2\sqrt{\lambda_2} + m_{22}^2\sqrt{\lambda_1} < 0\,, \quad m_{11}^2 < m_{22}^2\frac{\lambda_3}{\lambda_2}\,, \quad m_{22}^2 < m_{11}^2\frac{\lambda_3}{\lambda_1}$$

Effective potential at finite temperatures

Full one-loop and thermally corrected effective potential:

$$V_{1L} = V + V_{\text{CW}} + V_{\text{CT}} + V_T$$

with

- V_{CW}: T-independent one-loop Coleman-Weinberg potential
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Perturbative expansion becomes unreliable at high T

- Resum ‘Daisy‘ diagrams (‘Arnold-Espinosa‘ method) to recover perturbativity

$\Rightarrow$ Certain mass eigenvalues obtain T-dependent contributions

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One-loop thermal corrections for $T \to \infty$

$$V_T \overset{T\to\infty}{\approx} -\sum_k n_k \begin{cases} \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}\overline{m}_k^3 T & k = H^\pm, h, H, A, W_L, Z_L, \gamma_L \\ \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}m_k^3 T & k = W_T, Z_T \\ \frac{7\pi^2}{720}T^4 - \frac{1}{48}m_k^2T^2 & k = t, b, \tau \end{cases}$$

with

- n_k: number of d.o.f.s of field k
- $\overline{m}_k$ (m_k): mass eigenvalue for field k including (excluding) thermal Debye corrections from Daisy resummation

EW symmetry (non-)restoration

Is the EW symmetry always restored at $T \to \infty$? $\Rightarrow$ Check for minimum at origin

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Extract leading term $\propto T^2$ for $T \to \infty$:

$$\text{Hessian } H_{ij} \equiv \left.\frac{\partial^2 V_T}{\partial\bar{\omega}_i \partial\bar{\omega}_j}\right|_{\bar{\omega}_{i,j}=0} \quad \Rightarrow \quad \mathcal{H} \equiv \lim_{T\to\infty} \frac{H}{T^2} = \lim_{T\to\infty} \frac{1}{T^2} \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_{11} & 0 \\ 0 & \mathcal{H}_{22} \end{pmatrix}$$

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with

$$\mathcal{H}_{11} = c_1 - \frac{1}{16\pi}\left[\sqrt{2}\left(3g^3 + g'^3\right) + 4\left(3\sqrt{c_1}\lambda_1 + \sqrt{c_2}\left(2\lambda_3 + \lambda_4\right)\right)\right]$$
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EW symmetry (non-)restoration

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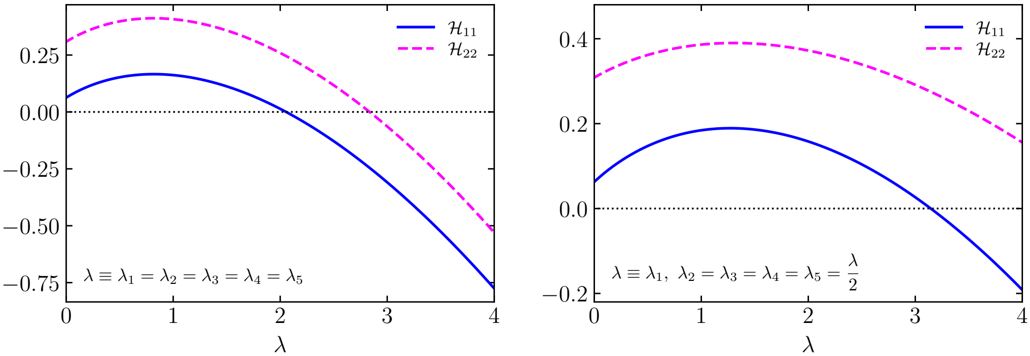
$\Rightarrow$ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

Electroweak symmetry (non-)restoration in the 2HDM



⇒ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

Scans of the 2HDM parameter space

Parameter	Scan range
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	$[0, 4\pi]$
λ_5	$[-4\pi, 4\pi]$
m_{11}^2, m_{22}^2	$[-10^6, 0]$ GeV^2
m_{12}^2	$[0, 10^6]$ GeV^2

- Random "smart" scan over parameter space
  $\rightarrow$ Get VEV v_0 and light CP-even Higgs mass $m_{h,0}$
- Rescale to get $v = 246.22$ GeV, $m_h = 125.09$ GeV:

$$m_{ij}^2 \rightarrow m_{ij}^2 \frac{m_h^2}{m_{h,0}^2}, \quad \lambda_k \rightarrow \lambda_k \frac{m_h^2}{m_{h,0}^2} \frac{v_0^2}{v^2}$$

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  - Bounded-from-below
  - Perturbativity ($|\lambda_{1,2,3,4,5}| < 4\pi$)
  - Perturbative unitarity *[Akeroyd, Arhrib, Naimi ’00]*
  - Absolute stability *[Barroso, Ferreira, Ivanov, Santos ’13]*

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$\Rightarrow$ **Seed points**

Numerical analysis of minima for one-loop potential

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  - Flavour physics
  - Higgs searches at colliders
  - Electroweak precision constraints (STU-parameters) *[Peskin, Takeuchi '92]*

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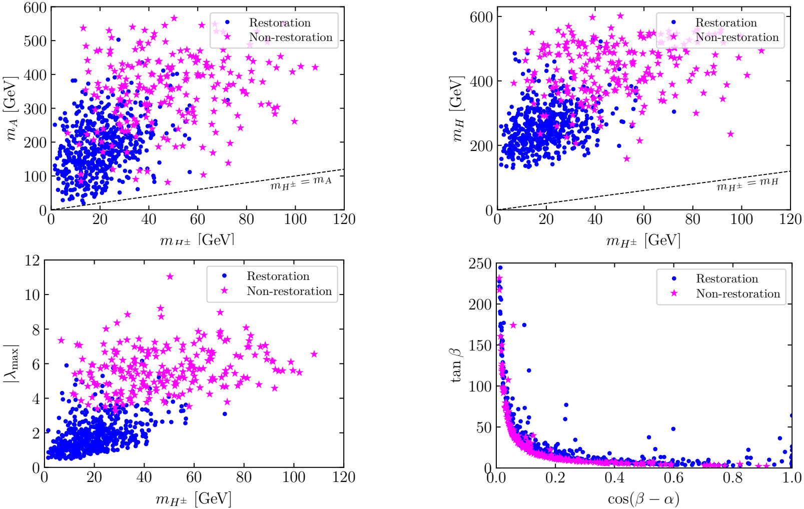
Two strategies for scans

- *Scan (i)*: intermediate CB phase in high-T approximation and in full one-loop treatment: experimental constraints exclude all points!
- *Scan (ii)*: intermediate CB phase in full one-loop treatment, enforce all experimental constraints

Results of scan *(i)* (no experimental constraints)



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